SPHERICAL GAUGE GRAVITATIONAL FIELD AND SPONTANEOUS SYMMETRY BREAKING*

GHEORGHE ZET[1], CAMELIA POPA[2]

[1] Department of Physics, "Gh. Asachi" Technical University, Iași 700050, Romania
[2] Faculty of Physics, "Al. I. Cuza" University, Iași 700506, Romania
E-mail: camelia_popa2002@yahoo.com

Received September 26, 2007

Using a gauge theory of the gravitational field we build a metric with spherical symmetry. We use the theory based on the gravitational gauge group G to obtain a spherical symmetric solution of the field equations for the gravitational potentials. We define the gravitational gauge group G and then we introduce the gauge covariant derivative D_μ. The strength tensor of the gravitational gauge field is obtained and a gauge invariant Lagrangian is then constructed. The field equations of the gauge potentials are written and a gravitational energy-momentum tensor $(T_g)_{\mu\nu}$ is determined. In such a theory the motion of a test particle may be assimilated with a spontaneous symmetry breaking field theory: the gauge field that mediates the gravitational interaction between the source field and the test particle spontaneously breaks the vacuum symmetry, generating a Reissner-Nordstrom type metric. All the calculations have been performed by GR Tensor II computer algebra package, running on the Maple 7 platform, along with several routines that we have written for our model.

Pacs numbers: 11.30.Qc, 11.15.-q, 04.20.Cv.

Key words: gravitational field, spontaneous symmetry breaking, gauge theory.

1. INTRODUCTION

The gauge theories are fundamental in the field theory and, in particular, in the elementary particle physics [1]. The three non-gravitational interactions (electromagnetic, weak, and strong) are completely described by means of gauge theory in the framework of the Standard Model (SM).

First of all, the gauge theory of the unitary groups SU(N) is of fundamental importance in elementary particle physics. The SM of strong and electroweak interactions is based on the gauge theory of $SU(3)\times SU(2)\times U(1)$ group. In addition, the "Grand Unification" is described by the gauging of SU(5) group [1].

* Paper presented at the 8th International Balkan Workshop on Applied Physics, 5–7 July 2007, Constanța, Romania.

Secondly, the Poincaré group (Lorentz transformations and space-time translations) is also of a fundamental importance in any field theory. After pioneering works of Utiyama [2], Sciama [3, 4], and Kibble [5] it was recognized that gravitation also can be formulated as a gauge theory. The gauge groups considered in gauge theory of gravitation are Poincaré group [6], de-Sitter group [7, 8], affine group [9, 10], etc. It is believed that the formulation of gravity as a gauge theory on a Minkowski space-time could lead to a consistent quantum theory of gravity.

Recently, N. Wu [11] proposed a gauge theory of General Relativity (GR) based on the gravitational gauge group (G). The gravitational interaction is considered in this theory as a fundamental interaction in a flat Minkowski space-time, and not as space-time geometry.

In this paper we use the theory based on the gravitational gauge group G to obtain a spherical symmetric solution of the field equations for the gravitational potentials. In Section 2 we define the gravitational gauge group G and then we introduce the gauge covariant derivative D_μ. The strength tensor of the gravitational gauge field is obtained and a gauge invariant Lagrangian is constructed. The field equations of the gauge potentials are written and a gravitational energy-momentum tensor $(T_g)_{\mu\nu}$ which is a conserved current is determined. It is shown that the theory of the gravitational field based on the gravitational gauge group G is equivalent to the General Relativity of Einstein.

Section 3 is devoted to the case of a model when the gravitational gauge potentials $A_\mu^\alpha(x)$ have spherical symmetry. The corresponding non-null components of the strength tensor $F_{\mu\nu}$ of the gravitational gauge field are obtained and then the gauge field equations are written. An analytical solution of these equations, which induce the Reissner-Nordstrom type metric on the gauge group space, is then determined. In Section 4, the motion of a test particle in a gravitational field with spherical symmetry, may be assimilated with a gauge field theory with a spontaneous symmetry breaking: the gauge gravitational field breaks spontaneously the symmetry of the vacuum state generating the kink solution.

2. THE TENSOR OF THE GAUGE POTENTIALS

The gravitational field is described in GR by the metric tensor of a curved space-time. In the gauge theory based on the gravitational gauge group G the gravity is treated as a physical interaction in a Minkowski (flat) space-time and the gravitational field is represented by gauge potentials.

In the following Sections we suppose that the gravitational gauge potentials have spherical symmetry and obtain a Reissner-Nordstrom type solution of the field equations.

The infinitesimal transformations of group G are of the form [11]:

$$U(\varepsilon) = 1 - \varepsilon^{\alpha} P_{\alpha}, \quad \alpha = 0, 1, 2, 3, \tag{2.1}$$

where ε_{α} are the infinitesimal parameters of the group, and $P_{\alpha} = -i\partial_{\alpha}$ are the generators of the gauge group. It is known that these generators commute each other

$$\left[P_{\alpha}, P_{\beta}\right] = 0. \tag{2.2}$$

However, this property of the generators does not means that the gravitational gauge group is an Abelian group, because its elements do not commute [11]

$$\left[U(\varepsilon_1), U(\varepsilon_2)\right] \neq 0. \tag{2.3}$$

There is a difference between the group T of space-time translations and the gravitational gauge group G. Space-time translations of T are coordinate transformations, that is, the objects or fields (physical systems) in space-time are fixed while the coordinates describing the motion of the physical system undergo transformation. But, under the transformations of the gravitational gauge group the space-time coordinate system is fixed, while the objects or fields undergo transformation. Therefore, the gravitational gauge group G contains all dynamical information of interactions and it is convenient to use it for studying the gravitational field.

Let us suppose now that $\phi(x)$ is a scalar field. Then its gravitational gauge transformation under G is:

$$\phi(x) \to \phi'(x) = U(\varepsilon)\phi(x). \tag{2.4}$$

Because $\partial_{\mu}U(\varepsilon) \neq 0,$ the partial derivative of $\phi(x)$ does not transform covariant under the gauge group G

$$\partial_{\mu}\phi(x) \to \partial_{\mu}\phi'(x) \neq U(\varepsilon)\left(\partial_{\mu}\phi(x)\right). \tag{2.5}$$

In order to construct an action which is invariant under local gauge transformations with parameters depending of coordinates $\varepsilon^{\alpha} = \varepsilon^{\alpha}(x),$ it is necessary to define a gauge covariant derivative [15, 16]

$$D_{\mu} = \partial_{\mu} - i g A_{\mu}(x), \tag{2.6}$$

where $A_{\mu}(x)$ is the gauge gravitational field (potential) with values in the Lie algebra of G and *g* is the gauge coupling constant of the gravitational interaction. The law of transformation under the gauge group of this potential is:

$$A_{\mu} \to A'_{\mu} = U A_{\mu} U^{-1} + \frac{i}{g} U\left(\partial_{\mu} U^{-1}\right). \tag{2.7}$$

The derivative D_μ has the property of gauge covariance under the gauge group:

$$D_\mu \to D'_\mu = U\, D_\mu\, U^{-1}. \tag{2.8}$$

The gravitational $A_\mu(x)$ gauge field can be expanded as a linear combination of generators P_α

$$A_\mu(x) = A_\mu^\alpha(x) P_\alpha, \tag{2.9}$$

where $A_\mu^\alpha(x)$ are the gravitational gauge potentials, *i.e.* they are the components of the gravitational gauge field.

We define now new gauge gravitational potentials

$$G_\mu^\alpha(x) = \delta_\mu^\alpha - g\, A_\mu^\alpha(x), \tag{2.10}$$

and introduce their inverses $\bar{G}_\alpha^\mu$ with the properties

$$\bar{G}_\alpha^\mu\, G_\mu^\beta = \delta_\alpha^\beta, \tag{2.11a}$$

$$\bar{G}_\alpha^\mu\, G_\nu^\alpha = \delta_\nu^\mu. \tag{2.11b}$$

Using these components, we can define a metric on the gauge group space as follows:

$$g_{\alpha\beta} = \eta_{\mu\nu}\, \bar{G}_\alpha^\mu\, \bar{G}_\beta^\nu, \tag{2.12a}$$

$$g^{\alpha\beta} = \eta^{\mu\nu}\, G_\mu^\alpha\, G_\nu^\beta, \tag{2.12b}$$

where $\eta_{\mu\nu} = diag(1,1,1,-1)$ is the metric on the Minkowski space-time and $\eta^{\mu\nu}$ denotes its inverse.

The strength tensor of the gravitational gauge field is given by the standard expression [12–14]

$$F_{\mu\nu} = \frac{i}{g}\left[D_\mu, D_\nu\right]. \tag{2.13}$$

Then, its components, defined by $F_{\mu\nu} = F_{\mu\nu}^\alpha\, P_\alpha$, are

$$F_{\mu\nu}^\alpha = \partial_\mu A_\mu^\alpha - \partial_\nu A_\mu^\alpha - g\, A_\mu^\beta\, \partial_\beta A_\nu^\alpha + g\, A_\nu^\beta\, \partial_\beta A_\mu^\alpha. \tag{2.14}$$

The strength tensor transforms covariant under the gravitational gauge transformations

$$F'_{\mu\nu} = U\, F_{\mu\nu}\, U^{-1}. \tag{2.15}$$

In his work [11] N. Wu chooses the Lagrangian of the gravitational gauge potentials in the form

$$L = \sqrt{-\bar{g}} L_0, \tag{2.16}$$

where $\bar{g} = \det(g_{\mu\nu})$ and

$$\begin{aligned} L_0 = & -\frac{1}{16} \eta^{\mu\rho} \eta^{\nu\sigma} g_{\alpha\beta} F^{\alpha}_{\mu\nu} F^{\beta}_{\rho\sigma} - \frac{1}{8} \eta^{\mu\rho} \bar{G}^{\nu}_{\beta} \bar{G}^{\sigma}_{\alpha} F^{\alpha}_{\mu\nu} F^{\beta}_{\rho\sigma} \\ & + \frac{1}{4} \eta^{\mu\rho} \bar{G}^{\nu}_{\alpha} \bar{G}^{\sigma}_{\beta} F^{\alpha}_{\mu\nu} F^{\beta}_{\rho\sigma}. \end{aligned} \tag{2.17}$$

This expression is quite special for gravitational interactions. Indeed, for ordinary SU(N) gauge field theory it is possible to construct only one invariant which is a quadratic form of field strength. In gauge theory of gravitation there are three different gauge invariant terms which are quadratic forms of field strength of gravitational gauge field. The integral of action associated to the Lagrangian L is defined as usually:

$$S = \int d^4 x\, L, \tag{2.18}$$

It can be verified that this action has gravitational gauge symmetry, *i.e.* it is invariant under the gravitational gauge transformations.

The first order variation of the gravitational gauge fields is

$$\delta A^{\alpha}_{\mu}(x) = -\varepsilon^{\beta} \partial_{\beta} A^{\alpha}_{\mu}(x), \tag{2.19}$$

and the corresponding first order variation of the action is

$$\delta S = \int d^4 x\, \varepsilon^{\alpha} \partial_{\mu} (T_i)^{\mu}_{\alpha}. \tag{2.20}$$

Here $(T_i)^{\mu}_{\alpha}$ is the inertial energy-momentum tensor, whose definition is

$$(T_i)^{\mu}_{\alpha} = \bar{g} \left(-\frac{\partial L_0}{\partial \partial_{\mu} A^{\beta}_{\nu}} \partial_{\alpha} A^{\beta}_{\nu} + \delta^{\mu}_{\alpha} L_0 \right). \tag{2.21}$$

The global gravitational gauge symmetry of the system gives out the conservation law of inertial energy-momentum tensor

$$\partial_{\mu} (T_i)^{\mu}_{\alpha} = 0. \tag{2.22}$$

This means that the inertial energy-momentum tensor is a conserved current.

The Euler-Lagrange equations for gravitational gauge field are

$$\frac{\partial L}{\partial A^{\alpha}_{\nu}} - \partial_{\mu} \left(\frac{\partial L}{\partial \partial_{\mu} A^{\alpha}_{\nu}} \right) = 0. \tag{2.23}$$

Introducing (2.16)–(2.17) into the Eqs. (2.23) we obtain the following field equations of the gravitational gauge fields $A^{\alpha}_{\mu}(x)$

$$\begin{aligned}&\partial_{\mu}(\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma}g_{\alpha\beta}F^{\sigma}_{\rho\sigma}-\frac{1}{4}\eta^{\nu\rho}F^{\mu}_{\rho\alpha}+\frac{1}{4}\eta^{\mu\rho}F^{\nu}_{\rho\alpha}\\&-\frac{1}{2}\eta^{\mu\rho}\delta^{\nu}_{\alpha}F^{\beta}_{\rho\beta}+\frac{1}{2}\eta^{\nu\rho}\delta^{\mu}_{\alpha}F^{\beta}_{\rho\beta})=-g\left[\left(T_{g}\right)^{\nu}_{\alpha}+\left(T_{m}\right)^{\nu}_{\alpha}\right]\end{aligned} \tag{2.24}$$

where $\left(T_{g}\right)^{\mu}_{\alpha}$ is the gravitational energy-momentum tensor, which is the source of gravitational gauge field [11] and $\left(T_{m}\right)^{\nu}_{\alpha}$ is the tensor of the field of point-like source Q positioned in the origin of the coordinate system.

3. SPHERICALLY SYMMETRIC GAUGE FIELD

We apply the previous results to the case when the gravitational gauge potentials $A^{\alpha}_{\mu}(x)$ have spherical symmetry. We choose these potentials in the form

$$A^{\alpha}_{\mu}(x)=\begin{pmatrix}A(r) & 0 & 0 & 0\\ 0 & \frac{r-1}{gr} & 0 & 0\\ 0 & 0 & \frac{r\sin\theta-1}{gr\sin\theta} & 0\\ 0 & 0 & 0 & -\frac{A(r)}{1-gA(r)}\end{pmatrix}, \tag{3.1}$$

where (r,θ,φ,t) are spherical coordinates on the gauge group space, and $A(r)$ is a function depending only of the variable *r*. Then the new gravitational gauge potentials $G^{\alpha}_{\mu}(x)$, defined by the Eqs. (2.10), become

$$G^{\alpha}_{\mu}=\begin{pmatrix}1-gA(r) & 0 & 0 & 0\\ 0 & \frac{1}{r} & 0 & 0\\ 0 & 0 & \frac{1}{r\sin\theta} & 0\\ 0 & 0 & 0 & \frac{1}{1-gA(r)}\end{pmatrix}. \tag{3.2}$$

and their inverse are

$$\bar{G}^{\mu}_{\alpha} = \begin{pmatrix} \frac{1}{1-gA(r)} & 0 & 0 & 0 \\ 0 & r & 0 & 0 \\ 0 & 0 & r\sin\theta & 0 \\ 0 & 0 & 0 & 1-gA(r) \end{pmatrix}. \tag{3.3}$$

The components $g_{\alpha\beta}$ of the metric tensor defined by the Eq. (2.12a) are given then by

$$g_{ab} = \begin{pmatrix} \frac{1}{\left(1-gA(r)\right)^2} & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2\sin^2\theta & 0 \\ 0 & 0 & 0 & -\left(1-gA(r)\right)^2 \end{pmatrix}, \tag{3.4}$$

The non-null components $F^{\alpha}_{\mu\nu}$ of the strength tensor field defined in the Eq. (2.13) are

$$\begin{aligned} &F^2_{12} = \sin\theta, \quad F^3_{13} = \frac{1-gA(r)}{gr^2} \\ &F^0_{10} = -\frac{A'(r)}{1-gA(r)}, \quad F^3_{23} = \frac{\cos\theta}{gr^2\sin^2\theta}, \end{aligned} \tag{3.5}$$

where $A'(r)$ denotes the derivative of $A(r)$ with respect to the spatial variable r.

Introducing (3.4) and (3.5) into the Eq. (2.24), we obtain the following gauge field equations

$$gA^2 + 2grAA' - 2A - 2rA' = 0. \tag{3.6}$$

Therefore, the gauge field equations (2.24) for the previous considered model reduce to only one independent equation for the unknown function (gauge potential) $A(r)$.

The unknown function $A(r)$ has therefore the form

$$A(r) = \frac{1 \pm \sqrt{1 - \frac{\alpha}{r} + \frac{\beta}{r^2}}}{g} \tag{3.7}$$

where α and β is an arbitrary constants of integration.

Now, introducing the solution (3.7) into the Eq. (3.4), we obtain the non-null components $g_{\alpha\beta}$ of the metric tensor on the gauge group space

$$g_{\alpha\beta}=\begin{pmatrix} \dfrac{1}{1-\dfrac{\alpha}{r}+\dfrac{\beta}{r^2}} & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2\sin^2\theta & 0 \\ 0 & 0 & 0 & -\left(1-\dfrac{\alpha}{r}+\dfrac{\beta}{r^2}\right) \end{pmatrix}. \tag{3.8}$$

The corresponding line element is

$$ds^2=\frac{dr^2}{1-\dfrac{\alpha}{r}+\dfrac{\beta}{r^2}}+r^2(d\theta^2+\sin^2\theta\, d\varphi^2)-\left(1-\frac{\alpha}{r}+\frac{\beta}{r^2}\right)c^2dt^2, \tag{3.9}$$

and it describes the gravitational field of our model.

If we chose $\alpha=\dfrac{2MG}{c^2}$, where G is the gravitational constant, M is the mass of the source of gravitational field and $\beta=\dfrac{Q^2G}{4\pi\varepsilon_0 c^4}$, then the metric $g_{\alpha\beta}$ corresponds to the Reissner-Nordstrom type solution for the Einstein equation of the gravitational field created by a point-like mass m_0 in vacuum.

We remember that this line element is defined on the gravitational gauge group space and that the space-time remains a Minkowski (flat) one.

In what follows we will write the metric (3.9) under the form:

$$ds^2=e^{\lambda}dr^2+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right)-e^{\nu}c^2dt^2, \tag{3.10}$$

where λ and ν are function only of the radial coordinate r, which in the case of Reissner-Nordstrom metric are given by the expression:

$$e^{\nu}=e^{-\lambda}=1-\frac{\alpha}{r}+\frac{\beta}{r^2}, \tag{3.11}$$

with α and β the constants above mentioned.

4. MATHEMATICAL MODEL WITH SPONTANEOUS SYMMETRY BREAKING

We consider the motion of a test particle of mass m_0 in the gravitational field with spherical symmetry produced by a source having the mass M. This gravitational field is supposed to have spherical symmetry and the square of the line element given by Eqs. (3.10)–(3.11).

The equation of motion for the test particle are given by geodesics [17]:

$$\frac{d^2x^\mu}{ds^2}+\Gamma^\mu_{\nu\rho}\frac{dx^\nu}{ds}\frac{dx^\rho}{ds}=0. \tag{4.1}$$

If we use the metric coefficients in Eq. (3.10), then we can obtain the equations of motion for the test particle. The differential equation for the coordinate θ has the form:

$$\frac{d^2\theta}{ds^2}+\frac{2}{r}\frac{dr}{ds}\frac{d\theta}{ds}-\cos\theta\left(\frac{d\varphi}{ds}\right)^2=0. \tag{4.2}$$

In order to simplify this equation we chose the central mass M (the source of the gravitational field) and the initial velocity vector in the coordinate plane $\theta=\frac{\pi}{2}$. Then, we have $\frac{d\theta}{ds}=0,$ and the Eq. (4.2) becomes:

$$\frac{d^2\theta}{ds^2}=0. \tag{4.3}$$

Therefore, the trajectory of the test particle is contained in the equatorial plane (the motion is plane).

The other equations of motion, resulting from (4.1), are:

$$\frac{d^2r}{ds^2}+\frac{1}{2}\frac{d\lambda}{dr}\left(\frac{dr}{ds}\right)^2-re^{-\lambda}\left(\frac{d\varphi}{ds}\right)^2+\frac{1}{2}e^{\nu-\lambda}\frac{d\nu}{dr}\left(c\frac{dt}{ds}\right)^2=0, \tag{4.4a}$$

$$\frac{d^2\varphi}{ds^2}+\frac{2}{r}\frac{dr}{ds}\frac{d\varphi}{ds}=0, \tag{4.4b}$$

$$\frac{d^2t}{ds^2}+\frac{d\nu}{dr}\frac{dr}{ds}\frac{dt}{ds}=0. \tag{4.4c}$$

The Eqs. (4.4b) and (4.4c) can be easy integrated and they give (after a first integration):

$$r^2\frac{d\varphi}{ds}=C_1, \tag{4.5a}$$

and respectively:

$$\frac{dt}{ds}=C_2e^{-\nu}, \tag{4.5b}$$

where C_1, C_2 are arbitrary constants of integration.

On the other hand, if we use the expression (3.10) of the line element, and the condition of spherical symmetry $e^{\lambda}=e^{-\nu}$, we obtain:

$$e^{-\nu}\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\varphi}{ds}\right)^2 + e^{\nu}\left(c\frac{dt}{ds}\right)^2 = -1. \qquad (4.6)$$

Introducing the Eqs. (4.5a)–(4.5b) into Eq. (4.6), we can write the equation of motion for the test particle m_0 in the form:

$$(C_1)^2 \frac{e^{-\nu}}{r^4}\left(\frac{dr}{d\varphi}\right)^2 + \frac{(C_2)^2}{r^2} - e^{-\nu}c^2(C_2)^2 - 1 = 0. \qquad (4.7)$$

Now, we will suppose that the metric (3.10) is of Reissner-Nordstrom type, *i.e.* the metric coefficients are given by Eq. (3.11).

Then, making the change of coordinates

$$\frac{1}{2}\phi^2 = \frac{\alpha}{r}, \xi = \frac{\varphi}{2}, \qquad (4.8)$$

we obtain from (4.7):

$$\left(\frac{d\phi}{d\xi}\right)^2 = -\left(1 + \frac{\beta}{C_1^2}\right)\phi^2 + \frac{1}{2}\phi^4 - \frac{\beta}{4\alpha^2}\phi^6 + \frac{2\alpha^2}{(C_1)^2}, \qquad (4.9)$$

where the constant C_2 is choosen equal to: $C_2 = \frac{1}{c}$ (*i.e.* $C_2 = 1$ in units $c = 1$).

If we take the derivative of (4.9) with respect to the new variable ξ, then we obtain:

$$\frac{d^2\phi}{d\xi^2} = -\left(1 + \frac{\beta}{C_1^2}\right)\phi + \phi^3 - \frac{3\beta}{4\alpha^2}\phi^5 \qquad (4.10)$$

But the Eq. (4.10) can be obtained from the variational principle

$$\delta S = \delta \int L dv = 0 \qquad (4.11)$$

(with dv the elementary volume) applied to the Lagrangian density

$$L = \frac{1}{2}\left(\frac{\partial\phi}{\partial\xi}\right)^2 - V(\phi), \qquad (4.12)$$

with the potential

$$V(\phi) = -\left(1 + \frac{\beta}{C_1^2}\right)\frac{\phi^2}{2} + \frac{\phi^4}{4} - \frac{\beta}{8\alpha^2}\phi^6 \qquad (4.13)$$

Now, if we choose $\frac{\beta}{C_1^2} = -0.2$ and $\frac{\beta}{8\alpha^2} = -0.1$, then Eq. (4.10) has the solution $\phi_v = 0, \phi_v = \pm 0.768$. By calculating the second derivative with respect to ϕ of the potential entering (4.13) and substituting the extreme values into the result of this differentiation we find $V_{\phi\phi}(0) = -0.8$, $V_{\phi\phi}(\pm 0.768) = 2.013 > 0$, *i.e.* the solutions $\phi = \pm 0.768$ are associated with the minimum energy [18]. Hence, the model under consideration has a double degenerated vacuum state (Fig. 1) and the system source test-particle has a spontaneous symmetry breaking (SSB).

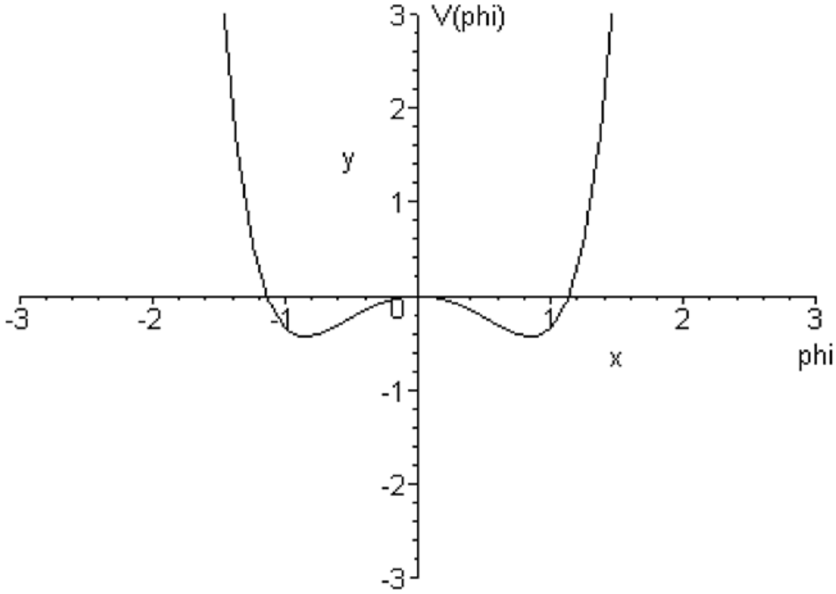

Fig. 1 – The potential energy for the case of SSB.

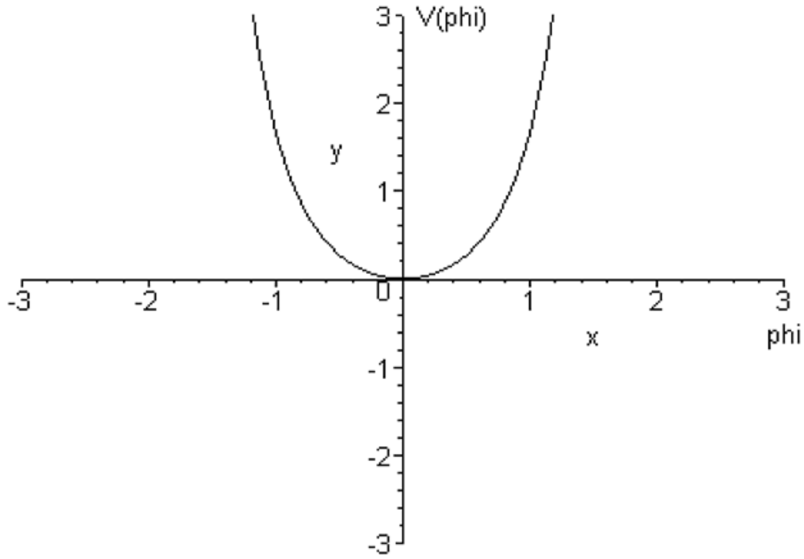

Fig. 2 – The potential energy for the case of ES.

If we chose $\frac{\beta}{C_1^2} = -1.8$ and $\frac{\beta}{8\alpha^2} = -0.1$, then the vacuum state is not degenerated (Fig. 2), and therefore we have an exact the symmetry (ES) of the Lagrangean (4.12).

These results give an extension of those obtained for the Schwarzschild metric [19] to the case of the Reissner-Nordstrom type metrics.

5. CONCLUDING REMARK

We constructed a metric with spherical symmetry using a gauge theory based on the gauge gravitational group. In this theory, the gravitational interaction is considered as a fundamental interaction in a flat Minkowski space-time, and not as space-time geometry. We define the gravitational gauge group G and then we introduced the gauge covariant derivative D_μ. The strength tensor of the gravitational gauge field is obtained and a gauge invariant Lagrangian is constructed. The field equations of the gauge potentials are written and a gravitational energy-momentum tensor $(T_g)_{\mu\nu}$ is introduced. The gauge field equations [see Eq. (2.24)] for the previous considered model reduce to only one independent equation for the unknown function (gauge potential) $A(r)$. We obtain the non-null components $g_{\alpha\beta}$ of the metric tensor on the gauge group space [see Eq. (3.8)].

The Reissner-Nordstrom type metric on the gauge group space, is then determined. The motion of a test particle in a gravitational field with spherical symmetry can be assimilated with a gauge field theory with a spontaneous symmetry breaking: the gauge gravitational field breaks spontaneously the symmetry of the vacuum state generating the kink solution.

The trajectory of the test particle is contained in the equatorial plane (the motion is plane). The model under consideration has a double degenerated vacuum state (see Fig. 1) and the system source test-particle has a *spontaneous symmetry breaking* (SSB).

REFERENCES

1. T. P. Cheng, L. F. Li, *Gauge Theory of Elementary Particle Physics*, Clarendon Press, Oxford p. 230 (1984).
2. R. Utiyama, Phys. Rev. **101**, 1597 (1956).
3. D.W. Sciama, Rev. Mod. Phys. **36** (1964).
4. D.W. Sciama, Rev. Mod. Phys. **36** 1103 (1964).
5. T. W. B. Kibble, J. Math. Phys. **2** 212 (1961).
6. M. Blagojević: *Three lectures on Poincaré gauge theory*, arXiv:gr-qc/0302040, 11 Feb 2003.
7. G. Zet, V. Manta, S. Babeti: *De-Sitter gauge theory of gravitation*, Int. J. Mod. Phys. **C14**, 41 (2003).

8. G. Zet, C. Popa, D. Partenie, *Schwarzschild-De-Sitter Solution in Quantum Gauge Theory of Gravity,* Commun. Theor. Phys. (Beijing, China), Vol. 47, No. 5, pp. 843–846 (2007).
9. F. Gronwald: *Metric-affine gauge theory of gravity. I. Fundamental structure and field equations*, Int. J. Modern, Physics **D6**, 263 (1997).
10. C. Wiesendanger, Class. Quant. Grav. **13**, 681(1996) and arXiv:gr-qc/9505049.
11. N. Wu, *Renormalizable Quantum Gauge General Relativity*, arXiv:gr-qc/0309041, September 8, 2003.
12. N. Wu, Commun. Theor. Phys. **38**, 151 (2002) and arXiv:hep-th/0207254.
13. N. Wu, Commun. Theor. Phys. **42**, 543 (2004).
14. N. Wu, Commun. Theor. Phys. **39**, 671(2003).
15. G. Zet, I. Gottlieb, V. Manta, Nuovo Cimento **B111**, 607 (1996).
16. G. Zet, Reports on Mathematical Physics Vol. **39**, 33 (1997).
17. L. Landau, F. Lifschitz, Théorie du champs, Ed. Mir, Moscou (1966).
18. M. Chaichian, N. F. Nelipa, *Introduction to Gauge Field Theories*. Springer Verlag, Berlin (1984).
19. M. Agop, M. Jarcău, A. Stroe, *El Naschie's instanton by means of the Schwarzschild's gravitational field*. Chaos, Solitons and Fractals **25**, 781–790 (2005).