A Two-Dimensional Single-Field FDTD Formulation for Oblique Incident Electromagnetic Simulations

***Gokhan Aydin*[1], *Atef Z. Elsherbeni*[2], *Ercument Arvas*[1], *and Jay K. Lee*[1]**

[1]Department of Electrical Engineering and Computer Science
Syracuse University, Syracuse, NY 13244-1240, USA
goaydin@syr.edu, earvas@syr.edu, leejk@syr.edu

[2]Department of Electrical Engineering
University of Mississippi, University, MS 38677-1848, USA
atef@olemiss.edu

Abstract

A set of general purpose two-dimensional single-field finite-difference time-domain updating equations for solving oblique incidence electromagnetic problems is derived. The traditional FDTD updating equations are based on Maxwell's curl equations whereas the single-field FDTD updating equations are based on the vector wave equation. Performance analyses of the single–field formulation in terms of CPU time and memory requirement are presented along with numerical validation. It was observed that the single-field method is more efficient than the traditional FDTD formulation in terms of speed and memory requirements for oblique incident problems.

1. Introduction

The first paper on finite-difference time-domain (FDTD) was published in 1966 by Yee [1]. Since then, the FDTD has become widely used in computational electromagnetics [2]. Extensive research has been reported to improve the accuracy and speed of the method and different absorbing boundary conditions (ABCs) are developed to provide more accurate results [3,4,5]. Several reported improvements in speed of the method were relying almost solely on the progress of computer hardware and software architecture. Merits of the single-field approach in two-dimensional problems are studied for normal incidence case [6]. For many practical applications, the excitation of plane wave is usually not at normal incidence with respect to the scattering object or objects. Therefore, in order to provide accurate assessment of the scattering properties of objects, one has to consider oblique incident conditions.

This paper investigates the single-field approach based on the vector wave equation (VWE) to derive the FDTD updating equations in a way that only one field component will be calculated and updated inside the iteration loop to eliminate iteration steps required to update the other field component. Since one field (E or H) can be calculated from the other field, whenever needed, the proposed method, hence, is able to provide simulation results similar to that obtained from traditional FDTD updating equations. To compare the proposed updating equations with the traditional ones [7], a 2D problem is constructed with three dielectric cylinders subject to obliquely incident plane wave, and the scattered field is calculated. Comparison of the single-field and the traditional formulations are performed in terms of CPU time and memory requirement.

2. Formulation

One can write Maxwell's equations for incident and total fields in free space as

$$\nabla \times \boldsymbol{E}_{inc}(t) = -\mu_0 \frac{\partial \boldsymbol{H}_{inc}(t)}{\partial t}, \nabla \times \boldsymbol{H}_{inc}(t) = \varepsilon_0 \frac{\partial \boldsymbol{E}_{inc}(t)}{\partial t} \tag{1, 2}$$

$$\nabla \times \boldsymbol{E}_{tot}(t) = -\mu \frac{\partial \boldsymbol{H}_{tot}(t)}{\partial t} - \sigma^m \boldsymbol{H}_{tot}(t), \nabla \times \boldsymbol{H}_{tot}(t) = \varepsilon \frac{\partial \boldsymbol{E}_{tot}(t)}{\partial t} + \sigma^e \boldsymbol{E}_{tot}(t) \tag{3, 4}$$

The total field is comprised of incident and scattered field components

978-1-4244-6051-9/11/$26.00 ©2011 IEEE

$$\boldsymbol{E}_{tot} = \boldsymbol{E}_{inc} + \boldsymbol{E}_{scat}, \boldsymbol{H}_{tot} = \boldsymbol{H}_{inc} + \boldsymbol{H}_{scat} \tag{5, 6}$$

Taking the curl of (1) and (3) and using (2) and (4), we have

$$\nabla \times \nabla \times \boldsymbol{E}_{inc} = -\mu_0 \varepsilon_0 \frac{\partial^2 \boldsymbol{E}_{inc}}{\partial t^2}, \nabla \times \nabla \times \boldsymbol{E}_{tot} = -\mu \frac{\partial}{\partial t}(\nabla \times \boldsymbol{H}_{tot}) - \sigma^m(\nabla \times \boldsymbol{H}_{tot}) \tag{7, 8}$$

Using (4), (5) and (7), Equation (8) reduces to

$$\left(-\mu_0 \varepsilon_0 \frac{\partial^2 \boldsymbol{E}_{inc}}{\partial t^2}\right) + \nabla \times \nabla \times \boldsymbol{E}_{scat} = -\mu \frac{\partial}{\partial t}\left(\varepsilon \frac{\partial \boldsymbol{E}_{tot}}{\partial t} + \sigma^e \boldsymbol{E}_{tot}\right) - \sigma^m \left(\varepsilon \frac{\partial \boldsymbol{E}_{tot}}{\partial t} + \sigma^e \boldsymbol{E}_{tot}\right) \tag{9}$$

Using the following vector identity $\nabla \times \nabla \times \boldsymbol{E} = \nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E}$, Equation (9) becomes

$$\nabla^2 \boldsymbol{E}_{scat} - \nabla(\nabla \cdot \boldsymbol{E}_{scat}) = (\mu\varepsilon - \mu_0\varepsilon_0)\frac{\partial^2 \boldsymbol{E}_{inc}}{\partial t^2} + \mu\varepsilon \frac{\partial^2 \boldsymbol{E}_{scat}}{\partial t^2} + (\mu\sigma^e + \varepsilon\sigma^m)\frac{\partial \boldsymbol{E}_{tot}}{\partial t} + \sigma^m \sigma^e \boldsymbol{E}_{tot} \tag{10}$$

To implement (10) with the FDTD method, we have to decompose it to its Cartesian components. Moreover, we assume no variation for field magnitude in z direction, but the variation in phase of the field can be obtained from the phase expression of the field. The phase expression of a time-harmonic incident plane wave can be written as $e^{j(\boldsymbol{k} \cdot \boldsymbol{r})}$ where the wave vector $\boldsymbol{k}$ and the position vector $\boldsymbol{r}$ are expressed in Cartesian coordinate as

$$\boldsymbol{k} = k_0(\hat{x} \sin\theta_{inc} \cos\varphi_{inc} + \hat{y} \sin\theta_{inc} \sin\varphi_{inc} + \hat{z}\cos\theta_{inc}), \boldsymbol{r} = \hat{x}x + \hat{y}y + \hat{z}z \tag{11, 12}$$

with $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$. Using the phase expression and Equations (11) and (12), one can derive the following identities for the variation in z direction

$$\frac{\partial}{\partial z} = jk_0 \cos\theta_{inc}, \frac{\partial^2}{\partial z^2} = (jk_0 \cos\theta_{inc})^2 \tag{13, 14}$$

Cartesian component of (10) incorporated with (13) and (14) in x direction can be written as

$$\begin{aligned}\frac{\partial^2 E_{scat,x}}{\partial y^2} + (jk_0 \cos\theta_{inc})^2 E_{scat,x} - \frac{\partial^2 E_{scat,y}}{\partial x \partial y} - (jk_0 \cos\theta_{inc})\frac{\partial E_{scat,z}}{\partial x} \\ = (\mu\varepsilon - \mu_0\varepsilon_0)\frac{\partial^2 E_{inc,x}}{\partial t^2} + \mu\varepsilon\frac{\partial^2 E_{scat,x}}{\partial t^2} + (\mu\sigma^e + \varepsilon\sigma^m)\frac{\partial E_{tot,x}}{\partial t} + \sigma^m\sigma^e E_{tot,x}\end{aligned} \tag{15}$$

To derive the FDTD updating equations for the electric fields, we have to evaluate all the spatial derivatives in equations (15) at the corresponding electric field node, e.g., for the E_x component. The time derivatives are evaluated at the n^{th} time step.

$$\begin{aligned}E_{scat,x}^{n+1}(i,j) = C_{ex}^{ex}(i,j)\left[E_{scat,x}^{n}(i,j)\right] + C_{ex}^{ex,nm1}(i,j)\left[E_{scat,x}^{n-1}(i,j)\right] \\ + C_{ex}^{ex,n,y}(i,j)\left[E_{scat,x}^{n}(i,j+1) + E_{scat,x}^{n}(i,j-1)\right] \\ + C_{ex}^{ey,n,xy}(i,j)\left[E_{scat,y}^{n}(i+1,j) - E_{scat,y}^{n}(i+1,j-1) - E_{scat,y}^{n}(i,j) \right. \\ \left. + E_{scat,y}^{n}(i,j-1)\right] + C_{ex}^{ez,n,\theta,x}(i,j)\left[E_{scat,z}^{n+1}(i+1,j) - E_{scat,z}^{n+1}(i,j)\right] \\ + C_{ex}^{exinc,n+1}(i,j)\left[E_{inc,x}^{n+1}(i,j)\right] + C_{ex}^{exinc,n}(i,j)\left[E_{inc,x}^{n}(i,j)\right] \\ + C_{ex}^{exinc,n-1}(i,j)\left[E_{inc,x}^{n-1}(i,j)\right]\end{aligned} \tag{16}$$

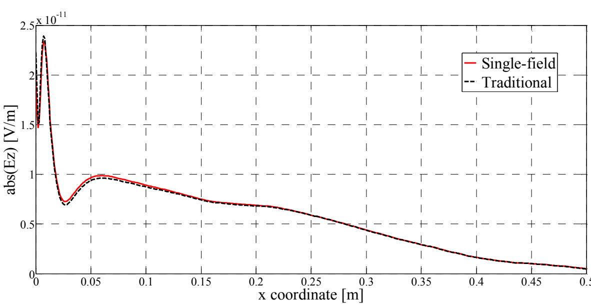
C terms are constant coefficients. The updating equations in y and z directions are given for the completeness.

$$\begin{aligned}E_{scat,y}^{n+1}(i,j) = {} & C_{ey}^{ey,n}(i,j)\left[E_{scat,y}^{n}(i,j)\right] + C_{ey}^{ey,n}(i,j)\left[E_{scat,y}^{n-1}(i,j)\right] \\ & + C_{ey}^{ey,n,x}(i,j)\left[E_{scat,y}^{n}(i+1,j) + E_{scat,y}^{n}(i-1,j)\right] \\ & + C_{ey}^{ex,n,xy}(i,j)\left[E_{scat,x}^{n}(i+1,j) - E_{scat,x}^{n}(i+1,j-1) - E_{scat,x}^{n}(i,j)\right. \\ & \left. + E_{scat,x}^{n}(i,j-1)\right] + C_{ey}^{ez,n,\theta,xt}(i,j)\left[E_{scat,z}^{n}(i,j+1) - E_{scat,z}^{n}(i,j)\right] \\ & + C_{ey}^{eincy,n+1}(i,j)\left[E_{inc,y}^{n+1}(i,j)\right] + C_{ey}^{eincy,n}(i,j)\left[E_{inc,y}^{n-1}(i,j)\right] \\ & + C_{ey}^{eincy,n-1}(i,j)\left[E_{inc,y}^{n}(i,j)\right]\end{aligned} \tag{17}$$

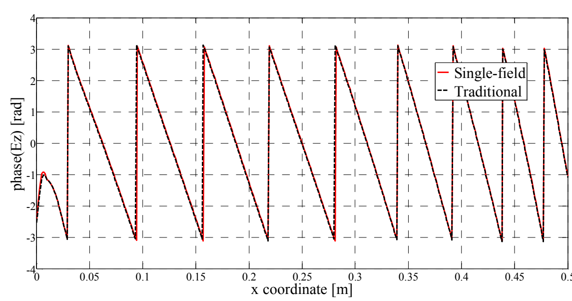
$$\begin{aligned}E_{scat,z}^{n+1}(i,j) = {} & C_{ez}^{ez,n}(i,j)\left[E_{scat,z}^{n}(i,j)\right] + C_{ez}^{ez,nm1}(i,j)\left[E_{scat,z}^{n-1}(i,j)\right] \\ & + C_{ez}^{ez,n,x}(i,j)\left[E_{scat,z}^{n}(i+1,j) + E_{scat,z}^{n}(i-1,j)\right] \\ & + C_{ez}^{ez,n,y}(i,j)\left[E_{scat,z}^{n}(i,j+1) + E_{scat,z}^{n}(i,j-1)\right] \\ & + C_{ez}^{ex,n,\theta,xt}(i,j)\left[E_{scat,x}^{n+1}(i,j) - E_{scat,x}^{n+1}(i-1,j)\right] \\ & + C_{ez}^{ey,n,\theta,yt}(i,j)\left[E_{scat,y}^{n+1}(i,j) - E_{scat,y}^{n+1}(i,j-1)\right] + C_{ez}^{eincz,n+1}(i,j)\left[E_{inc,z}^{n+1}(i,j)\right] \\ & + C_{ez}^{eincz,n}\left[E_{tot,z}^{n}(i,j)\right] + C_{ez}^{eincz,n-1}(i,j)\left[E_{tot,z}^{n-1}(i,j)\right]\end{aligned} \tag{18}$$

3. Numerical Validation

Problem geometry is constructed as three dielectric cylinders located on *y* axis, as a plane wave is obliquely incident towards -*x* direction. Scattered fields are sampled at 500 points on the *x*-axis (0-0.5m, 0). The dielectric cylinders have radius of 1 cm and dielectric constant of 4. Center-to-center distance is 3 cm. The incident wave has a Gaussian shape with maximum frequency of 15 GHz, the azimuth angle (φ_{inc}) is 0 with respect to the *x* axis and its angle of incidence (θ_{inc}) is 30 degrees with respect to the *z* axis. Figure 1 shows magnitude and pahse comparison for the near field results of the single-field and the traditional formulations. In both formulations, k_0 value is set according to the following formula $k = \omega\sqrt{\mu\varepsilon}\cos\theta = \frac{2\pi 10^{10}\cos 30}{3\times 10^{8}} = 181.38$. Figure 1 shows a good agreement between the results generated based on the single-field and the traditional formulations.



(a)



(b)

Fig. 1 Electric field comparison for f = 10 GHz; (a) magnitude, (b) phase.

4. Comparison of Memory Usage

Memory requirements for both formulations can be compared by counting the number of coefficients used in the updating equations in addition to the scattered and incident field terms needed. The traditional formulation has six updating equations, such as

$$\begin{aligned}E_{scat,x}^{n+1}(i,j) = {} & C_1(i,j)\left[E_{scat,x}^{n}(i,j)\right] + C_2(i,j)\left[H_{scat,z}^{n+0.5}(i,j) - H_{scat,z}^{n+0.5}(i,j-1)\right] + C_3(i,j)\left[H_{scat,y}^{n+0.5}(i,j)\right] \\ & + C_4(i,j)\left[E_{inc,x}^{n+1}(i,j)\right] + C_5(i,j)\left[E_{inc,x}^{n}(i,j)\right]\end{aligned} \tag{19}$$

Table 1 shows the number of floating-point addition operations per node (FLAOPn), floating-point multiplication operations per node (FLMOPn) and memory allocation required for field terms per node (MAFTn) required for the single-field and the traditional formulations. Given the memory allocations are for updating equations only, problem domain and material related memory allocations are not mentioned here since they apply in both formulations. The single-field formulation seems to require less memory; therefore it can handle bigger problems than the traditional one with the same amount of available memory.

Table 1. Required FLAOPn, FLMOPn and MAFTn for oblique incidence case.

Formulations	# FLAOPn	# FLMOPn	# MAFTn
Single-field	36	25	25 coefficients + 15 fields (6 scat. + 9 inc.) = 40
Traditional	30	30	30 coefficients + 18 fields (6 scat. + 12 inc.) = 48

5. Comparison of CPU Time

Figure 2 shows the CPU time required to complete a simulation of a corresponding domain size. The *x* axis represents the number of cells in the domain. Though in oblique incidence case the single-field formulation has to use three updating equations as opposed to the normal incidence case where only one updating equation is used to solve 2D TE_z or TM_z problem [6], the single-field formulation is still faster than the traditional one as shown in Figure 3. This result reinforces the argument of the single-field formulation to be faster than the traditional one for any 2D problem for both normal and oblique incidence cases [8].

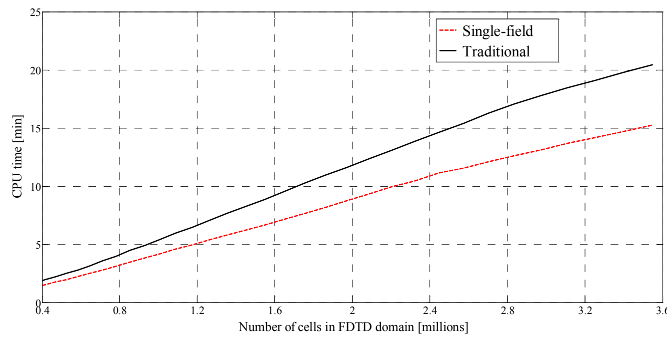


Fig. 2 CPU time comparison for 2D oblique case.

6. Conclusion

Oblique incidence FDTD updating equations are derived and compared with the traditional formulation in terms of accuracy, speed and memory requirements. The single-field formulation is advantageous in terms of speed and memory requirements in oblique incidence case as is the case with normal incidence [6].

7. References

[1] K. Yee, “Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media,” *IEEE Trans. Antennas Propagation*, vol. AP-14, pp. 302-307, May 1966.

[2] K. L. Shlager and J.B. Schneider, “A selective survey of the finite-difference time-domain literature,” *IEEE Antennas and Propagation Magazine,* vol. 37, pp. 39-57, August 1995.

[3] G. Mur, “Absorbing boundary conditions for the finite-difference approximations of the time-domain electromagnetic field equations,” *IEEE Trans. Electromagnetic Compatibility*, vol. EMC-23, pp. 1073-1077, Nov. 1981.

[4] Z. P. Liao, H. L. Wong, B. Yang, and Y. Yuan, “A transmitting boundary for transient wave analysis," *Sci. Sin., Ser. A*, vol. 27, pp. 1063–1076, 1984.

[5] J. P. Berenger, “A perfectly matched layer for the absorption of electromagnetic waves,” *Journal of Computational Physics*, vol. 114, pp. 185–200, 1994.

[6] G. Aydin, A. Z. Elsherbeni, E. Arvas, “A single-field FDTD formulation for electromagnetic simulations," *ACES Journal*. (in review process)

[7] B. H. Henin, *Electromagnetic Scattering from Metamaterial and Applications*, Ph.D. dissertation, The University of Mississippi, 2009.

[8] G. Aydin, *A Single-Field Finite-Difference Time-Domain Formulation for Electromagnetic Simulations*, Ph.D. dissertation, Syracuse University, 2011.