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# Three-Body Decays: Structure, Decay Mechanism and Fragment Properties 

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#### Abstract

We discuss the three-body decay mechanisms of many-body resonances. $R$-matrix sequential description is compared with full Faddeev computation. The role of the angular momentum and boson symmetries is also studied. As an illustration we show the computed $\alpha$-particle energy distribution after the decay of ${ }^{12} \mathrm{C}\left(1^{+}\right)$resonance at 12.7 MeV .


## 1 Introduction

The three-body decay of many-body resonances can be accurately measured in complete kinematics. Information about the decaying state and the decay mechanism is usually extracted from the measurement of the three fragments after the decay. Although this is a common practice, the situation is ambiguous. The experimental analyses of these processes are based on the $R$-matrix formalism which inherently assumes two successive two-body decays. The input are the properties of the intermediate two-body states and the population of the nuclear many-body initial state approximated as a three-body system.

Occasionally, in principle contrary descriptions are able to explain the observed distributions making the understanding of the underlying physics difficult. An example demonstrating the difficulties is the $3 \alpha$ decay of $1^{+}$state in ${ }^{12} \mathrm{C}$ which was successfully described by two opposite mechanisms: a sequential decay via the $2^{+}$ state in ${ }^{8} \mathrm{Be}$ [1], and a direct decay into the three-body continuum [2]. This requires an explanation. What information is contained in a full-kinematics measurement of three-body decay? Apparently unique information can only be extracted under favorable conditions. The crux of the matter is that the decay mechanism is related to a "decay path", an intermediate structure, which in contrast to the final state signal in the detector is not an observable. We shall in this contribution compare the results from three-body calculations and experimental $R$-matrix analyses.

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## 2 Energy Distributions

The large-distance observable structure of the many-body initial state is a three-body continuum state, therefore we compute the resonance structure in a three-body cluster model [3]. We use the complex scaled hyperspherical adiabatic expansion method to solve the Faddeev equations which describe the 3-body system. The appropriate coordinates are the so-called hyperspherical coordinates and consist of the hyperradius $\rho^{2}=4 \sum_{i=1}^{3}\left(\mathbf{r}_{i}-\mathbf{R}\right)^{2}$, and five hyperangles. In the adiabatic hyperspherical expansion method the angular part of the Faddeev equations is solved first and the angular eigenfunctions $\Phi_{n J M}$ are then used as a basis to expand the total wave function $\Psi^{J M}$.

We include short-range [4] and Coulomb potentials. The many-body effects that are present at short distances are assumed to be unimportant except for the resonance energy. This is taken into account by using a structureless 3-body interaction that fits the position of the resonance.

The resonance wave-function contains information about the decay mechanism, and the large-distance properties reflect directly the measurable fragment momentum distributions. The single particle probability distributions are obtained after integration of the absolute square of the wave function over the four hyperangles describing the directions of the momenta.

The many-body initial state resonance evolves into three clusters at large distances. The total angular momentum and parity $J^{\pi}$ is conserved in the process. This symmetry combined with Bose-statistics imposes constraints on the resulting momentum distributions. An early example of these effects applied to three pion decays can be found in [5]. Figure 1 shows the regions of the Dalitz plot where the density must vanish for the decay of a $1^{+}$state into three $\alpha$-particles. This gives rise to the minima in the single $\alpha$-particle energy distribution. The Dalitz plot computed within the Faddeev framework is also shown and is in agreement with these symmetry constraints.

Figure 2 shows the single- $\alpha$ energy distributions, i.e. the probability for emergence of one $\alpha$ particle with a given energy divided by its maximum allowed, for the $1^{+}$state of ${ }^{12} \mathrm{C}$ computed with $R$-matrix analysis $[1,6]$. This corresponds to the projection of the Dalitz plot in Fig. 1 on the $y$-axis. The decay is assumed to be sequential via ${ }^{8} \mathrm{Be}\left(2^{+}\right)$since angular momentum forbids the decay via ${ }^{8} \mathrm{Be}\left(0^{+}\right)$. We have varied the two-body energy and width. When both the two-body energy and width are small (left) a narrow peak corresponding to the emission of the first $\alpha$ arises. The other two $\alpha$ 's are related to the broad peaks. By increasing the twobody energy and width the three-peak distribution becomes rather pronounced and insensitive to the two-body parameters when either $E_{2 r} / E_{3 r}$ is larger than about 0.5 or the two-body width is large. The same figure contains the curves corresponding to the case where the boson symmetry is omitted. For a low and narrow two-body state the effect of this symmetry seems to be unimportant, but an increase on the width leads to a two-peak (not three-peak) distribution.

Figure 3 shows the results from the full three-body computation and the computation from the lowest continuum three-body wave function $(K=8)$ from [7,8] (democratic decay). This is the simplest assumption with the correct symmetries. We have varied the three-body energy and consequently the three-body width. Two rotation angles have been considered: one of them is large enough to accumulate the contribution of sequential decay through the ${ }^{8} \mathrm{Be}$ resonance in a single adiabatic potential, while the other is not. The calculations include the boson symmetry of the $\alpha$-particles. The results from the large rotation angle do not include the contribution from the decay via ${ }^{8} \mathrm{Be}\left(2^{+}\right)$and are very close to the democratic decay. In the result from the full Faddeev computation the three peaks are closer to each other and this approaches better the experiment. The fractions of population at large distance are given in Table 1 for the different values of $E_{3 r}$ shown in Fig. 3. We can observe that the sequential decay probability increases as we increase the three-body energy.


Fig. 1 Regions of the $3 \alpha$ Dalitz plot where the density must vanish (in black) for a $1^{+}$state (left). Dalitz plot for the $1^{+}$state of ${ }^{12}$ C. $x$-axis corresponds to $\left(E_{\alpha 1} / 2 E_{\alpha 2}\right) / \sqrt{(3)}$ and $y$-axis to $E_{\alpha 1}$ in MeV (Color online)




Fig. 2 Single $\alpha$ energy distributions from $R$-matrix analysis for the decay into three $\alpha$-particles of the ${ }^{12} \mathrm{C}\left(1^{+}\right)$resonance at 12.7 MeV of excitation energy. The energies and widths of the intermediate ${ }^{8} \mathrm{Be}\left(2^{+}\right)$state are varied as specified in the panels. The green dashed curve corresponds to the case where the symmetrization of the wave function is omitted. The middle panel corresponds to the measured resonance energy (Color online)


Fig. 3 Single $\alpha$ energy distributions from Faddeev computation for direct decay of the ${ }^{12} \mathrm{C}\left(1^{+}\right)$resonance. The three-body energy is varied by changing the strength of the three-body potential. The relative energies and widths of the intermediate ${ }^{8} \mathrm{Be}\left(2^{+}\right)$ resonance are specified in the panels. The solid (black) and dashed (green) curves correspond to complex rotation angles of $\theta=0.25$ and 0.1 respectively. The dotted (blue) curve corresponds to democratic decay $[7,8]$. For $\theta=0.25$ only direct decay is shown (Color online)

Table 1 The probability $P_{\text {seq }}$ for populating the component related to the decay via ${ }^{8} \mathrm{Be}\left(2^{+}\right)$at large distances in the computation of the $1^{+}$resonance of ${ }^{12} \mathrm{C}$ for a complex rotation angle $\theta=0.25$

| ${ }^{12} \mathrm{C}\left(J^{\pi}\right)$ | $E_{2 r} / E_{3 r}$ | $\Gamma_{2 r} / E_{3 r}$ | $E_{3 r}(\mathrm{MeV})$ | $\Gamma_{3 r}(\mathrm{MeV})$ | $l_{y}$ | $P_{\mathrm{seq}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0.86 | 0.43 | 3.5 | 0.005 | 2 | 0.001 |
| $1^{+}$ | 0.56 | 0.28 | 5.4 | 0.09 | 2 | 0.12 |
|  | 0.39 | 0.19 | 7.8 | 1.15 | 2 | 0.89 |

The three-body energy $\left(E_{3 r}\right)$ is varied by adjusting the strength of the three-body potential. The ${ }^{8} \mathrm{Be}\left(2^{+}\right)$two-body energy is maintained $E_{2 r}=2.7 \mathrm{MeV}$. The energies are referred to the $3 \alpha$ or $2 \alpha$ separation threshold

## 3 Conclusions

We have computed the observable momentum distributions from decay of three-body resonances by use of $R$-matrix simulations and from full Faddeev calculations. We have considered the example of the $1^{+}$resonance of ${ }^{12} \mathrm{C}$. The angular momentum and boson symmetries constrain the resulting momentum distributions. The same measured momentum distributions can be described in different complete basis sets, e.g. either direct products of two-body states and their center-of-mass motion relative to the third particle ( $R$-matrix) or three-body continuum wave functions (Faddeev). The fact that different descriptions seem to work indicates that the same wave function could be described in different ways. Extracting information of both structure and decay mechanism can then be misleading and requires model interpretations. Full Faddeev computations successfully reproduce the measured distributions.

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