ON THE UPLINK-DOWNLINK DUALITY FOR GAUSSIAN VECTOR CHANNELS WITH COLORED NOISE AND APPLICATIONS TO CDMA TRANSMITTER ADAPTATION

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ABSTRACT

In this paper we study the uplink-downlink duality for Gaussian vector channels with colored noise, and we derive the duality transformations (downlink-to-uplink and vice versa) that imply transmit covariance matrices for which user rates in uplink and downlink are identical. These transformations are then applied for transmitter adaptation in Code Division Multiple Access (CDMA) systems to obtain an ensemble of downlink CDMA codewords from an optimal ensemble of uplink codewords.

1. INTRODUCTION

In a wireless communication system the uplink (also referred to as the reverse link) describes the information flow from a set of wireless terminals to a central receiver corresponding to a base station, and the downlink (also referred to as the forward link) describes the flow of information from a central transmitter corresponding to a base station to a set of wireless terminals. The two mathematical models used to describe these two scenarios are the multiple access channel (MAC) [1, Sec. 15.3] for the uplink and the broadcast channel (BC) [1, Sec. 15.6] for the downlink, and several duality results have been established recently between various MAC and BC scenarios [2–5].

We note that a common assumption in all these references is that the statistics of the noise at all receivers in both the uplink and the downlink scenarios is the same, and all of them take noise corrupting transmitted signals in a vector Gaussian channel to be white with a scaled identity covariance matrix. However, this assumption may not be satisfied in many practical scenarios where noise at different receivers can have different statistics. For example, in the downlink scenario the user receivers are operating in different locations where the additive noise corrupting received signals can have different statistics, or, even when the noise statistics is similar and has identity covariance matrix at all receivers, the receiver processing can color it through various operations such as channel equalization for example. This motivates the work presented in this paper where we study the uplink-downlink duality under the explicit consideration of different noise statistics at different receivers in the uplink and the downlink.

The paper is organized as follows: in Section II we present the system model followed by derivation of the duality transformations for symmetric rates in Section III. In Section IV we present application of the duality transformations in the context of CDMA systems and discuss how downlink CDMA codewords can be obtained from optimal uplink CDMA codewords In Section V we present final remarks and conclusions.

2. SYSTEM MODEL

2.1. The Downlink Scenario

In the downlink scenario the base station acts as transmitter and sends information signal $\hat{\mathbf{x}}$ to a set of K wireless terminals acting as receivers. The transmitted signal is received at the corresponding receivers through distinct vector channels characterized by channel matrices $\mathbf{H}_1, \ldots, \mathbf{H}_K$ of dimension $N \times N$ and is corrupted by N-dimensional independent additive Gaussian noise vectors $\hat{\mathbf{n}}_1, \ldots, \hat{\mathbf{n}}_K$ with covariance matrices $\hat{\mathbf{W}}_1, \ldots, \hat{\mathbf{W}}_K$. We note that all channel matrices are assumed to be constant and known at the transmitter and at their corresponding receivers.

This scenario is illustrated in Figure 1 and is described mathematically by a Gaussian vector BC model [6] for which the corresponding N-dimensional received signal vector at a given receiver k is expressed as

$$\hat{\mathbf{r}}_k = \mathbf{H}_k \hat{\mathbf{x}} + \hat{\mathbf{n}}_k \qquad k = 1, \ldots, K. \tag{1}$$

The transmitter is subject to an average power constraint expressed in terms of the transmit covariance matrix $\hat{\mathbf{X}} = E[\hat{\mathbf{x}}\hat{\mathbf{x}}^\dagger]$ (where $(\cdot)^\dagger$ represents the Hermitian operator, that is, the transposed and complex conjugate) as

$$\text{Trace}\left[\hat{\mathbf{X}}\right] \leq P \tag{2}$$

Using dirty paper coding techniques [7] as discussed in [4,6] the transmitted signal $\hat{\mathbf{x}}$ with covariance matrix $\hat{\mathbf{X}}$ is obtained by adding up the signals $\hat{\mathbf{x}}_k$, with covariance matrices $\hat{\mathbf{X}}_k$, $k = 1, \ldots, K$, intended for individual users. These are encoded in a specified order such that, when the signal intended for a given user k is obtained, the

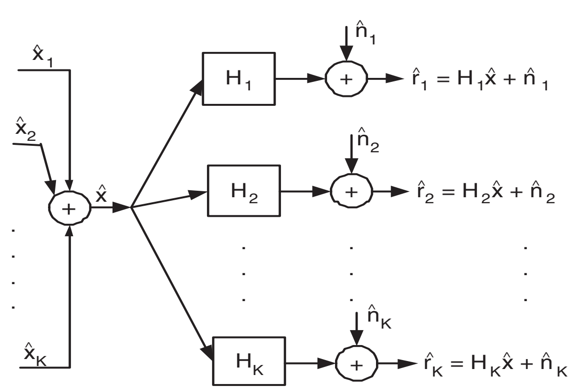

Fig. 1. Block Diagram for Downlink Scenario.

transmitter has full (noncausal) knowledge of all the signals encoded before $\hat{\mathbf{x}}_1, \ldots, \hat{\mathbf{x}}_{k-1}$, and does not see these as interference. This dirty paper coding procedure yields statistically independent signals $\hat{\mathbf{x}}_k$, $k = 1, \ldots, K$, which implies that the input transmit covariance matrix can be written as

$$\hat{\mathbf{X}} = \hat{\mathbf{X}}_1 + \ldots + \hat{\mathbf{X}}_K \ , \tag{3}$$

and the power constraint (2) is equivalent to

$$\sum_{k=1}^{K} \text{Trace}\left[\hat{\mathbf{X}}_k\right] \leq P \tag{4}$$

The downlink rate of user k is the rate corresponding to user k in the vector BC model with the specified order of encoding users and is given by [6, Th. 1]

$$\hat{R}_k = \frac{1}{2} \log \frac{\det\left(\sum_{i=k}^{K} \mathbf{H}_k \hat{\mathbf{X}}_i \mathbf{H}_k^{\dagger} + \hat{\mathbf{W}}_k\right)}{\det\left(\sum_{i=k+1}^{K} \mathbf{H}_k \hat{\mathbf{X}}_i \mathbf{H}_k^{\dagger} + \hat{\mathbf{W}}_k\right)} \tag{5}$$

for $k = 1, \ldots, K-1$ and

$$\hat{R}_K = \frac{1}{2} \log \frac{\det\left(\mathbf{H}_K \hat{\mathbf{X}}_K \mathbf{H}_K^{\dagger} + \hat{\mathbf{W}}_K\right)}{\det \hat{\mathbf{W}}_K} \tag{6}$$

2.2. The Uplink Scenario

In the dual uplink scenario the set of K wireless terminals act as transmitters and send information signals $\check{\mathbf{x}}_1, \ldots, \check{\mathbf{x}}_K$ to the base station which acts as a central receiver. The transmitted signals are received at the base station receiver through reciprocal physical channels characterized by "flipped" channel matrices [4] $\mathbf{H}_1^{\dagger}, \ldots, \mathbf{H}_K^{\dagger}$, and are corrupted by the N-dimensional additive Gaussian noise vector $\check{\mathbf{n}}$ with covariance matrix $\check{\mathbf{W}}$. Similar to the downlink scenario, we assume that the channel matrices are known at their corresponding transmitters and at the receiver.

This scenario is illustrated in Figure 2 and is described mathematically by a Gaussian vector MAC model [8] for which the corresponding N-dimensional received signal vector at the base station receiver is expressed as

$$\check{\mathbf{r}} = \sum_{k=1}^{K} \mathbf{H}_k^{\dagger} \check{\mathbf{x}}_k + \check{\mathbf{n}} \tag{7}$$

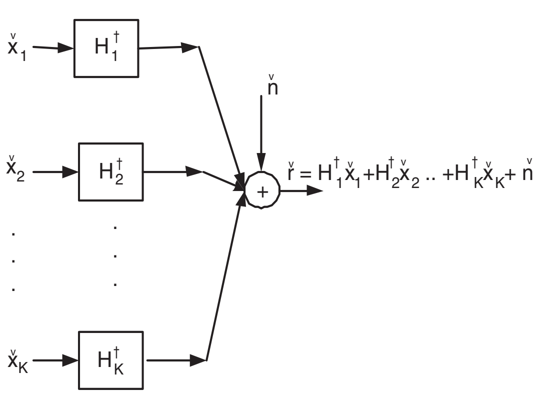

Fig. 2. Block Diagram for Uplink Scenario.

In this case each transmitter is subject to an individual power constraint expressed in terms of the transmit covariance matrices $\check{\mathbf{X}}_k = E[\check{\mathbf{x}}_k \check{\mathbf{x}}_k^{\dagger}]$ as

$$\text{Trace}\left[\check{\mathbf{X}}_k\right] \leq P_k, \qquad k = 1, \ldots, K \tag{8}$$

In order to study the duality between the downlink scenario presented in the previous section and the uplink scenario we note that the decoding of user information in the uplink must be performed in the opposite order of the downlink user encoding [4]. That is, if downlink encoding order is such that user 1 is encoded first, followed by user 2 second, user 3 third, and so on up to user K last, in uplink one must start by decoding information for the last user K first followed by user $K-1$ second, user $K-2$ third, and so on, up to user 1 last. The main reason for encoding/decoding in reverse order is due to the fact that the encoding order determines the amount of interference that a given user's signal is aware of in the downlink transmission, while the decoding order determines the amount of interference a given user's signal is aware of in the uplink reception. That is, in the downlink the signal encoded first has no interference knowledge, the signal encoded second knows the interference only from the signal encoded first, and so on. In the dual uplink scenario, the user decoded last has full interference knowledge as all the other users' information has already been decoded, then the user decoded before the last one has knowledge of all the other users' signals except that of the last user whose information has not been decoded yet, and so on.

The reverse uplink decoding order $K, K-1, \ldots, 2, 1$, implies that when the signal from a given user k is decoded, the transmitter has knowledge of all the signals decoded before $\check{\mathbf{x}}_K, \check{\mathbf{x}}_{K-1}, \ldots, \check{\mathbf{x}}_{k+1}$. The uplink rate of user k is the rate corresponding to user k in the vector MAC model with the specified decoding order of users and is given by [4]

$$\check{R}_k = \frac{1}{2} \log \frac{\det\left(\sum_{i=1}^{k} \mathbf{H}_i^{\dagger} \check{\mathbf{X}}_i \mathbf{H}_i + \check{\mathbf{W}}\right)}{\det\left(\sum_{i=1}^{k-1} \mathbf{H}_i^{\dagger} \check{\mathbf{X}}_i \mathbf{H}_i + \check{\mathbf{W}}\right)} \tag{9}$$

for $k = K, K-1, \ldots, 2$ and

$$\check{R}_1 = \frac{1}{2} \log \frac{\det\left(\mathbf{H}_1^{\dagger} \check{\mathbf{X}}_1 \mathbf{H}_1 + \check{\mathbf{W}}\right)}{\det \check{\mathbf{W}}} \tag{10}$$

3. DUALITY TRANSFORMATIONS FOR SYMMETRIC RATES

Our goal is to study the uplink-downlink duality such that given a set of uplink transmit covariance matrices $\check{\mathbf{X}}_1, \ldots, \check{\mathbf{X}}_K$, we obtain the corresponding set of downlink transmit covariances $\hat{\mathbf{X}}_1, \ldots, \hat{\mathbf{X}}_K$, and viceversa, that imply the same rates in uplink and downlink, that is $\hat{R}_k = \check{R}_k$, $k = 1, \ldots, K$, under equal power constraints

$$\sum_{k=1}^{K} P_k = P \tag{11}$$

By defining the correlation of the interference+noise corresponding to a given user k in the downlink scenario similar to [4]

$$\mathbf{A}_k = \begin{cases} \hat{\mathbf{W}}_k + \mathbf{H}_k \left(\sum_{i=k+1}^{K} \hat{\mathbf{X}}_i\right) \mathbf{H}_k^{\dagger} & k = 1, 2, \ldots, K-1 \\ \hat{\mathbf{W}}_K & k = K \end{cases} \tag{12}$$

we can rewrite the downlink rate expressions (5) and (6) compactly as

$$\hat{R}_k = \frac{1}{2}\log\det\left(\mathbf{I} + \mathbf{A}_k^{-1/2}\mathbf{H}_k\hat{\mathbf{X}}_k\mathbf{H}_k^\dagger\mathbf{A}_k^{-1/2}\right) \quad \forall k. \qquad (13)$$

We note that when the noise at all receivers in the downlink scenario is white, that is $\hat{\mathbf{W}}_k = \mathbf{I}$, $\forall k$, the interference+noise correlation matrix $\hat{\mathbf{A}}_k$ and has the same expression as in [4].

For the uplink scenario we define the correlation matrix of the interference+noise corresponding to a given user k also similar to [4]

$$\mathbf{B}_k = \begin{cases} \check{\mathbf{W}} & k=1 \\ \check{\mathbf{W}} + \sum_{i=1}^{k-1}\mathbf{H}_i^\dagger\check{\mathbf{X}}_i\mathbf{H}_i & k=2,\ldots,K-1,K \end{cases} \qquad (14)$$

and rewrite the uplink rate expressions (9) and (10) compactly as

$$\check{R}_k = \frac{1}{2}\log\det\left(\mathbf{I} + \mathbf{B}_k^{-1/2}\mathbf{H}_k^\dagger\check{\mathbf{X}}_k\mathbf{H}_k\mathbf{B}_k^{-1/2}\right) \quad \forall k. \qquad (15)$$

The equivalent rate expressions for downlink (13) and uplink (15) allow the use of the result in Appendix A of [4] to obtain the duality transformations for symmetric rates which should satisfy the condition

$$\hat{R}_k = \check{R}_k, \quad k=1,\ldots,K \qquad (16)$$

Using the downlink rate expression (13) in which we introduce the extra factor $\mathbf{B}_k^{-1/2}\mathbf{B}_k^{1/2} = \mathbf{B}_k^{1/2}\mathbf{B}_k^{-1/2} = \mathbf{I}$ to the left and to the right of $\hat{\mathbf{X}}_k$ along with the uplink rate expression (15) in which we introduce the extra factor $\mathbf{A}_k^{-1/2}\mathbf{A}_k^{1/2} = \mathbf{A}_k^{1/2}\mathbf{A}_k^{-1/2} = \mathbf{I}$ to the left and to the right of $\check{\mathbf{X}}_k$ we rewrite (16) as

$$\det\left(\mathbf{I} + \mathbf{A}_k^{-1/2}\mathbf{H}_k\mathbf{B}_k^{-1/2}\mathbf{B}_k^{1/2}\hat{\mathbf{X}}_k\mathbf{B}_k^{1/2}\mathbf{B}_k^{-1/2}\mathbf{H}_k^\dagger\mathbf{A}_k^{-1/2}\right) =$$

$$\det\left(\mathbf{I} + \mathbf{B}_k^{-1/2}\mathbf{H}_k^\dagger\mathbf{A}_k^{-1/2}\mathbf{A}_k^{1/2}\check{\mathbf{X}}_k\mathbf{A}_k^{1/2}\mathbf{A}_k^{-1/2}\mathbf{H}_k\mathbf{B}_k^{-1/2}\right) \qquad (17)$$

Similar to [4] we consider the term $\mathbf{A}_k^{-1/2}\mathbf{H}_k\mathbf{B}_k^{-1/2}$ as user k effective channel with singular value decomposition (SVD) [9]

$$\mathbf{A}_k^{-1/2}\mathbf{H}_k\mathbf{B}_k^{-1/2} = \mathbf{U}_k\mathbf{D}_k\mathbf{V}_k^\dagger \qquad (18)$$

in order to apply the result in Appendix A of [4].

3.1. Downlink-to-Uplink Duality Transformation

In this case the transmit covariance matrices $\hat{\mathbf{X}}_1,\ldots,\hat{\mathbf{X}}_K$ for the downlink scenario are given along with the channel matrices $\mathbf{H}_1,\ldots,\mathbf{H}_K$ and we need to determine the corresponding uplink transmit covariance matrices $\check{\mathbf{X}}_1,\ldots,\check{\mathbf{X}}_K$ such that (17) is satisfied, and we get [4, Appendix A]

$$\check{\mathbf{X}}_k = \mathbf{A}_k^{-1/2}\mathbf{U}_k\mathbf{V}_k^\dagger\mathbf{B}_k^{1/2}\hat{\mathbf{X}}_k\mathbf{B}_k^{1/2}\mathbf{V}_k\mathbf{U}_k^\dagger\mathbf{A}_k^{-1/2} \qquad (19)$$

We note that the duality result in [4, Appendix A] guarantees that

$$\text{Trace}\left[\check{\mathbf{X}}_k\right] \le \text{Trace}\left[\hat{\mathbf{X}}_k\right] \qquad (20)$$

which implies that as long as the downlink transmit covariance matrices satisfy that Trace $\left[\hat{\mathbf{X}}_k\right] \le P_k$, $\forall k$, the uplink power constraints (8) will be satisfied.

We also note that the presence of the term $\mathbf{B}_k$ in equation (19) requires knowledge of the transmit covariance matrices $\check{\mathbf{X}}_1,\ldots,\check{\mathbf{X}}_{k-1}$ in order to calculate $\check{\mathbf{X}}_k$. Thus, the order in which the uplink transmit covariance matrices are calculated using equation (19) is starting from user 1 for which $\mathbf{B}_1 = \check{\mathbf{W}}$ in the order in which the users are encoded in the downlink.

3.2. Uplink-to-Downlink Duality Transformation

In this case the transmit covariance matrices $\check{\mathbf{X}}_1,\ldots,\check{\mathbf{X}}_K$ for the uplink scenario are given along with the channel matrices $\mathbf{H}_1,\ldots,\mathbf{H}_K$ and we need to determine the corresponding downlink transmit covariance matrices $\hat{\mathbf{X}}_1,\ldots,\hat{\mathbf{X}}_K$ such that (17) is satisfied, and using the same result [4, Appendix A] we get

$$\hat{\mathbf{X}}_k = \mathbf{B}_k^{-1/2}\mathbf{U}_k\mathbf{V}_k^\dagger\mathbf{A}_k^{1/2}\check{\mathbf{X}}_k\mathbf{A}_k^{1/2}\mathbf{V}_k\mathbf{U}_k^\dagger\mathbf{B}_k^{-1/2} \qquad (21)$$

Since the duality result in [4, Appendix A] implies also that

$$\text{Trace}\left[\hat{\mathbf{X}}_k\right] \le \text{Trace}\left[\check{\mathbf{X}}_k\right] \qquad (22)$$

and the uplink transmit covariances are such that Trace $\left[\check{\mathbf{X}}_k\right] \le P_k$, $\forall k$, this along with (11) implies that the downlink power constraint (4) is also be satisfied.

The order in which the downlink transmit covariance matrices are calculated is the order in which users are decoded in the uplink as the presence of the term $\mathbf{A}_k$ in equation (21) requires knowledge of the transmit covariance matrices $\hat{\mathbf{X}}_K,\ldots,\hat{\mathbf{X}}_{k+1}$ in order to calculate $\hat{\mathbf{X}}_k$. Thus, in this case we start by computing $\hat{\mathbf{X}}_K$ first for which $\mathbf{A}_K = \hat{\mathbf{W}}_k$, then continue with $\hat{\mathbf{X}}_{K-1}$, and so on, ending with $\hat{\mathbf{X}}_1$.

4. APPLICATION: DOWNLINK CDMA CODEWORDS

As application of the uplink-downlink duality transformations presented in the previous section, we discuss in this section how these can be used in the context of CDMA systems to obtain a downlink codeword ensemble starting from an uplink one. We note that while many algorithms are available for obtaining uplink CDMA codewords (see [10–13] and references therein) only a few algorithms have been proposed for downlink scenarios [14–16]. We also note that the uplink CDMA codeword ensembles can be designed to optimize various performance measures such as individual or sum rates and Signal-to-Interference+Noise-Ratios (SINR). Thus, if specific rates are needed in a downlink CDMA scenario, one could design an ensemble of uplink CDMA codewords that achieves the specified rates and the apply the duality transformations for symmetric rates as outlined in the previous section to obtain a corresponding ensemble of downlink CDMA codewords. We discuss the procedure in this section and illustrate it with numerical examples.

We start with an optimal uplink CDMA codeword ensemble $\check{\mathbf{S}} = [\check{\mathbf{s}}_1 \ldots \check{\mathbf{s}}_K]$ and power matrix $\check{\mathbf{P}} = \text{diag}\{\check{p}_1,\ldots,\check{p}_K\}$ which maximizes the sum capacity of the corresponding vector MAC. For general scenarios with non-ideal channels and colored noise, these may be obtained by applying the interference avoidance algorithms discussed in [11–13], and the corresponding uplink transmit covariance for a given user in this case is given by

$$\check{\mathbf{X}}_k = \check{p}_k\check{\mathbf{s}}_k\check{\mathbf{s}}_k^\dagger \qquad (23)$$

Using the duality transformation given in equation (21) we first calculate the downlink transmit covariance matrix of user k, $\hat{\mathbf{X}}_k$, that corresponds to the uplink transmit covariance (23 for which the uplink and downlink rates of user k are the same. We note that since $\check{\mathbf{X}}_k$ is a rank one matrix the resulting $\hat{\mathbf{X}}_k$ matrix will also be rank one. This implies that it has only one singular value that is different from zero, and by selecting the downlink user power $\hat{p}_k$ to be equal to the non-zero singular value of $\hat{\mathbf{X}}_k$ and the downlink user codeword $\hat{\mathbf{s}}_k$ to be equal to the singular vector of $\hat{\mathbf{X}}_k$ that corresponds to the non-zero singular value $\hat{p}_k$ we have

$$\hat{\mathbf{X}}_k = \hat{p}_k \hat{\mathbf{s}}_k \hat{\mathbf{s}}_k^\dagger, \qquad k = 1, \ldots, K. \tag{24}$$

Putting together the resulting vectors and singular values we obtain the corresponding downlink codeword and power matrices as $\hat{\mathbf{S}} = [\hat{\mathbf{s}}_1 \ldots \hat{\mathbf{s}}_K]$, respectively $\hat{\mathbf{P}} = \text{diag}[\hat{p}_1, \ldots, \hat{p}_K]$.

We illustrate the above procedure with numerical examples obtained for a system with $K = 4$ users operating in a signal space with $N = 3$ dimensions. The uplink power matrix is $\check{\mathbf{P}} = \mathbf{I}_4$ and in the first example we consider ideal channel matrices $\mathbf{H} = \mathbf{I}_4$ for all users and the same colored noise $\mathbf{W} = \text{diag}\{0.9501, 0.2311, 0.6068\}$ at all the receivers. An optimal uplink codeword matrix (obtained using the interference avoidance algorithm [13]) is:

$$\check{\mathbf{S}}_1 = \begin{bmatrix} 0.1200 & -0.1172 & -0.9494 & 0.2230 \\ 0.9030 & -0.9131 & 0.2112 & -0.0668 \\ 0.4125 & 0.3905 & 0.2324 & 0.9725 \end{bmatrix}$$

Using the uplink-downlink duality transformation followed by the SVD factorization of resulting downlink transmit covariance matrices we obtain the downlink codeword and power matrices

$$\hat{\mathbf{S}}_1 = \begin{bmatrix} -0.0905 & -0.0898 & -0.9750 & 0.2654 \\ 0.9344 & -06144 & 0.1210 & -0.0445 \\ 0.3445 & 0.7839 & 0.1866 & -0.9631 \end{bmatrix}$$

$$\hat{\mathbf{P}}_1 = \text{diag}[1.6193, 0.7660, 0.9597, 0.6678]$$

for which the corresponding downlink and uplink rates are equal $\hat{R} = \check{R} = [0.7867, 0.4763, 0.3652, 0.3652]$.

In a second example we consider non-ideal channels and different colored noise at each receiver. The optimal uplink codeword matrix (obtained also using the interference avoidance algorithm [13]) is

$$\check{\mathbf{S}}_2 = \begin{bmatrix} 0.0115 & -0.0022 & -0.0056 & -0.1307 \\ 0.0366 & -0.4205 & 0.0273 & -0.1180 \\ 0.9993 & 0.9073 & 0.9996 & 0.9844 \end{bmatrix}$$

for which the corresponding downlink codeword and power matrices are

$$\hat{\mathbf{S}}_2 = \begin{bmatrix} -0.8949 & -0.9827 & -0.8732 & -0.6901 \\ 0.1172 & 0.1423 & 0.4386 & -0.0272 \\ 0.4305 & 0.1183 & 0.2124 & 0.7232 \end{bmatrix}$$

$$\hat{\mathbf{P}}_2 = \text{diag}[1.0213, 0.4701, 0.6466, 0.7815]$$

for which we note again that the rates for downlink and uplink are identical $\hat{R} = \check{R} = [0.3498, 0.0258, 0.0700, 0.1480]$.

5. CONCLUSION

In this paper we studied the uplink-downlink duality for Gaussian vector channels with different noise statistics at different receivers in the uplink and downlink scenarios. We have extended the approach in [4] and derived expressions for the duality transformations that allow us to determine transmit covariance matrices in the downlink given a set of uplink transmit covariances. We have applied the proposed approach to determine optimal donwlink CDMA codewords.

6. REFERENCES

[1] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, Wiley-Interscience, Hoboken, NJ, 2006.

[2] P. Viswanath and D. N. C. Tse, "Sum Capacity of the Vector Gaussian Broadcast Channel and Uplink-Downlink Duality," *IEEE Transactions on Information Theory*, vol. 49, no. 8, pp. 1912–1921, August 2003.

[3] N. Jindal, S. Vishwanath, and A. Goldsmith, "On the Duality of Gaussian Multiple-Access and Broadcast Channels," *IEEE Transactions on Information Theory*, vol. 50, no. 5, pp. 768–783, May 2004.

[4] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, Achievable Rates, and Sum-Rate Capacity of Gaussian MIMO Broadcast Channels," *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2658–2668, October 2003.

[5] W. Yu, "Uplink-Downlink Duality via Minimax Duality," *IEEE Transactions on Information Theory*, vol. 52, no. 2, pp. 361–374, February 2006.

[6] W. Yu and J. M. Cioffi, "Sum-Capacity of Gaussian Vector Broadcast Channels," *IEEE Transactions on Information Theory*, vol. 50, no. 9, pp. 1875–1892, September 2004.

[7] M. H. M. Costa, "Writing on Dirty Paper," *IEEE Transactions on Information Theory*, vol. IT-29, no. 13, pp. 439–441, May 1983.

[8] W. Yu, W. Rhee, S. Boyd, and J. M. Cioffi, "Iterative Water-Filling for Gaussian Vector Multiple-Access Channels," *IEEE Transactions on Information Theory*, vol. 50, no. 1, pp. 145–152, January 2004.

[9] G. Strang, *Linear Algebra and Its Applications*, Harcourt Brace Jovanovich College Publishers, San Diego, CA, third edition, 1988.

[10] P. Viswanath, V. Anantharam, and D. Tse, "Optimal Sequences, Power Control and Capacity of Spread Spectrum Systems with Multiuser Linear Receivers," *IEEE Transactions on Information Theory*, vol. 45, no. 6, pp. 1968–1983, September 1999.

[11] G. S. Rajappan and M. L. Honig, "Signature Sequence Adaptation for DS-CDMA with Multipath," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 2, pp. 384–395, February 2002.

[12] D. C. Popescu and C. Rose, "Codeword Optimzation for Uplink CDMA Dispersive Channels," *IEEE Transactions on Wireless Communications*, vol. 4, no. 4, pp. 1563–1574, July 2005.

[13] D. C. Popescu, O. Popescu, and C. Rose, "Interference Avoidance and Multiaccess Vector Channels," *IEEE Transactions on Communications*, vol. 55, no. 8, pp. 1466–1471, August 2007.

[14] H. Bi and M. L. Honig, "Power and Signature Optimization for Downlink CDMA," in *Proceedings 2002 IEEE International Conference on Communications – ICC'02*, New York, NY, May 2002, vol. 3, pp. 1758–1762.

[15] D. C. Popescu, "Interference Avoidance for Downlink CDMA Systems," in *Proceedings* 64^{th} *IEEE Vehicular Technology Conference – VTC 2006 Fall*, Montreal, Canada, September 2006, vol. 4, pp. 1563–1566.

[16] D. C. Popescu, S. Abraham, and O. Popescu, "Downlink Transmitter Adaptation Based on Greedy SINR Maximization," in *Proceedings* 22^{nd} *IEEE Canadian Conference on Electrical and Computer Engineering – CCECE 2009*, St. John's, Newfoundland, Canada, May 2009, pp. 94–98.