Rent-Seeking Contests with Private Values and Resale*

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Abstract

This paper studies rent-seeking contests with private values and resale possibilities. With a stochastic success function, the resulting possible inefficiency creates a motive for aftermarket trade. Players' valuations are endogenously determined when there is an opportunity of resale. We characterize symmetric equilibria. We assume that the winner has full bargaining power, however, the results extend to other resale mechanisms. We show that resale enhances allocative efficiency ex post at the expense of more wasted social resources since players compete more aggressively with resale possibilities.

Keywords: rent-seeking contest, resale, efficiency

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1 Introduction

Situations in which competitors expend costly and irreversible resources to win a limited number of prizes are ubiquitous. Since Tullock's (1975, 1980) seminal contribution, the theory of rent-seeking contests has advanced considerably.[1] Most literature studying Tullock's rent-seeking contest assumes stochastic success function, in the sense that the winning probability of a player is proportional to her expenditure relative to the total expenditure. The more one spends, the more likely he will win the prize. But he can never guarantee winning even if he spends the most. Therefore, the allocation of the prize is stochastic and thus inefficient ex post with positive probability. This is true even if competitors are ex ante identical and follow symmetric strategies in equilibrium.

The possibility of ex post inefficiency lies in the stochastic winning probability and it leaves space for potential gain through aftermarket trade–resale. Indeed, many realistic characteristics of rent-seeking contests cannot be captured by a static Tullock model. For instance, the prize in rent-seeking contests could be a patented innovation. If a cost-reducing innovation is to be patented, the incumbent monopolist holding related or substitutable technology has more incentive to acquire the patent than a potential entrant. Then the winner could benefit from selling the patent to the incumbent monopolist. If the rent is certain operating license that is usually not allowed to sell, the loser could obtain the license by taking over the whole company holding it. If the rent relates to government contracts, like defense contracts, the winner could benefit through subcontracting with the losing rival. In all above situations, the winner may not have the highest valuation due to the stochastic success function. Hence, the winner has incentive to resell the prize to those having higher valuations in order to seek additional profit.

The purpose of this paper is to investigate the effect of allowing resale on players' strategic behaviors and seller's expected revenue. Malueg and Yates (2004) firstly studies rent-seeking

[1] See Nitzan (1994) for an excellent survey.

contests with two-sided private information. Using a two-player-two-value model, they characterize the equilibrium strategies and provide sufficient conditions for the existence of a symmetric equilibrium. This paper follows the same model and introduce resale possibility into the standard rent-seeking contests with private information.

Resale possibilities introduce an endogenous element to players' valuations. Upon winning, a low-value player could resell the prize to his rival who may have higher valuation with positive probability. Similarly, a high-value loser may benefit from trading with a low-value winner, depending on the distribution of bargaining power. This changes both players' strategic behaviors since they will incorporate this additional opportunities of buying and selling when they formulate their outlay of effort. We characterize the symmetric equilibrium and find an interesting proportional difference property. At symmetric equilibrium, both players compete more aggressively and thus increase expected revenue for the seller. Moreover, if the seller could commit to publicly disclosing players' private values, s/he could further increase his or her expected revenue.

Introducing resale possibility to the standard rent-seeking contests will improve allocative efficiency ex post. This is straightforward if both players have two valuations. If both players have the same valuation about the prize, the allocation is always efficient. Whenever they value the prize differently, there will be positive probability that the ex post allocation is inefficient. Indeed, if a low-value player competes with a high-value rival and wins the prize, there is potential gain if he resells the prize to the high-value rival. The resale price will be determined by their relative bargaining power. For analytical simplicity, we assume that the winner has full bargaining power. However, we can show that our results are robust to other resale mechanisms, like monopsony pricing resale or probabilistic k-double auctions that will be defined in Section 6.

More and more theoretical literature studies resale in standard auctions, such as sealed-bid and English auctions.[2] This paper is the first one that studies resale in rent-seeking contest environment.[3] The main focus of this paper is to investigate how resale changes players' strategic behaviors in rent-seeking contests and its effect on expected revenue and allocative efficiency. We hope that our study shed some light on this interesting question.

The rest of the paper is organized as following. Section 2 presents the model. Section 3 studies rent-seeking contests with private values and resale possibility and Section 4 contains parallel study when values become public. Section 5 derives a general revenue ranking result. Section 6 examines two other resale mechanisms and Section 7 concludes. All proofs are contained in the appendix.

2 The Model

The rent-seeking contest proceeds as following. Two risk-neutral players, 1 and 2, compete for an indivisible prize to be awarded by a seller. Each player privately learns her valuation of the prize, v_1 and v_2. We assume that the possible realizations of valuation could be either low (v_L) or high (v_H). The prior probability distribution of (v_1, v_2) is given by

$$f(v_1, v_2) = \begin{cases} \frac{1}{2}\sigma, & \text{if } v_1 = v_2 \\ \frac{1}{2}(1-\sigma), & \text{if } v_1 \neq v_2 \end{cases} \tag{1}$$

The distribution is symmetric, but players' values could be different and correlated. For instance, $\sigma = 0$ refers to perfect negative correlation, $\sigma = 1$ to perfect positive correlation, and $\sigma = \frac{1}{2}$ to independence. From (1), we have the following conditional probabilities.

$$Pr(v_2 = v_L | v_1 = v_L) = Pr(v_2 = v_H | v_1 = v_H) = \sigma \tag{2}$$

[2] See Haile (2003) for a thorough analysis.

[3] Sui (2006) studies resale through a different setting: all-pay auctions. The resale results from new entrants to the market, instead of possible inefficient allocation.

and

$$Pr(v_2 = v_L | v_1 = v_H) = Pr(v_2 = v_H | v_1 = v_L) = 1 - \sigma \tag{3}$$

After learning their private valuations, both players simultaneously submit nonnegative bids, b_1 and b_2, which could also be considered as effort levels. Since both players are ex ante identical, without loss of generality, we analyze the game from the standpoint of player 1. The probability of player 1 wins the prize is given by $p(b_1, b_2)$ defined as

$$p_1(b_1, b_2) = \frac{b_1}{b_1 + b_2} \tag{4}$$

The winning probability of player 2 is $1 - p_1(b_1, b_2)$. For given values and bids, the expected utility for player 1 is

$$U_1(b_1, b_2) = \frac{b_1}{b_1 + b_2} v_1 - b_1 \tag{5}$$

Similarly, we could define $U_2(b_1, b_2)$. The utility functions and the probability distribution of values are common knowledge.

After the prize is awarded, there is possibility of resale if the low-value player wins the prize. With positive probability, the loser has high value. Thus there is potential gain resulting from resale for the low-value winner. If the winner has high value, there is no resale since no potential gain exists. For resale mechanism, we assume the winner possess full bargaining power, so he will propose a take-it-or-leave-it offer to the loser. The winner tries to extract as much surplus as possible, so he will ask for v_H and the high-value loser will accept the offer in equilibrium.

A *pure strategy* β_1 for player 1 specifies a bid contingent on the realization of his private value. Formally, $\beta_1 = (\beta_L, \beta_H)$ specifies bids β_L if $v_1 = v_L$, β_H if $v_1 = v_H$. A *Bayesian equilibrium* is a pair of strategies (β_1, β_2) such that β_1 maximizes player 1's expected payoff conditional on his realizations of value and player 2 using β_2; and β_2 maximizes player 2's expected payoff conditional on his realizations of value and player 1 using β_1. The Bayesian equilibrium is *symmetric* if $\beta_1 = \beta_2 = \beta$.

3 Equilibrium with Private Values and Resale

First we characterize a symmetric equilibrium and then provide the sufficient conditions under which this equilibrium exists.

Suppose player 2 follow strategy β, player 1 learns his private value as v_L and submits b_L, then his expected utility is

$$EU_1(b_L, \beta) = \sigma \frac{b_L}{b_L + \beta_L} v_L + (1-\sigma) \frac{b_L}{b_L + \beta_H} v_H - b_L \tag{6}$$

With probability $1-\sigma$, he competes with a high-value player and resells the prize at price equal to player 2's valuation v_H; with probability σ, the competitor has low value and there will be no resale. Maximizing (6) with respect to b_L yields

$$\frac{\sigma \beta_L}{(b_L + \beta_L)^2} v_L + \frac{(1-\sigma)\beta_H}{(b_L + \beta_H)^2} v_H = 1 \tag{7}$$

At the symmetric equilibrium, $b_L = \beta_L$, then

$$\frac{\sigma}{4\beta_L} v_L + \frac{(1-\sigma)\beta_H}{(\beta_L + \beta_H)^2} v_H = 1 \tag{8}$$

Multiplying both sides of (8) by β_L, we have

$$\frac{\sigma}{4} v_L + \frac{(1-\sigma)\beta_L \beta_H}{(\beta_L + \beta_H)^2} v_H = \beta_L \tag{9}$$

By symmetry, we also have

$$\frac{\sigma}{4}v_H + \frac{(1-\sigma)\beta_L\beta_H}{(\beta_L+\beta_H)^2}v_H = \beta_H \tag{10}$$

(9) and (10) imply that

$$\beta_H - \beta_L = \frac{\sigma}{4}(v_H - v_L) \tag{11}$$

Since the objective functions are globally concave, solutions to (9) and (10) determine a unique symmetric equilibrium. Due to analytical complexity, we cannot derive the closed-form equilibrium strategies. However, we do know that there exists one symmetric equilibrium determined by (9) and (10).

Proposition 1. *The symmetric equilibrium* β *of the rent-seeking contest with resale is given by the solutions to* (9) *and* (10). *In addition, the equilibrium efforts satisfy* $\frac{\beta_H-\beta_L}{v_H-v_L} = \frac{\sigma}{4}$.

Malueg and Yates (2004) fully characterize equilibrium strategies for rent-seeking contests with private values but without resale. Indeed, the equilibrium strategies have proportionality property. For the convenience of comparative study, we summarize their result in the following proposition as a reference.

Proposition 2. *Let* $\rho = \frac{v_L}{v_H}$, *then the symmetric equilibrium of the rent-seeking contest without resale is given by* $\tilde{\beta}_L = \tilde{\theta}v_L$, $\tilde{\beta}_H = \tilde{\theta}v_H$ *where* $\tilde{\theta} = \frac{\sigma}{4} + \frac{1-\sigma}{(\rho^{-1/2}+\rho^{1/2})^2}$.

From (11), we have

$$\frac{\beta_H - \beta_L}{v_H - v_L} = \frac{\sigma}{4} \tag{12}$$

By Proposition 2, we have

$$\frac{\tilde{\beta}_H - \tilde{\beta}_L}{v_H - v_L} = \frac{\sigma}{4} + \frac{1-\sigma}{(\rho^{-1/2}+\rho^{1/2})^2} \geq \frac{\sigma}{4} = \frac{\beta_H - \beta_L}{v_H - v_L} \tag{13}$$

Remark 1. *The introduction of resale possibility increases low-value player's valuation upon winning the contest since he has full bargaining power. Hence the low-value player has incentive to bid more aggressively to increase the winning probability and his expected payoff upon winning. As for the high-value player, the situation is as if she is now competing with a rival possessing higher valuation than before. Standard wisdom in contest literature suggests that a player with high valuation bids more aggressively if he learns his rival's valuation is high rather than low. However, the high-value player does not bid as more aggressively as the low-value player does. This is the intuition behind* (13).

The same intuition could explain the comparative statics of σ. Let $\delta = \beta_H - \beta_L$, and $\tilde{\delta} = \tilde{\beta}_H - \tilde{\beta}_L$, then we have

$$\frac{\partial \delta}{\partial \sigma} = \frac{1}{4}(v_H - v_L) \tag{14}$$

and

$$\frac{\partial \tilde{\delta}}{\partial \sigma} = \left(\frac{1}{4} - \frac{1}{\rho + \rho^{-1} + 2}\right)(v_H - v_L) \leq \frac{1}{4}(v_H - v_L) \tag{15}$$

Increases in σ implies greater ex post similarity in realized values, thus evens out the contest. But the effects of this evening out on equilibrium effort levels are different with or without resale. Without resale, evening out the contest leads to closer competition; however, resale opportunities lead to more aggressive competition for both players, thus distort the effect of evening out.

In rent-seeking contests, the overall welfare with resale possibilities is hard to tell. It is good that resale promotes allocative efficiency ex post, however, more aggressive bids means more social waste since more resources are devoted to unproductive activities. Our analysis may be justified if revenue is one of goals when politicians allocate the rent.

4 Equilibrium with Public Values and Resale

Consider the rent-seeking contests with public values and resale possibilities. We assume that the realizations of (v_1, v_2) are revealed publicly to both players before they submit their bids. We first characterize the pure-strategy Nash equilibrium and then compare it with the symmetric equilibrium we characterize in previous section.

According to the combination of values, we have four cases: (v_L, v_L), (v_L, v_H), (v_H, v_L), and (v_H, v_H). For the first and last cases, there is no potential gain of resale, so resale indeed does not take place for those two cases. Resale only happens when a low-value player competes with a high-value rival.

Using similar arguments as in Section 3, we have the following result.

Proposition 3. *The Nash equilibrium of rent-seeking contest with public values and resale possibilities is given by* β *defined as*

$$\beta_{LL} = \frac{v_L}{4},\ \beta_{LH} = \beta_{HL} = \beta_{HH} = \frac{v_H}{4}$$

Nti (1999) characterizes the equilibrium for contests with public values without considering resale possibilities. We summarize his result in the following proposition:

Proposition 4. *The unique pure-strategy Nash equilibrium of the rent-seeking contest with public values and without resale is given by* $(\tilde{\beta}_1, \tilde{\beta}_2)$ *defined as*

$$\tilde{\beta}_i(v_i, v_j) = \frac{v_i^2 v_j}{(v_i + v_j)^2} \tag{16}$$

From Proposition 4, we have $\tilde{\beta}_{LL} = \frac{v_L}{4}$, $\tilde{\beta}_{HH} = \frac{v_H}{4}$, $\tilde{\beta}_{LH} = \frac{v_L^2 v_H}{(v_L+v_H)^2}$, and $\tilde{\beta}_{HL} = \frac{v_H^2 v_L}{(v_L+v_H)^2}$.

Again resale opportunities make the low-value player compete more aggressively than otherwise, in turn makes the high-value player compete more aggressively as well. This is clearly seen from that

$$\tilde{\beta}_{LH} \leq \tilde{\beta}_{HL} \leq \beta_{LH} = \beta_{HL} \tag{17}$$

5 Revenue and Welfare

Now let us compare expected revenues resulting from rent-seeking contests with or without resale.

Let R_C denote ex ante expected revenue resulting from contests with public values and resale, then we have

$$R_C = \sigma(\beta_{LL} + \beta_{HH}) + (1-\sigma)(\beta_{LH} + \beta_{HL}) = \frac{1}{4}(\sigma v_L + (2-\sigma)v_H) \tag{18}$$

Let $\tilde{R}_C$ denote ex ante expected revenue resulting from contests with public values and no resale, then we have

$$\tilde{R}_C = \sigma(\tilde{\beta}_{LL} + \tilde{\beta}_{HH}) + (1-\sigma)(\tilde{\beta}_{LH} + \tilde{\beta}_{HL}) = \frac{1}{4}[\sigma(v_L + v_H) + 4(1-\sigma)\frac{v_L v_H}{v_L + v_H}] \tag{19}$$

Therefore, we have

$$R_C - \tilde{R}_C = \frac{(1-\sigma)v_H(v_H - v_L)}{2(v_L + v_H)} \geq 0 \tag{20}$$

Proposition 5. *For rent-seeking contests with public values, the expected revenue with resale exceeds that without revenue.*

Malueg and Yates (2004) documents a revenue equivalent result for rent-seeking contest without resale. They show that, ex ante expected revenue in rent-seeking contests with private information equals that in rent-seeking contests with public information. This could be verified as following.

Let $\tilde{R}_I$ denote ex ante expected revenue resulting from contests with incomplete information and no resale, and $\tilde{R}_C$ with complete information and no resale. Then we have

$$\tilde{R}_I = \sigma(\tilde{\beta}_L + \tilde{\beta}_H) + (1-\sigma)(\tilde{\beta}_L + \tilde{\beta}_H) = \tilde{\theta}(v_L + v_H) \tag{21}$$

and

$$\tilde{R}_C = \sigma(\tilde{\beta}_{LL} + \tilde{\beta}_{HH}) + (1-\sigma)(\tilde{\beta}_{LH} + \tilde{\beta}_{HL}) = \tilde{\theta}(v_L + v_H) \tag{22}$$

As we know, the existence of resale possibilities introduces an endogenous element for low-value player's valuation. Therefore, both players will compete more aggressively, which implies that $\tilde{R}_I \leq R_I$. But the extent for this upward change of valuation is different under different informational regimes. With public information, the low-value player's valuation upon winning is v_H if competing with a high-value rival. With private information, however, it only becomes $\sigma v_L + (1-\sigma)v_H$ since he does not know whether he competes with low-value or high-value rival. It is this uncertainty that decreases low-value player's incentive to bid more aggressively, hence reduces seller's expected revenue. Therefore, $R_I \leq R_C$. This argument shows that the endogenous element for low-value player's valuation upon winning explains why rent-seeking contest with public information is revenue superior. Without resale possibility, there is no such element. Therefore, revenue equivalence follows.

Given the above results, we could derive a general revenue ranking for rent-seeking contests with or without resale and with private values or public values. Therefore, the general revenue ranking result follows.

Proposition 6. *The ex ante expected revenues resulting from rent-seeking contests could be ranked as*

$$\tilde{R}_C = \tilde{R}_I \leq R_I \leq R_C \tag{23}$$

As for the efficiency of allocation, with resale possibility the ex post allocation of the prize is always efficient by construction of the equilibria. This is true regardless of informational regimes.

Proposition 7. *For a two-player-two-value rent-seeking contest with resale possibilities, ex post allocation is always efficient regardless of informational regimes. Moreover, ex post allocation is inefficient with positive probability without resale. Hence, introducing resale possibility enhances allocative efficiency ex post.*

Without resale possibilities, Malueg and Yates (2004) shows that private-information and public-information contests are identical in terms of allocative efficiency. Indeed, for each possible realization of players' values (v_1, v_2), the prize is awarded to a player with the highest value with the same probability in private-information contests as in public-information contests. However, in both informational regimes, ex post allocation is not efficient with positive probability. This inefficiency will disappear if resale possibilities are introduced.

We must admit the limitation of Proposition 7. The ex post efficiency is only restored for the simple case with two players and each player has two possible values. Once we deviate from this simple model, Proposition 7 no longer holds. Consider the following example.

Example 1. *Two players compete for one indivisible prize. Each player's valuation is independent draw from $\{v_L, v_M, v_H\}$ with equal probability, where $v_L \leq v_M \leq v_H$.*

If the realized values are the same, ex post allocation is efficient no matter who wins the prize. If the realized values are different, there is no scheme ensuring ex post efficiency. If the winner has high value, there will no resale. If the winner has medium value, he will ask v_H, and the offer will be accepted in equilibrium if his rival has high value. If the winner has low value, there is no optimal bargaining scheme that ensures efficiency ex post. If the low-value asks v_M, his expected valuation will be $\frac{v_L+2v_M}{3}$; if the low-value asks v_H, his expected valuation will be $\frac{2v_L+v_H}{3}$. Depending on different parameter values, the low-value winner's optimal asking price may be different. For instance, if $v_M < \frac{v_L+v_H}{2}$, he will ask v_H. However, with positive probability his rival may have medium valuation. Hence, the final allocation may not be efficient with positive probability.

Intuitively, the more values both players have, the more difficult to ensure ex post efficiency through resale. If both players have continuous private valuation, Myerson and Satterthwaite (1983) show that there is no incentive-compatible individually rational bargaining mechanism can be ex post efficient.

6 Other Resale Mechanisms

Until now we just consider only one possibility of resale mechanism: the winner makes a take-it-or-leave-it offer to the loser. We refer to this as monopoly resale since the winner has full bargaining power. Actually in practice the resale buyer may share the bargaining power with the seller or even have full bargaining power. In this Section, we consider other possible resale mechanisms: monopsony pricing and probabilistic k-double auctions. Specifically, monopsony pricing means that the buyer has full bargaining power and makes a take-it-or-leave-it offer to the seller. For probabilistic k-double auctions, we refer to the case in which with probability k, the seller makes a take-it-or-leave-it offer to the buyer and with probability $1-k$, the buyer makes a take-it-or-leave-it offer to the seller.[4]

6.1 Monopsony Resale

For monopsony resale, the loser of contest has full bargaining power and can make a take-it-or-leave-it offer to and buy the prize from the winner. As before, resale only takes place when the realized valuations of both players are different. Then the high-value loser exerts his bargaining power by offering a price equals to the low-value winner's valuation, v_L, and extracts all the surplus.

[4]As Hafalir and Krishna (2006) points out, the term k-double auction usually refers to a situation in which the price is weighted average of the price demanded by the seller and that offered by the buyer.

Suppose player 2 follow strategy μ, player 1 learns his private value as v_L and submits b_L, the expected payoff for him is

$$EU_1(b_L,\mu) = [\sigma\frac{b_L}{b_L+\mu_L} + (1-\sigma)\frac{b_L}{b_L+\mu_H}]v_L - b_L \tag{24}$$

Similarly, the expected payoff for player 1 if his private value is v_H and he bids b_H:

$$EU_1(b_H,\mu) = \sigma\frac{b_H}{b_H+\mu_H}v_H + (1-\sigma)[\frac{b_H}{b_H+\mu_L}v_H + \frac{\mu_L}{b_H+\mu_H}(v_H-v_L)] - b_H \tag{25}$$

By manipulating the first-order conditions, we have

$$\mu_H - \mu_L = \frac{\sigma}{4}(v_H - v_L) \tag{26}$$

The above result is quite interesting since we have exactly the same relationship for monopoly resale mechanism. In other words, no matter who has the full bargaining power, the ratio between difference in equilibrium efforts and difference in valuation remains the same: $\frac{\sigma}{4}$.

Similarly, as before we can derive the equilibrium strategies with public values and monopsony resale:

$$\mu_{LL} = \mu_{LH} = \mu_{HL} = \frac{v_L}{4},\ \mu_{HH} = \frac{v_H}{4} \tag{27}$$

Hence the ex ante expected revenue is

$$\hat{R}_C = \frac{1}{4}(\sigma v_H + (2-\sigma)v_L) \tag{28}$$

Recall that $\tilde{R}_C = \frac{1}{4}[\sigma(v_L+v_H) + 4(1-\sigma)\frac{v_L v_H}{v_L+v_H}]$, then we have

$$\hat{R}_C - \tilde{R}_C = 2(1-\sigma)\frac{v_L(v_L-v_H)}{v_L+v_H} \leq 0 \tag{29}$$

Proposition 8. *For rent-seeking contests with public values, if the losing player possesses full bargaining power, resale opportunities decrease expected revenue for the seller.*

Remark 2. *The intuition underlying Proposition* 8 *is as follows. Assigning full bargaining power to the loser only benefits the high-value player. This makes high-value player's expected payoff upon losing the contest positive. This decreases his incentive to compete as aggressively as what he does without resale opportunities. Moreover, the low-value player has incentive to bid less aggressively since he has no bargaining power upon winning, no additional benefit upon losing. Therefore, the overall equilibrium outlay is less than that without resale.*

6.2 Probabilistic k-Double Auctions

In this mechanism, resale takes place as follows. With probability k, the winner of contest makes a take-it-or-leave-it offer to the loser and with probability $1-k$ the loser makes a take-it-or-leave-it offer to the winner. Again resale takes place ex post only if players have different valuations. Obviously if $k = 1$, this reduces to the monopoly resale mechanism considered earlier. If $k = 0$, it reduces to the monopsony resale mechanism. In this subsection, we consider the case $0 < k < 1$.

Let us first characterize the symmetric equilibrium strategies. Suppose player 2 follow τ, player 1 learns his valuation as v_L and submits b_L, his expected payoff will be

$$EU_1(b_L,\tau) = \sigma \frac{b_L}{b_L+\tau_L} v_L + (1-\sigma)\frac{b_L}{b_L+\tau_H}\tilde{v} - b_L \tag{30}$$

where $\tilde{v} = kv_H + (1-k)v_L$ is the expected valuation upon winning when player 1 competes with a high-value rival.

Similarly, the expected payoff for player 1 if his private value is v_H and he bids b_H:

$$EU_1(b_H,\tau) = \sigma\frac{b_H}{b_H+\tau_H}v_H + (1-\sigma)[\frac{b_H}{b_H+\tau_L}v_H + \frac{\tau_L}{b_H+\tau_L}\check{v}] - b_H \tag{31}$$

where $\check{v} = (1-k)(v_H - v_L)$ is the expected valuation upon losing when player 1 competes with a low-value rival.

By manipulating the first-order conditions, we have

$$\tau_H - \tau_L = \frac{\sigma}{4}(v_H - v_L) \tag{32}$$

From previous analysis, we already know that such relationship holds in cases $k = 1$ and $k = 0$. It is not surprising to observe the same relationship when $0 < k < 1$. Let us summarize this interesting finding as follows.

Proposition 9. *For rent-seeking contests with resale, the ratio between difference in equilibrium effort and difference in valuation remains a constant* ($\frac{\sigma}{4}$) *independent of the distribution of bargaining power. Furthermore, this ratio is less than that without resale* ($\frac{\sigma}{4} + \frac{1-\sigma}{\rho+\rho^{-1}+2}$).

To figure out the expected revenue if values are public information, we need to derive the equilibrium strategies first. It is trivial to show that

$$\tau_{LL} = \frac{v_L}{4},\ \tau_{HH} = \frac{v_H}{4},\ \tau_{LH} = \tau_{HL} = \frac{(kv_H + (1-k)v_L)}{4} \tag{33}$$

Therefore, the ex ante expected revenue with public information ($\bar{R}_C$) is

$$\bar{R}_C = \frac{1}{4}(mv_H + (2-m)v_L) \tag{34}$$

where $m = \sigma + 2(1-\sigma)k$.

Recall the expected revenue with public values and without resale $\tilde{R}_C = \frac{1}{4}[\sigma(v_L + v_H) +$

$4(1-\sigma)\frac{v_L v_H}{v_L+v_H}]$, then we have

$$\bar{R}_C - \tilde{R}_C = 2(1-\sigma)\frac{(v_H - v_L)[k(v_L+v_H) - v_L]}{v_L + v_H} \tag{35}$$

Hence, $\bar{R}_C \geq \tilde{R}_C$ if and only if $k \geq \frac{v_L}{v_L+v_H}$.

It is interesting to examine how the ex ante expected revenue will change if the distribution of bargaining power varies. Given $\bar{R}_C = \frac{1}{4}(mv_H + (2-m)v_L)$, we have

$$\frac{\partial \bar{R}_C}{\partial k} = \frac{1-\sigma}{2}(v_H - v_L) \geq 0 \tag{36}$$

The intuition is quite straightforward. The resale possibility introduces an endogenous element to the winner's valuation upon winning. The more bargaining power the winner has, the more surplus he can extract from resale.[5] Hence, the low-value player will bid more aggressively. This means the high-value player is competing against a rival with higher valuation, thus he will also bid more aggressively. The unconditional payment rule of rent-seeking contests renders the expected revenue to become larger as more bargaining power goes to the winner. Indeed, we have[6]

$$\hat{R}_C \leq \bar{R}_C \leq R_C \tag{37}$$

It is trivial to show that $\beta_{LL} = \mu_{LL} = \tau_{LL}$, $\beta_{HH} = \mu_{HH} = \tau_{HH}$, $\beta_{LH} \geq \tau_{LH} \geq \mu_{LH}$, and $\beta_{HL} \geq \tau_{HL} \geq \mu_{HL}$.[7] Again the intuition is as before. Although the high-value player loses more bargaining power as k increases, he still bids more aggressively since the low-value

[5] When $v_L = v_H$ or $\sigma = 1$, the expected revenue is independent of the distribution of bargaining power. Under these two extremes, both players' valuations are perfectly aligned, hence there will be no resale.

[6] It is trivial to show that $R_C \geq \hat{R}_C$. Note $\bar{R}_C - \hat{R}_C = \frac{r}{4}(v_H - v_L)(m-\sigma) \geq 0$ since $m = 2k - 2k\sigma + \sigma \geq \sigma$. Moreover, $R_C - \bar{R}_C = \frac{r}{4}(v_H - v_L)(2-m-\sigma) \geq 0$ since $2-m-\sigma = 2(1-\sigma)(1-k) \geq 0$.

[7] The ex ante bids submitted by the low-value player are $\frac{v_L}{4}$, $\sigma\frac{v_L}{4} + (1-\sigma)\frac{(kv_H+(1-k)v_L)}{4}$ and $\sigma\frac{v_L}{4} + (1-\sigma)\frac{v_H}{4}$ for $k=0$, $0<k<1$, $k=1$ respectively. Similarly, the ex ante bids submitted by the high-value player are $\sigma\frac{v_H}{4} + (1-\sigma)\frac{v_L}{4}$, $\sigma\frac{v_H}{4} + (1-\sigma)\frac{(kv_H+(1-k)v_L)}{4}$, and $\frac{v_H}{4}$ for $k=0$, $0<k<1$, $k=1$ respectively.

player bids more aggressively as k increases. It is as if the high-value player is competing against with a rival with higher and higher valuation. Simple manipulation of equilibria strategies shows that the ex ante expected bids are getting bigger as the winner gets more bargaining power. This is true for both the low-value player and the high-value player.

A general revenue ranking among all possible situations is not available. It can be shown that the general ranking will depend on specific realization of parameter values.

7 Discussion and Conclusion

We have introduced into a standard rent-seeking contest with private values the interesting feature that resale may take place whenever there is potential gain by implementing it. We characterize the symmetric equilibrium and find an interesting proportional difference property. We show that resale enhances both ex post allocative efficiency and seller's ex ante expected revenue. By comparing both private and public information regimes, we derive a general revenue ranking for rent-seeking contest. It turns out that the highest expected revenue goes to the case with resale and public information. Depending on different parameter values, ex ante a player may prefer values to be private, public or indifferent. Similarly, whether resale possibility benefits a player ex ante depends on different parameter values.

For analytic simplicity, we focus on a two-player-two-value model and assume that the winner has full bargaining power in resale. The two-player-two-value setting ensures that most of our results are robust to variation of resale mechanisms. Indeed, they survive for any resale mechanism in which the resale price is somewhere between both players' valuations. But they do not survive if each player has more than two values. For example, if each player has three values, resale cannot lead to efficient allocation ex post. Depending on different configurations of values, allocation may not be always efficient ex post. Suppose that possible values are low, intermediate or high, the low-value player wins the contest and has full bargaining power. After updating his posterior belief, the low-value winner makes a take-it-or-leave-it offer in order to maximize his expected payoff. If values are private information, there is no general optimal pricing scheme that ensures efficiency ex post.[8]

More general analysis needs to be addressed with more than two players and continuous type space. Unfortunately there is no theoretical benchmark for rent-seeking contests with private information and more than two players or continuous type space. Actually it remains open to characterize equilibrium strategies for just two players whose valuations' supports have three points. The characterization of equilibrium strategies for these situations with or without resale seems a challenging exercise.

[8] I am indebted to Andreas Blume for pointing out to me this arguments, which lead to an interesting generalization for the current model.

A Proofs of Results

Proof of Proposition 1

Proof. From (6), we have

$$\frac{\partial EU_1(b_L,\beta)}{\partial b_L} = \frac{\sigma\beta_L}{(b_L+\beta_L)^2}v_L + \frac{(1-\sigma)\beta_H}{(b_L+\beta_H)^2}v_H - 1$$

Consider the second-order condition, then

$$\frac{\partial^2 EU_1(b_L,\beta)}{\partial b_L^2} = \frac{-2\sigma v_L\beta_L}{(b_L+\beta_L)^3} + \frac{2(\sigma-1)v_H\beta_H}{(b_L+\beta_H)^3} < 0$$

Similarly, we have $\frac{\partial^2 EU_1(b_H,\beta)}{\partial b_H^2} < 0$. Therefore, the objective function is globally concave. Hence, the first-order condition is both necessary and sufficient to characterize the symmetric equilibrium which is determined by (9) and (10). □

Proof of Proposition 2

Proof. See Malueg and Yates (2004). □

Proof of Proposition 3

Proof. If the realized values are (v_L, v_L) or $(v_H, v_H$, it is trivial to show that $\beta_{LL} = \frac{v_L}{4}$ and $\beta_{HH} = \frac{v_H}{4}$. Now let us look at the cases (v_L, v_H) and (v_H, v_L). With resale possibilities, the expected valuation upon winning is v_H for both players, since the low-value player could resell the prize to his high-value rival with price equal to v_H. Therefore, existence of resale possibilities symmetrizes valuations for both players. It remains easy to show that $\beta_{LH} = \beta_{HL} = \frac{v_H}{4}$. □

Proof of Proposition 4

Proof. See Nti (1999) or Malueg and Yates (2004). □

Proofs of Proposition 5 and 6 are contained in the text, so are omitted.

Proof of Proposition 7

Proof. This is trivial by our equilibrium construction. If the realized valuations for both players are the same, the final allocation is always efficient. If the low-value player wins the prize, he will resell it to the high-value rival and the latter will accept the offer in equilibrium. Therefore, the final allocation is always efficient. □

Proof of Proposition 8

Proof. See Section 6.1. □

Proof of Proposition 9

Proof. From (9), (10), we have

$$\frac{\beta_H - \beta_L}{v_H - v_L} = \frac{\sigma}{4}$$

From Proposition 2, we have

$$\frac{\tilde{\beta}_H - \tilde{\beta}_L}{v_H - v_L} = \frac{\sigma}{4} + \frac{1-\sigma}{(\rho^{-1/2} + \rho^{1/2})^2}$$

From (24), (25), we have

$$\frac{\mu_H - \mu_L}{v_H - v_L} = \frac{\sigma}{4}$$

From (30), (31), we have

$$\frac{\tau_H - \tau_L}{v_H - v_L} = \frac{\sigma}{4}$$

□

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