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Analysis of Two Robust Learning Control Schemes in the Presence of Random Iteration-Varying Noise

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***Abstract*— This paper deals with the design problem of robust iterative learning control (ILC), in the presence of noise that is varying randomly from iteration to iteration. Two ILC schemes are considered: one adopts the previous iteration tracking error (PITE) and the other adopts the current iteration tracking error (CITE), in the updating law. For both schemes, the convergence results are obtained by using a frequency-domain approach, and a comparison between them is presented from the viewpoints of the convergence condition, robustness against plant uncertainty, and delay compensation. It shows that sufficient conditions can be derived to bound the tracking error and make its expectation monotonically convergent in the sense of $\mathscr{L}_2$-norm, which work effectively with robustness for all admissible plant uncertainties. Furthermore, the sufficient conditions for both schemes can also be formulated in terms of two complementary functions, which do not depend on the delay time as well as the plant uncertainty and, thus, make them convenient to be checked and solved using the frequency-domain tools. Numerical simulations are included to illustrate the effectiveness of the two proposed ILC schemes.**

***Index Terms*— Iterative learning control, monotonic convergence, previous iteration tracking error, current iteration tracking error, random iteration-varying noise, delay compensation.**

I. Introduction

Iterative learning control (ILC) is found to be an attractive technique when it comes to addressing systems that perform the same task repetitively over a finite time interval ([1], [2]). The key feature of ILC is to incorporate information from the previous and/or current iterations in the updating law design, such that the control objective is achieved finally in the sense that the improvement of learning proceeds from one iteration to the next. Owing to its simplicity and effectiveness, ILC has generated considerable interest over the past two decades, in many areas and applications.

Robustness has been considered one of the most important issues in ILC, as argued and demonstrated in [3]. With regard to robust ILC, the model uncertainty has been widely studied. It has been shown that using the frequency-domain approach, the analysis and design of ILC can be achieved, guaranteeing the robust convergence for all admissible model uncertainties (see, e.g., [4]-[6]). One advantage of this approach to ILC is that the convergence condition can be checked and/or solved with many existing tools such as Bode plots, μ-synthesis and Youla parameterization [7]. For the discrete-time counterpart, see [8]. In addition to the model uncertainty, external (load and/or measurement) disturbances have been investigated as a practically important issue in robust ILC design (e.g., [9]), as well as reinitialization errors (see [10], [11]). However, the aforementioned results of robust ILC are derived for repeated (or iteration-invariant) uncertainty. In ILC, the robustness issue on iteration-varying uncertainties has also been discussed with many schemes. For example, using a super vector-based approach, an adaptive ILC has been studied in [12], a higher-order ILC in [13], and an H_∞-optimal ILC in [14]. From the stochastic point of view, a class of promising ILC algorithms have been investigated in, e.g. [15]. For more details, see [16] for ILC with forgetting factor, [17] for ILC with wavelet filter and [18] for ILC with iteration-varying filter, based on which a statistical analysis of robust ILC design has been presented very recently in [19]. Despite of all these existing results, the theory for robust ILC is far from complete, even in the field of linear plants as claimed in [3].

In this paper, two ILC schemes are addressed for uncertain linear plants that are subject to noise varying randomly from iteration to iteration. More specifically the first ILC scheme is considered using the previous iteration tracking error (PITE) in the control updating law, resulting in an open-loop strategy (see [1] and denote it shortly by ILC-1). In contrast to ILC-1, the second ILC scheme$-$denoted by ILC-2$-$is a closed-loop strategy (see also [1]) that uses the current iteration tracking error (CITE) to update the control law. In both ILC schemes, there are two design parameters: the performance weighting function and learning gain function, with different selections of which new properties of ILC-1 and ILC-2 can be obtained. The issues for convergence, implementation, robustness with respect to external noise and model uncertainties, as well as delay effect related to both ILC schemes, are discussed using a frequency-domain analysis approach. It shows that for both ILC schemes, monotonic convergence results can be obtained and certain implementations can be derived to not only show robustness against all admissible model uncertainties but also to achieve the time delay compensation. Simulation tests are finally provided to verify the theoretical study.

Throughout this paper, the ∞-norm and the 2-norm of any

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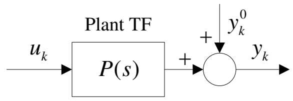

Fig. 1: Control system considered in the presence of noise $y_k^0(t)$.

given function $G(s)$ are defined, respectively, as

$$\|G(s)\|_\infty = \sup_\omega |G(j\omega)| \tag{1}$$

$$\|G(s)\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} |G(j\omega)|^2 \, d\omega}. \tag{2}$$

II. Problem Statement

Let us consider the system shown in Fig. 1, where $P(s)$ is the plant transfer function (TF), and $u_k(t)$, $y_k(t)$, and $y_k^0(t)$ are the control input, system output, and zero-input response, respectively, for $t \in [0, T]$ at the iteration k. From this figure, the output is clearly given by

$$Y_k(s) = P(s)U_k(s) + Y_k^0(s) \tag{3}$$

where $Y_k(s) = \mathscr{L}[y_k(t)]$, $U_k(s) = \mathscr{L}[u_k(t)]$, and $Y_k^0(s) = \mathscr{L}\left[y_k^0(t)\right]$. It can be easily seen that $Y_k^0(s)$ can represent the noise including the reinitialization errors and the load and/or measurement disturbances (like, e.g., [12], [18], [19]).

In the sequel, the Laplace variable s and the time variable t will be omitted when this does not lead to any confusion.

In this paper, the objective is to examine two ILC schemes in the presence of the random iteration-varying noise arising from $Y_k^0(s)$ and make a comparison between results obtained for them. The following two ILC schemes are considered:

- ILC-1: The first ILC adopts the tracking error from the previous iteration in the updating law which is given by

$$U_{k+1}(s) = W_{11}(s)\left[U_k(s) + K_1(s)E_k(s)\right]. \tag{4}$$

- ILC-2: The second ILC uses the tracking error from the current iteration in the updating law which is given by

$$U_{k+1}(s) = W_{12}(s)U_k(s) + K_2(s)E_{k+1}(s). \tag{5}$$

In both schemes, W_{11} and W_{12} are the stable performance weighting functions, K_1 and K_2 are the learning gains, $E_k = Y_d - Y_k = \mathscr{L}[e_k]$, and $Y_d = \mathscr{L}[y_d]$. The variable y_d denotes the bounded reference trajectory, $e_k = y_d - y_k$ denotes the tracking error at the iteration k, and U_0 denotes the arbitrarily selected initial control input that is assumed to be bounded.

Assumptions: Assume that Y^0 denotes noise that is varying randomly from iteration to iteration. Thus, for each iteration k, Y_k^0 can be viewed as a sample selected from the population Y^0. Let $\mathcal{E}$ denote the expectation operator with respect to the iteration domain for a fixed s. Then, for this population Y^0, it is considered that

1) The mean of the noise Y^0 is given by

$$\mathcal{E}\left(Y^0\right) = Y_e^0. \tag{6}$$

2) The $\mathscr{L}_2$-norm of the error $Y^0 - Y_e^0$ is not more than Λ, i.e.,

$$\left\|Y^0 - Y_e^0\right\|_2 \le \Lambda. \tag{7}$$

Remark 1: For each iteration k, the mean of the noise Y_k^0 can be obtained from the issue 1), i.e., $\mathcal{E}\left(Y_k^0\right) = \mathcal{E}\left(Y^0\right) = Y_e^0$, $\forall k$. While through the issue 2), the bound of the variance of the noise Y^0 (hence Y_k^0, $\forall k$) can be provided. In particular, let $\left[y^0(t) - y_e^0(t)\right]^2 \le \lambda^2(t)$ and $\|\lambda(t)\|_2 \le \Lambda$. Thus, the fact (7) is ensured, since $\left\|Y^0(s) - Y_e^0(s)\right\|_2 = \left\|y^0(t) - y_e^0(t)\right\|_2$. Moreover, the variance satisfies

$$\mathrm{Var}\left[y^0(t)\right] = \mathcal{E}\left[y^0(t) - y_e^0(t)\right]^2 \le \lambda^2(t), \ \forall t$$

which leads to

$$\left\|\sqrt{\mathrm{Var}\left[y^0(t)\right]}\right\|_2 \le \Lambda. \tag{8}$$

That is, the standard derivation of the noise can be bounded by Λ, in the sense of $\mathscr{L}_2$-norm.

III. Analysis and Comparison Results

A. Convergence Analysis

First of all, let us consider the ILC-1. Define

$$\mathcal{F}_1 = W_{11}\left(1 - PK_1\right) \tag{9}$$

$$E_* = \frac{1 - W_{11}}{1 - W_{11} + W_{11}PK_1}\left(Y_d - Y_e^0\right) \tag{10}$$

and $e_*(t) = \mathscr{L}^{-1}[E_*(s)]$. Then, the following theorem can be obtained for ILC-1:

Theorem 1: Consider the system in Fig. 1 under the facts of (6) and (7), and let the ILC law (4) be applied. If

$$\|\mathcal{F}_1\|_\infty < 1 \tag{11}$$

then 1) the expected tracking error $\mathcal{E}[e_k(t)]$ is bounded for all k and converges uniformly to $e_*(t)$ as $k \to \infty$, in the sense of $\mathscr{L}_2$-norm, and 2) the tracking error $e_k(t)$ is bounded for all k and satisfies (12) as $k \to \infty$, where

$$\limsup_{k\to\infty} \|e_k(t) - e_*(t)\|_2 \le \frac{1 + \|W_{11}\|_\infty}{1 - \|\mathcal{F}_1\|_\infty}\Lambda. \tag{12}$$

Proof: See Appendix I. ■

Next, let us consider the ILC-2 and correspondingly, define

$$\mathcal{F}_2 = \frac{W_{12}}{1 + PK_2}, \ \mathcal{F}_3 = \frac{1}{1 + PK_2} \tag{13}$$

$$E_\diamond = \frac{1 - W_{12}}{1 - W_{12} + PK_2}\left(Y_d - Y_e^0\right) \tag{14}$$

and $e_\diamond(t) = \mathscr{L}^{-1}[E_\diamond(s)]$. In the same way as in the proof of Theorem 1, one can now show the following theorem related to ILC-2:

Theorem 2: Consider the system in Fig. 1 under the facts of (6) and (7), and let the ILC law (5) be applied. If

$$\|\mathcal{F}_2\|_\infty < 1 \tag{15}$$

then 1) the expected tracking error $\mathcal{E}[e_k(t)]$ is bounded for all k and converges uniformly to $e_\diamond(t)$ as $k \to \infty$, in the sense of $\mathscr{L}_2$-norm, and 2) the tracking error $e_k(t)$ is bounded for all k and satisfies (16) as $k \to \infty$, where

$$\limsup_{k\to\infty} \|e_k(t) - e_\diamond(t)\|_2 \le \frac{1 + \|W_{12}\|_\infty}{1 - \|\mathcal{F}_2\|_\infty}\|\mathcal{F}_3\|_\infty \Lambda. \tag{16}$$

Proof: See Appendix II. ■

Now, the following *Remarks* are in order:

Remark 2: Obviously, the convergence results derived for ILC-1 and ILC-2 are similar to each other. More specifically, for both ILC schemes, the expected tracking error converges monotonically to a certain limit trajectory that can be predefined, and the tracking error varies around this limit trajectory within a bound that is proportional to the bound on noise.

Remark 3: The trajectories E_* and $E_\diamond$ are described in a very similar way, which depend on the learning parameters, the desired trajectory, and the mean of the noise. In particular, if one takes $W_{11} = 1$ in (4) and $W_{12} = 1$ in (5), then one can prove that for all t, $e_*(t) = 0$ and $e_\diamond(t) = 0$. In this case, the expected tracking error for both schemes converges to zero, and the tracking error varies around zero along the time axis within a bound.

Remark 4: From (11) and (15), it is clear that convergence conditions of two ILC schemes both require that the ∞-norm of the transfer function ($\mathcal{F}_1$ or $\mathcal{F}_2$) should be less than 1. This is the standard frequency-domain condition (see, e.g. [4]-[6]). But, $\mathcal{F}_1$ and $\mathcal{F}_2$ are developed in different ways because the previous and current iteration tracking errors are respectively used by ILC-1 and ILC-2 to update their learning laws.

B. Robustness on Plant Uncertainty

Let us consider the plant TF P described by the following uncertain form:

$$P = (1 + \Delta W_2) G_n \tag{17}$$

where G_n is the nominal plant model, W_2 is a known stable TF as the weight, and Δ is an unknown stable TF satisfying $\|\Delta\|_\infty \leq 1$. For formulation, let us take

$$W_{11} = W_1^{-1},\ K_1 = -C \tag{18}$$

in ILC-1 of (4) (like [6]), and take

$$W_{12} = W_1,\ K_2 = C \tag{19}$$

in ILC-2 of (5) (like [5]), where W_1 and C are now learning parameters to be determined. To this end two complementary functions associated with the nominal system are introduced as:

$$S = \frac{1}{1 + G_n C},\ T = 1 - S = \frac{G_n C}{1 + G_n C}. \tag{20}$$

Following the same steps used in [6], one can obtain from (9), (17), (18) and (20) that

$$\begin{aligned} \mathcal{F}_1 &= W_1^{-1} \left[1 + (1 + \Delta W_2) G_n C\right] \\ &= \frac{1}{W_1 (1 + G_n C)^{-1}} \left(1 + \frac{\Delta W_2 G_n C}{1 + G_n C}\right) \\ &= \frac{1 + \Delta W_2 T}{W_1 S} \end{aligned} \tag{21}$$

and that

$$\begin{aligned} E_* &= \frac{1 - W_1^{-1}}{1 - W_1^{-1} - W_1^{-1} (1 + \Delta W_2) G_n C} \left(Y_d - Y_e^0\right) \\ &= \frac{1 - W_1}{1 - W_1 + (1 + \Delta W_2) G_n C} \left(Y_d - Y_e^0\right) \\ &= \frac{(1 - W_1) S}{1 + \Delta W_2 T - W_1 S} \left(Y_d - Y_e^0\right). \end{aligned} \tag{22}$$

Using (21), the convergence condition (11) becomes

$$\left\| \frac{1 + \Delta W_2 T}{W_1 S} \right\|_\infty < 1,\ \forall \Delta \tag{23}$$

which is equivalent to (see [6, Proposition 3])

$$\left\| \frac{1 + |W_2 T|}{W_1 S} \right\|_\infty < 1. \tag{24}$$

That is, the convergence condition for the ILC-1 can be given by (24) which exactly provides a condition to determine both learning parameters W_1 and C in (18). Next, we consider the ILC-2. Using (13), (17), (19) and (20), one can derive that

$$\begin{aligned} \mathcal{F}_2 &= \frac{W_1}{1 + (1 + \Delta W_2) G_n C} \\ &= \frac{W_1 S}{1 + \Delta W_2 T} \end{aligned} \tag{25}$$

and that

$$\begin{aligned} E_\diamond &= \frac{1 - W_1}{1 - W_1 + (1 + \Delta W_2) G_n C} \left(Y_d - Y_e^0\right) \\ &= \frac{(1 - W_1) S}{1 + \Delta W_2 T - W_1 S} \left(Y_d - Y_e^0\right). \end{aligned} \tag{26}$$

Based on (25), the condition of (15) becomes

$$\left\| \frac{W_1 S}{1 + \Delta W_2 T} \right\|_\infty < 1,\ \forall \Delta \tag{27}$$

which, together with $\|W_2 T\|_\infty < 1$, holds if and only if ([7])

$$\|W_2 T\|_\infty < 1,\ \left\| \frac{W_1 S}{1 - |W_2 T|} \right\|_\infty < 1. \tag{28}$$

Thus, (28) can give the convergence condition for the ILC-2, and also provides the condition to design W_1 and C in (19).

With the above analysis, the following *Remarks* are stated:

Remark 5: From (22) and (26), it can be clearly seen that with the aid of S and T, the trajectories E_* and $E_\diamond$ are given in the same way, which further explains the aforementioned fact of Remark 3. Moreover, it can be easily seen that both of the trajectories are well defined if (23) and (27) are satisfied, since they can ensure that for all ω, $|1 + \Delta W_2|$ is not equal to $|W_1 S|$.

Remark 6: As mentioned in Remark 4, $\mathcal{F}_1$ and $\mathcal{F}_2$ are of different formulation ways. From (21) and (25), it is obvious that one can be formulated in form of the inverse of the other. However, both $\mathcal{F}_1$ and $\mathcal{F}_2$ can be expressed in terms of two complementary parameters S and T. Benefiting from such an expression, the convergence condition for both ILC schemes can be simplified into (24) and (28), respectively, which are independent of the uncertainty and can be easily checked via using the frequency-domain tools, e.g., Bode plots.

C. Effect of Delay Factor

Now, let us consider the plant TF P having a delay factor, which is given by

$$P = G e^{-\theta s} \tag{29}$$

where G and θ are unknown TF and delay, respectively. For the plant TF of (29), a nominal model is described by

$$\hat{P} = G_n e^{-\hat{\theta} s} \tag{30}$$

where G_n is a known TF, and $\hat{\theta}$ is an estimate of θ. Assume that P is perturbed and described in the following uncertain form:

$$P = (1 + \Delta W_2)\,\hat{P} \tag{31}$$

where W_2 is a known stable TF, and Δ is an unknown stable TF satisfying $\|\Delta\|_\infty \leq 1$.

In order to deal with the delay factor, different approaches are employed by the ILC-1 and ILC-2. For the first ILC, an anticipatory approach is used, which takes learning parameters of (4) as follows (see also [6]):

$$W_{11} = W_1^{-1},\ K_1 = -Ce^{\hat{\theta}s}. \tag{32}$$

As argued in [6], the anticipatory form of ILC-1 is available, since a feedforward action is required, in essence, to generate the current control input U_{k+1}. For the second ILC, a Smith predictor-based approach is used (see also [4]). To this end, the learning parameters of (5) are taken as

$$W_{12} = \frac{W_1}{M},\ K_2 = \frac{C}{M} \tag{33}$$

where $M = 1 + \left(G_n - \hat{P}\right)C$ is generated from the structure of the Smith predictor.

Considering (9) and inserting (30)-(32), one can obtain for the ILC-1 that

$$\begin{aligned}\mathcal{F}_1 &= W_1^{-1}\left[1 - (1 + \Delta W_2)\,\hat{P}\left(-Ce^{\hat{\theta}s}\right)\right]\\ &= W_1^{-1}\left[1 + (1 + \Delta W_2)G_nC\right]\end{aligned} \tag{34}$$

which, by following the same steps of (21), yields

$$\mathcal{F}_1 = \frac{1 + \Delta W_2 T}{W_1 S}.$$

Similarly, one can also obtain

$$\begin{aligned}E_* &= \frac{\left(1 - W_1^{-1}\right)\left(Y_d - Y_e^0\right)}{1 - W_1^{-1} + W_1^{-1}(1 + \Delta W_2)\,\hat{P}\left(-Ce^{\hat{\theta}s}\right)}\\ &= \frac{1 - W_1^{-1}}{1 - W_1^{-1} - W_1^{-1}(1 + \Delta W_2)G_nC}\left(Y_d - Y_e^0\right)\end{aligned} \tag{35}$$

which, in view of (22), leads to

$$E_* = \frac{(1 - W_1)S}{1 + \Delta W_2 T - W_1 S}\left(Y_d - Y_e^0\right).$$

Hence $\mathcal{F}_1$, as well as E_*, has the same formulation with that derived in the case without delay, which benefits from using the anticipation $\hat{\theta}$ in the time axis. For more details, see [6]. Consequently, the convergence condition for the ILC system (3), (4), and (30)-(32) can be given by (24).

For the ILC-2, using (13) and by inserting (30), (31) and (33), it yields

$$\begin{aligned}\mathcal{F}_2 &= \frac{(W_1/M)}{1 + (1 + \Delta W_2)\,\hat{P}\,(C/M)}\\ &= \frac{W_1}{M + (1 + \Delta W_2)\,\hat{P}C}\\ &= \frac{W_1}{1 + G_nC + \Delta W_2 G_n C e^{-\hat{\theta}s}}\\ &= \frac{W_1 S}{1 + \Delta W_2 T e^{-\hat{\theta}s}}\end{aligned} \tag{36}$$

and

$$\begin{aligned}E_\diamond &= \frac{1 - (W_1/M)}{1 - (W_1/M) + (1 + \Delta W_2)\,\hat{P}\,(C/M)}\left(Y_d - Y_e^0\right)\\ &= \frac{M - W_1}{M - W_1 + (1 + \Delta W_2)\,\hat{P}C}\left(Y_d - Y_e^0\right)\\ &= \frac{1 + G_nC - G_nCe^{-\hat{\theta}s} - W_1}{1 + G_nC + \Delta W_2 G_n C e^{-\hat{\theta}s} - W_1}\left(Y_d - Y_e^0\right)\\ &= \frac{1 - Te^{-\hat{\theta}s} - W_1 S}{1 + \Delta W_2 T e^{-\hat{\theta}s} - W_1 S}\left(Y_d - Y_e^0\right).\end{aligned} \tag{37}$$

Using (36) and following [4], one can show that the condition of (15) is still satisfied, i.e.,

$$\left\|\frac{W_1 S}{1 + \Delta W_2 T e^{-j\omega\hat{\theta}}}\right\|_\infty < 1,\ \forall\Delta \tag{38}$$

if (28) holds. That is, the convergence condition for the ILC system (3), (5), (30), (31) and (33) can be given by (28).

Now, the following *Remarks* are given:

Remark 7: From the above development, one can find that if the learning gains of (32) are taken in the ILC-1, and those of (33) are taken in the ILC-2, then the convergence condition for both schemes can be derived independent of not only the plant uncertainty but also the delay factor. That is, conditions designed for the nominal LTI model can work with sufficient robustness with respect to the uncertainties arising from both plant model and delay time.

Remark 8: Due to its structure, the ILC-1 can be designed with an action using anticipation in time to compensate delay. The anticipatory property of ILC has already been discussed, e.g., in [1], [6]. Clearly, it can be seen that one advantage of this property is that the design of ILC for an uncertain delay plant can be achieved by considering related nominal model that does not include delay. In contrast to [6], here the ILC is extended to more general situations, which relaxes restricted requirements on the initial conditions and disturbances.

Likewise, the ILC-2, designed with (33), can compensate delay, since a Smith predictor is incorporated. In comparison with the existing results (e.g., [4]), random iteration-varying noise is taken into account, which makes the obtained results more applicable.

Remark 9: For the ILC-1 the trajectory E_* keeps the same for both cases with and without delay, and becomes the zero one when $W_1 = 1$ is taken. Moreover, E_* is independent of the delay θ and its estimation $\hat{\theta}$. Hence, this limit trajectory for TDS is determined in the delay-free case, rather than the delay-dependent case.

For the ILC-2, it is obvious from (37) that $E_\diamond$ depends on the estimation $\hat{\theta}$. If $W_1 = M$ is used, thus yielding $W_{12} = 1$, then $E_\diamond$ still becomes zero (see Remark 3).

Remark 10: Particularly, if $\hat{\theta} = 0$ is set, then the results of this subsection will become those of the previous subsection. For this case, the ILC-1 and ILC-2 offer standard open-loop and closed-loop schemes for linear plants, respectively. As a matter of fact, if θ is small, then the delay plant of (29) can be treated by embedding P in a family where the representative

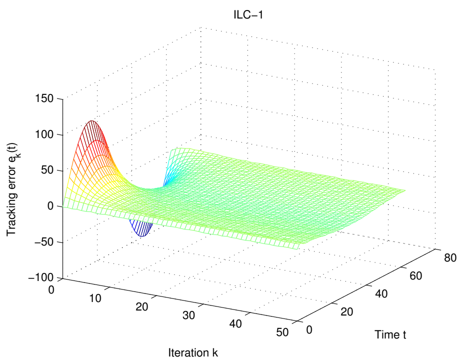

Fig. 2: Process of the tracking error for ILC-1.

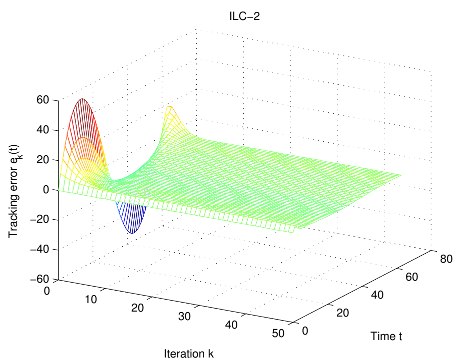

Fig. 3: Process of the tracking error for ILC-2.

element is described by (17) (or equivalently, (31) with $\hat{\theta} = 0$). For the details of such a design approach, one can refer to [7, pp. 46-48] which is based on the Bode magnitude plots.

IV. Simulation Results

In this example, the delay plant of (29) is considered where (see also the example in [4])

$$G_n = \frac{1.2}{1.3s+1},\ W_2 = \frac{2s+1}{0.7s+5}$$

the noise $y_k^0(t)$ is considered to be varying randomly within $[-0.1, 0.1]$, along both time axis t and iteration axis k, and the simulation test is performed with $y_d(t) = 100\sin(0.1t)$, $t \in [0, 20\pi]$. For both ILC schemes, the zero initial input is adopted, i.e., $u_0(t) = 0$ for all t.

To implement the ILC-1, the design parameters of (32) are used, and more specifically,

$$W_{11} = \frac{1}{0.1s+1},\ K_1 = \frac{s^2+2s+2}{0.1s^2+12s+10}e^{\hat{\theta}s}$$

which leads to

$$\left\|\frac{1+|W_2T|}{W_1S}\right\|_\infty = 0.9706.$$

For the ILC-2, the design parameters of (33) are used, i.e.,

$$W_{12} = \frac{1}{0.1s+1}\cdot\frac{1}{M},\ K_2 = \frac{s^2+2s+1}{0.1s^2+10s+1}\cdot\frac{1}{M}$$

which leads to

$$\left\|\frac{W_1S}{1-|W_2T|}\right\|_\infty = 0.9787.$$

Clearly, the convergence conditions (24) and (28) hold. Then the simulation results are shown in Figs. 2 and 3, respectively for the ILC-1 and the ILC-2. In both figures the evolution of the tracking error $e_k(t)$ along time axis t and iteration axis k is described. It can be seen that the nice iteration-to-iteration error convergence is possible. This illustrates that both ILC-1 and ILC-2 can work effectively when the uncertain delay plant is subject to random iteration-varying noise.

V. Conclusions

In this paper, two ILC schemes have been discussed, with respect to uncertain linear systems in the presence of random iteration-varying noise. It has been proved that under certain conditions, monotonic convergence of both ILC schemes can be guaranteed in the sense of expectation, which ensures that the expected tracking error is monotonically convergent to a limit trajectory that can be predefined, and the tracking error is convergent to a neighborhood of this limit trajectory, with an error bound proportional to the bound on noise. Numerical simulations have been provided to illustrate the effectiveness of these theoretical results, as well as robustness against the model uncertainty and good property of delay compensation.

Appendix I: Proof of Theorem 1

Proof: For the ILC system (3) and (4), it can be easily shown that the tracking errors, at two sequential iterations k and $k+1$, satisfy

$$\begin{aligned} E_{k+1} &= Y_d - PU_{k+1} - Y_{k+1}^0 \\ &= Y_d - W_{11}PU_k - W_{11}PK_1E_k - Y_{k+1}^0 \\ &= (1-W_{11})\,Y_d + \mathcal{F}_1E_k + W_{11}Y_k^0 - Y_{k+1}^0 \end{aligned} \quad (39)$$

which, by taking the operation $\mathcal{E}$ and inserting (6), leads to

$$\begin{aligned} \mathcal{E}\left(E_{k+1}\right) &= (1-W_{11})\,Y_d + \mathcal{F}_1\mathcal{E}\left(E_k\right) \\ &\quad + W_{11}\mathcal{E}\left(Y_k^0\right) - \mathcal{E}\left(Y_{k+1}^0\right) \\ &= (1-W_{11})\left(Y_d - Y_e^0\right) + \mathcal{F}_1\mathcal{E}\left(E_k\right). \end{aligned} \quad (40)$$

If one defines

$$E_* = \frac{1-W_{11}}{1-\mathcal{F}_1}\left(Y_d - Y_e^0\right) \quad (41)$$

(due to (9), (41) is the same with (10)), then it can be derived from (10) and (40) that

$$\mathcal{E}\left(E_{k+1}\right) - E_* = \mathcal{F}_1\left[\mathcal{E}\left(E_k\right) - E_*\right] \quad (42)$$

which results in

$$\left\|\mathcal{E}\left[e_{k+1}(t)\right] - e_*(t)\right\|_2 \le \left\|\mathcal{F}_1\right\|_\infty \left\|\mathcal{E}\left[e_k(t)\right] - e_*(t)\right\|_2 \quad (43)$$

and thus

$$\left\|\mathcal{E}\left[e_k(t)\right] - e_*(t)\right\|_2 \le \left\|\mathcal{F}_1\right\|_\infty^k \left\|\mathcal{E}\left[e_0(t)\right] - e_*(t)\right\|_2. \quad (44)$$

With the fact that $y_d(t)$ and $u_0(t)$ are bounded, one can easily conclude that $e_0(t)$ (hence, $\mathcal{E}\left[e_0(t)\right]$) is bounded. Since W_{11} is stable, one can also conclude that if (11) is satisfied, then $\mathcal{E}\left[e_k(t)\right]$ is bounded for all k, and

$$\lim_{k\to\infty}\left\|\mathcal{E}\left[e_k(t)\right] - e_*(t)\right\|_2 = 0 \quad (45)$$

(monotonic convergence in the sense of $\mathscr{L}_2$-norm). Moreover subtracting (40) from (39) yields

$$E_{k+1} - \mathcal{E}\left(E_{k+1}\right) = \mathcal{F}_1\left[E_k - \mathcal{E}\left(E_k\right)\right] + W_{11}\left(Y_k^0 - Y_e^0\right) - \left(Y_{k+1}^0 - Y_e^0\right) \quad (46)$$

which, under the fact of (7), leads to

$$\left\|e_{k+1}(t) - \mathcal{E}\left[e_{k+1}(t)\right]\right\|_2 \le \left\|\mathcal{F}_1\right\|_\infty \left\|e_k(t) - \mathcal{E}\left[e_k(t)\right]\right\|_2 + \left(1 + \left\|W_{11}\right\|_\infty\right)\Lambda. \quad (47)$$

Hence, if (11) holds, then it can be obtained from (47) that

$$\limsup_{k\to\infty} \left\|e_k(t) - \mathcal{E}\left[e_k(t)\right]\right\|_2 \le \frac{1 + \left\|W_{11}\right\|_\infty}{1 - \left\|\mathcal{F}_1\right\|_\infty}\Lambda \quad (48)$$

and consequently, (12) can be shown by using (45) and (48). This proof is completed. ■

Appendix II: Proof of Theorem 2

Proof: Similar to (39), the tracking errors E_k and E_{k+1} for the ILC system (3) and (5) satisfy

$$\begin{aligned} E_{k+1} &= Y_d - PU_{k+1} - Y_{k+1}^0 \\ &= Y_d - W_{12}PU_k - PK_2E_{k+1} - Y_{k+1}^0 \\ &= \left(1 - W_{12}\right)Y_d + W_{12}E_k - PK_2E_{k+1} \\ &\quad + W_{12}Y_k^0 - Y_{k+1}^0 \end{aligned} \quad (49)$$

and hence

$$\begin{aligned} E_{k+1} &= \frac{1 - W_{12}}{1 + PK_2}Y_d + \frac{W_{12}}{1 + PK_2}E_k \\ &\quad + \frac{W_{12}}{1 + PK_2}Y_k^0 - \frac{1}{1 + PK_2}Y_{k+1}^0 \\ &= \left(\mathcal{F}_3 - \mathcal{F}_2\right)Y_d + \mathcal{F}_2E_k + \mathcal{F}_2Y_k^0 - \mathcal{F}_3Y_{k+1}^0 \end{aligned} \quad (50)$$

from which one can obtain

$$\begin{aligned} \mathcal{E}\left(E_{k+1}\right) &= \left(\mathcal{F}_3 - \mathcal{F}_2\right)Y_d + \mathcal{F}_2\mathcal{E}\left(E_k\right) \\ &\quad + \mathcal{F}_2\mathcal{E}\left(Y_k^0\right) - \mathcal{F}_3\mathcal{E}\left(Y_{k+1}^0\right) \\ &= \left(\mathcal{F}_3 - \mathcal{F}_2\right)\left(Y_d - Y_e^0\right) + \mathcal{F}_2\mathcal{E}\left(E_k\right). \end{aligned} \quad (51)$$

Moreover, if let

$$E_\diamond = \frac{\mathcal{F}_3 - \mathcal{F}_2}{1 - \mathcal{F}_2}\left(Y_d - Y_e^0\right) \quad (52)$$

which is equivalent to (14), then it can be obtained that

$$\mathcal{E}\left(E_{k+1}\right) - E_\diamond = \mathcal{F}_2\left[\mathcal{E}\left(E_k\right) - E_\diamond\right] \quad (53)$$

which leads to

$$\begin{aligned} \left\|\mathcal{E}\left[e_k(t)\right] - e_\diamond(t)\right\|_2 &\le \left\|\mathcal{F}_2\right\|_\infty \left\|\mathcal{E}\left[e_{k-1}(t)\right] - e_\diamond(t)\right\|_2 \\ &\le \left\|\mathcal{F}_2\right\|_\infty^k \left\|\mathcal{E}\left[e_0(t)\right] - e_\diamond(t)\right\|_2. \end{aligned} \quad (54)$$

Thus, one can conclude that if (15) is satisfied, then $\mathcal{E}\left[e_k(t)\right]$ is bounded for all k, and $\lim_{k\to\infty}\left\|\mathcal{E}\left[e_k(t)\right] - e_\diamond(t)\right\|_2 = 0$ (monotonic convergence in the sense of $\mathscr{L}_2$-norm). Clearly, from (50) and (51), one has

$$E_{k+1} - \mathcal{E}\left(E_{k+1}\right) = \mathcal{F}_2\left[E_k - \mathcal{E}\left(E_k\right)\right] + \mathcal{F}_2\left(Y_k^0 - Y_e^0\right) - \mathcal{F}_3\left(Y_{k+1}^0 - Y_e^0\right) \quad (55)$$

which, together with (7), (13) and (15), yields

$$\limsup_{k\to\infty} \left\|e_k(t) - \mathcal{E}\left[e_k(t)\right]\right\|_2 \le \frac{1 + \left\|W_{12}\right\|_\infty}{1 - \left\|\mathcal{F}_2\right\|_\infty}\left\|\mathcal{F}_3\right\|_\infty\Lambda. \quad (56)$$

From this and the convergence of $\mathcal{E}\left[e_k(t)\right]$ to $e_\diamond(t)$, (16) is immediate. This proof is completed. ■

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