Adaptive Lattice Filters for Band-Inverse Covariance Matrix Approximations

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I. EXTENDED ABSTRACT

Theory of adaptive lattice filters, developed originally for stationary processes with Toeplitz covariance matrices, was well-established by the pioneering contributions of Burg [1], Kailath [2], Friendlander [3]. The lattice filters are intimately related to the well-known Levinson algorithm, that requires $O(N^2)$ operations for $N \times N$ Toeplitz covariance matrix inversion, and exploit Burg technique for adaptive estimation of reflection coefficients that naturally preserves positive definiteness of the estimated Toeplitz covariance matrix [3].

It has been known since 1981 at least, that the Levinson (and Shur) algorithm can be applied to a non-stationary process with its associated arbitrary non-Toeplitz covariance matrix. However, in general case this generalized Levinson algorithm involves $O(N^3)$ computations, so that there are no particular advantages over the usual methods of Choleski decomposition or of matrix inversion. Therefore, the main attention devoted to a special class of non-stationary processes with covariance matrices that have a finite "displacement rank" (or equivalently "Toeplitz distance"). For this class of non-stationary processes, adaptive lattice filters retained most of their computational and structural advantages.

Another type of approximations for an arbitrary Hermitian covariance matrices that is based upon the so-called band-inverse extension, developed by H. Dym and I. Gohberg in [4] has been recently explored for adaptive filter (antenna) applications in [5].

Yet, long before these results of H. Dym and I. Gohberg were considered for applications in adaptive processing in [5]–[7] and in a large number of subsequent papers, the theory of generalized lattice filters for an arbitrary Hermitian covariance matrix has been suggested. Now it turns out that there is intimate connection between the class of these generalized lattice filters, and the band inverse approximation (extension) of an arbitrary Hermitian covariance matrix. In the presented paper we demonstrate that the number of stages of the generalized lattice structure corresponds to the width of a band inverse approximation. While practical recommendations on limiting the number of these stages have been implemented in [8], [9] it is now made clear that in fact these technical solutions implement maximum entropy band extension as proven by H. Dym and I. Gohberg in [4]. We demonstrate, that in the way the classical lattice filter with a finite number of stages m, implements autoregressive VAR(m) approximation of an arbitrary Toeplitz matrix, the generalized lattice filter with a finite number of stages m implements the time varying TVAR(m) approximation of a arbitrary Hermitian covariance matrix. Moreover, this approach could be extended over a vector process, that is important for two-dimensional TVARM(m) or TVARM($m|n$) approximations of space-time covariance matrices in STAP processing. Finally, we demonstrate some results of practical implementation of the generalized adaptive lattice filters for clutter mitigation in air traffic control (ATC) radars [10]. Comparison with conventional MTL systems demonstrates significant performance improvement as illustrated by the radar screen shots attached 1.

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REFERENCES

[1] J. Burg, "Maximum entropy spectral analysis," Ph.D. dissertation, Stanford University, California, 1975, Dept of Electrical Engineering.

[2] T. Kailath, "An innovation approach to least squares estimation. Part I: Linear filtering in additive white noise," *IEEE Trans. Autom. Control*, vol. 13, pp. 646–655, 1968.

[3] B. Friedlander, "Lattice filters for adaptive processing," *Proc. IEEE*, vol. 70, pp. 829–867, 1982.

[4] H. Dym and I. Gohberg, "Extensions of band matrices with band inverses," *Linear Algebra Appl.*, vol. 36, pp. 1–24, Mar. 1981.

[5] Y.I. Abramovich, N.K. Spencer, and M.D.E. Turley, "Time-varying autoregressive (TVAR) models for multiple radar observations," *IEEE Trans. Sig. Proc.*, vol. 55, no.4, pp. 1298–1311, Apr. 2007.

[6] Y. Abramovich, B. Johnson, and N. Spencer, "Two-dimensional matrix-valued parametric models for radar applications – Part I: Maximum-entropy extensions for Toeplitz-block matrices," *IEEE Trans. Sig. Proc.*, vol. 56, no. 11, pp. 5509–5526, Nov. 2008.

[7] D.I. Lekhovytskiy, V.I. Zaritskiy, I.D. Rakov, B.G. Sverdlov, and M.V. Ratynskiy, *Adaptive lattice filtering methods for space-time processing problems*, Academy of Science USSR, Ac A. L. Mintsa Radiotechnical Institute, Preprint 8610, Moscow, USSR, 1987, in Russian.

[8] D.I. Lekhovytskiy, "Generalized Levinson algorithm and universal lattice filters," *ProcVUZ Radiophysics*, vol. 35, no. 9-10, pp. 790–808, 1992, in Russian.

[9] D.I. Lekhovytskiy, G.A. Zhuga B.M. Vovshin, and V.V. Lavrukevich, "Special features of inter-period signal processing at the clutter background in non-coherent pulse radars," in *Proc. IRS 2007*, Cologne, Germany, Sept. 2007.

[10] D. Lekhovytskiy, V. Ryabykha, G. Zhuga, V. Lavrent'ev "Results of the experimental studies of the MTD systems based on adaptive lattice filters in the pulse radar with batch-to-batch staggering of pulse repetition intervals," *Applied radio electronics*, Kharkiv, Ukraine, vol. 7, no. 1, pp. 11–24, in Russian.