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Abstract—Hydrocarbons, carbon monoxide, and other polluting emissions produced by diesel engines are usually much lower than those from gasoline engines. However, higher combustion temperature in diesel engines cause substantially larger percentage of nitrogen oxides (NO_x) emissions. Selective catalyst reduction (SCR) is a well proven technology for reducing NO_x emissions from automotive sources and in particular, heavy-duty truck diesel engines. In this paper, we develop a linear parameter varying (LPV) control design method for the urea-SCR aftertreatment system to minimize the NO_x emissions and ammonia slippage downstream the catalyst. Performance of the closed-loop system obtained from the interconnection of the SCR system and the output feedback LPV control strategy is then compared with other control design methods including sliding mode, and observer-based static state feedback methods.

I. INTRODUCTION

The lean burn conditions of diesel combustion that yield improved efficiency produce an exhaust containing an excess of oxygen (up to 10%). While net-oxidizing exhaust enables the comparatively straightforward oxidation of hydrocarbons and carbon monoxide (CO) on precious metal catalysts, it complicates the chemical reduction of nitrogen oxides (NO_x) to N_2 . This scenario has led to the vigorous development of technologies for NO_x reduction to meet the stringent NO_x limit mandated by the EPA. The NO_x emissions are one of the main air pollutants which are responsible for ozone depletion, photochemical smog formation causing severe respiratory problems to humans. Selective catalyst reduction (SCR) is a well proven technology used in power generation for more than 30 years. Power generation involves very slow variation of operating conditions, allowing simple open-loop controllers to efficiently tackle the task of control. However, automobile engines work in a broad envelope of fast varying conditions, making the use of advanced control techniques desirable.

The SCR system operates as follows: the urea injector, driven by a command signal from the controller, pumps the mixture of compressed air and aqueous urea solution into the exhaust stream through the nozzle. Ammonia (NH₃) and carbon dioxide are formed as a result of urea decomposition and HNCO hydrolysis in the exhaust pipe [3]. The mixture of ammonia, CO₂, remaining urea, and the exhaust then enters the SCR catalyst, where NH₃ reacts with NO_x from the exhaust producing pure nitrogen and water. A sensor placed at the catalyst outlet measures concentration of the unreacted NO_x and supplies this information to the controller, thereby closing the loop. The control problem consists of achieving the appropriate regulation of the urea injection to minimize NO_x emissions without significant ammonia slip.

There have been some efforts in lumped parameter modeling of the SCR reactions in the literature. These simplified models are appropriate for model-based control, since they reduce the complexity of the design. Tronconi and Forzatti [1] develop one- and two-dimensional steady-state isothermal models of SCR for different geometries of the catalyst. Upadhyay and Nieuwstadt [6] derive lumped parameter model of SCR by first assuming that the catalyst behaves as an isothermal continuously stirred tank reactor (ICSTR) and neglecting mass transfer, and next using method of weighted residuals. The reaction mechanism involves the $DeNO_x$ reaction, adsorption/desorption of NH_3 and NH_3 oxidation. This model considers the reduction of only NO by ammonia. Since, in Fe-zeolite catalyst NO₂ based reactions are highly favored, and NO₂ is more toxic compared to NO, Devarakonda et al. [10] present a set of ordinary differential equations (ODEs) to model the SCR reactions considering both fast SCR reaction involving NO₂ and the standard SCR reaction.

Model-based control and optimization of the SCR system have been the focus of few recent published papers. Upadhyay and Nieuwstadt [8] present a model-based control strategy using sliding mode control. A nonlinear observer is designed using the measured NO_x concentration downstream the SCR catalyst for estimation of surface coverage fraction and ammonia slip concentration. A similar control design method which incorporates both the NO and NO₂ conversion efficiency in addition to the ammonia slip is proposed in [11]. Schar et al. [9] use a similar model as in [6] and design a model-based feedforward controller to limit the ammonia slip and a PI feedback controller for disturbance rejection purposes. Chi et al. [7] present a more advanced SCR model where the catalyst channel is discretized axially and radially. A simplified first-order model of the system is then used in [7] for control design purposes, where the parameters are estimated in real-time using a model reference adaptive controller.

In the present paper, we use a 3-state nonlinear model developed in [6] and design a reduced-order adaptive LPV controller to maximize the NO conversion efficiency and minimize the NH_3 slip. The simulation results of this paper illustrate a comparison between the performance of developed output feedback controller to that of static state feedback and also sliding mode control, where the latter methods require design of an observer for state estimation.

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II. SCR MODEL

The key requirement of an SCR catalyst is to selectively reduce NO_x to N_2 in the presence of ammonia (NH₃). Typical SCR washcoats contain base metals such as Cu, Fe, and zeolites that store ammonia to enable NO_x reduction. Ammonia is obtained by the thermal decomposition and hydrolysis of externally supplied aqueous urea. The two key reactions involved in the SCR NO_x reduction process are as follows:

$$4NH_3 + 4NO + O_2 \longrightarrow 4N_2 + 6H_2O$$

$$4NH_3 + 2NO + 2NO_2 \longrightarrow 4N_2 + 6H_2O.$$

The second reaction, which involves both NO and NO₂ at equimolar amounts, is much faster than the first reaction with only NO. The NO_x conversion is improved by the presence of NO₂ except at high temperatures where ammonia oxidation limits NO_x conversion.

The model we use in the present paper is borrowed from [6], in which the NO and NH_3 concentrations, as well as, surface coverage fraction are three states of the developed differential equations. The model is associated with the first reaction described above. The complete model is as follows

$$\begin{bmatrix} \dot{C}_{NO} \\ \dot{\theta} \\ \dot{C}_{NH_3} \end{bmatrix} = \begin{bmatrix} f_1(C_{NO}, \theta, C_{NH_3}) \\ f_2(C_{NO}, \theta, C_{NH_3}) \\ f_3(C_{NO}, \theta, C_{NH_3}) \end{bmatrix} + \begin{bmatrix} F \\ 0 \\ 0 \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ 0 \\ F \\ V \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} C_{NO} \\ \theta \\ C_{NH_3} \end{bmatrix}$$
(1)
$$f_1 = -C_{NO}(\Theta_{SC}R_{red}\theta + \frac{F}{V}) + R_{ox}\Theta_{SC}\theta$$

$$f_2 = -\theta(R_{ads}C_{NH_3} + R_{des} + R_{red}C_{NO} + R_{ox}) + R_{ads}C_{NH_3}$$

$$f_3 = -C_{NH_3}\left(\Theta_{SC}R_{ads}(1 - \theta) + \frac{F}{V}\right) + \Theta_{SC}R_{des}\theta$$

where C_{NO} and C_{NH_3} are concentrations of NO and NH₃ in mole/m³, respectively. The reaction rate for reaction *i* is represented by $R_i = k_i exp(\frac{-E_i}{RT})$, i = ads, des, red, ox where E_i is the activation energy and k_i is the pre-exponential term for the corresponding reaction. Θ_{SC} is the maximum ammonia storage, and θ is the surface coverage fraction. *F*, *V*, *R*, and *T* represent the constant flow rate through catalyst, catalyst volume, universal gas constant, and temperature, respectively. The input *u* is the concentration of ammonia entering the catalyst and is the only controllable parameter. The input *d* is the NO concentration upstream the catalyst treated as an external disturbance. The only measurement available is the concentration of NO downstream the catalyst denoted by *y*.

The above system can be transformed into the quasi-LPV

form [4] with x as the state variable vector as follows

$$\begin{bmatrix} \dot{x}_1\\ \dot{x}_2\\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\Theta_{SC}R_{red}x_2 - \frac{F}{V} & R_{ox}\Theta_{SC}\\ -R_{red}x_2 & -(R_{ox} + R_{des})\\ 0 & \Theta_{SC}R_{des} \end{bmatrix}$$

$$\begin{bmatrix} 0\\ R_{ads}(1-x_2)\\ -\Theta_{SC}R_{ads}(1-x_2) - \frac{F}{V} \end{bmatrix} \begin{bmatrix} x_1\\ x_2\\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{F}{V}\\ 0\\ 0 \end{bmatrix} d + \begin{bmatrix} 0\\ 0\\ \frac{F}{V} \end{bmatrix} u. \quad (2)$$

As observed, the system matrix A has an affine dependence on the second state, *i.e.*, surface coverage fraction θ , and therefore θ is considered as the scheduling parameter in the quasi-LPV model and needs to be known in real-time. In the system state-space representation, the system matrix A can be represented as $A(\theta) = A_0 + \theta A_1$ and rest of the system matrices are parameter-independent.

III. LPV STATE FEEDBACK CONTROL DESIGN APPROACH

In the SCR system the only measurement available is the NO concentration downstream the catalyst. The control design objective is to maximize the NO_x conversion efficiency and minimize the ammonia slip, as well as, the amount of urea to be injected. For the design purposes of this paper, we consider ammonia as the control input; however, it is noted that in an actual SCR system, the concentration of urea is the control variable. We propose to design an LPV state feedback controller combined with an LPV observer. The controller is designed to minimize the H_{∞} norm of the closed-loop system as the performance measure. Observability and controllability of the system (1) are discussed in [8]. The model introduced in (2) can be rewritten as

$$\dot{x} = A(\theta)x + B_1d + B_2u$$

$$y = C_2x$$

$$z = C_1x.$$
(3)

The following state-space representation for the dynamics of the LPV observer is considered

$$\dot{\hat{x}} = A(\theta)\hat{x} + B_1d + B_2u + L(y - \hat{y})$$
$$\hat{y} = C_2\hat{x}$$
$$\hat{z} = C_1\hat{x}$$
(4)

where L is the observer gain and is designed such that the error system is stable and satisfies the specified performance criterion. Substituting $u = K(\theta)\hat{x}$ and $e = x - \hat{x}$ in the above equations result in

$$\dot{\hat{x}} = (A(\theta) + B_2 K(\theta))\hat{x} + B_1 d + LC_2 e \qquad (5)$$
$$\dot{e} = (A - LC_2)e.$$

The augmented system of the plant and the observer becomes

$$\begin{aligned} \zeta &= \mathcal{A}\zeta + \mathcal{B}d \\ \hat{z} &= \mathcal{C}\zeta \end{aligned} \tag{6}$$

where

$$\mathcal{A} = \begin{bmatrix} A(\theta) + B_2 K(\theta) & LC_2 \\ 0 & A(\theta) - LC_2 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$$
$$\mathcal{C} = \begin{bmatrix} C_1 & 0 \end{bmatrix}, \qquad \zeta = \begin{bmatrix} \hat{x} \\ e \end{bmatrix}.$$

Remark 1: It is noted that the disturbance term d is also included in the observer state-space representation (4) since it is possible to measure NO_x concentration upstream the catalyst.

The following lemma presents the synthesis conditions to design the observer and a parameter-dependent state feedback controller to guarantee the closed-loop system stability and H_{∞} performance.

Lemma 1: For the LPV system represented by (3) and the stability margin ν , there exist an LPV observer (4) and a state feedback controller such that the closed-loop system is stable and a prescribed level of H_{∞} performance γ is guaranteed if parameter-dependent matrices $R(\theta) > 0$, $P_1(\theta) > 0$, $P_2(\theta) > 0$, $S(\theta)$, and $Q(\theta)$ exist that satisfy the following set of LMIs.

$$A^{T}R + RA - C_{1}^{T}S^{T} - SC_{1} + 2\nu R < 0 \quad (7)$$

$$\begin{bmatrix} AP_{1} + B_{2}Q + (*) - \dot{P}_{1} & LC_{2}P_{2} \\ (*) & AP_{2} - LC_{2}P_{2} + (*) - \dot{P}_{2} \\ (*) & (*) \\ (*) & (*) \\ (*) & (*) \\ \end{bmatrix}$$

$$B_{1} \quad P_{1}C_{1}^{T} \\ 0 \quad 0 \\ -\gamma I \quad 0 \\ (*) & -\gamma I \end{bmatrix} < 0 \quad (8)$$

$$\begin{bmatrix} P_{1} & 0 \\ 0 & P_{2} \end{bmatrix} > 0. \quad (9)$$

Then, the LPV observer gain and the feedback controller gain are determined by

$$L(\theta) = R^{-1}(\theta)S(\theta) \tag{10a}$$

$$K(\theta) = Q(\theta)P_1^{-1}(\theta).$$
(10b)

In the above formulation (*) denotes the transpose of the terms it proceeds and (*) is used to denote the sub-matrices lying under the main diagonal.

Proof of this lemma is based on the Bounded Real Lemma LMI formulation corresponding to the H_{∞} norm of the closed-loop system (6). To avoid the bilinear terms in the matrix inequality associated with the closed-loop system, a block-diagonal structure is enforced on the Lyapunov matrix as in (9). The algebra led to the LMIs (7) and (8) is omitted for brevity.

Remark 2: Due to the cross-product of the observer gain L and the Lyapunov matrix in (8), the LMI (7) is first solved independently to determine the observer gain. This is due to separation principle in designing observer and controller for a linear system.

IV. LPV OUTPUT FEEDBACK CONTROL DESIGN APPROACH

The state feedback control approach proposed in Lemma 1 needs the estimation of all states, which is not desirable for implementation purposes. Therefore, we next consider the dynamic output feedback control design. The gain-scheduled output feedback controller is represented with the following state-space representation

$$\dot{x}_k = A_k(\theta)x_k + B_k(\theta)y$$
(11)
$$u = C_k(\theta)x_k + D_k(\theta)y.$$

As discussed earlier, the surface coverage fraction θ is assumed to be the scheduling parameter. However, the ratio θ is not measurable in the actual system and must be estimated in real-time. In this paper, we estimate θ using its steadystate value to prevent the use of a full-state observer. From the first equation in (1), we have the following at steadystate:

$$-C_{NO}(\Theta_{SC}R_{red}\theta + \frac{F}{V}) + R_{ox}\Theta_{SC}\theta + \frac{F}{V}d = 0.$$

Therefore, θ can be estimated by

$$\bar{\theta} = \frac{\frac{F}{V}(C_{NO} - d)}{-C_{NO}\Theta_{SC}R_{red} + R_{ox}\Theta_{SC}}$$
(12)

which implies that the knowledge of inlet and outlet NO concentrations is sufficient to estimate θ at each time instant.

We use the basic characterization of the gain-scheduled control design with guaranteed stability and H_{∞} performance as presented in [2] for the quasi-LPV system represented by (2). Since, the dependence on the scheduling parameter θ is affine the synthesis LMIs need to be solved only at the two corners (*i.e.*, maximum and minimum allowable quantities for θ). The range of variation for θ is between 0 and 1. For simplicity we have considered constant basis functions for Lyapunov functions R(.) and S(.) and the auxiliary controller matrices \hat{A}_k , \hat{B}_k , \hat{C}_k , and D_k in the corresponding synthesis LMIs [2]. Therefore, we have a finite number of decision variables to optimize. Taking into account the above structure for the decision variables, the only parameter-dependent controller matrix is A_k characterized by

$$A_k(\theta) = A_{k_0} + \theta A_{k_1}. \tag{13}$$

In the controller representation (11), we have forced the matrix D_k to be zero. The obtained controller is a fullorder one; however, it was observed that the dominant pole of this system is constant for a fixed temperature and over the range of variation of the LPV parameter θ . Therefore, we determine a reduced-order controller that has the same DC gain as the original full-order controller. We determined that the reduced-order controller is a first-order transfer function and since D_k is zero, it can be represented by

$$K(s) = \frac{\alpha(\theta)}{s+p} \tag{14}$$



Fig. 1. Schematic of the adaptive control strategy for urea-SCR system

where p is the dominant pole of the obtained controller. In order to have equal DC gains for both full-order and reduced-order controllers, the following relation holds

$$\frac{\alpha(\theta)}{p} = -C_k A_k^{-1}(\theta) B_k \tag{15}$$

which indicates that the knowledge of p provides the parameter α scheduled based on θ . Interestingly, it was observed that in the temperature range of SCR operation $(200-500^{\circ}C)$ a straight line well represents the dependence of the DC gain α on scheduling parameter θ . This will be demonstrated in the next section.

To improve the steady-state performance of the closedloop system, a feedforward term is added to the feedback control output (as shown in Figure 1) by keeping the ammonia slip at a desired level at steady-state using the following law

$$u_{ff} = \frac{V}{F} [C_{NH_3,desired} \left(\Theta_{SC} R_{ads} (1 - \bar{\theta}) + \frac{F}{V} \right) -\Theta_{SC} R_{des} \bar{\theta}] \qquad (16)$$

where $C_{NH_3,desired}$ is the desired ammonia slip and $\bar{\theta}$ is the real-time estimate of the LPV parameter using (12).

V. SIMULATION RESULTS

In order to satisfy the dual goals of maximizing NO conversion efficiency and minimizing ammonia slip, we chose $z = a_1x_1 + a_3x_3$ as the control output in (3). The profile of NO concentration upstream the catalyst is shown in Figure 2 with the temperature fixed at $300^{\circ}C$. For the simulation purposes, measurement noise was added to the NO measurement downstream the catalyst. The feedforward part of the output feedback controller is designed to keep the ammonia slip around 10 ppm in all of the performed simulations.

Figure 3 illustrates the result of state feedback controller integrated with the observer designed using Lemma 1. The three plots shown in Figure 3 correspond to NO concentration downstream the catalyst, the released ammonia, and injected ammonia.

Next, we demonstrate the results of using the designed reduced-order LPV output feedback controller. After solving the synthesis LMIs, the dominant pole is determined to be at -16.5. Figure 4 shows the dependence of the controller DC gain versus θ as in (15). It is interesting to note that the gain is a linear function of θ , and hence $\alpha(\theta)$ in (14) can be adapted in real-time to changes of θ . The determined



Fig. 2. NO concentration in SCR inlet



Fig. 3. LPV state feedback control performance

reduced-order controller in (14) is applied to the SCR system using the design configuration of Figure 1. Shown in Figure 5 is the closed-loop simulation result where an improvement in NO reduction is observed compared to the state feedback control. The ammonia slip of around 10 ppm is quickly achieved due to the use of feedforward term resulting in the improved steady-state performance. The reduced-order controller (14) is scheduled based on the parameter θ = θ as in (12). In Figure 6 the approximation of θ by its steady-state value $\bar{\theta}$ is depicted which shows that the steadystate approximation is a very good estimation of the actual coverage ratio to be used for control gain adaptation. To compare the proposed LPV H_∞ controllers of this paper with the nonlinear control methods, we design a sliding mode controller as described in [5]. In Figure 7 the result of closing the feedback loop using sliding mode control following the approach in [8], [11] is shown.

Next, we apply the proposed reduced-order output feedback control strategy in this paper for different exhaust tem-



Fig. 4. Controller DC gain as a function of θ



Fig. 5. Reduced-order LPV output feedback control performance



Fig. 6. Surface coverage fraction estimate



Fig. 7. Sliding mode control performance



Fig. 8. NO concentration upstream the catalyst corresponding to different exhaust temperatures

peratures. We evaluate the closed-loop system performance in terms of the NO conversion efficiency in Figure 8 for temperatures fixed at 220, 300, and $400^{\circ}C$. As expected, the SCR catalyst performs better at higher temperatures. This trend, however, changes at very high temperatures where ammonia oxidation limits the NO_x conversion.

Finally, we compare the performance (in terms of NO conversion efficiency) of the closed-loop systems using a sliding mode controller, an LPV state feedback controller, and an adaptive reduced-order feedback/feedforward controller. To design feedforward controller, the ammonia slip is kept around 10 ppm and the NO conversion efficiency for the three control design methods are compared in Table I. The following metric is used to evaluate the conversion efficiency:

$$DeNO = \frac{\sum_{i} C_{NO,in}(i) - \sum_{i} C_{NO,out}(i)}{\sum_{i} C_{NO,in}(i)}$$
(17)

The NO concentration profile (upstream the catalyst) used



Fig. 9. Reduced-order LPV output feedback control performance



Fig. 10. Control input

to evaluate the NO conversion efficiency is shown by solid line in Figure 9. This emission profile corresponds to an EPA Urban Dynamometer Driving Schedule (UDDS) which has been developed for chassis dynamometer testing of heavyduty vehicles. The basic parameters of the cycle are duration for 1060 seconds, distance of 5.55 miles, average speed of 18.86 mile/h, and maximum speed of 58 mile/h. The dashed line in Figure 9 illustrates the NO concentration downstream the catalyst using the adaptive control strategy proposed in the paper. Figure 10 shows the ammonia injected (output of the adaptive controller) to satisfy the dual objective of keeping ammonia slip around 10 ppm and maximizing the conversion efficiency, which in this case was calculated to be close to 92%.

VI. CONCLUDING REMARKS

In this paper we addressed the LPV control design method for urea-selective catalyst reduction aftertreatment system. Both the state feedback and output feedback controllers were developed and compared. The first method required an

TABLE I Comparisons between different control methods

Control Strategy	DeNO %
Sliding mode control	91.90
LPV state feedback	90.97
Adaptive output feedback	91.97

observer for state estimation, but the latter one only used the concentration of NO upstream and downstream the catalyst. Elimination of the fast modes in the full-order LPV output feedback controller resulted in a simple first-order control strategy gain-scheduled as a function of the LPV parameter (the surface coverage fraction at steady-state). Inspired by the gain-scheduling control strategy proposed in this paper, the authors are currently investigating the impact of exhaust temperature on the structure of the reduced-order controller as an additional LPV parameter. The proposed method can also be readily extended to apply to the modified 3-state and 4-state models developed in [10].

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