Comparison of Wavelet Transform and Fourier Transform based methods of Phasor Estimation for Numerical Relaying

V.S.Kale S.R.Bhide P.P.Bedekar
Department of Electrical Engineering, VNIT
Nagpur, India

Abstract—**Estimation of fundamental frequency component of current and / or voltage signals is essential for relaying decision in most of the numerical algorithms used for the protection of power system components. Modern numerical relays generally use discrete Fourier transform for extracting the fundamental component from the post fault signals consisting of fundamental as well as non-fundamental frequency signals. With the advent of wavelet analysis, many researchers used wavelet analysis for the phasor estimation with the claims of better performance. The paper compares the performance of discrete wavelet transform method with discrete Fourier transform method for phasor estimation, considering the effect of decaying dc offset. MATLAB software was used for programming and simulation.**

Keywords-Decaying dc offset, fundamental frequency component, phasor estimation, numerical relaying, wavelet transform

I. INTRODUCTION

Numerical relays are preferred over their electromechanical and static counterparts for economic as well as technical reasons. The process of numerical relaying consists of several steps. Currents and voltages from instrument transformers are band-limited using low pass anti-aliasing analog filters. The analog signals are then discritized and quantized to get digital signals. The acquired information is then processed by the relay algorithm. The algorithm uses signal processing techniques to estimate the magnitudes and angles of the corresponding current and voltage phasors. These estimates are used to calculate other quantities, such as impedances. The computed quantities are compared with pre-specified thresholds to decide whether the power system is experiencing a fault condition or not. If it is, relay sends a command to open one or more circuit breakers to isolate the faulted section of power system [1].

In phasor estimation process, the desired frequency component of the signal received is converted to a representative phasor. This process is called estimation because the true value of the desired component is not known upfront. All the methods which attempt to estimate the parameters of the signal are based on some assumptions and accuracy of phasor estimated depends on method used [2].

One of the important areas of application of wavelet transform is the measurements of various power system signals [3]. An algorithm for phasor calculation based on wavelet analysis was proposed in [4] and effect of frequency deviation on the accuracy of the method was considered. The digital distance protection schemes for the transmission line distance protection based on wavelet transform were presented in [5] and [6]. However, the complete removal of decaying dc offset from the signal by some means was required. It may be noted that the complete removal of dc offset is still an open issue as can be seen from the recent publication [7]. The method of calculating effective values of voltage and current using wavelet transform was illustrated in [8], but the method computed only rms values and not phase angle and therefore, has limited application. Hilbert transform and wavelet packet transform was used to extract the rms value and phase angle of fundamental harmonic in [9]. The equation based and experimental data was used for validation but the effect of dc offset was not considered.

This paper attempts to compare the performances of DFT and DWT methods of phasor calculations. Fault waveforms were generated from the equations as well as simulated using MATLAB/ Simulink software package.

II. REVIEW OF ALGORITHMS

A. DFT Algorithm

The most commonly used method of calculating phasors from sampled data is that of Discrete Fourier Transform (DFT). It is performed by the implementation of following equation:

$$X_k = \frac{\sqrt{2}}{N}\sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi kn}{N}} \tag{1}$$

In this equation, 'X' is the phasor of a signal, 'k' is the order of harmonic, 'x' is the instantaneous value of sinusoidal signal such as voltage and current, 'n' is the n^{th} sample of data window and 'N' is the number of samples in data window.

Fast Fourier Transform (FFT) is a numerical technique to make the calculation of DFT faster. It is orders of magnitude faster than the calculations implied by (1), if all the X_k are desired and if N is large. In relaying applications, however, N is generally small (from 4 to 20 for most algorithms) and only a few of the F_k are required. Hence, FFT has found little application in digital relaying.

Proc. of the International Conference on Science and Engineering (ICSE 2011)
Copyright © 2011 RG Education Society
ISBN: 978-981-08-7931-0

Recursive implementation of DFT is a more computationally efficient method which computes the estimated phasor recursively by adding the contribution made by the new sample, and subtracting the contribution made by the oldest sample. Phasor Measurement Units, which are being deployed on power systems as a measurement tool in many countries, use recursive DFT algorithm. This paper has used this implementation.

B. DWT based Algorithm

The algorithm [4] is briefly described here. The magnitude and phase of a signal at fundamental frequency can be computed by constructing a unity amplitude sinusoidal 50 Hz reference signal (R1). For each window of samples, reference signal and measured signal are decomposed into two levels using suitable mother wavelet and their approximation coefficient vectors of the second level are used for phasor estimation. The reason for this can be explained using Fig. 1, which shows the filter bank model for a discrete signal sampled at 1 kHz. Since the phasor is a component at fundamental frequency and fundamental frequency lies in the middle of approximation vector at level 2, these coefficients .

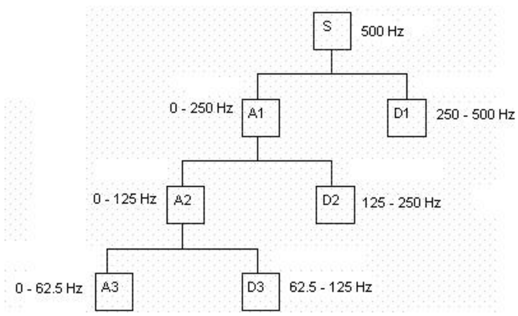


Fig.1 Filter bank model

Using vector mathematics, angle between reference and measured signal is given by:

$$\theta = \cos^{-1} \frac{(A_{2R1} \bullet A_{2S})}{\|A_{2R1}\| \|A_{2S}\|} \quad (2)$$

where, A_{2R1} is the approximation coefficient vector at second level of decomposition of reference signal and A_{2S} is approximation coefficient vector at second level of decomposition of measured signal. Numerator is the dot product of two vectors while $\|A_{2R1}\|$ and $\|A_{2S}\|$ are the norms of the corresponding vectors.

Now a new unity magnitude sinusoidal reference signal (R2) with phase shift equal to calculated angle θ is constructed. The new signal is decomposed as before. Then the magnitude M of measured signal is given by:

$$M = \frac{\|A_{2S}\|}{\|A_{2R2}\|} \quad (3)$$

C. Selection of mother wavelet

Performance of signal extraction depends on the mother wavelet used. To select the appropriate mother wavelet, a typical fault waveform was analyzed with different wavelets. The results of phasor estimation using commonly used wavelet functions viz. Daubechies 4 (db 4), Daubechies 8 (db 8), Biorthogonal 1.1 (bior 1.1) and Symlet 5 (sym 5) are shown in Fig. 2. As can be seen from the fig. 2, all the waveforms reach the final value at the same time but waveforms obtained by using db4, db8, sym5 wavelets oscillates about the final value for a longer period. Therefore, bior 1.1 was selected as mother wavelet.

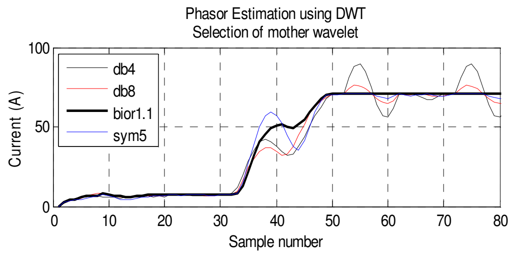


Fig. 2. Selection of mother wavelet

III. MODIFICATIONS IN IMPLEMENTATION OF ALGORITHMS

Considering that the phasor estimation is a continuous process, it is necessary to consider algorithms which will update the phasor estimate as newer data samples are acquired. For example, in non-recursive DFT algorithm, unlike recursive DFT algorithm, all the calculations are performed fresh for each window without using any data from the earlier estimates. It may be noted that recursive DFT is only computational efficient. It still requires full cycle window for phasor estimation. However, speed in estimation might be achieved, if, all the non-fault samples of window were removed on detection of fault and a new window of only fault samples was considered for phasor estimation. With this in view, both DFT and DWT algorithms were modified in their implementations for updating phasor estimates and results were compared. The faults were detected using DWT as given below.

A. Detection of fault

Measured current signals were decomposed using db1 wavelet. This level contains the high frequencies that are associated with faults. Norm is a measure of amount of energy content of a signal, which increases with fault. By calculating the norms of detail coefficients at level 1 for all the current signals, fault was detected. This detection was used to adjust the samples in the data window. To illustrate how a fault can be detected using DWT, a single line to ground fault AG on 400 KV line was simulated using MATLAB. The norm values of detail coefficients at level 1 for three phase currents are shown in Fig. 3. It can be seen that norm for AG fault, 'A' phase current is much higher than the corresponding values for the other two phases.

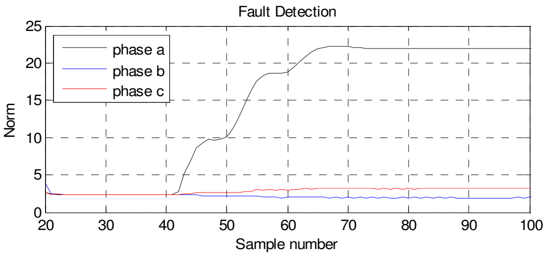


Fig. 3. Norm of detail coefficients before and after AG fault

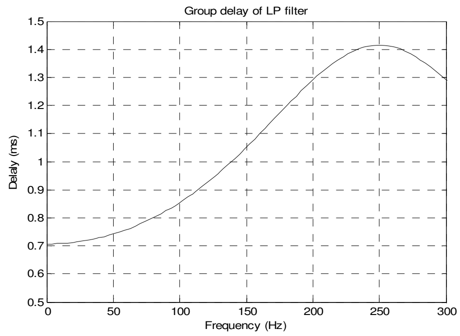


Fig. 4. Group delay of low pass second order Butterworth filter

IV. SOURCES OF ERRORS IN PHASOR ESTIMATION

A. *DC offset*

The major component of a power system signal is its fundamental frequency component. However, post fault current and voltage signals fail to be pure fundamental frequency sinusoids for a variety of reasons. The most predictable non-fundamental term is the decaying exponential, which can be present in the current signal. For the series RL model of the line, under some assumptions, the instantaneous current for a fault at a time t is given by:

$$i(t)=\frac{V_m}{|Z|}\sin(wt+\alpha-\theta)+\frac{V_m}{|Z|}\sin(\theta-\alpha)e^{-(R/L)t} \quad (4)$$

where, parameter α controls the instant on the voltage wave when short circuit occurs, and θ is the impedance angle of transmission line. The second term in (4) decays exponential with the time constant of the line and is the main cause of transient overreach in high speed relays. Some algorithms require that the offset be removed prior to processing while some algorithms do not require its elimination. Traditional method of reducing the dc component in current signal is with the use of mimic impedance which can be implemented as r+j x burden on CT secondary or can be implemented in software.

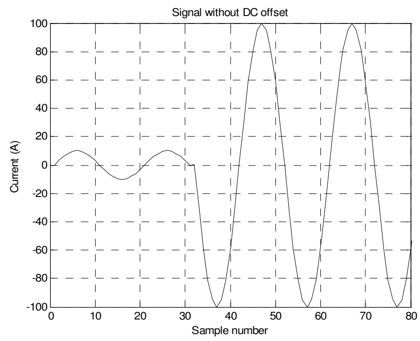
B. *High frequency noise*

The contributors to the high frequency noise or harmonics include (i) transient response of capacitive voltage transformer to the abrupt change in voltage (ii) non linear behavior of the fault arc (iii) reflections of fault generated traveling waves between bus and fault. The anti-aliasing low pass filter which is a part of numerical relay will remove most of these error signals. It will add a small delay to the overall operating time of the relay. However, it can be seen from the fig.4 that phase lag at the fundamental frequency of 50 Hz is about 11^{o}, which corresponds to the phase delay of 0.75 ms. Considering that the low pass filter has been designed for a sampling frequency of 1 KHz, the phase delay produced by it is less than the sampling period of 1 ms.

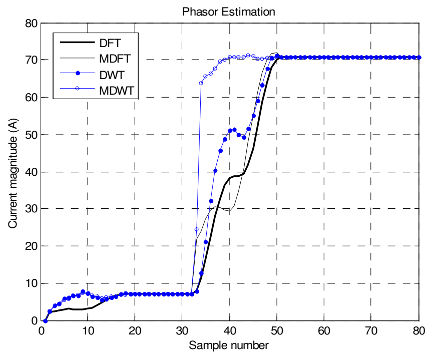
V. RESULTS AND ANALYSIS

A. *Results of Phasor Extraction for equation based signals*

The results of phasor extraction by applying algorithms with and without the modifications as mentioned in section 2 and using equation based signals are discussed below. A typical fault current waveform, assuming no harmonics, can be modeled using equation (4), which is an expression of current in time domain when ac voltage is imposed on a R-L circuit representing transmission line. Taking α =θ, the waveform was devoid of dc offset as shown in fig.5. The waveform was sampled at 1 KHz and its phasor estimation was obtained using four algorithms viz. (i) DFT, (ii) modified DFT (MDFT), (iii)DWT and (iv) modified DWT (MDWT). As explained in section III, modification was done in their implementation to update the phasor estimation when fault occurs. From the fig. 5, it can be seen that DWT output rises faster than DFT and MDFT, but output of remaining three methods stabilizes at the same time, exactly after one cycle. However, MDWT reaches the final value in less than half cycle and its performance looked promising.



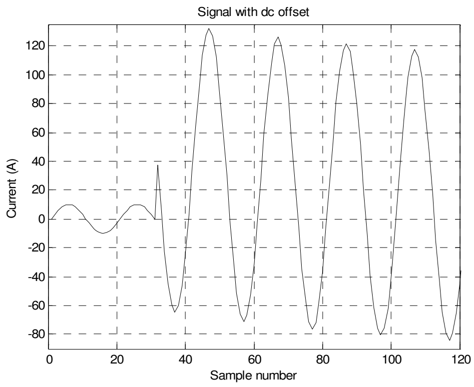
(a)



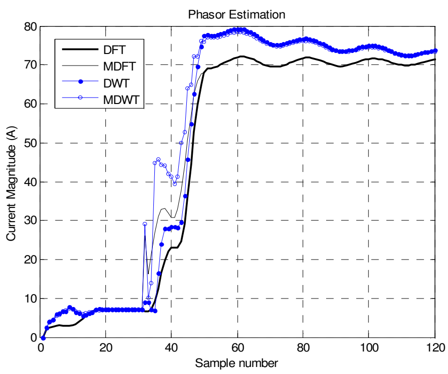
(b)

Fig. 5. (a) Fault current without dc offset (b) its phasor estimation

Now consider a second fault signal with dc offset with fault occurring at 30 ms, and the results of phasor estimation using all the four algorithms as shown in Fig. 6. It can be observed that all the algorithms performed poorly. Both DFT and MDFT remain oscillatory over a narrow band around the true phasor magnitude, while DWT and MDWT overshoot and produced large errors in addition to being oscillatory. There was hardly any difference between the performances of DFT and MDFT in both the cases described above.



(a)



(b)

Fig. 6. (a) Fault current with dc offset (b) its phasor estimation

The above analysis was based on signals generated from equation and these signals do not represent the fault signals in terms of decaying dc offset, transients and harmonics. Therefore, the results of simulated waveform based on the double end fed power system model are discussed next.

B. R esults of Phasor Extraction fo rsimulated signals

The results of phasor extraction by applying DFT and MDWT algorithms as mentioned in section 2 and using MATLAB/Simulink based signals are discussed below.A double end fed 400 KV power system model consisting of 100 Km transmission line section was selected for fault simulations. The transmission line was simulated using distributed parameter model with parameters as follows.

$Z_0 = 0.151 + j\ 1.1655\ \Omega$ / km and $C_0 = 0.0009\ \mu F/km$
$Z_1 = 0.0134 + j\ 0.3144\ \Omega$ / km and $C_1 = 0.0017\ \mu F/km$

Phasor for the fault current in phase ‘a’ for AG fault was estimated using DFT and MDWT methods. As seen from Fig. 7, for Fault Inception Angle (FIA) of 0 deg., MDWT output rises faster to final value but overshoots to a greater extent as compared to output of DFT method due the effect of dc offset. Both the methods take longer time (more than a cycle) to settle. In Fig. 8, fault current had no dc offset and both methods stabilize at the same time with DWT output rising faster. The graph of error in the estimation as a function of fault inception angle for both the methods was plotted in Fig. 9 for AG fault. Thus, the method based on wavelet analysis does not seem have any advantage over traditional Fourier based method.

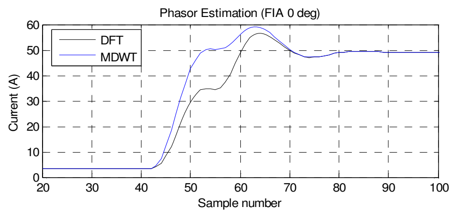


Fig. 7 Phasor estimation (FIA 0 deg.)

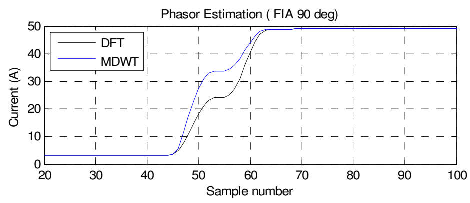


Fig. 8 Phasor estimation (FIA 90 deg.)

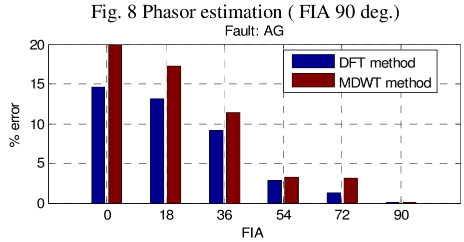


Fig. 9. Test results for AG fault

VI. CONCLUSIONS

The protective relays based on principles such as over-current protection, differential protection, distance protection and directional protection require that the phasors of current and/or voltage signals be extracted. DC offset which may be present in the post fault current signal can adversely affect this estimation.

It was found that DWT method performed better in absence of dc offset, in terms of speed of measurement. With the existence of exponentially decaying DC offset, the amplitude of the fundamental frequency component calculated by both DFT and DWT methods, deviated from the expected value. However, DWT method performed more badly in presence of dc offset and harmonics. It can not be assumed that post fault signals will be devoid of non fundamental frequency components and the error in estimation would subsequently reduce the accuracy of protective relays. Therefore, though wavelet analysis has many other useful applications in power system protection, it has limited role in phasor estimation.

REFERENCES

[1] Paithankar, Y.G., Bhide, S.R., Fundamentals of power system protection, Prentice-Hall of India, 2004.
[2] Phadke, A.G., Thorp, J.S., Computer relaying for power systems, Second Edition, John Wiley, 2009.
[3] Kim C.H., Aggarwal, R., "Wavelet transforms in power systems," IET Power Engineering Journal, vol. 15, pp. 193-200, Aug. 2001.
[4] Chi-kong Wong, Ieng-tak Leong, Chu-san Lei, Jing-tao Wu, Ying-duo Ham, A novel algorithm for phasor calculation based on wavelet analysis, IEEE Power Engineering Society Summer Meeting, 2001, vol. 3, pp. 1500-1503.
[5] Osman A.H., Malik O.P., Transmission line distance protection based on wavelet transform, IEEE Trans. on Power Delivery, vol. 19, no. 2, April 2004, pp.515-523.
[6] Liang F., Jeyasurya B., Transmission line distance protection using wavelet transform algorithm, IEEE Trans. on Power Delivery, vol. 19, no. 2, April 2004, pp. 545 -553.
[7] Sang-Hee Kang, , Dong-Gyu Lee, , Soon-Ryul Nam, Peter A. Crossley, and Yong-Cheol Kang, 'Fourier Transform-Based Modified Phasor Estimation Method Immune to the Effect of the DC Offsets', IEEE Trans. on Power Delivery, vol. 24, no. 3, July 2009, pp. 1104-1111.
[8] Zhu T.X., Effective value calculation in wavelet domain, IEEE Trans. on Power Deliver, vol. 19, no. 1, Jan. 2004, pp. 400-4004.
[9] Hamid E.Y., Kawasaki Z.I., R. Mardiana, Wavelet packet transform for rms and power measurements, IEEE Power Engineering Society Summer Meeting, vol.2, 2001, pp. 1243-1245.
[10] MATLAB 7.1 User's Guides for SimPowerSystem, Wavelet Toolbox.