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Research Article

Stability of Stochastic Discrete-Time Neural Networks with Discrete Delays and the Leakage Delay

Liyuan Hou and Hong Zhu

School of Automation Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China

Correspondence should be addressed to Liyuan Hou; liyuanhou@163.com

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This paper investigates the stability of stochastic discrete-time neural networks (NNs) with discrete time-varying delays and leakage delay. As the partition of time-varying and leakage delay is brought in the discrete-time system, we construct a novel Lyapunov-Krasovskii function based on stability theory. Furthermore sufficient conditions are derived to guarantee the global asymptotic stability of the equilibrium point. Numerical example is given to demonstrate the effectiveness of the proposed method and the applicability of the proposed method.

1. Introduction

Neural networks (NNs) have received much interest owing to their wide application in the areas of signal processing, pattern recognition, and static image processing in [1, 2]. One of the most important and challenging questions in theoretical analysis of neural networks (NNs) is dynamical behaviors of the neural networks, for example, stability, instability, periodic oscillatory, and chaos. Among them analysis of stability has received much attention and various stability conditions have been obtained in [3–10]. The main problem of stability analysis is how to construct appropriate Lyapunov functions which are widely used in various fields [11–18]. Then the construction of Lyapunov functions is determined by the structure of neural networks. The study of neural networks generally considers the following factors.

It is well known that time delay is inherent in various systems, including artificial neural networks, owing to the finite speed of signal transmission [3]. Delays in a system may cause oscillation and divergence and may degrade the performance. Hence, stability analysis of systems with time delay is widespread. Scholars classify the analysis of stability into delay-independent and delay-dependent ones. It has been proved in [4, 5] that the delay-dependent criterion is less conservative than the delay-independent one, especially for smaller values of delay. Delay-dependent stability condition for continuous-time NNs with time-varying delays has been reported in literatures [4–10]. The approaches to handle the time-varying delay in most literatures are based on introducing free weighting matrices [19], model transformation method [20], and linear matrix inequality (LMI) approach and employing the delay partitioning approach in [21, 22].

In the digital life, most of the signals including the continuous-time NNs need to be processed, experimentalized, or computed by the computer, such that we must discretize the continuous-time signals before delivering them to the computer. Therefore, the stability of discrete-time neural networks is necessary, and more and more literature about it was published [22–28].

As pointed out in [29], discrete-time delays were introduced into bidirectional associative memory (BAM) neural networks which were known as leakage (or forgetting) terms. From literatures [30–36] it can be found that the leakage terms had a tendency to destabilize the neural networks. In the literatures [30, 31], the system with leakage delays had been studied. However, different from [29], the leakage delays changed to be time varying. Not only that, different kinds of neural networks with time delays in the leakage terms were studied, especially the effect of leakage term on the dynamical behavior of various kinds of neural networks [32–42]. In [33], passivity analysis of neural networks with time-varying delays and leakage delay was considered. And they used the free-weighting matrix method and stochastic analysis technique. Furthermore, Li and others in [34, 35] researched stability of various differential systems including neural networks in leakage terms, by using contraction mapping theorem, Brouwers fixed point theorem, Lyapunov-Krasovskii functional method, and free weighting matrix technique. Unfortunately, neural networks with the leakage terms in most literatures are continuous systems, and there is not any thesis about discrete neural networks with the leakage delay. On the other hand, it has now been well recognized that stochastic disturbances are mostly inevitable owing to noise in electronic implementations. It has also been revealed that certain stochastic inputs could make a neural network unstable.

In this paper, the stability problem is considered for discrete-time neural networks with time-varying delays in the leakage terms. Firstly, the mathematical models are established. Secondly, a less conservative and new stability criterion is derived by using a novel Lyapunov-Krasovskii functional which depends on the circumstance of the delay partition. Thirdly, a numerical example is provided to show the effectiveness of the main result.

Notation. Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ denotes the set of all real matrices. The superscript T denotes matrix transposition and the notation $X \geq Y$ ($X > Y$, resp.), where X and Y are symmetric matrices, means that $X-Y$ is positive semidefinite (positive definite, resp.). In symmetric block matrices, the symbol $*$ is used as an ellipsis for terms induced by symmetry. $|\cdot|$ stands for the Euclidean vector norm in $\mathbb{R}^n$. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x \mid y\}$ denote the expectation of x and the expectation of x conditional on y. $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space, and $\mathcal{P}$ is the probability measure on $\mathcal{F}$.

2. Preliminaries

Discretizing the continuous-time NNs with leakage terms [39], we will consider the following discrete-time systems with time-varying delays in leakage terms:

$$\begin{aligned} x(k+1) &\\ &= x(k) - Cx(k-\sigma) + AF(x(k)) + BG(x(k-\tau(k))), \end{aligned} \tag{1}$$

where $x(k) = [x_1(k), x_2(k), \ldots, x_n(k)]^T$ is the state vector at time k; $C = \mathrm{diag}[c_1, c_2, \ldots, c_n]$ with $C > 0$ is the state feedback coefficient matrix; the $n \times n$ matrices $A = [a_{ij}]_{n\times n}$ and $B = [b_{ij}]_{n\times n}$ are the connection weight matrix and the discretely delayed connection weight matrix, respectively; $F(x(k))$, $G(x(k))$ are the neuron activation functions, which satisfy $F(x(k)) = [F_1(x_1(k)), F_2(x_2(k)), \ldots, F_n(x_n(k))]^T$, $G(x(k)) = [G_1(x_1(k)), G_2(x_2(k)), \ldots, G_n(x_n(k))]^T$; $\tau(k)$ denotes the discrete time-varying delay; σ means the leakage delays.

Assumption 1. For any $x, y \in R$, $(x \neq y)$, $i \in \{1, 2, \ldots, n\}$, the activation functions satisfy

$$\widehat{f}_i^- \leq \frac{F_i(x) - F_i(y)}{x-y} \leq \widehat{f}_i^+; \qquad \widehat{g}_i^- \leq \frac{G_i(x) - G_i(y)}{x-y} \leq \widehat{g}_i^+, \tag{2}$$

where $\widehat{f}_i^-$, $\widehat{f}_i^+$, $\widehat{g}_i^-$, and $\widehat{g}_i^+$ are constants.

Remark 2. The condition on the activation functions in Assumption 1 was originally employed in [6, 7] and has been subsequently used in recent papers with the problem of stability of neural networks; see [21–28].

The system (1) has equilibrium points under Assumption 1. Let x^* be the equilibrium point of system (1), and shift it to the origin by letting $y(k) = x(k) - x^*$, and then system (1) with stochastic disturbances can be rewritten as

$$\begin{aligned} y(k+1) &= y(k) - Cy(k-\sigma) + Af(y(k)) \\ &\quad + Bg(y(k-\tau(k))) + \delta(k, y(k))\,\omega(k), \end{aligned} \tag{3}$$

where $y(k) = [y_1(k), y_2(k), \ldots, y_n(k)]^T$ is the state vector of the transformed system, $y(k-\sigma) = x(k-\sigma) - x^*$, and the transformed neuron activation functions are $f(y(k)) = F(x(k)) - F(x^*)$, $g(y(k)) = G(x(k)) - G(x^*)$, and $\omega(k)$ is a scalar Wiener process on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with $\mathbb{E}\{\omega(k)\} = 0$, $\mathbb{E}\{\omega^2(k)\} = 1$, $\mathbb{E}\{\omega(i)\omega(j)\} = 0$ $(i \neq j)$. Then $\widehat{f}_i^- \leq (f_i(x) - f_i(y))/(x-y) \leq \widehat{f}_i^+$; $\widehat{g}_i^- \leq (g_i(x) - g_i(y))/(x-y) \leq \widehat{g}_i^+$ can be verified from Remark 2.

Assumption 3. The noise intensity function vector $\delta(\cdot,\cdot) : \mathbb{N} \times \mathbb{N}^n \rightarrow \mathbb{N}^n$ satisfies the Lipschitz condition; that is, the nonlinear function $\delta(k, x)$ satisfies the following inequality:

$$\delta(k, x)^T \delta(k, x) \leq \xi x^T x, \tag{4}$$

where ξ is a known scalar constant.

Assumption 4. The time-varying delay $\tau(k)$ is bounded, $0 < \tau_m \leq \tau(k) \leq \tau_M$, and its probability distribution can be observed. Assume that integer $\tau(k)$ is satisfied in $\tau_i \leq \tau(k) \leq \tau_{i+1}$, $d_i = \tau_i - \tau_{i-1} \neq 0$, $i = 1, 2, \ldots, l$, which means we divide $[\tau_m, \tau_M]$ into l parts, and $\mathrm{Prob}\{\tau(k) \in [\tau_{i-1}, \tau_i)\} = \rho_i = 1 - \widetilde{\rho}_i$, where $0 \leq \rho_i \leq 1$, $\sum_i \rho_i = 1$, $i = 1, \ldots, l$, and $\tau_0 = \tau_m$, $\tau_l = \tau_M$.

To describe the probability distribution of time-varying delay, we define the following set $\mathcal{A}_i = (\tau_{i-1}, \tau_i]$, $i = 1, 2, \ldots, l$. Define mapping functions

$$\tau_i(k) = \begin{cases} \tau(k), & \tau(k) \in \mathcal{A}_i \\ \tau_{i-1}, & \text{else}, \end{cases} \qquad \rho_i(k) = \begin{cases} 1, & \tau(k) \in \mathcal{A}_i \\ 0, & \text{else}. \end{cases} \tag{5}$$

Remark 5. Consider $\mathrm{Prob}\{\rho_i(k) = 1\} = \mathbb{E}\{\rho_i(k)\} = \rho_i$, $\mathrm{Prob}\{\rho_i(k) = 0\} = \widetilde{\rho}_i$, $\mathbb{E}\{\rho_i(k) * \rho_j(k)\} = \begin{cases} \rho_i, & i=j \\ 0, & \text{else}. \end{cases}$

Remark 6. In the literature [21], the delay-partitioning projection technique is used. In [21], they partition $\tau(t)$ into several components, that is, $\tau(t) = \sum_{i=1}^{m} \tau_i(t)$, where m is a positive integer, in order to derive some less restrictive stability criteria. Different from the literature [21], we use the knowledge of probability to describe the partition of time delay. This way has the advantage of reducing the amount of calculation.

Then the system (3) can be rewritten as

$$\begin{aligned} y(k+1) &= y(k) - Cy(k-\sigma) + Af(y(k)) \\ &\quad + \sum_{i=1}^{l} \rho_i(k) Bg(y(k-\tau_i(k))) + \delta(k, y(k))\omega(k). \end{aligned} \tag{6}$$

3. New Stability Criteria

In this section, we will establish new stability criteria for system (1). The following lemma is needed in order to derive our main results.

Lemma 7 (Zhu and Yang [23] discrete Jenson inequality). *For any constant matrix $M \in R^{n\times n}$, $M = M^T > 0$, integers $\gamma_2 \geq \gamma_1$, vector function $\omega : [\gamma_1, \gamma_1+1, \ldots, \gamma_2] \to R^n$ such that the sums in the following are well defined, and then*

$$\begin{aligned} &-(\gamma_2 - \gamma_1 + 1) \sum_{i=\gamma_1}^{\gamma_2} \omega^T(i) M\omega(i) \\ &\leq -\left(\sum_{i=\gamma_1}^{\gamma_2} \omega(i)\right)^T M \left(\sum_{i=\gamma_1}^{\gamma_2} \omega(i)\right). \end{aligned} \tag{7}$$

The next theorem can be obtained if the stochastic term $\omega(k)$ is removed in the system (6).

Theorem 8. *Under Assumptions 1 and 4, the system (6) with $\omega(k) = 0$ is globally asymptotically stable, if there exist positive matrices P, R_1, R_2, R_3, Q, S_i ($i = 1, \ldots, l$), $W =$ diag$[w_1, w_2, \ldots, w_l]$, $\mathbf{U} =$ diag$[u_1, u_2, \ldots, u_l]$, and free matrix $T_1, T_2, \ldots, T_l + 1$, V_1, V_2, V_3 with any appropriate dimensions, such that the following LMI holds:*

$$\begin{bmatrix} \Gamma_1 & \Gamma_{1,2} & 0 & \Gamma_{1,4} & \Gamma_{1,5} & \Gamma_{1,6} & 0 & \Gamma_{1,8} & \Gamma_{1,9} & 0 & 0 \\ * & \Gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_{2,11} \\ * & * & \Gamma_3 & 0 & 0 & \Gamma_{3,6} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Gamma_4 & 0 & \Gamma_{4,6} & 0 & 0 & 0 & \Gamma_{4,10} & 0 \\ * & * & * & * & \Gamma_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Gamma_6 & 0 & 0 & 0 & \Gamma_{6,10} & 0 \\ * & * & * & * & * & * & \Gamma_7 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Gamma_8 & \Gamma_{8,9} & \Gamma_{8,10} & 0 \\ * & * & * & * & * & * & * & * & \Gamma_9 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\Gamma_{10} & 0 \\ * & * & * & * & * & * & * & * & * & * & \Gamma_{11} \end{bmatrix} \leq 0,$$

$$\Gamma_1 = C^TPC + \sigma^2 R_3 - R_1 - UF_1 - WG_1 - V_1^T - V_1,$$

$$\Gamma_{1,2} = R_1, \qquad \Gamma_{1,4} = -C^TPA + UF_2,$$

$$\Gamma_{1,5} = WG_2, \qquad \Gamma_{1,6} = \left(-\rho_1 C^TPB, \ldots, -\rho_l C^TPB\right),$$

$$\Gamma_{1,8} = V_1 - V_2^T, \qquad \Gamma_{1,9} = V_1 - V_3^T,$$

$$\Gamma_2 = \begin{bmatrix} -2R_1 & R_1 & & & \\ R_1 & -2R_1 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & -2R_1 & R_1 \\ & & & R_1 & -R_1 \end{bmatrix} - \begin{bmatrix} 0 & & & \\ & WG_1 & & & \\ & & \ddots & & \\ & & & WG_1 & \\ & & & & 0 \end{bmatrix} + \sum_{i=1}^{l+1} \left(\Phi_i + \Phi_i^T\right),$$

$$\Gamma_{2,6} = \mathrm{diag}\,[0, WG_2, \ldots, WG_2, 0],$$

$$\Gamma_{2,11} = \mathrm{diag}\,[T_1, T_2, \ldots, T_{l+1}],$$

$$\Gamma_3 = -R_3, \qquad \Gamma_4 = A^TPA - U,$$

$$\Gamma_{4,6} = \left(\rho_1 A^TPB, \ldots, \rho_l A^TPB\right), \qquad \Gamma_{4,10} = A^T\Gamma_{10},$$

$$\Gamma_5 = \sum_{i=1}^{l} (\tau_i - \tau_{i-1} + 1) S_i - W,$$

$$\Gamma_6 = \Psi_1 - \mathrm{diag}\,[W + S_1, \ldots, W + S_l],$$

$$\Psi_1 = \mathrm{diag}\left[\rho_1 B^TPB, \ldots, \rho_l B^TPB\right],$$

$$\Gamma_{6,10} = \left(\rho_1 \Gamma_{10}^T B, \rho_2 \Gamma_{10}^T B, \ldots, \rho_l \Gamma_{10}^T B\right)^T, \qquad \Gamma_7 = -P,$$

$$\Gamma_8 = V_2 + V_2^T, \qquad \Gamma_{8,9} = V_2 + V_3^T,$$

$$\Gamma_9 = V_3 + V_3^T - \frac{1}{\sigma}Q,$$

$$\Gamma_{10} = (\tau_M - \tau_m) hR_1 + (\tau_M - \tau_m) R_2 + \sigma Q,$$

$$\Gamma_{11} = -\frac{1}{h}\mathrm{diag}\,[R_2, R_2, \ldots, R_2],$$

$$\Phi_i = T_i \widetilde{I}_i, \qquad F_1 = \mathrm{diag}\left[\widehat{f}_1^- \widehat{f}_1^+, \widehat{f}_2^- \widehat{f}_2^+, \ldots, \widehat{f}_n^- \widehat{f}_n^+\right],$$

$$F_2 = \mathrm{diag}\left[\frac{\widehat{f}_1^- + \widehat{f}_1^+}{2}, \frac{\widehat{f}_2^- + \widehat{f}_2^+}{2}, \ldots, \frac{\widehat{f}_n^- + \widehat{f}_n^+}{2}\right],$$

$$G_1 = \mathrm{diag}\,[\widehat{g}_1^- \widehat{g}_1^+, \widehat{g}_2^- \widehat{g}_2^+, \ldots, \widehat{g}_n^- \widehat{g}_n^+],$$

$$G_2 = \mathrm{diag}\left[\frac{\widehat{g}_1^- + \widehat{g}_1^+}{2}, \frac{\widehat{g}_2^- + \widehat{g}_2^+}{2}, \ldots, \frac{\widehat{g}_n^- + \widehat{g}_n^+}{2}\right],$$

$$h = \max_{i=2,3,\ldots,l} \{\tau_i - \tau_{i-2}\}, \tag{8}$$

where $\widetilde{I}_i \in \mathbb{R}^{n\times(l+2)n}$, $(i = 1, 2, \ldots, l, l+1)$ denotes a matrix with the ith element $I_{n\times n}$ while the $i+1$th element is $-I_{n\times n}$, and other elements are zero matrixes. For example, $\Phi_2 = [0 \ \ I \ \ -I \ \ 0 \ \ \cdots \ \ 0]$.

Proof. For convenience, we denote $\Delta(y(k)) = y(k+1) - y(k)$; that is, $\Delta\{y(k) - C\sum_{i=k-\sigma}^{k-1} y(i)\} = y(k+1) - y(k) - Cy(k) + Cy(k-\sigma)$, and then systems (6) with $\omega(k) = 0$ can be rewritten as follows:

$$\zeta(k+1) = \Delta\left\{y(k) - C\sum_{i=k-\sigma}^{k-1} y(i)\right\} = -Cy(k) + Af(y(k)) + \sum_{i=1}^{l}\rho_i(k) Bg(y(k-\tau_i(k))). \tag{9}$$

We construct a new Lyapunov-Krasovskii functional $V(y_k, k)$ as

$$V(y_k, k) = \sum_{i=1}^{6} V_i(y_k, k), \tag{10}$$

where

$$\begin{aligned} V_1(y_k, k) &= \zeta^T(k) P\zeta(k), \\ V_2(y_k, k) &= h\sum_{i=-\tau_M}^{-1}\sum_{j=k+i}^{k-1}\eta^T(j) R_1\eta(j), \\ V_3(y_k, k) &= \sum_{i=-\tau_M}^{-\tau_m-1}\sum_{j=k+i}^{k-1}\eta^T(j) R_2\eta(j), \\ V_4(y_k, k) &= \sigma\sum_{i=-\sigma}^{-1}\sum_{j=k+i}^{k-1} y^T(j) R_3 y(j), \\ V_5(y_k, k) &= \sum_{i=1}^{l}\sum_{j=k-\tau_i(k)}^{k-1} g^T(y(j)) S_i g(y(j)) \\ &\quad + \sum_{i=1}^{l}\sum_{j=\tau_{i-1}}^{\tau_i-1}\sum_{m=k-j}^{k-1} g^T(y(m)) S_i g(y(m)), \\ V_6(y_k, k) &= \sum_{i=-\sigma}^{-1}\sum_{j=k+i}^{k-1}\eta^T(j) Q\eta(j), \\ \eta(k) &= y(k+1) - y(k). \end{aligned} \tag{11}$$

Taking the difference of the functional along the solution of (9), we obtain

$$\mathbb{E}\{\Delta V(y_k, k)\} = \sum_{i=1}^{6}\mathbb{E}\{\Delta V_i(y_k, k)\}, \tag{12}$$

$$\begin{aligned} &\mathbb{E}\{\Delta V_1(y_k, k)\} \\ &\quad = \mathbb{E}\{V_1(y_{k+1}, k+1) - V_1(y_k, k)\} \\ &\quad = \mathbb{E}\left\{\zeta^T(k+1) P\zeta(k+1) - \zeta^T(k) P\zeta(k)\right\} \\ &\quad = \mathbb{E}\left\{ y^T(k) C^T PCy(k) - 2y^T(k) C^T PAf(y(k)) \right. \\ &\qquad - 2\sum_{i=1}^{l}\rho_i(k) y^T(k) C^T PBg(y(k-\tau_i(k))) \\ &\qquad + f^T(y(k)) A^T PAf(y(k)) \\ &\qquad + 2\sum_{i=1}^{l}\rho_i(k) f^T(y(k)) A^T PBg(y(k-\tau_i(k))) \\ &\qquad + \left(\sum_{i=1}^{l}\rho_i(k) Bg(y(k-\tau_i(k)))\right)^T \\ &\qquad \cdot P\left(\sum_{j=1}^{l}\rho_j(k) Bg\left(y\left(k-\tau_j(k)\right)\right)\right) \\ &\qquad \left. - \zeta^T(k) P\zeta(k)\right\}. \end{aligned} \tag{13}$$

According to Remark 5, it is easy to get

$$\begin{aligned} &\mathbb{E}\{\Delta V_1(y_k, k)\} \\ &\quad = y^T(k) C^T PCy(k) - 2y^T(k) C^T PAf(y(k)) \\ &\qquad - 2\sum_{i=1}^{l}\rho_i y^T(k) C^T PBg(y(k-\tau_i(k))) \\ &\qquad + f^T(y(k)) A^T PAf(y(k)) \\ &\qquad + 2\sum_{i=1}^{l}\rho_i f^T(y(k)) A^T PBg(y(k-\tau_i(k))) \\ &\qquad + \alpha_2^T(k)\Psi_1\alpha_2(k) - \zeta^T(k) P\zeta(k), \end{aligned} \tag{14}$$

where $\alpha_2(k) = [g^T(y(k-\tau_1(k))) \ \ \cdots \ \ g^T(y(k-\tau_l(k)))]^T$.

Consider

$$\begin{aligned} &\mathbb{E}\{\Delta V_2(y_k, k)\} \\ &\quad = \mathbb{E}\{V_2(y_{k+1}, k+1) - V_2(y_k, k)\} \\ &\quad = \mathbb{E}\left\{ h\sum_{i=-\tau_M}^{-1}\sum_{j=k+i+1}^{k}\eta^T(j) R_1\eta(j) \right. \\ &\qquad \left. - h\sum_{i=-\tau_M}^{-1}\sum_{j=k+i}^{k-1}\eta^T(j) R_1\eta(j)\right\} \end{aligned}$$

$$= \mathbb{E}\left\{h\tau_M \eta^T(k) R_1 \eta(k) - h\sum_{i=-\tau_M}^{-1} \eta^T(k+i) R_1 \eta(k+i)\right\},$$

$$\mathbb{E}\{\Delta V_3(y_k,k)\}$$
$$= \mathbb{E}\{V_3(y_{k+1},k+1) - V_3(y_k,k)\}$$
$$= \mathbb{E}\left\{\sum_{i=-\tau_M}^{-\tau_m-1}\sum_{j=k+i+1}^{k} \eta^T(j) R_2 \eta(j) - \sum_{i=-\tau_M}^{-\tau_m-1}\sum_{j=k+i}^{k-1} \eta^T(j) R_2 \eta(j)\right\}$$
$$= \mathbb{E}\left\{(\tau_M-\tau_m)\eta^T(k) R_2 \eta(k) - \sum_{i=-\tau_M}^{-\tau_m-1} \eta^T(k+i) R_2 \eta(k+i)\right\},$$

$$\mathbb{E}\{\Delta V_4(y_k,k)\}$$
$$= \mathbb{E}\{\mathbb{E}\{V_4(y_{k+1},k+1) \mid y_k\} - V_4(y_k,k)\}$$
$$= \mathbb{E}\left\{\sigma\sum_{i=-\sigma}^{-1}\sum_{j=k+i+1}^{k} y^T(j) R_3 y(j) - \sigma\sum_{i=-\sigma}^{-1}\sum_{j=k+i}^{k-1} y^T(j) R_3 y(j)\right\}$$
$$= \sigma^2 y^T(k) R_3 y(k) - \sigma\sum_{i=k-\sigma}^{k-1} y^T(i) R_3 y(i),$$

$$\Delta V_5(y_k,k)$$
$$= V_5(y_{k+1},k+1) - V_5(y_k,k)$$
$$= \sum_{i=1}^{l}\sum_{j=k+1-\tau_i(k+1)}^{k} g^T(y(j)) S_i g(y(j)) + \sum_{i=1}^{l}\sum_{j=\tau_{i-1}}^{\tau_i-1}\sum_{m=k-j+1}^{k} g^T(y(m)) S_i g(y(m))$$
$$- \left[\sum_{i=1}^{l}\sum_{j=k-\tau_i(k)}^{k-1} g^T(y(j)) S_i g(y(j)) + \sum_{i=1}^{l}\sum_{j=\tau_{i-1}}^{\tau_i-1}\sum_{m=k-j}^{k-1} g^T(y(m)) S_i g(y(m))\right]$$
$$= \sum_{i=1}^{l}\Big(g^T(y(k)) S_i g(y(k)) - g^T(y(k-\tau_i(k))) S_i g(y(k-\tau_i(k)))\Big)$$
$$+ \sum_{j=k-\tau_i(k+1)+1}^{k-1} g^T(y(j)) S_i g(y(j)) - \sum_{j=k-\tau_i(k)+1}^{k-1} g^T(y(j)) S_i g(y(j))$$
$$+ \sum_{j=\tau_{i-1}}^{\tau_i-1}\Big(g^T(y(k)) S_i g(y(k)) - g^T(y(k-j)) S_i g(y(k-j))\Big)$$
$$\le \sum_{i=1}^{l}(\tau_i-\tau_{i-1}+1) g^T(y(k)) S_i g(y(k)) - g^T(y(k-\tau_i(k))) S_i g(y(k-\tau_i(k))),$$

$$\Delta V_6(y_k,k)$$
$$= V_6(y_{k+1},k+1) - V_6(y_k,k)$$
$$= \sum_{i=-\sigma}^{-1}\sum_{j=k+1+i}^{k} \eta^T(j) Q\eta(j) - \sum_{i=-\sigma}^{-1}\sum_{j=k+i}^{k-1} \eta^T(j) Q\eta(j)$$
$$= \sum_{i=-\sigma}^{-1}\left(\eta^T(k) Q\eta(k) + \sum_{j=k+1+i}^{k-1} \eta^T(j) Q\eta(j) - \sum_{j=k+1+i}^{k-1} \eta^T(j) Q\eta(j)\right) - \eta^T(k+i) Q\eta(k+i)$$
$$= \sigma\eta^T(k) Q\eta(k) - \sum_{i=-\sigma}^{-1}\eta^T(k+i) Q\eta(k+i). \tag{15}$$

Using the algebraic expression $-h\sum_{i=-\tau_M}^{-1}\eta^T(k+i)R_1\eta(k+i)$ in $\Delta V_2(y_k,k)$ and by using Lemma 7, we can get

$$-h\sum_{i=-\tau_M}^{-1}\eta^T(k+i) R_1\eta(k+i)$$
$$= -h\sum_{i=k-\tau_m}^{k-1}\eta^T(i) R_1\eta(i) - h\sum_{i=k-\tau_1(k)}^{k-\tau_m-1}\eta^T(i) R_1\eta(i) - h\sum_{i=k-\tau_2(k)}^{k-\tau_1(k)-1}\eta^T(i) R_1\eta(i) - \cdots - h\sum_{i=k-\tau_M}^{k-\tau_l(k)-1}\eta^T(i) R_1\eta(i)$$
$$\le -\left(\sum_{i=k-\tau_m}^{k-1}\eta(i)\right)^T R_1\left(\sum_{i=k-\tau_m}^{k-1}\eta(i)\right) - \left(\sum_{i=k-\tau_1(k)}^{k-\tau_m-1}\eta(i)\right)^T R_1\left(\sum_{i=k-\tau_1(k)}^{k-\tau_m-1}\eta(i)\right)$$

$$-\left(\sum_{i=k-\tau_2(k)}^{k-\tau_1(k)-1}\eta(i)\right)^T R_1\left(\sum_{i=k-\tau_2(k)}^{k-\tau_1(k)-1}\right)$$
$$-\cdots-\left(\sum_{i=k-\tau_M}^{k-\tau_l(k)-1}\eta(i)\right)^T R_1\left(\sum_{i=k-\tau_M}^{k-\tau_l(k)-1}\eta(i)\right)$$
$$\begin{aligned}=&-\left(y(k)-y(k-\tau_m)\right)^T R_1\left(y(k)-y(k-\tau_m)\right)\\&-\left(y(k-\tau_m)-y(k-\tau_1(k))\right)^T\\&\cdot R_1\left(y(k-\tau_m)-y(k-\tau_1(k))\right)\\&-\left(y(k-\tau_1(k))-y(k-\tau_2(k))\right)^T\\&\cdot R_1\left(y(k-\tau_1(k))-y(k-\tau_2(k))\right)\\&-\cdots-\left(y(k-\tau_l(k))-y(k-\tau_M)\right)^T\\&\cdot R_1\left(y(k-\tau_l(k))-y(k-\tau_M)\right).\end{aligned}\tag{16}$$

Let $\alpha_1(k) = (y^T(k-\tau_m), y^T(k-\tau_1(k)), y^T(k-\tau_2(k)),\ldots, y^T(k-\tau_l(k)), y^T(k-\tau_M))^T$; then

$$\begin{aligned}&-h\sum_{i=k-\tau_M}^{k-1}\eta^T(i)R_1\eta(i)\\&\le\left[y^T(k)\ \ \alpha_1^T(k)\right]\\&\cdot\begin{bmatrix}-R_1 & R_1 & 0 & \cdots & 0\\ * & -2R_1 & R_1 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ * & * & * & -2R_1 & R_1\\ * & * & * & * & -R_1\end{bmatrix}\begin{bmatrix}y(k)\\ \alpha_1(k)\end{bmatrix}\\&\triangleq\left[y^T(k)\ \ \alpha_1^T(k)\right]\Psi_2\begin{bmatrix}y(k)\\ \alpha_1(k)\end{bmatrix}.\end{aligned}\tag{17}$$

As is well known, for any vectors x, y and any symmetric matrix $Z>0$, the following inequality holds:

$$-2x^Ty\le x^TZ^{-1}x+y^TZy.\tag{18}$$

Denoting $x=T_{l+1}^T\alpha_1(k)$, $y=\sum_{j=k-\tau_M}^{k-\tau_l(k)-1}\eta(j)$, $Z=R_2/(\tau_l-\tau_l(k))$, where T_{l+1} is free matrix, then the last inequality can be expressed as follows:

$$\begin{aligned}&-2\alpha_1^T(k)T_{l+1}\sum_{i=k-\tau_M}^{k-\tau_l(k)-1}\eta(i)\\&\le\alpha_1^T(k)T_{l+1}Z^{-1}T_{l+1}\alpha_1^T(k)+\left(\tau_l-\tau_l(k)\right)\\&\cdot\left(\sum_{i=k-\tau_M}^{k-\tau_l(k)-1}\eta(i)\right)^T R_2\left(\sum_{i=k-\tau_M}^{k-\tau_l(k)-1}\eta(i)\right)\\&\le\alpha_1^T(k)T_{l+1}Z^{-1}T_{l+1}^T\alpha_1^T(k)+\sum_{i=k-\tau_M}^{k-\tau_l(k)-1}\eta(i)^TR_2\eta(i)\end{aligned}\tag{19}$$

which is equal to

$$\begin{aligned}&-\sum_{i=k-\tau_M}^{k-\tau_l(k)-1}\eta(i)^TR_2\eta(i)\\&\le\left(\tau_l-\tau_l(k)\right)\alpha_1^T(k)T_{l+1}R_2^{-1}T_{l+1}^T\alpha_1^T(k)\\&\quad+2\alpha_1^T(k)T_{l+1}\sum_{j=k-\tau_M}^{k-\tau_l(k)-1}\eta(j).\end{aligned}\tag{20}$$

Similarly, we can get

$$\begin{aligned}&-\sum_{j=k-\tau_{i+1}(k)}^{k-\tau_i(k)-1}\eta(j)^TR_2\eta(j)\\&\le\left(\tau_{i+1}(k)-\tau_i(k)\right)\alpha_1^T(k)T_{i+1}^{-1}R_2T_{i+1}^T\alpha_1^T(k)\\&\quad+2\alpha_1^T(k)T_{i+1}\sum_{j=k-\tau_{i+1}(k)}^{k-\tau_i(k)-1}\eta(j).\end{aligned}\tag{21}$$

Combining those inequalities, we obtain

$$\begin{aligned}&-\sum_{i=-\tau_M}^{-\tau_m-1}\eta^T(k+i)R_2\eta(k+i)\\&=-\sum_{i=k-\tau_1(k)}^{k-\tau_m-1}\eta^T(i)R_2\eta(i)-\sum_{i=k-\tau_2(k)}^{k-\tau_1(k)-1}\eta^T(i)R_2\eta(i)\\&\quad-\cdots-\sum_{i=k-\tau_M}^{k-\tau_l(k)-1}\eta^T(i)R_2\eta(i)\\&\le\left(\tau_l-\tau_l(k)\right)\alpha_1^T(k)T_{l+1}R_2^{-1}T_{l+1}^T\alpha_1^T(k)\\&\quad+2\alpha_1^T(k)T_{l+1}\sum_{j=k-\tau_M}^{k-\tau_l(k)-1}\eta(j)\\&\quad+\sum_{i=1}^{l-1}\left\{\left(\tau_{i+1}(k)-\tau_i(k)\right)\alpha_1^T(k)T_{i+1}R_2^{-1}T_{i+1}^T\alpha_1^T(k)\right.\\&\qquad\left.+2\alpha_1^T(k)T_{i+1}\sum_{j=k-\tau_{i+1}(k)}^{k-\tau_i(k)-1}\eta(j)\right\}\\&\quad+\left(\tau_1(k)-\tau_m\right)\alpha_1^T(k)T_1R_2^{-1}T_1^T\alpha_1^T(k)\\&\quad+2\alpha_1^T(k)T_1\sum_{j=k-\tau_1(k)}^{k-\tau_m-1}\eta(j).\end{aligned}\tag{22}$$

According to the equation in Assumption 1, we have

$$\begin{gathered}\left(f(y(k))-\widehat{f}_i^- y(k)\right)\left(f(y(k))-\widehat{f}_i^+ y(k)\right)\leq 0,\\ \left(g\left(y\left(k-\tau_i(k)\right)\right)-\widehat{g}_i^- y\left(k-\tau_i(k)\right)\right)\\ \cdot\left(g\left(y\left(k-\tau_i(k)\right)\right)-\widehat{g}_i^+ y\left(k-\tau_i(k)\right)\right)\leq 0.\end{gathered}\tag{23}$$

It can be deduced that there exist $\mathbf{U}=\operatorname{diag}[u_1,u_2,\ldots,u_n]>0$ and $W=\operatorname{diag}[w_1,w_2,\ldots,w_n]>0$, such that

$$\begin{aligned}&\sum_{i=1}^{n}u_i\begin{bmatrix}y(k)\\ f(y(k))\end{bmatrix}^T\begin{bmatrix}\widehat{f}_i^-\widehat{f}_i^+e_ie_i^T & -\dfrac{\widehat{f}_i^-+\widehat{f}_i^+}{2}e_ie_i^T\\ -\dfrac{\widehat{f}_i^-+\widehat{f}_i^+}{2}e_ie_i^T & e_ie_i^T\end{bmatrix}\\ &\quad\cdot\begin{bmatrix}y(k)\\ f(y(k))\end{bmatrix}\\ &=\begin{bmatrix}y(k)\\ f(y(k))\end{bmatrix}^T\begin{bmatrix}UF_1 & -UF_2\\ -UF_2 & U\end{bmatrix}\begin{bmatrix}y(k)\\ f(y(k))\end{bmatrix}\leq 0,\\ &\sum_{i=1}^{n}w_i\begin{bmatrix}y(k)\\ g(y(k))\end{bmatrix}^T\begin{bmatrix}\widehat{g}_i^-\widehat{g}_i^+e_ie_i^T & -\dfrac{\widehat{g}_i^-+\widehat{g}_i^+}{2}e_ie_i^T\\ -\dfrac{\widehat{g}_i^-+\widehat{g}_i^+}{2}e_ie_i^T & e_ie_i^T\end{bmatrix}\\ &\quad\cdot\begin{bmatrix}y(k)\\ g(y(k))\end{bmatrix}\\ &=\begin{bmatrix}y(k)\\ g(y(k))\end{bmatrix}^T\begin{bmatrix}WG_1 & -WG_2\\ -WG_2 & W\end{bmatrix}\begin{bmatrix}y(k)\\ g(y(k))\end{bmatrix}\leq 0,\\ &\sum_{i=1}^{n}w_i\begin{bmatrix}y\left(k-\tau_j(k)\right)\\ g\left(y\left(k-\tau_j(k)\right)\right)\end{bmatrix}^T\begin{bmatrix}\widehat{g}_i^-\widehat{g}_i^+e_ie_i^T & -\dfrac{\widehat{g}_i^-+\widehat{g}_i^+}{2}e_ie_i^T\\ -\dfrac{\widehat{g}_i^-+\widehat{g}_i^+}{2}e_ie_i^T & e_ie_i^T\end{bmatrix}\\ &\quad\cdot\begin{bmatrix}y\left(k-\tau_j(k)\right)\\ g\left(y\left(k-\tau_j(k)\right)\right)\end{bmatrix}\\ &=\begin{bmatrix}y\left(k-\tau_j(k)\right)\\ g\left(y\left(k-\tau_j(k)\right)\right)\end{bmatrix}^T\begin{bmatrix}WG_1 & -WG_2\\ -WG_2 & W\end{bmatrix}\\ &\quad\cdot\begin{bmatrix}y\left(k-\tau_j(k)\right)\\ g\left(y\left(k-\tau_j(k)\right)\right)\end{bmatrix}\leq 0,\end{aligned}\tag{24}$$

where e_i denotes the unit column vector having one element on its rth row and zeros elsewhere. Derived from $\sum_{i=k-\sigma}^{k-1}\eta(i)=y(k)-y(k-\sigma)$, we can get the following equation for any matrices $V=(V_1^T,V_2^T,V_3^T)^T$ with appropriate dimensions:

$$\begin{aligned}\mu(k)&=2\left[y^T(k)\quad y^T(k-\sigma)\quad\sum_{i=k-\sigma}^{k-1}\eta^T(i)\right]\\ &\quad\cdot V\left(-y(k)+y(k-\sigma)+\sum_{i=k-\sigma}^{k-1}\eta(i)\right)=0.\end{aligned}\tag{25}$$

Combining (12)–(25), we obtain

$$\begin{aligned}&\mathbb{E}\left\{\Delta V(y(k),k)\right\}\\ &\leq\mathbb{E}\Bigg\{y^T(k)C^TPCy(k)-2y^T(k)C^TPAf(y(k))\\ &\quad-2\sum_{i=1}^{l}\rho_iy^T(k)C^TPBg\left(y\left(k-\tau_i(k)\right)\right)\\ &\quad+f^T(y(k))A^TPAf(y(k))\\ &\quad+2\sum_{i=1}^{l}\rho_if^T(y(k))A^TPBg\left(y\left(k-\tau_i(k)\right)\right)\\ &\quad+\alpha_2^T(k)\Psi_1\alpha_2(k)-\zeta^T(k)P\zeta(k)-\alpha_1^T(k)\Psi_2\alpha_1(k)\\ &\quad+\sum_{i=1}^{l}\left[\left(\tau_i-\tau_{i-1}+1\right)g^T(y(k))S_ig(y(k))\right.\\ &\qquad\left.-g^T\left(y\left(k-\tau_i(k)\right)\right)S_ig\left(y\left(k-\tau_i(k)\right)\right)\right]\\ &\quad+\sum_{i=1}^{l}\left[\left(\tau_i-\tau_{i-1}+1\right)y^T(k)Q_iy(k)\right.\\ &\qquad\left.-y^T\left(k-\tau_i(k)\right)Q_iy\left(k-\tau_i(k)\right)\right]\\ &\quad+\left(\tau_l-\tau_l(k)\right)\alpha_1^T(k)T_{l+1}R_2^{-1}T_{l+1}^T\alpha_1(k)\\ &\quad+\sum_{i=1}^{l-1}\left(\tau_{i+1}(k)-\tau_i(k)\right)\alpha_1^T(k)T_{i+1}R_2^{-1}T_{i+1}^T\alpha_1(k)\\ &\quad+\left(\tau_1(k)-\tau_m\right)\alpha_1^T(k)T_1R_2^{-1}T_1^T\alpha_1(k)\\ &\quad+\sum_{i=1}^{l-1}2\alpha_1^T(k)T_{i+1}\left(y\left(k-\tau_i(k)\right)-y\left(k-\tau_{i+1}(k)\right)\right)\\ &\quad+2\alpha_1^T(k)T_1\left(y\left(k-\tau_m\right)-y\left(k-\tau_1(k)\right)\right)\\ &\quad+2\alpha_1^T(k)T_{l+1}\left(y\left(k-\tau_l(k)-y\left(k-\tau_M\right)\right)\right)\\ &\quad+\eta^T(k)\left[\left(\tau_M-\tau_m\right)hR_1+\left(\tau_M-\tau_m\right)R_2+\sigma Q\right]\eta(k)\\ &\quad-\frac{1}{\sigma}\sum_{i=-\sigma}^{-1}\eta^T(k+i)Q\sum_{i=-\sigma}^{-1}\eta(k+i)+\mu(k)\Bigg\}\end{aligned}$$

$$\leq \theta^T(k)\Bigg(\Xi + \Phi^T\Gamma_{10}\Phi + (\tau_M - \tau_l(k))\widetilde{T}_{l+1}R_1\widetilde{T}_{l+1}^T + \sum_{i=1}^{l-1}(\tau_{i+1}(k) - \tau_i(k))\widetilde{T}_{i+1}R_1\widetilde{T}_{i+1}^T + (\tau_1(k) - \tau_m)\widetilde{T}_1R_1\widetilde{T}_1^T\Bigg)\theta(k), \tag{26}$$

$$\theta(k) = \left(y^T(k), \alpha_1^T(k), \sum_{i=k-\sigma}^{k-1} y(i), f^T(y(k)), g^T(y(k)), \alpha_2^T(k), \zeta^T(k), y^T(k-\sigma), \sum_{i=k-\sigma}^{k-1}\eta^T(i)\right)^T,$$

$$\Xi = \begin{bmatrix} \Gamma_1 & 0 & 0 & \Gamma_{1,4} & \Gamma_{1,5} & \Gamma_{1,6} & 0 & \Gamma_{1,8} & \Gamma_{1,9} \\ * & \Gamma_2 & 0 & 0 & 0 & \Gamma_{2,6} & 0 & 0 & 0 \\ * & * & \Gamma_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Gamma_4 & 0 & \Gamma_{4,6} & 0 & 0 & 0 \\ * & * & * & * & \Gamma_5 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Gamma_6 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Gamma_7 & 0 & 0 \\ * & * & * & * & * & * & * & \Gamma_8 & \Gamma_{8,9} \\ * & * & * & * & * & * & * & * & \Gamma_9 \end{bmatrix},$$

$$\Phi = (0,0,0,A,0,\rho_1 B, \rho_2 B, \ldots, \rho_l B, 0, -C, 0),$$

$$\widetilde{T}_i = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \end{bmatrix}^T T_i. \tag{27}$$

Then if

$$\Xi + \Phi^T\Gamma_{10}\Phi + (\tau_l - \tau_l(k))\widetilde{T}_{l+1}R_1\widetilde{T}_{l+1}^T + \sum_{i=1}^{l-1}(\tau_{i+1}(k) - \tau_i(k))\widetilde{T}_{i+1}R_1\widetilde{T}_{i+1}^T + (\tau_1(k) - \tau_m)\widetilde{T}_1R_1\widetilde{T}_1^T \leq 0 \tag{28}$$

we can conclude that $\sum_{k=1}^{+\infty}\mathbb{E}\{\| y(k) \|^2\}$ is convergent and $\lim_{k\to+\infty}\mathbb{E}\{\| y(k) \|^2\} = 0$, which implies that the system (6) is globally asymptotically stable. Using Schur complement and the boundary condition $\tau_i(k) \in [\tau_{i-1}, \tau_i]$, we can get the inequality in this theorem.

This completes the proof of the theorem. □

Remark 9. The stability analysis problem of a general class of discrete-time neural networks with leakage delays is dealt with in Theorem 8. The stability condition can be expressed by a set of LMIs, which can be checked by resorting to MATLAB LMI Toolbox.

Remark 10. A similar study to this paper has been investigated in [21, 22, 33]. We note that our new stability criterion for stochastic discrete system benefits from the idea of delay partitioning. Based on the general assumption of time delay, we represent $\tau(k)$ as l parts, such that the condition is not only delay dependent but also dependent on the partitioning number.

Remark 11. From the proof of Theorem 8, it can be easily found that we enlarged $(\tau_1(k)-\tau_m)$, $\tau_{i+1}(k)-\tau_i(k)$, $(\tau_l-\tau_l(k))$ into h. We can get the next theorem in a different way to handle $(\tau_1(k)-\tau_m)$, $\tau_{i+1}(k)-\tau_i(k)$, $(\tau_l-\tau_l(k))$.

Corollary 12. *Under Assumptions 1 and 4, the system (6) with $\omega(k) = 0$ is globally asymptotically stable, if there exist positive matrices P, R_1, R_2, R_3, Q, S_i $(i = 1,\ldots,l)$, $W = \mathrm{diag}[w_1, w_2, \ldots, w_l]$, $\mathbf{U} = \mathrm{diag}[u_1, u_2, \ldots u_l]$, and free matrix T_i, $(i = 1,2,\ldots,2l)$, V_1, V_2, V_3 with any appropriate dimensions, such that the following LMI holds:*

$$\Pi_i = \begin{bmatrix} \Gamma_1 & \Gamma_{1,2} & 0 & \Gamma_{1,4} & \Gamma_{1,5} & \Gamma_{1,6} & 0 & \Gamma_{1,8} & \Gamma_{1,9} & 0 & 0 & 0 \\ * & \Gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Gamma_3 & 0 & 0 & \Gamma_{3,6} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Gamma_4 & 0 & \Gamma_{4,6} & 0 & 0 & 0 & 0 & \Gamma_{4,11} & 0 \\ * & * & * & * & \Gamma_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Gamma_6 & 0 & 0 & 0 & 0 & \Gamma_{6,11} & 0 \\ * & * & * & * & * & * & \Gamma_7 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Gamma_8 & \Gamma_{8,9} & 0 & \Gamma_{8,11} & 0 \\ * & * & * & * & * & * & * & * & \Gamma_9 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Gamma_{10} & 0 & \Gamma_{10,12} \\ * & * & * & * & * & * & * & * & * & * & -\Gamma_{11} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \Gamma_{12} \end{bmatrix} \leq 0, \tag{29}$$

$$\Xi_i = \begin{bmatrix} \Gamma_1 & \Gamma_{1,2} & 0 & \Gamma_{1,4} & \Gamma_{1,5} & \Gamma_{1,6} & 0 & \Gamma_{1,8} & \Gamma_{1,9} & 0 & 0 & 0 \\ * & \Gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Gamma_3 & 0 & 0 & \Gamma_{3,6} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Gamma_4 & 0 & \Gamma_{4,6} & 0 & 0 & 0 & 0 & \Gamma_{4,11} & 0 \\ * & * & * & * & \Gamma_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Gamma_6 & 0 & 0 & 0 & 0 & \Gamma_{6,11} & 0 \\ * & * & * & * & * & * & \Gamma_7 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Gamma_8 & \Gamma_{8,9} & 0 & \Gamma_{8,11} & 0 \\ * & * & * & * & * & * & * & * & \Gamma_9 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Omega_{10} & 0 & \Omega_{10,12} \\ * & * & * & * & * & * & * & * & * & * & -\Gamma_{11} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \Gamma_{12} \end{bmatrix} \leq 0 \quad (30)$$

$$\Gamma_{10} = \sum_{j=1}^{l} T_{2j-1}\widehat{I}_j + \sum_{j=1}^{l} \widehat{I}_j^T T_{2j-1}^T,$$

$$\Gamma_{10,12} = \operatorname{diag}\left[T_1, T_3, \ldots, T_{2l-1}\right],$$

$$\Omega_{10} = \sum_{j\neq i}^{l} T_{2j-1}\widehat{I}_j + \sum_{j\neq i}^{l} \widehat{I}_j^T T_{2j-1}^T + T_{2i}\widehat{I}_i + \widehat{I}_i^T T_{2i}^T, \quad (31)$$

$$\Omega_{10,12} = \operatorname{diag}\left[T_1, T_3, \ldots, T_{2i-1}, T_{2i}, T_{2i+1}, \ldots, T_{2l-1}\right],$$

$$\Gamma_{12} = -\operatorname{diag}\left[\frac{R_2}{d_1}, \frac{R_2}{d_2}, \ldots, \frac{R_2}{d_l}\right],$$

and the definition of $\widehat{I}_i$ *is the same as the definition in Theorem 8, and* Γ_{11}, $\Gamma_{4,11}$, $\Gamma_{6,11}$, $\Gamma_{8,11}$ *in inequality (29) are* Γ_{10}, $\Gamma_{4,10}$, $\Gamma_{6,10}$, $\Gamma_{8,10}$ *in Theorem 8, respectively.*

Proof. $-\sum_{i=-\tau_M}^{-\tau_m-1} \eta^T(k+i)R_2\eta(k+i)$ in $\Delta V_3(k)$ is handled in the following way:

$$\begin{aligned} &-\sum_{i=-\tau_M}^{-\tau_m-1} \eta^T(k+i) R_2 \eta(k+i) \\ &= -\sum_{i=1}^{l}\left(\sum_{j=k-\tau_i(k)}^{k-\tau_{i-1}-1} + \sum_{j=k-\tau_i}^{k-\tau_i(k)-1}\right)\eta^T(i) R_2\eta(i) \\ &\leq \sum_{i=1}^{l}\left[\left(\tau_i - \tau_i(k)\right)\alpha_3^T(k) T_{2i-1}R_2^{-1}T_{2i-1}^T\alpha_3(k)\right. \\ &\quad + 2\alpha_3^T(k) T_{2i-1}\left(y(k-\tau_i(k)) - y(k-\tau_i)\right) \\ &\quad + \left(\tau_i(k) - \tau_{i-1}\right)\alpha_3^T(k) T_{2i}R_2^{-1}T_{2i}^T\alpha_3(k) \\ &\quad \left.+ 2\alpha_3^T(k) T_{2i}\left(y(k-\tau_{i-1}) - y(k-\tau_i(k))\right)\right], \\ &\alpha_3^T(k) = \left(y(k-\tau_m), y(k-\tau_1), \ldots, y(k-\tau_M)\right). \end{aligned} \quad (32)$$

When $\tau(k) \in \mathscr{A}_i$, $\tau_i(k) = \tau(k)$, $\tau_j(k) = \tau_{j-1}$, $(j \neq i)$, the last inequality is equivalent to

$$\begin{aligned} &-\sum_{i=-\tau_M}^{-\tau_m-1} \eta^T(k+i) R_2\eta(k+i) \\ &\leq \left[\left(\tau_i - \tau_i(k)\right)\alpha_3^T(k) T_{2i-1}R_2^{-1}T_{2i-1}^T\alpha_3(k)\right. \\ &\quad + 2\alpha_3^T(k) T_{2i-1}\left(y(k-\tau_i(k)) - y(k-\tau_i)\right) \\ &\quad + \left(\tau_i(k) - \tau_{i-1}\right)\alpha_3^T(k) T_{2i}R_2^{-1}T_{2i}^T\alpha_3(k) \\ &\quad \left.+ 2\alpha_3^T(k) T_{2i}\left(y(k-\tau_{i-1}) - y(k-\tau_i(k))\right)\right] \\ &\quad + \sum_{j\neq i}^{l}\left(\tau_j - \tau_{j-1}\right)\alpha_3^T(k) T_{2j-1}R_2^{-1}T_{2j-1}^T\alpha_3(k) \\ &\quad + 2\alpha_3^T(k) T_{2j-1}\left(y\left(k-\tau_{j-1}\right) - y\left(k-\tau_j\right)\right). \end{aligned} \quad (33)$$

Then

$$\begin{aligned} &\mathbb{E}\left\{\Delta V\left(y(k), k\right)\right\} \\ &\leq \theta^T(k)\left(\Xi + \Phi^T\Gamma_{11}\Phi\right)\theta(k) \\ &\quad + \left[\left(\tau_i - \tau_i(k)\right)\alpha_3^T(k) T_{2i-1}R_2^{-1}T_{2i-1}^T\alpha_3(k)\right. \\ &\quad + 2\alpha_3^T(k) T_{2i-1}\left(y(k-\tau_i(k)) - y(k-\tau_i)\right) \\ &\quad + \left(\tau_i(k) - \tau_{i-1}\right)\alpha_3^T(k) T_{2i}R_2^{-1}T_{2i}^T\alpha_3(k) \\ &\quad \left.+ 2\alpha_3^T(k) T_{2i}\left(y(k-\tau_{i-1}) - y(k-\tau_i(k))\right)\right] \\ &\quad + \sum_{j\neq i}^{l}\left(\tau_j - \tau_{j-1}\right)\alpha_3^T(k) T_{2j-1}R_2^{-1}T_{2j-1}^T\alpha_3(k) \\ &\quad + 2\alpha_3^T(k) T_{2j-1}\left(y\left(k-\tau_{j-1}\right) - y\left(k-\tau_j\right)\right). \end{aligned} \quad (34)$$

By solving the convex LMI condition at boundary conditions $\tau_i(k) \in [\tau_{i-1}, \tau_i]$, we can get the LMIs in Corollary 12. This completes the proof of Corollary 12. □

Remark 13. The difference between Theorem 8 and Corollary 12 is obvious. The LMIs in Corollary 12 are derived by property of $\tau_i(k)$, which is more simple than the LMIs in Theorem 8.

The next theorem can be obtained if the stochastic term $\omega(k)$ is not removed in (6).

Corollary 14. *Under Assumptions 1 and 4, the system (6) with $\omega(k) = 0$ is globally asymptotically stable, if there exist positive matrices $P = P^T$, R_1, R_2, Q_i $(i = 1, \dots, l)$, S_i $(i = 1, \dots, l)$, $W = \operatorname{diag}(w_1, w_2, \dots, w_l)$, $\mathbf{U} = \operatorname{diag}[u_1, u_2, \dots u_l]$, and free matrix $T_1, T_2, \dots, T_l$ with any appropriate dimensions, such that the following LMI holds:*

$$P + \tau_M h R_1 + (\tau_M - \tau_m) R_2 < \lambda I, \tag{35}$$

$$\Pi_i = \begin{bmatrix} \Gamma_1 & \Gamma_{1,2} & 0 & \Gamma_{1,4} & \Gamma_{1,5} & \Gamma_{1,6} & 0 & \Gamma_{1,8} & \Gamma_{1,9} & 0 & 0 & 0 \\ * & \Gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Gamma_3 & 0 & 0 & \Gamma_{3,6} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Gamma_4 & 0 & \Gamma_{4,6} & 0 & 0 & 0 & 0 & \Gamma_{4,11} & 0 \\ * & * & * & * & \Gamma_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Gamma_6 & 0 & 0 & 0 & 0 & \Gamma_{6,11} & 0 \\ * & * & * & * & * & * & \Gamma_7 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Gamma_8 & \Gamma_{8,9} & 0 & \Gamma_{8,11} & 0 \\ * & * & * & * & * & * & * & * & \Gamma_9 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Gamma_{10} & 0 & \Gamma_{10,12} \\ * & * & * & * & * & * & * & * & * & * & -\Gamma_{11} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \Gamma_{12} \end{bmatrix} \leq 0, \tag{36}$$

$$\Xi_i = \begin{bmatrix} \Gamma_1 & \Gamma_{1,2} & 0 & \Gamma_{1,4} & \Gamma_{1,5} & \Gamma_{1,6} & 0 & \Gamma_{1,8} & \Gamma_{1,9} & 0 & 0 & 0 \\ * & \Gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Gamma_3 & 0 & 0 & \Gamma_{3,6} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Gamma_4 & 0 & \Gamma_{4,6} & 0 & 0 & 0 & 0 & \Gamma_{4,11} & 0 \\ * & * & * & * & \Gamma_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Gamma_6 & 0 & 0 & 0 & 0 & \Gamma_{6,11} & 0 \\ * & * & * & * & * & * & \Gamma_7 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Gamma_8 & \Gamma_{8,9} & 0 & \Gamma_{8,11} & 0 \\ * & * & * & * & * & * & * & * & \Gamma_9 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Omega_{10} & 0 & \Omega_{10,12} \\ * & * & * & * & * & * & * & * & * & * & -\Gamma_{11} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \Gamma_{12} \end{bmatrix} \leq 0,$$

$$\Gamma_{10} = \sum_{j=1}^{l} T_{2j-1} \widehat{I}_j + \sum_{j=1}^{l} \widehat{I}_j^T T_{2j-1}^T,$$

$$\Gamma_{10,12} = \operatorname{diag}(T_1, T_3, \dots, T_{2l-1}),$$

$$\Omega_{10} = \sum_{j \neq i}^{l} T_{2j-1} \widehat{I}_j + \sum_{j \neq i}^{l} \widehat{I}_j^T T_{2j-1}^T + T_{2i} \widehat{I}_i + \widehat{I}_i^T T_{2i}^T,$$

$$\Omega_{10,12} = \operatorname{diag}(T_1, T_3, \dots, T_{2i-1}, T_{2i}, T_{2i+1}, \dots, T_{2l-1}),$$

$$\Gamma_{12} = -\operatorname{diag}\left(\frac{R_2}{d_1}, \frac{R_2}{d_2}, \dots, \frac{R_2}{d_l}\right), \tag{37}$$

and the definition of $\widehat{I}_i$ is the same as it in Theorem 8, and Γ_{11}, $\Gamma_{4,11}$, $\Gamma_{6,11}$, $\Gamma_{8,11}$ are Γ_{10}, $\Gamma_{4,10}$, $\Gamma_{6,10}$, $\Gamma_{8,10}$, respectively, in Theorem 8.

Proof. As $\omega(k) \neq 0$, $\Delta\{V_1(y_k,k)\}$, and $\Delta\{V_3(y_k,k)\}$ are different from Theorem 8,

$$\begin{aligned}
&\mathbb{E}\{\Delta V_1(y_k,k)\} \\
&\quad = \mathbb{E}\{\mathbb{E}\{V_1(y_{k+1},k+1) \mid y_k\} - V_1(y_k,k)\} \\
&\quad = \mathbb{E}\left\{\zeta^T(k+1)P\zeta(k+1) - \zeta^T(k)P\zeta(k)\right\} \\
&\quad = y^T(k)C^TPCy(k) - 2y^T(k)C^TPAf(y(k)) \\
&\qquad - 2\sum_{i=1}^{l}\rho_i y^T(k)C^TPBg(y(k-\tau_i(k))) \\
&\qquad + f^T(y(k))A^TPAf(y(k)) \\
&\qquad + 2\sum_{i=1}^{l}\rho_i f^T(y(k))A^TPBg(y(k-\tau_i(k))) \\
&\qquad + \alpha_2^T(k)\Psi_1\alpha_2(k) \\
&\qquad - \zeta^T(k)P\zeta(k) + \delta^T(k,y(k))P\delta(k,y(k)).
\end{aligned} \tag{38}$$

Derived from Assumption 3, we can get

$$\begin{aligned}
&\delta^T(k,y(k))(P+\tau_M hR_1+(\tau_M-\tau_m)R_2)\delta(k,y(k)) \\
&\leq \lambda_{\max}(P+\tau_M hR_1+(\tau_M-\tau_m)R_2)\delta^T(k,y(k))\delta(k,y(k)) \\
&\leq \lambda\xi y^T(k)y(k).
\end{aligned} \tag{39}$$

Based on Theorem 8, the desired condition can be obtained. This is the end of proof. □

4. Examples

In this section, a numerical example is given to illustrate the effectiveness and benefits of the developed methods.

Example 1. We consider the delayed stochastic DNNs (6) with the following parameters, which have been considered in [33]:

$$C = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.3 \end{bmatrix}, \qquad A = \begin{bmatrix} 0.5 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, \qquad B = \begin{bmatrix} 0.4 & -0.1 \\ 0.1 & 0.2 \end{bmatrix}. \tag{40}$$

And the activation functions satisfy Assumption 1 with

$$\begin{gathered}
F_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}; \qquad G_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\
G_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}.
\end{gathered} \tag{41}$$

For the parameters listed above, we can obtain the following feasible solution. Due to the limitation of the length of this paper, we only provide a part of the feasible solution here:

$$\begin{aligned}
Q_1 &= \begin{bmatrix} 0.7557 & 0.0283 \\ 0.0283 & 0.0014 \end{bmatrix}, \qquad & Q_2 &= \begin{bmatrix} 0.5832 & 0.0217 \\ 0.0217 & 0.0011 \end{bmatrix}, \\
S_1 &= \begin{bmatrix} 22.5229 & 1.5164 \\ 1.5164 & 0.1059 \end{bmatrix}, \qquad & S_2 &= \begin{bmatrix} 0.5903 & 0.0055 \\ 0.0055 & 0.0010 \end{bmatrix}.
\end{aligned} \tag{42}$$

5. Conclusions

The stability analysis of stochastic discrete-time NNs with leakage delay has been investigated in this paper via the Lyapunov functional method. By employing delay partition and introducing a new Lyapunov functional, a delay-dependent stability condition is proposed. What is more, more general LMIs conditions on the stability of the stochastic discrete-time NNs are established by choosing a new novel Lyapunov-Krasovskii functional. Finally, the feasibility and effectiveness of the developed methods have been shown by the numerical simulation example. The foregoing results have the potential to be useful for the study of leakage delay. And the results can be extended to complex networks with mixed time-varying delays.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] T. Yang, "Fuzzy cellular neural networks and their applications to image processing," *Advances in Imaging and Electron Physics*, vol. 109, pp. 265–446, 1999.

[2] Y. Yao, W. J. Freeman, B. Burke, and Q. Yang, "Pattern recognition by a distributed neural network: an industrial application," *Neural Networks*, vol. 4, no. 1, pp. 103–121, 1991.

[3] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*, Birkhäuser, Boston, Mass, USA, 2003.

[4] H. Shao, "New delay-dependent stability criteria for systems with interval delay," *Automatica*, vol. 45, no. 3, pp. 744–749, 2009.

[5] J. Sun, G. P. Liu, J. Chen, and D. Rees, "Improved delay-range-dependent stability criteria for linear systems with time-varying delays," *Automatica*, vol. 46, no. 2, pp. 466–470, 2010.

[6] Y. Liu, Z. Wang, and X. Liu, "Global exponential stability of generalized recurrent neural networks with discrete and distributed delays," *Neural Networks*, vol. 19, no. 5, pp. 667–675, 2006.

[7] Z. Wang, H. Shu, Y. Liu, D. W. C. Ho, and X. Liu, "Robust stability analysis of generalized neural networks with discrete and distributed time delays," *Chaos, Solitons & Fractals*, vol. 30, no. 4, pp. 886–896, 2006.

[8] D. K. Das, S. Ghosh, and B. Subudhi, "Stability analysis of linear systems with two delays of overlapping ranges," *Applied Mathematics and Computation*, vol. 243, pp. 83–90, 2014.

[9] O. M. Kwon, M. J. Park, J. H. Park, S. M. Lee, and E. J. Cha, "On stability analysis for neural networks with interval time-varying delays via some new augmented Lyapunov-Krasovskii functional," *Communications in Nonlinear Science and Numerical Simulation*, vol. 19, no. 9, pp. 3184–3201, 2014.

[10] J. Xia, J. H. Park, H. Zeng, and H. Shen, "Delay-difference-dependent robust exponential stability for uncertain stochastic neural networks with multiple delays," *Neurocomputing*, vol. 140, pp. 210–218, 2014.

[11] Y. Liu, S. M. Lee, O. M. Kwon, and J. H. Park, "Delay-dependent exponential stability criteria for neutral systems with interval time-varying delays and nonlinear perturbations," *Journal of the Franklin Institute*, vol. 350, no. 10, pp. 3313–3327, 2013.

[12] Y. Liu, S. M. Lee, O. M. Kwon, and J. H. Park, "Robust delay-dependent stability criteria for time-varying delayed Lur'e systems of neutral type," *Circuits, Systems, and Signal Processing*, 2014.

[13] D. Zhang and L. Yu, "Exponential state estimation for Markovian jumping neural networks with time-varying discrete and distributed delays," *Neural Networks*, vol. 35, pp. 103–111, 2012.

[14] D. Zhang, W. Cai, L. Xie, and Q. Wang, "Non-fragile distributed filtering for T-S fuzzy systems in sensor networks," *IEEE Transactions on Fuzzy Systems*, 2014.

[15] J. Cheng, H. Zhu, S. Zhong, Q. Zhong, and Y. Zeng, "Finite-time H_∞ estimation for discrete-time Markov jump systems with time-varying transition probabilities subject to average dwell time switching," *Communications in Nonlinear Science and Numerical Simulation*, vol. 20, no. 2, pp. 571–582, 2015.

[16] C. Yin, S. Zhong, and W. Chen, "Design of sliding mode controller for a class of fractional-order chaotic systems," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 1, pp. 356–366, 2012.

[17] J. Cao, "Improved delay-dependent stability conditions for MIMO networked control systems with nonlinear perturbations," *The Scientific World Journal*, vol. 2014, Article ID 196927, 4 pages, 2014.

[18] C. Yin, Y. Chen, and S. Zhong, "Fractional-order sliding mode based extremum seeking control of a class of nonlinear systems," *Automatica*, vol. 50, no. 12, pp. 3173–3181, 2014.

[19] J. Ma, J. Li, X. Gao, and Z. Wang, "Research on time-delay stability upper bound of power system wide-area damping controllersbased on improved free-weighting matrices and generalized eigenvalue problem," *Electrical Power and Energy Systems*, vol. 64, pp. 476–482, 2015.

[20] J. Cheng, H. Zhu, S. Zhong, F. Zheng, and Y. Zeng, "Finite-time filtering for switched linear systems with a mode-dependent average dwell time," *Nonlinear Analysis: Hybrid Systems*, vol. 15, pp. 145–156, 2015.

[21] J. Duan, M. Hu, Y. Yang, and L. Guo, "A delay-partitioning projection approach to stability analysis of stochastic Markovian jump neural networks with randomly occurred nonlinearities," *Neurocomputing*, vol. 128, pp. 459–465, 2014.

[22] Y. Shao, X. Liu, X. Sun, and Q. Zhang, "A delay decomposition approach to H_∞ admissibility for discrete-time singular delay systems," *Information Sciences*, vol. 279, pp. 893–905, 2014.

[23] X.-L. Zhu and G.-H. Yang, "Jensen inequality approach to stability analysis of discrete-time systems with time-varying delay," in *Proceedings of the American Control Conference (ACC '08)*, pp. 1644–1649, Seattle, Wash, USA, June 2008.

[24] R. Geiselhart, R. H. Gielen, M. Lazar, and F. R. Wirth, "An alternative converse Lyapunov theorem for discrete-time systems," *Systems & Control Letters*, vol. 70, pp. 49–59, 2014.

[25] X. L. Zhu and G. H. Yang, "New delay-dependent stability results for neural networks with time-varying delay," *IEEE Transactions on Neural Networks*, vol. 19, no. 10, pp. 1783–1791, 2008.

[26] B. Zhang, S. Xu, and Y. Zou, "Improved delay-dependent exponential stability criteria for discrete-time recurrent neural networks with time-varying delays," *Neurocomputing*, vol. 72, no. 1–3, pp. 321–330, 2008.

[27] C. Song, H. Gao, and W. X. Zheng, "A new approach to stability analysis of discrete-time recurrent neural networks with time-varying delay," *Neurocomputing*, vol. 72, no. 10–12, pp. 2563–2568, 2009.

[28] Y. Tang, J.-A. Fang, M. Xia, and D. Yu, "Delay-distribution-dependent stability of stochastic discrete-time neural networks with randomly mixed time-varying delays," *Neurocomputing*, vol. 72, no. 16–18, pp. 3830–3838, 2009.

[29] K. Gopalsamy, "Leakage delays in BAM," *Journal of Mathematical Analysis and Applications*, vol. 325, no. 2, pp. 1117–1132, 2007.

[30] R. Sakthivel, P. Vadivel, K. Mathiyalagan, A. Arunkumar, and M. Sivachitra, "Design of state estimator for bidirectional associative memory neural networks with leakage delays," *Information Sciences*, vol. 296, pp. 263–274, 2015.

[31] K. Li, L. Yang, and W. Wu, "Anti-periodic solution for impulsive BAM neural networks with time-varying leakage delays ontime scales," *Neurocomputing*, vol. 149, pp. 536–545, 2015.

[32] B. Liu, "Global exponential stability for BAM neural networks with time-varying delays in the leakage terms," *Nonlinear Analysis: Real World Applications*, vol. 14, no. 1, pp. 559–566, 2013.

[33] Z. Zhao, Q. Song, and S. He, "Passivity analysis of stochastic neural networks with time-varying delays and leakage delay," *Neurocomputing*, vol. 125, pp. 22–27, 2014.

[34] X. Li, X. Fu, and R. Rakkiyappan, "Delay-dependent stability analysis for a class of dynamical systems with leakage delay and nonlinear perturbations," *Applied Mathematics and Computation*, vol. 226, pp. 10–19, 2014.

[35] X. Li and X. Fu, "Effect of leakage time-varying delay on stability of nonlinear differential systems," *Journal of the Franklin Institute. Engineering and Applied Mathematics*, vol. 350, no. 6, pp. 1335–1344, 2013.

[36] Q. Zhu, R. Rakkiyappan, and A. Chandrasekar, "Stochastic stability of Markovian jump BAM neural networks with leakage delays and impulse control," *Neurocomputing*, vol. 136, pp. 136–151, 2014.

[37] S. Lakshmanan, J. H. Park, T. H. Lee, H. Y. Jung, and R. Rakkiyappan, "Stability criteria for BAM neural networks with leakage delays and probabilistic time-varying delays," *Applied Mathematics and Computation*, vol. 219, no. 17, pp. 9408–9423, 2013.

[38] M. J. Park, O. M. Kwon, J. H. Park, S. M. Lee, and E. J. Cha, "Synchronization criteria for coupled stochastic neural networks with time-varying delays and leakage delay," *Journal of the Franklin Institute*, vol. 349, no. 5, pp. 1699–1720, 2012.

[39] X. Li and J. Cao, "Delay-dependent stability of neural networks of neutral type with time delay in the leakage term," *Nonlinearity*, vol. 23, no. 7, pp. 1709–1726, 2010.

[40] C. Li and T. Huang, "On the stability of nonlinear systems with leakage delay," *Journal of the Franklin Institute. Engineering and Applied Mathematics*, vol. 346, no. 4, pp. 366–377, 2009.

[41] L. Duan and L. Huang, "Global exponential stability of fuzzy BAM neural networks with distributed delays and time-varying delays in the leakage terms," *Neural Computing and Applications*, vol. 23, pp. 171–178, 2013.

[42] S. Haykin, *Neural Networks*, Prentice Hall, Upper Saddle River, NJ, USA, 1999.