

ROCK FAILURE

INFLUENCE OF THE BLOCK-HIERARCHICAL STRUCTURE OF ROCKS ON THE PECULIARITIES OF SEISMIC WAVE PROPAGATION

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The analysis is performed for the parameters of long pendulum-type waves in a one-dimensional periodic system with an arbitrary hierarchical structure. A case study is carried out into the wave propagation peculiarities in an impact-excited system of equal rigid blocks with parting layers differing in rigidity. The pendulum wave velocity in the automodel block-hierarchical system is determined.

Impact, block medium, pendulum-type waves, parting layers, elasticity

Recently the geomechanics and geophysics researchers tend to describing deformation of a rock mass as a block structure with a complex hierarchy. Under this conception, a rock mass is a structure of nested blocks different in scales [1]. By analyzing dimensions of blocks, starting from scales of crystals to fractions of rock mass and geoblocks of the earth's crust, it has been found that a ratio of sizes of blocks at neighboring scale levels, $a = l_{N+1} / l_N$, exhibits a certain stability, viz, $a \approx 1.4$ [2]. One more critical, obtained experimentally statistic invariant of a block structure is equation ratio of the parting width between blocks of the same scale to the typical size of a block, μ . It was determined that $\mu \approx (0.5 - 2) \cdot 10^{-2}$ for rocks at Norilsk mines [2]. Seismic approach to studying the deformation properties of partings between blocks with different dimensions showed that rigidity of the parties changes inversely to the dimensions of blocks [3]. The block-to-block partings are often composed of weak jointy rocks. Owing to the presence of these soft partings, a block rock mass deformation takes place, both in statics and in dynamics, due to the deformation of the partings.

The theoretical and experimental studies into the waveguide properties of one-dimensional block models in the form of a chain of elastic blocks with soft partings showed that wave propagation in such media is well-described by an approximation that block displace as non-deformable bodies [4–9]. At that, rather accurate description is obtained for low-frequency pendulum-type waves induced by an impact action. The experiments exhibited that high-frequency waves that are characteristic for the eigenmodes of blocks, attenuate fairly quickly. The authors devoted this study to the influence exerted by internal structure of blocks on propagation regularities of non-stationary waves in periodic block-hierarchical systems.

BLOCK HIERARCHICAL MODELS OF WAVEGUIDES

Let a periodic system have a block-hierarchical structure consisting of equal structural blocks each containing a nested system of sub-blocks as is in Fig. 1.

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Fig. 1. A schematic segment of a period of the 16th order one-dimensional block-hierarchical model of masses (light color rectangles) and partings (dark color rectangles)

We denote the system as a n th order system if its generator has n degrees of freedom. Figure 1 depicts the 16th order system after 4 steps of nesting.

Consider systems of discrete elements — masses and inertialess linkages — springs. The simplest periodic system presented by a chain of spring-connected masses (CMS) studied earlier [7–12] is the first-order system for which:

— stationary propagation of waves is only possible in the frequency band of $0 < \omega < \Omega$, where $\Omega = 2\sqrt{k/m}$; k is the rigidity of springs; m is the mass;

— phase velocity of long low-frequency waves ($\omega = cq \rightarrow 0$, $q \rightarrow 0$) is described by an asymptotics:

$$c = C_*(1 - \alpha q^2 + O(q^4)) \quad (1)$$

with integral parameters:

$$C_* = L\sqrt{\frac{k}{m}}, \quad \alpha = \frac{L^2}{24}, \quad (2)$$

where L is the distance between masses.

An impact-excited wave propagates along CMS and its asymptotic description at large times involves the following formulae for displacements $u(x, t)$, strains $\varepsilon_x(x, t)$, mass velocities $v(x, t)$ and accelerations $a(x, t)$:

$$u \approx IC_* \left[\frac{1}{3} - \int_0^\eta Ai(z) dz \right], \quad I = \int_0^{t_0} f(t) dt, \quad \eta = \frac{x - C_*t}{(3C_*\alpha t)^{1/3}}, \quad Ai(\eta) = \frac{1}{\pi} \int_0^\infty \cos(\eta z + z^3/3) dz,$$

$$\varepsilon_x = -\frac{v}{C_*} \approx -\frac{C_*I}{(3C_*\alpha t)^{1/3}} Ai(\eta), \quad a \approx \frac{C_*^3 I}{(3C_*\alpha t)^{2/3}} \frac{dAi(\eta)}{d\eta}. \quad (3)$$

Here, I is an impulse of an external force $f(t)$; Ai is the Airy function.

The outcome has the following physical sense. The velocity C_* is a movement velocity of a quasifront $x = C_*t$ with a cluster of low-frequency oscillations in its vicinity. Behind the front, with a decreasing velocity, higher-frequency modes move. Wave propagating from the area of influence are oscillating and diffuse with time (distance from the source of effect) as $t^{1/3}$ ($x^{1/3}$). The displacements oscillate relative to an average $U = IC_*$, the oscillation amplitude and period are maximal near the quasifront, and such are the velocities and strains there, but the latter, unlike the displacements, decrease with time as $t^{1/3}$. The accelerations go down with time as $t^{2/3}$ and have the maximal amplitudes at a certain distance (increasing as $t^{1/3}$) from the quasifront.

We think our goal is to describe collateral wave effects in high-level waveguides. The so-called Born chain exemplifies the second-order system with a cell of the length L , that is a chain of four elements represented by two masses and two linkages (Fig. 2).

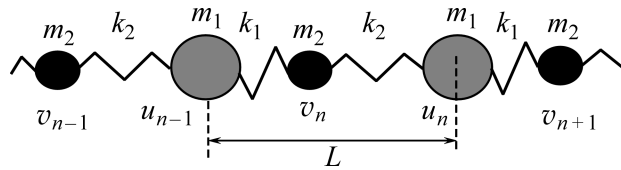


Fig. 2. Born chain

This system motion equations:

$$m_1 \ddot{u}_n + k_1(u_n - v_{n+1}) + k_2(u_n - v_n) = 0, \quad m_2 \ddot{v}_n + k_1(v_n - u_{n-1}) + k_2(v_n - u_n) = 0$$

define two oscillation modes by the number of the degrees of freedom the generator has, namely, the acoustic *I* and optic *II* modes (Fig. 3).

The system has three resonant frequencies, that are Ω_1 and Ω_2 at a point $q = \pi$ for short-wave oscillations, and Ω_3 at the point $q = 0$ for long-wave oscillations. With no restrictions imposed on generality, we introduce the expressions for the frequencies for the case when $m_1 = m_2 = m$:

$$\Omega_{1,2} = \Omega_0 \sqrt{1 \pm \sqrt{1 - \beta}}, \quad \Omega_3 = \sqrt{\frac{2(k_1 + k_2)}{m}}, \quad \Omega_0 = \sqrt{\frac{k_1 + k_2}{m}}, \quad \beta = \frac{4k_1 k_2}{(k_1 + k_2)^2}. \quad (4)$$

The bands $\Omega_1 \leq \omega \leq \Omega_2$ and $\omega \geq \Omega_3$ correspond to blocking bands as propagation of harmonic waves with such frequencies is impossible.

The phase velocity of the long low-frequency waves is described by asymptotics (1) with integral parameters:

$$C_*^I = L \sqrt{\frac{k_1 k_2}{2(k_1 + k_2)m}}, \quad \alpha = \frac{L^2}{24} \left(1 - \frac{3}{4} \beta \right). \quad (5)$$

An asymptotic analysis of the long-wave longitudinal excitations near the quasifront, $nL = C_*^I t$, shows that they are described by asymptotic (3) with parameters (5).

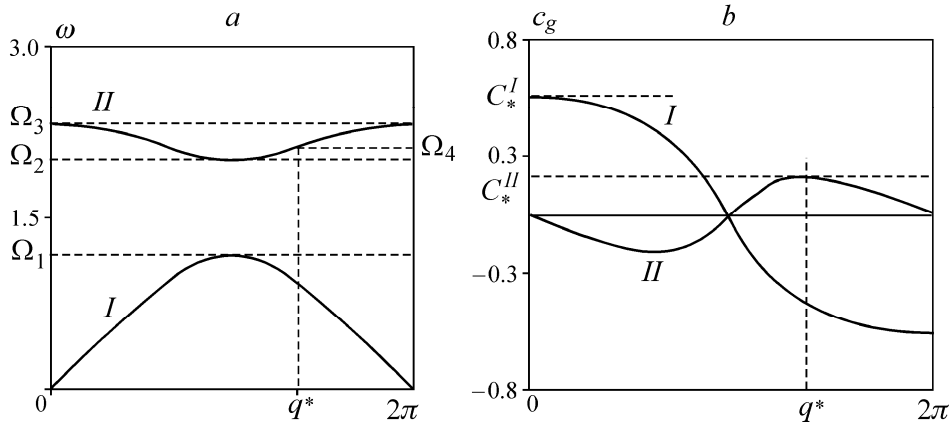


Fig. 3. Branches of the modes *I* and *II* of dispersion characteristics for the frequencies ω and group velocities c_g , calculated for $m_1 = m_2 = 1$, $L = 1$, $k_1 = 0.34$, $k_2 = 1$

Figure 4 presents the acceleration oscillograms for the 40th and 80th masses in the Born chain under the impact action by a half-sine pulse with a duration $T = 0.02765$, obtained by the finite difference method. As against the system CMS, in the Born chain, the local impulse-generated excitations that propagate to the periphery have a pronounced two-wave structure. The low-frequency pendulum wave running with the velocity C_*^I is the same in nature as in CMS. But in the case discussed, this wave is followed by the high-frequency constituent with an envelope velocity that equals the second mode group velocity maximum $C_*^{II} = \max c_g^{II} = c_g^{II}(q^*)$ and the frequency $\Omega_4 = \omega(q^*)$ (Fig. 3). The vertical dashed lines in Fig. 4 indicate the time moments $t_*^I = nL / C_*^I$ and $t_*^{II} = nL / C_*^{II}$ of the quasifronts of the long-wave and short-wave excitations, respectively.

GENERAL CASE OF THE PERIODIC BLOCK-STRUCTURED MODEL

Consider the general case of the periodic block-hierarchical system composed on equal blocks each having structures of different types (with the simplest cells of CMS, hierarchical systems as is in Fig. 1 and, for instance, three-dimensional finite-size bodies). Joint movement of the system is directed long-wise.

Structure of blocks is arbitrary, that means inertialess linkages discrete structure with many degrees of freedom, or continual elements, the only requirement is to have possibility of associating “displacement” $U_n(t)$ ($n = 0, \pm 1, \pm 2, \dots$) of a block with a displacement of any material point of it.

So, we assume having a system of dynamic equations for the elements of a block and a required set of boundary conditions. We solve the non-stationary problem using the Laplace transform of time with the transform parameter p (denoted by L). Unloading the linear equations for the Laplace images from the displacements of internal elements of the block yields equation that connect $U_n^L, U_{n+1}^L, U_{n-1}^L$ as they occur in the boundary conditions for the motion equation of the n th block.

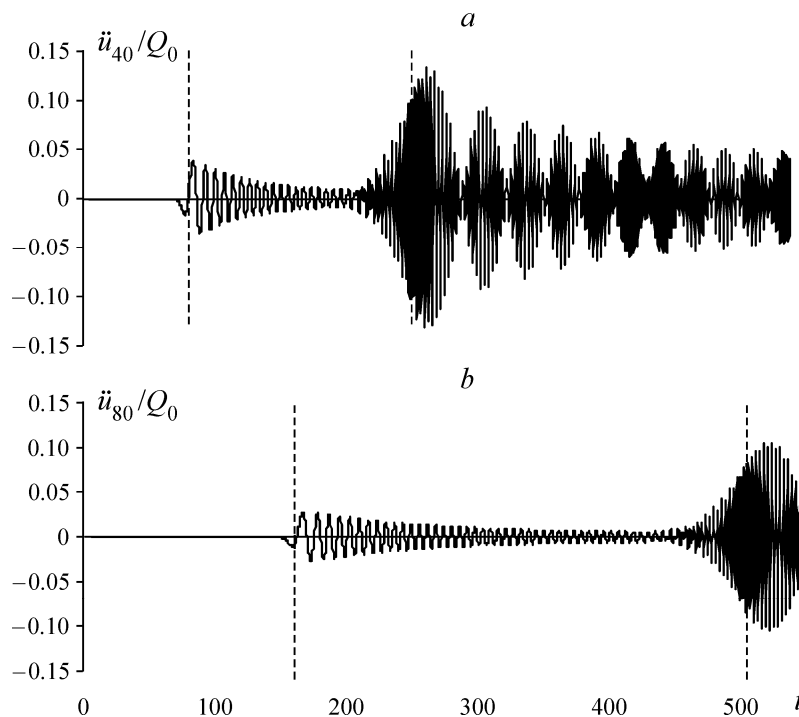


Fig. 4. Wave propagation in the Born chain under impulse load. The acceleration oscillograms for the 40th and 80th masses are calculated for $m_1 = m_2 = 1$, $L = 1$, $k_1 = 0.34$, $k_2 = 1$

Now, the problem is reduced to solving an infinite system of equations:

$$\Phi(U_{n-1}^L, U_n^L, U_{n+1}^L; p) = Q_n^L,$$

where Φ is a linear function of U_n^L , U_{n+1}^L , U_{n-1}^L , and Q_n^L is the Laplace image of external force. After that, the discrete Fourier transform is used, with the parameter q (denoted by F_d). The formal solution has the form:

$$U^{LF_d} = \frac{Q^{LF_d} A(p, q, K)}{D(p, q, K)}, \quad (6)$$

where A is an operator fitting the type of linkages between the elements of the blocks; K is all parameters (constants) of the problem; D is the dispersion operator of the system:

$$D(p, q, K) = \sin^2(qL/2) - \beta(p, K); \quad (7)$$

here, generally, β is a transcendental function.

A formal solution to the non-stationary problem is:

$$U_n(t) = \frac{1}{4\pi^2 i} \int_{-\pi\alpha-i\infty}^{\pi\alpha+i\infty} \int U^{LF_d}(p, q) e^{pt-iqn} dp dq. \quad (8)$$

Unfortunately, an exact converse of (8) is only possible in some simplest case studies. To this end, we use a technique described in [11] for the asymptotic inversion of a double Laplace and Fourier image as $p \rightarrow 0$ and $q \rightarrow 0$, which corresponds to the long-wave excitations in the origin space at an infinite large time from the loading start ($t \rightarrow \infty$).

Let us analyze the dispersion equation derived after replacement of $p = iq\omega = i\omega$ in the dispersion equation:

$$D(i\omega, q, K) = \sin^2(qL/2) - \beta(i\omega, K) = 0.$$

We can show that the low-frequency approximation of the dispersion equation is:

$$\sin^2(qL/2) = \alpha_1(K)\omega^2 - \alpha_2(K)\omega^4 + O(\omega^6) \quad (\alpha_1, \alpha_2 \in R; \alpha_1, \alpha_2 > 0),$$

and its long-wave asymptotics:

$$c = C_* (1 - \alpha q^2 + O(q^4))$$

exactly matches the structure of CMS asymptotics (1) but with its own constants, which are the long-waves velocity C_* and the dispersion parameter α being integral representatives of properties of a particular structure:

$$C_* = \frac{L}{2\sqrt{\alpha_1}}, \quad \alpha = \frac{L^2 \alpha_2}{8\alpha_1^2}.$$

Following the method [1] for the double transform near a ray $x = C_* t$, assume that the subintegral expression $p = s + iq(C_* + c')$ includes $|q| \leq \varepsilon$ (ε is small), where $s \rightarrow 0$, $c' \rightarrow 0$. We obtain asymptotic expansion of the dispersion operator:

$$D(s, q) \approx 2iqC_*(s + iq c' + i\alpha C_* q^3).$$

By doing so for the numerator $A(s + iqC_*, q, K) = A_U \approx O(1)$, we arrive at the asymptotics for the subintegral expression:

$$U^{LF_d} \approx \frac{Q^{LF_d} A_U}{2iqC_*(s + iqC_*' + i\alpha C_* q^3)}.$$

Let in the section $x=0$, a concentrated half-sine load is applied with ω_* : $Q(t) = Q_0 \sin(\omega_* t) H_0(t) H_0(\pi - \omega_* t)$, where H_0 is the Heaviside functions, whence:

$$Q^{LF_d} = \frac{Q_0 \omega_* (1 + e^{-p\pi/\omega_*})}{p^2 + \omega_*^2}, \quad Q^{LF_d} \approx \frac{2Q_0}{\omega_*} \quad (p \rightarrow 0).$$

The result of converse on the ray is the low-frequency asymptotics of the long-wave excitations at $t \rightarrow \infty$:

$$\dot{U}_n = \frac{Q_0 A_U}{\omega_* (3\alpha C_* t)^{1/3}} Ai(\eta), \quad \ddot{U}_n = \frac{Q_0 A_U C_*}{\omega_* (3\alpha C_* t)^{2/3}} \frac{dAi(\eta)}{d\eta}, \quad \eta = \frac{nL - C_* t}{(3\alpha C_* t)^{1/3}}.$$

This solution structure matches with (3) for CMS. So, response of any long discrete-periodic structure to a non-stationary action is described by the asymptotic solution (3), and differing are only the coefficients α and C_* . Having calculated them, we can find an equivalent CMS for a complex block-hierarchical structure.

For the block-hierarchical system as is in Fig. 1, the long wave propagation velocity C_* in asymptotics (3) is possible to be calculated as a sound speed in an equivalent continual elastic medium with an averaged deformation modulus E_0 . For the simplicity, we analyze the case with rigid blocks and soft parting layers. Let the main period of a one-dimensional block system have a length L , cross section s , mass m and its boundary partings rigidity k . Should the period has no inner parting layers, than the averaged modulus is $E_0 = kL/s$ and the wave propagation velocity is $V_0 = \sqrt{E_0/\rho} = L\sqrt{k/m}$ that complies with the value of C_* in CMS.

Divide the main block into two equal sub-blocks and introduce in-between a parting layer k_1 . According to the experimental data on the ratio of properties of parting layers between blocks between different scale levels [2, 3], we assume that $k_1 = 2k$. Then, averaging yields $E_1 = (2/3)E_0$ and $V_1 = V_0\sqrt{2/3}$, and the latter expression coincides with the value of C_* for the Born chain with $m_1 = m_2 = m$, $k_2 = 2k_1 = 2k$.

If nesting has j steps, then we have:

$$E_j = \frac{2}{2+j} E_0, \quad V_j = V_0 \sqrt{\frac{2}{2+j}}.$$

Wherefrom, in the analyzed hierarchical system of the n th order, the long-waves propagate with the velocity:

$$C_* = L \sqrt{\frac{k}{m}} \sqrt{\frac{2}{2 + \log_2 n}}.$$

This formula implies that when the hierarchical system order n grows, the velocity of long pendulum waves decreases rather slow.

In a similar way, when calculating E , we can take into account compressibility of blocks, other division into sub-blocks, other law of the rigidity variation for parting layers.

CONCLUSIONS

We have studied the 2nd order block-hierarchical model (Born chain), calculated non-steady wave processes, found occurrence of low-frequency pendulum-type waves and high-frequency waves, and determined their propagation velocities.

Analysis of the long waves in a 1D periodic system with an arbitrary hierarchy has shown that the pendulum wave structure (length, velocity, acceleration) are determined by an asymptotically equivalent model of a chain of spring-separated masses with two integral parameters depending on properties of a particular system.

We have found the velocity of a pendulum wave in the automodel block-hierarchical system with the set nesting.

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