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Opportunistic Preventive Maintenance Scheduling Based on Theory of Constraints

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Abstract In recent years, to improve system reliability and economy, machine maintenance strategies have been paid more and more intention by researchers. This paper aims to integrate the concept of theory of constraints (TOC) into multi-machine opportunistic maintenance policy. Based on the preventive maintenance algorithm, an improved model containing bottleneck strategy which influences opportunistic maintenance has been developed. By maximizing total cost saving, an optimal maintenance schedule of all machines in a series product line can be obtained. The results of a case study show that this model is valid for planning a comprehensive optimal maintenance schedule. Furthermore, by comparing with other models, this model has been proven to be more effectively especially in series product lines with bottleneck.

Keywords
Opportunistic maintenance, Theory of constraints, Preventive maintenance, Cost saving, Optimal maintenance schedule

1. Introduction
With the growing high-precision and high complexity trend of modern manufacturing equipment, to avoid high cost and potential risk of sudden failures, preventive maintenance has been widely applied in manufacturing industry and has become a research hotspot for years [1]. Some researchers focus on single machine preventive maintenance. Xia proposed an improved preventive maintenance model based on health index [2]. Lim presented an optimal periodic preventive maintenance schedule which influenced by times of maintenances [3]. Some studies pay attention on multi-machine opportunistic preventive maintenance. A dynamic opportunistic preventive maintenance model for multi-unit series systems was developed by Zhou [4]. A system preventive maintenance model which considered spares inventory subsystem was proposed by P. Lynch [5]. However, traditional opportunistic maintenance models do not distinguish the importance level of different machines, which can influence the opportunistic maintenance timing of multi-machine, especially in series product lines. Therefore, this paper integrates the theory of constraints' concept into opportunistic maintenance to solve the problem.

Theory of Constraints (TOC) is a kind of decision-making techniques, which was first proposed by Dr. Eliyahu M. Goldratt in 1980s. It has been applied in many manufacturing areas, such as production scheduling, supply chain management, project control and even in some service industries like banks and hospitals. Kirkwood introduced the concept of TOC (including lead and lag measures and bottlenecks) to opportunistic maintenance [6]. Chakravorty used TOC concept to give a maintenance order which determined by equipment's failure rate

[7]. Ju used TOC concept to schedule maintenance order which determined bottleneck by equipment residual life [8]. However, these studies did not notice the principal aspect of bottleneck in a product line. A machine which influences the total productivity is the bottleneck. Productivity determines the bottleneck which is also quite important to decide whether to apply opportunistic maintenance. Therefore, this paper focuses on building an opportunistic preventive maintenance model by considering productivity bottleneck. This improved model not only saves maintenance cost but also reduces work delay caused by maintenance; it meets the requirements in actual product lines.

According to the actual product line situation, to describe the problem clearly and to better state the modeling steps, this study has the following assumptions: 1) The machines are in series, when one machine is in repair, the others can continue producing. 2) There are two kinds of maintenance: scheduled maintenance costs less and can be brought forward but not be delayed; unscheduled maintenance is used to deal with sudden failure, it costs more and its repairing time can be ignored. 3) Two kinds of repairing are both imperfect maintenance: unscheduled maintenance do not influence machine health index, while machine health index becomes new level after scheduled maintenance. 4) The product line is in full production with no free time.

2. Single-machine preventive maintenance model

In the single-machine preventive maintenance model, this paper introduce age decline factor and failure increase factor to model the imperfect maintenance, initially add average cost rate of work delay in it, and then form the preventive maintenance strategy based on machine health index [9,10]. The health index representing reliability of a machine is:

$$H(t) = \exp\left[-F(t)\right] \tag{1}$$

In which, the cumulative risk function $F(t)$ refers to machine cumulative risk to breakdown in [0, t], it is defined as $F(t) = \int_0^t f(t)dt$. $f(t)$ is machine hazard function and defined as $f(t) = \lim\limits_{\Delta t \to 0} \frac{prob(t < T \le t + \Delta t | T > t)}{\Delta t}$. T is continuous random variable meaning machine's duration of work without failure. And then, health index equation can be deducted:

$$\exp\left[-\int_0^{T_1} f_1(t)dt\right] = \exp\left[-\int_0^{T_2} f_2(t)dt\right] = ... = \exp\left[-\int_0^{T_i} f_i(t)dt\right] = H_0 \tag{2}$$

H_0 represents health index threshold value. To accurately establish imperfect maintenance model, age decline factor a and failure increase factor b are introduced：

$$f_{i+1}(t) = bf_i(t + aT_i) \quad t \in (0, T_{i+1}) \tag{3}$$

In formula (3), i is maintenance interval and the factors satisfy $(0 < a < 1, b > 1)$. Because of actual monitoring data is discrete, formula (2), (3) can be discretized as followings:

$$\left[-\sum_{k=1}^{\frac{T_1}{\Delta t}} f_1(k)\Delta t\right] = \exp\left[-\sum_{k=1}^{\frac{T_2}{\Delta t}} f_2(k)\Delta t\right] = ... = \exp\left[-\sum_{k=1}^{\frac{T_i}{\Delta t}} f_i(k)\Delta t\right] = H_0 \tag{4}$$

$$f_{i+1}(k) = bf_i\left(k + \tfrac{aT_i}{\Delta t}\right) \quad for \;\; k = 0,1,2...,\frac{T_{i+1}}{\Delta t} \tag{5}$$

Assuming that after N maintenance intervals, machine need to renewal due to high maintenance frequency. Then, the total maintenance cost rate in the N intervals is:

$$C_{Er}=\frac{N\left[(C_{up}+C_{ub})(1-H_0)+(C_{sp}+C_{sb})H_0+C_{od}t_{pm}\right]+Cost_{rs}}{\sum_{i=1}^{N}T_i+Nt_{pm}} \tag{6}$$

Let C_{Er} be the total maintenance cost rate, C_{up} be the unscheduled maintenance cost, C_{ub} be the unscheduled breakdown cost, C_{sp} be the scheduled maintenance cost, C_{sb} be the scheduled breakdown cost, C_{od} be the work delay cost rate caused by maintenance, $Cost_{rs}$ be the equipment renewal cost and t_{pm} be the maintenance time, assuming unscheduled and scheduled maintenance cost the same time. By solving $\min C_{Er}$, the optimal health index H_0, each maintenance intervals T_i and maintenance time N can be obtained. Thus, the health index based single-machine preventive maintenance strategy can be worked out.

3. Multi-machine opportunistic maintenance model based on TOC

After getting the maintenance strategy of each machine, let us focus on opportunistic maintenance strategy of multi-machine in series. Put all machines' preventive maintenance intervals together, apply maintenance chronologically and meanwhile find if other machines can apply opportunistic maintenance with the initiative machine. This paper follows the rules: 1) Different from other opportunistic maintenance strategy, a time zone is given according to actual production and allows machine to have more than one time maintenance in it. Time zone in this paper is a relative long time which may contains several maintenance intervals of machines, thus there are more chances to apply opportunistic maintenance. In other words, time zone is just schedule making interval. 2) After an opportunistic maintenance, each machine which applies opportunistic maintenance updates its next interval due to its own preventive maintenance interval. 3) Calculate the productivity of each machine, the one with the lowest productivity has the highest priority, and so forth. 4) Opportunistic maintenance follows TOC, which means only machines with lower priority can apply opportunistic maintenance with the initiative machine at this interval; the ones with higher priority continue working to next interval. Assuming machines can work normally when some of them are applying maintenance. 5) Set a time window T_w, only when the time difference between two machines' scheduled maintenance is smaller than T_w, the opportunistic maintenance can be applied.

Costs save is used to value the model. Assuming machine m can apply opportunistic maintenance together with machine n, and thus the costs saved by the ahead of schedule maintenance of machine m can be defined as:

$$CostSave_{mn}=SBC_{mn}+SPC_{mn}-SEC_{mn} \tag{7}$$

Let SBC_{mn} be the breakdown costs save, which mainly includes maintenance's influence to work in process and machines' start and stop costs caused by maintenance. When machine m running to the health index H_{mn} ($H_{mn}>H_0$, H_{mn} can be calculated by formula(4), in which T is the maintenance interval of machine *n*; H_0 is machine m's originally scheduled health index), it applies opportunistic maintenance together with machine n, the SBC_{mn} is:

$$SBC_{mn}=\underbrace{(1-H_0)\ C_{ub}+H_0C_{sb}}_{\text{Old breakdown cost}}-\underbrace{\left[(1-H_{mn})\ C_{ub}+H_{mn}C_{sb}\right]}_{\text{New breakdown cost}} \tag{8}$$

Let SPC_{mn} be the maintenance costs save; it includes costs of maintenance staff, equipment and appointment. Assuming an opportunistic maintenance of multi-machine just need maintenance cost once. The SPC_{mn} is:

$$SPC_{mn}=\underbrace{(1-H_0)\ C_{up}+H_0C_{sp}}_{\text{Old maintenance cost}}-\underbrace{(1-H_{mn})\ C_{up}}_{\text{New maintenance cost}} \tag{9}$$

Let SEC_{mn} be the extra costs of ahead of schedule maintenance, which is caused by the cutting down of machine's total service time in an update cycle. It can be defined as:

$$SEC_{mn} = C_{Er} * \Delta T = C_{Er} * (T_{iold} - T_{inew}) \quad (10)$$

Thus costs save of machine m one time opportunistic maintenance with machine n can be calculated. Then the total costs save of opportunistic maintenance in the time zone is:

$$CostSave_{total} = \sum CostSave_{mn} \quad (11)$$

Finally, because only when the time difference between two machines' scheduled maintenance is smaller than T_w, the opportunistic maintenance can be applied, different T_w may causes different opportunistic maintenance chances; by comparing $CostSave_{total}$ under different T_w, the relatively optimal opportunistic maintenance schedule can be obtained.

4. Case study and discussion

4.1 Single machine preventive maintenance schedule

This paper focuses on a series product line with three machines *A*, *B*, *C*. The production sequence is *A*→*B*→*C*. Assuming machines' current failure rate distribution accords with Weibull Proportional Hazards Model [11], that is $f_1(t) = \frac{m}{\eta}\left(\frac{t}{\eta}\right)^{m-1} \exp\left(\gamma z(t)\right)$, where the characteristic life parameter $\eta = 100$, the shape parameter $m = 3$, the concomitant variable $\gamma z(t) = 0.006t$. Let machine's characteristic detection period be $\Delta t = 0.1$, thus $f_1(k) = \frac{3}{100}\left(\frac{0.1k}{100}\right)^2 \exp\left(0.006k * 0.1\right)$. Assuming the age decline factor $a = 0.15$ and the failure increase factor $b = 1.15$, each machine's maintenance parameter values are shown in Table 1.

Table 1: Machines' maintenance parameter values

Machine	C_{sp}	C_{sb}	C_{up}	C_{ub}	C_{od}	$Cost_{rs}$	t_{pm}
A	7	5	105	75	7	100	2
B	10	6	200	120	10	200	2
C	8	6	120	90	8	80	2

Because of the complexity of function C_{Er}, this paper uses a search algorithm to solve it. Let variable optimization scale be $\{H_0 \in (0.00, 1.00), N \in (0, 10)\}$, and step range of H_0 is 0.01, step range of N is 1. By solving $\min C_{Er}$, the optimal health index and maintenance time combination (H_0^*, N^*) can be worked out. This paper uses VBA to program and calculate. The preventive maintenance results of each machine are shown in Table 2.

Table 2: Machines' preventive maintenance results

Machine	N	H_0	T_1	T_2	T_3	T_4	T_5	T_6	……
A	4	0.85	50.6	40.9	33	26.6	50.6	40.9	……
B	5	0.89	45.6	36.9	29.7	24	19.5	45.6	……

C	3	0.87	48.2	39	31.4	48.2	39	31.4	……

4.2 Multi-machine opportunistic maintenance schedule

Assuming the priority of machines is *B*>*C*>*A*, machine *A*, *B*, *C* corresponding to priority 3, 1, 2. According to actual conditions, the time zone to apply opportunistic maintenance strategy is 300. Using search algorithm to get the optimal time window T_w which can have max $CostSave_{total}$, variable optimization scale is [0, 25] and step range is 1. Different $CostSave_{total}$ values under different T_w are shown in Table 3.

Table 3: $CostSave_{total}$ values under different T_w

T_w	0	1	2	3	4	5	6	7	8
$CostSave_{total}$	0	12.18	12.18	34.43	49.03	51.85	51.85	50.23	50.23
T_w	9	10	11	12	13	14	15	16	17
$CostSave_{total}$	50.23	50.23	50.23	50.23	50.23	50.23	31.6	31.6	31.6
T_w	18	19	20	21	22	23	24	25	
$CostSave_{total}$	10.45	10.45	10.45	4.51	4.51	4.51	-27.6	-88.1	

From Table 3, it can be seen that when $T_w = 5$ or $T_w = 6$, $CostSave_{total}$ =51.85 is the maximum value. Take $T_w = 6$ and then opportunistic maintenance schedule of each machine are shown in Table 4. M_n represents the *n*th times opportunistic maintenance, its corresponding value is the time from last maintenance, if the value is red the machine applies opportunistic maintenance and vice versa.

Table 4: Opportunistic maintenance schedule under $T_w = 6$ (Ma represents machine and Pr represents priority)

Ma	Pr	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}	M_{13}	M_{14}	M_{15}
A	3	45.6	36.9	29.7	24	19.5	31.1	7.9	6.6	24.8	12.1	20.9	8.8	6.4	11.4	6.2
B	1	45.6	36.9	29.7	24	19.5	31.1	7.9	6.6	24.8	12.1	20.9	8.8	6.4	11.4	6.2
C	2	45.6	36.9	29.7	24	19.5	31.1	7.9	6.6	24.8	12.1	20.9	8.8	6.4	11.4	6.2

By comparing with other maintenance strategy, this paper's TOC based opportunistic maintenance strategy not only has obvious effect in costs saving, but also has superiority in reducing work delay. Results of different maintenance strategies under the same parameters' value are shown in Figure 1. TOC refers to this paper's strategy, Zhou refers to Zhou's strategy [4] and single refers to single machine preventive maintenance without opportunistic maintenance.

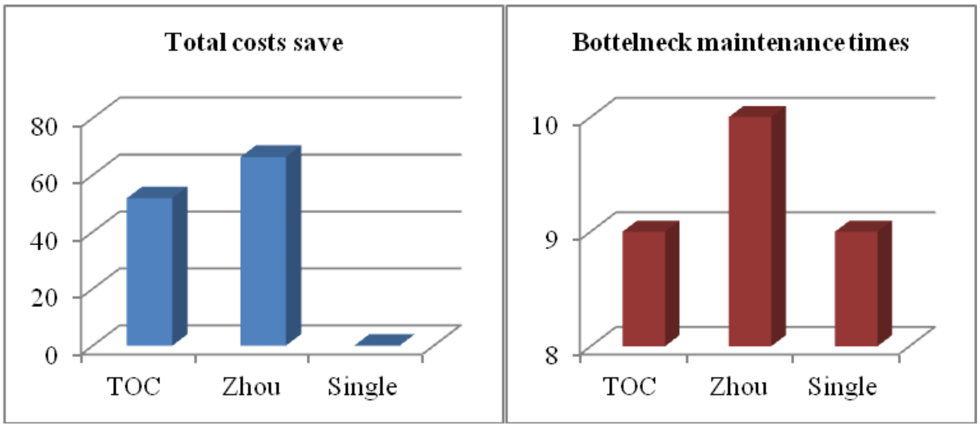

Figure 1: Comparing of different strategy

It can be seen that though TOC's total costs save is a little less than Zhou's, the bottleneck maintenance is one time less than Zhou's. According to the theory of constraints, the bottleneck determines the total production efficiency. So the less bottleneck maintenance times means less work delay, it is another form of costs saving. Including work delay costs saving, this paper's strategy is better than Zhou's and let alone the single machine preventive maintenance without opportunistic maintenance.

5. Conclusion

This paper proposed an opportunistic maintenance strategy combined with theory of constraints (TOC). Based on single machine's health index preventive maintenance strategy, opportunistic maintenance rules including concept of bottleneck are formed. A total costs saving model is built to get the optimal opportunistic maintenance schedule. From the case study, it can be found that, the improved strategy not only can have relatively high costs saving, but also can reduce the bottleneck maintenance times which can lessen work delay. Especially to product lines which require no work delay or whose bottleneck machine's maintenance time is long, this paper's strategy has its advantages in lessen work delay. Besides, how to use such strategy in more complex production systems and how to put work delay costs saving into total costs saving model will be investigated in future studies.

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