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A fuzzy risk approach for performance evaluation of an irrigation reservoir system

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Abstract

In this paper, a model for fuzzy risk of low yield of a crop is developed to study the implications of a reservoir operating policy model. When an optimal operating policy is derived based on a known objective, the policy itself does not, in general, indicate a measure of the system performance unless a criterion to this effect is embedded in the objective function. While a systems analyst is interested in the nature of the objective function used in arriving at a policy, the irrigation decision maker would look for the implications of using the policy through answers to the questions such as, how often the system will fail and how quickly it will recover from a failure. It is, therefore, important that the implications of reservoir operation with a given policy be studied keeping in view the interests of the decision makers. Some earlier studies on reservoir operation models for irrigation have considered reliability, resiliency and productivity index, as the performance indicators of the operating policy. In this paper, fuzzy risk of low yield of a crop is considered as another performance indicator to address uncertainties due to both randomness and fuzziness. Uncertainty due to randomness arises primarily because of the random variations of hydrologic variables such as reservoir inflows and rainfall in the command area. Uncertainty due to imprecision or fuzziness arises because of uncertain crop yield response to various factors (such as farm practices and climatic variables) other than to the applied water. Two important concepts are introduced in this paper with respect to irrigation reservoir system. The first one is related to viewing the low yield of a crop, as a fuzzy event. The second concept is related to the definition of fuzzy risk of low yield of a crop. The fuzzy risk of low yield is derived using the concept of probability of a fuzzy event. Application of the methodology is demonstrated with a case study in India.

Keywords: Fuzzy risk; Fuzzy event; Low crop yield; Crisp risk

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Nomenclature

A_{c}	area in which crop c is grown
c	crop index
d	class interval to which available water w in period t belongs
d_i	number of discrete class intervals of available water
$\mathrm{DM}_{\mathrm{c}}^{t}$	demand from crop c in period t
ER_t	effective rainfall in period t
$\mathrm{ET}_{\mathrm{ac}}^{t}$	actual evapotranspiration of crop c in period t
$\mathrm{ET}_{\mathrm{pc}}^{t}$	potential evapotranspiration of a crop c in period t
F_{c}	farm practices for crop c
FETDM	fuzzy evapotranspiration deficit model
$G(k, i, l, M, p, t)$	system performance measure corresponding to storage class interval k in period t and l in period $t+1$, inflow class i, rainfall class p and soil moisture vector $\boldsymbol{M}$ in period t
i	class interval to which the inflow in period t belongs
j	class interval to which the inflow in period $t+1$ belongs
k	class interval to which storage at the beginning of period t belongs
l	class interval to which storage at the beginning of period $t+1$ belongs
l^*	optimal value of l
$l^*\ (k, i, p, \boldsymbol{M}, t)$	optimal end of period storage, for a given initial storage class interval k, inflow class interval i, rainfall class interval p, and initial soil moisture vector of crops $\boldsymbol{M} = \{m_1, m_2, \ldots, m_{\mathrm{NC}}\}$
L_{c}	crisp set of low relative yield of a crop c
m_{c}	class interval to which soil moisture of crop c at the beginning of period t belongs
$\boldsymbol{M}$	soil moisture vector in period t, $m_1, \ldots, m_{\mathrm{NC}}$, representing the initial soil moisture class intervals of crops in period t
$\boldsymbol{M} = \{m_1, \ldots, m_{\mathrm{NC}}\}$	soil moisture vector representing the soil moisture class intervals of crops in period t
nk	class interval to which the relative yield of a crop belongs
NC	number of crops
NC_t	number of crops in time period t
p	class interval to which the rainfall in period t belongs
$\tilde{P}$	probability of a fuzzy event
P_{ij}^{t}	probability that the inflow Q_{t+1} in time period $t+1$ is in class interval j, given the inflow Q_t in time period t is in class interval i

P_{ij}^{t}	probability that the inflow in period $t+1$ lies in the class interval j given that the inflow in period t lies in the class interval i
P_{p}^{t}	probability of rainfall being in class interval p in period t
$R_{\mathrm{c}}^{\mathrm{a}}$	acceptable value of the relative yield of crop c
$R_{\mathrm{c}}^{\mathrm{m}}$	minimum value of the relative yield of crop c
R_{kilt}	release from the reservoir in period t when the storage at the beginning of the period is S_{k}^{t}, inflow during the period is Q_{i}^{t} and storage at the beginning of period $t+1$ is S_{l}^{t+1}
RAIN_{p}^{t}	representative value of rainfall in class interval p in period t
t	time period (decision interval)
T_e	temperature
w	amount of water available at a given stage in the allocation problem
w_d	representative value of water corresponding to class interval d
W_{c}	fuzzy set of low relative yields of a crop
x_{c}^{t}	optimal irrigation allocation to crop c in period t
X_t	amount of water available at the field for allocation in period t corresponding to R_{kilt}
$X_t = \beta R_{kilt}$	known amount of water available at the field for allocation in period t, corresponding to R_{kilt}
α	reliability
β	loss factor accounting for all losses in R_{kilt}
γ	resiliency
μ	membership function
Φ_{c}^{t}	defuzzified value of weighted evapotranspiration deficit of crop c in period t
Ψ_{c}	relative yield of crop c
Ω_{c}	crisp risk of crop c
Υ_{c}	fuzzy risk of low relative yield of crop c

1. Introduction

Irrigation reservoir operation problems are charecterised by uncertainties due to randomness and imprecision (or fuzziness). Uncertainty due to randomness associated with inflows to the reservoir has received much attention (Dudley and Burt, 1973; Dudley and Musgrave, 1988; Dudley, 1988,b; Karamouz and Houck, 1982; Vedula and Mujumdar, 1992; Vedula and Nagesh Kumar, 1996). Random nature of rainfall in the command area has also been considered in some studies (Vedula and Nagesh Kumar, 1996). Uncertainty due to imprecision is as prominent as randomness and is little addressed in the area of irrigation reservoir management.

Uncertainties due to imprecision in objectives and model parameters in water resources problems have been modelled with fuzzy sets in some recent work (Bardossy and Disse, 1993; Bardossy and Duckstein, 1995; Bogardi et al., 1983; Fontane et al., 1997; Shrestha et al., 1996; Tilmant et al., 2002; Teegavarapu and Simonovic, 1999). Panigrahi and Mujumdar (2000) developed a fuzzy rule based model for the operation of a single purpose reservoir. The approach they adopted is essentially the same as that of Russel and Campbell (1996), Shrestha et al. (1996), with the difference that the expert knowledge for framing the fuzzy rules is derived from an explicit stochastic model. Efforts have been made for simultaneous treatment of randomness and fuzziness in water quality management of river systems (Mujumdar and Sasikumar, 2002; Sasi Kumar and Mujumdar, 2000). Sasi Kumar and Mujumdar (2000) have presented a theoretical framework to include both randomness and fuzziness in river water quality management models. The concept of probability of a fuzzy event is used to link probability with fuzzy sets.

When an optimal operating policy is derived based on a known objective, the policy itself does not, in general, indicate a measure of the system performance unless a criterion to this effect is embedded in the objective function. While a system analyst is interested in the details of the objective function used in arriving at a policy, the irrigation decision maker would look for the implications of using the policy through answers to the questions such as how often the system will fail and how quickly it will recover from a failure. It is, therefore, important that the implications of reservoir operation with a given policy be studied keeping in view the interests of the irrigation decision maker. It must be noted that in most cases, the irrigation decision maker is also the systems analyst and this distinction is only theoretical.

Performance indicators used to study the implications of the operating policy, such as reliability and resiliency (with reference to the adequacy of water supply to meet the irrigation requirement) and productivity index (with reference to crop yield) provide useful information on the overall performance of the system (Mujumdar and Vedula, 1992). The productivity index is derived based on the assumption that the crop yield is affected by randomness of water supply only. For an irrigation reservoir system, randomness is not the only relevant uncertainty but imprecision (fuzziness) may also be of considerable importance. Uncertainty due to randomness arises primarily because of the random variations of hydrologic variables such as reservoir inflows and rainfall in the command area. Uncertainty due to imprecision or fuzziness arises because of uncertain crop yield response to various factors (such as farm practices and climatic variables) other than to the applied water. For example, crop yield response to temperature changes during the crop period and the farm practices followed by the farmers is not quantifiable precisely, and thus introduces uncertainty due to imprecision. The yield of a crop, which is realised at the end of the season depends on these factors in addition to water allocation. There is thus a fuzziness associated with quantification of the crop yield. This necessitates a performance measure, which considers both types of uncertainty.

In this paper, a model for the evaluation of the fuzzy risk of low yield of a crop is developed with respect to optimal allocations to individual crops. A schematic diagram for the computation of fuzzy risk of low yield of a crop is shown in Fig. 1. A steady state reservoir operating policy, such as that derived in Mujumdar and Vedula (1992), which integrates reservoir release decisions with water utilisation by crops, is used to simulate the reservoir operation over a long period of time. Synthetically generated inflows are used for simulation. From the simulation results, the actual evapotranspiration, ET_{ac}^{t} for each crop c in period t is determined. Using a dated production function, with simulated ET_{ac}^{t} values as

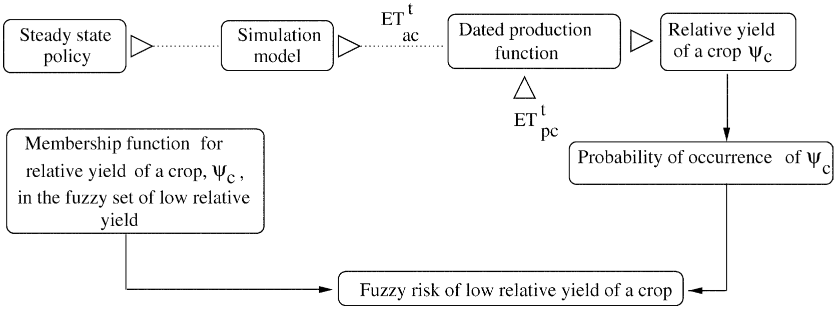

Fig. 1. Evaluation of fuzzy risk of low yield of a crop. ET^t_{ac}, actual evapotranspiration of crop c in period t; ET^t_{pc}, potential evapotranspiration of crop c in period t; Ψ_c, relative yield of a crop.

inputs, the yield of crop c, Ψ_c is determined for all years of simulation. The probability of occurrence of the yield of crop c, Ψ_c, corresponding to a number of discrete class intervals is estimated with a relative frequency approach. In the estimation of the fuzzy risk of low crop yield, subjectivity in classification of the yield of a crop into a *satisfactory (working)* state and a *nonsatisfactory (failed)* state is addressed by defining the low yield of a crop as a fuzzy event. A fuzzy set with an appropriate membership function is defined to describe not only the working state or the failed state but any intermediate state also.

Two important concepts are introduced in this paper. The first one is related to viewing the low yield of a crop, as a fuzzy event. The second concept is related to the definition of fuzzy risk of low yield of a crop. Risk of low yield is derived using the concept of probability of a fuzzy event (Zadeh, 1968). Fuzzy risk of low yield of a crop for an irrigation system is evaluated using these concepts. For convenience in presentation, the relative yield of a crop (ratio of actual yield to maximum yield) is simply referred to as ‘yield’ in this paper.

2. Low yield as a fuzzy event

The yield of a crop is realised only at the end of the season. Success or failure of a crop depends on the yield. In the classical crisp terminology, a crop is a failure if Ψ_c is less than some minimum yield, R^m_c. In other words, a crop yield lower than R^m_c represents a *non-satisfactory (failure)* state. On the other hand, a crop yield Ψ_c greater than R^m_c represents a *satisfactory (working)* state. Linguistically, the farmers’ goal may be expressed as making the crop yield as close to the maximum yield (i.e., $\Psi_c = 1$) as possible. Depending on the type of crop, the cost of expenditure and the prevailing market price, the farmer may be fully satisfied if he gets an acceptable value of the yield, R^a_c. Fixing the acceptable value of the yield of a crop, R^a_c involves many considerations. These acceptable values may be fixed based on the expert input. As a thumb rule, if the benefit:cost ratio is more than 2 for a given crop, the farmer is fully satisfied (Yellamanda Reddy and Sankara Reddi, 2001). The income for the maximum yield is estimated based on the prevailing market rates.

The acceptable R^a_c depends on the socio- economic conditions of a farmer. Based on the experience, agricultural experts with knowledge on local conditions may prescribe R^a_c values for each crop. Depending on Ψ_c, the state of the yield may be described for that crop. In the crisp definition, a crop yield is acceptable when it is greater than or equal to the acceptable value R_c^a. On the other hand, if it is less than R_c^a, it represents a failed state of the system and termed as low yield for that crop. This crisp definition of the low yield of a crop may be expressed mathematically using a characteristic function of Ψ_c as follows:

$$\mu(\Psi_c) = \begin{cases} 1, & (\Psi_c < R_c^a) \\ 0, & (\Psi_c > R_c^a) \end{cases} \quad (1)$$

The crisp set L_c of the yield values that belong to low yield for crop c is defined as

$$L_c = \Psi_c : \mu(\Psi_c) = 1 \quad (2)$$

Characteristic functions of L_c can be represented by a step function as shown in Fig. 2. Under this definition of low yield of a crop, any yield value that is greater than, but very close to, R_c^a is not considered as low yield. This leads to a very stringent definition of low yield. In real situations, it may be more advantageous to describe the state of the yield of a crop Ψ_c with degrees of failure or success state, rather than with the binary classification of either a failure or success. In other words, a system may be described as being in partly working or partly failed state. A situation like this may effectively be treated using fuzzy sets. A fuzzy set with an appropriate membership function is defined to describe both the working and failed states of a crop. The set of low yields of a crop is defined as a fuzzy set, W_c, instead of crisp set, L_c. Each value of yield in the set W_c is assigned a membership value that lies in the closed interval [0,1]. Mathematically, the fuzzy set W_c of low yield of a crop is expressed as follows:

$$W_c = \{\Psi_c : 0 \leq \mu_{W_c}(\Psi_c) \leq 1\} \quad (3)$$

where $\mu_{W_c}(\Psi_c)$ is the degree of membership function of the yield in the fuzzy set W_c. In the crisp definition of low yield, each value of yield in the crisp set L_c is assigned a membership value of either one or zero depending on whether that value corresponds to low yield or not. On the other hand, the fuzzy set W_c allows partial membership also for the yield values. The fuzzy membership value indicates the degree of compatibility of the crop yield with the notion of low yield. The membership function that assigns membership values to the elements of the fuzzy set of low yield thus modifies the conventional definition of low yield and makes it more flexible and realistic.

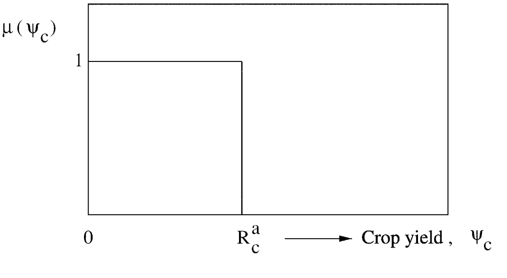

Fig. 2. Characteristic function for the crisp set, L_c. R_c^a, acceptable value of the relative yield of crop c coresponding to which the farmer was fully satisfied; Ψ_c, relative yield of a crop; $\mu(\Psi_c)$, degree of membership of relative yield of crop c in the crisp set L_c.

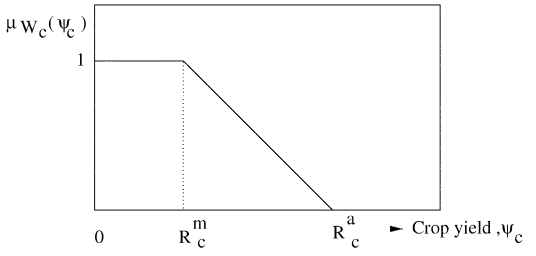

Fig. 3. Membership function for the fuzzy set of low yield of a crop. R_c^a, acceptable value of the relative yield of crop c corresponding to which the farmer was fully satisfied; R_c^m, minimum desired value of the relative yield of crop c; Ψ_c, relative yield of a crop; $\mu_{W_c}(\Psi_c)$, degree of membership function of the relative yield in the fuzzy set W_c.

It may be seen from Fig. 3 that the yield values greater than R_c^m, also correspond to low yield but to a lesser degree than those less than R_c^m. Mathematically, the linear membership function shown in Fig. 3 may be expressed as follows:

$$\mu_{W_c}(\Psi_c) = \begin{cases} 1, & (\Psi_c < R_c^m) \\ \dfrac{(R_c^a - \Psi_c)}{(R_c^a - R_c^m)}, & (R_c^m < \Psi_c < R_c^a) \\ 0, & (\Psi_c > R_c^a) \end{cases} \tag{4}$$

Eq. (4) is interpreted as follows: yield values less than R_c^m represent a low yield value with a membership grade equal to 1 in the fuzzy set of low yield, W_c and those greater than R_c^a represent a low yield value with a membership grade equal to 0. Yield values between R_c^m and R_c^a represent low yield with a membership grade varying from 1 to 0, in the fuzzy set, W_c. All yield values are thus mapped to the fuzzy set of low yield, with varying degrees of membership.

2.1. Membership function for low yield of a crop

The aspirations and requirements of the decision maker may be taken as the deciding factors in setting the guidelines for selection of appropriate membership functions for the fuzzy sets of low yield of a crop. In the present case, to demonstrate the applicability of the model, simple linear membership functions are considered. In order to develop a membership function for low yield of a crop, the following steps are followed:

1. From the field surveys, the amount of seedlings, amount of fertilisers and manure used, type of pest control and the type of weedicides, per unit crop area, are known.
2. With the above inputs for each crop, the total cost of expenditure from the time of sowing to harvesting is determined, using local market rates.
3. From the field surveys and available literature, the expected maximum and minimum yield (in weight) for each crop per unit area is assessed (Handbook of Agriculture, 2001; Yellamanda Reddy and Sankara Reddi, 2001).
4. With the prevailing market rates, the expected income for each crop is determined for maximum and minimum yields.

5. Income corresponding to the maximum yield for each crop serves as a bench mark. The yield corresponding to an income equal to the cost of expenditure is determined and is taken as the lower limit (R_c^m) in the membership function.
6. If the income is equal to or less than expenditure, the farmer is under loss and his satisfaction is zero. The yield of a crop is then low to a degree 1. On the other hand, if he gets a yield equal to or more than the acceptable yield R_c^a, he will be fully satisfied. Then, crop yield is low to a degree 0. The acceptable yield for crop c, R_c^a is fixed based on expert input and is different for different crops. For any intermediate yield values, the degree of membership varies from zero to one. The fuzzy membership function (Fig. 3) is thus fully constructed.

Field surveys are conducted for each crop by interviewing a number of farmers. The average response of all farmers for each crop is considered. Different membership functions are defined for different crops.

2.2. Fuzzy risk of low yield

The definition of low yield of a crop as a fuzzy event necessitates use of the concept of probability of a fuzzy event (Zadeh, 1968). The fuzzy risk of low yield is defined as the probability of occurrence of the fuzzy event of low crop yield.

Mathematically, this can be stated as follows:

$$\text{fuzzy risk} = P[\text{fuzzy event of low relative yield of a crop}] \tag{5}$$

$$\text{fuzzy risk} = \tilde{P}[\text{low relative yield of a crop}] \tag{6}$$

where $\tilde{P}$ denotes the probability of a fuzzy event.

Zadeh (1968) has defined the probability of a fuzzy event A_f as follows:

$$\tilde{P}(A_f) = \int_{R^n} \mu_{A_f}(y) dP \tag{7}$$

where R^n is Euclidean n-space, $\mu_{A_f}(y)$ is the membership function of the fuzzy event A_f, and P is a probability measure over R^n. A point in R^n is denoted by y. $\tilde{P}(A_f)$ may be rewritten using $dP = f(y)dy$ as follows:

$$\tilde{P}(A_f) = \int_{R^n} \mu_{A_f}(y) f(y)\, \mathrm{d}y \tag{8}$$

where $f(y)$ is the probability density function of the random variable Y. In the present case, R^n is a one-dimensional space ($n = 1$) of yield values defined by [0, R_c^a]. The fuzzy risk of low yield of a crop γ_c, can then be defined as follows:

$$\gamma_c = \int_0^{R_c^a} \mu_{W_c}(\Psi_c) f(\Psi_c) d(\Psi_c) \tag{9}$$

where $\mu_{W_c}(\Psi_c)$ is the degree of membership function of yield of a crop Ψ_c in the fuzzy set W_c of low yield of crop c, and $f(\Psi_c)$ is the probability density function of yield of crop c. Depending on the nature of probability density function $f(\Psi_c)$ and the membership function $\mu_{W_c}(\Psi_c)$, direct or numerical integration may be performed to evaluate the fuzzy risk γ_c.

It may be noted that if the crisp definition of the low yield, Eq. (1) is substituted in Eq. (9) and the fuzzy set W_c is replaced by L_c, we obtain the conventional definition of risk of low yield (which is, simply the probability of yield being less than R_c^a). Thus, the crisp definition of low yield may be considered as a particular case of the more general fuzzy set based definition. This definition of probability of a fuzzy event given by Zadeh (1968) acts as an effective tool in linking the uncertainty due to imprecision in the definition of low yield and the uncertainty due to randomness of crop yield arising because of uncertain supplies.

2.2.1. Discrete yield states

The fuzzy risk of low yield of crop c, γ_c, defined by Eq. (9) is for continuous state of crop yield. For discrete states of yield, the fuzzy risk is given by:

$$\gamma_c = \sum_{nk} \mu_{W_c}(\Psi_c^{nk}) P(\Psi_c^{nk}) \tag{10}$$

where $P(\Psi_c^{nk})$ is the probability of occurrence of the nk^{th} element of the set Ψ_c. Each element of the set Ψ_c corresponds to an elementary fuzzy event of low yield with the degree of membership of $\mu_{W_c}(\Psi_c^{nk})$. The expected degree of failure represents the fuzzy risk of low yield of a crop.

The crisp risk for a crop c is defined as the probability that the yield of the crop Ψ_c is less than the acceptable value of the yield of the crop R_c^a. That is,

$$\Omega_c = P[\Psi_c < R_c^a] \tag{11}$$

The fuzzy risk (Eq. (10)) and the crisp risk (Eq. (11)) are both determined for a given operating policy of an irrigation reservoir. The operating policy should be ideally derived considering the stochastic nature of reservoir inflows and optimal crop water allocations in the command area (e.g., (Bardossy and Disse, 1993)5).

3. Application to the case study

The fuzzy risk of low yield of a crop discussed in the above sections is applied to the case study of the Malaprabha reservoir project in the Krishna basin of Karnataka state, India. It is a single purpose irrigation project which has been in operation since 1973. There is no upstream storage structures and the inflows to the reservoir are virgin flows. Located in the northern region of Karnataka state, a major portion of the irrigated area in the reservoir command is in black cotton soil. The major crops grown in the command area are cotton, wheat, sorghum, maize, safflower, and pulses (peas, beans, and legume). The reservoir has a gross storage capacity of 1070 $\mathrm{M\,m^3}$ and a live storage capacity of 870 $\mathrm{M\,m^3}$The mean annual flow is 1349 $\mathrm{M\,m^3}$. The mean annual rainfall in the reservoir command area is 576 mm. The upper catchment area of the reservoir is 2564 $\mathrm{km^2}$. The Malaprabha left bank canal (MLBC) irrigates an area of 53137 ha consisting mainly of red laterite soil. The Malaprabha right bank canal (MRBC) irrigates an area of 128634 hectares, comprising mainly of black cotton soil.

A water year (1 June to 31 May) is divided into 36 ten-day-periods. The duration of the last few periods was increased by one day each to compensate for the additional number

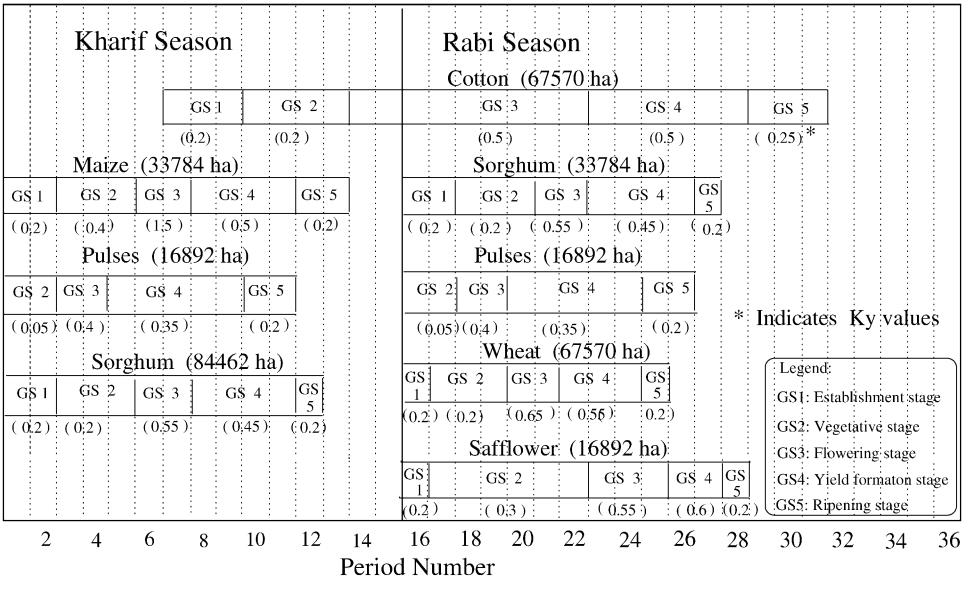

Fig. 4. Crop calender from 1 June, period number 1 to 31 May, period number 36. GS_1, establishment stage; GS_2, vegetative stage; GS_3, flowering stage; GS_4, yield formation stage; GS_5, ripening stage; K_y, yield factor.

of days over 360 in a given year. Fig. 4 shows the crop calender for these crops with their areas. The growth stages (Doorenbos and Kassam, 1979)are adjusted marginally wherever necessary in order that each growth stage is an integral multiple of the decision interval (of 10 days).

3.1. Methodology

A steady state operating policy is derived by treating the inflow into the reservoir and the rainfall in the command area as independent variables, whereas, crop response to climatic factors and farm practices is treated as fuzzy. Details of the operating policy model are given in Suresh (2002). A brief description of the operating policy model is given here. It must be noted that the procedure developed for performance evaluation in the paper is independent of the operating policy model presented here and may be used for any other operating policy also. The outline of the steady state operating model is depicted in Fig. 5. In the first phase, the crop water allocation model uses the fuzzy evapotranspiration deficit model (FETDM) to address the fuzziness associated with the response of the crop due to variation in temperature, T_e and farm practices for crop c, F_c (Suresh and Mujumdar, 2003). The fuzzy evapotranspiration deficit model is formulated to determine the evapotranspiration deficit Φ_c^t of a crop c in a given period t for a given amount of allocated water, known initial soil moisture, rainfall in the command area, in time period t, temperature T_e during the period and the farm practice followed by the farmers for crop c, F_c. Fuzzy Inference System using *Fuzzy Logic Tool Box (MATLAB, Version 5.2)* is used to solve the FETDM. The allocation model is formulated to allocate a given amount of irrigation water optimally among the crops present

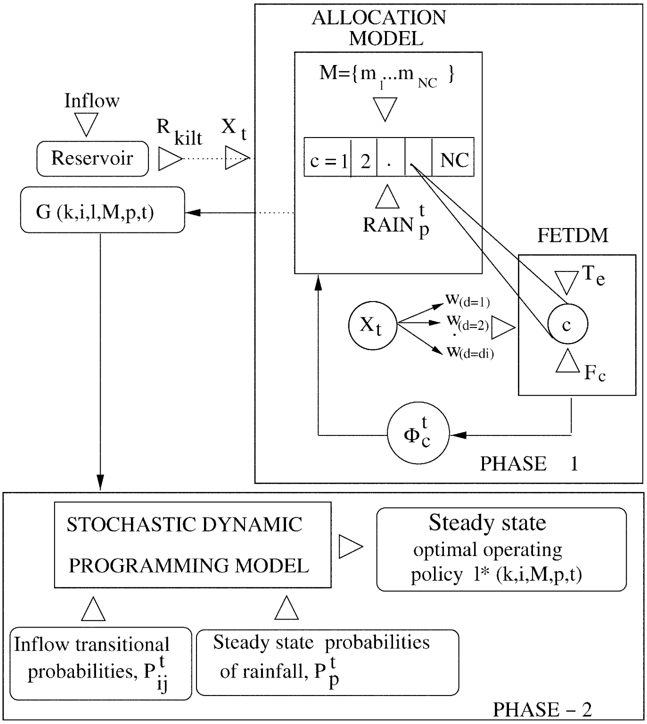

Fig. 5. Block diagram of the steady state operating policy model. w_d, representative value of available water corresponding to class interval d; d_i, number of discrete class intervals of available water; T_e, temperature; F_c, farm practices followed by the farmers for crop c; c, crop index; FETDM, fuzzy evapotranspiration deficit model; Φ_t^c, defuzzified value of the evapotranspiration deficit of a crop c in period t.

in a time period. Dynamic programming (*DP*) is used as an optimisation tool. The objective function of the allocation model uses the defuzzified value of the evapotranspiration deficit of a crop (Φ_c^t), obtained from the FETDM for a given amount of allocated water. In the second phase, a stochastic dynamic programming (SDP) model is used to derive the optimal steady state operating policy of the reservoir using the results obtained by solving the allocation model for each period. The SDP model considers reservoir storage at the beginning of the time period, rainfall in the command area during the period, inflow to the reservoir during the period, and the initial soil moisture of the crops at the beginning of the time period as state variables. The cropped areas and crop calender are assumed to be fixed. The model incorporates uncertainty due to randomness through inflow transition probabilities and rainfall steady state probabilities. In irrigation reservoir systems, where the command areas are far removed from the catchment area of the reservoir, the rainfall in the command area and the inflow to the reservoir may be assumed to be stochastically independent. In the absence of such an assumption on independence, joint probability distribution of reservoir inflow and rainfall would be needed, in the SDP model, accurate estimation of which is rather difficult. The assumption of independence of reservoir inflow and rainfall in the command area greatly simplifies the SDP model and has been used in the earlier applications for the case study (Mujumdar and Vedula, 2003; Vedula and Nagesh Kumar, 1996). Also, the assumption of a Markov chain for the rainfall and the concept of transition probabilities are not relevant in case of rainfall in intra-seasonal periods in a monsoon driven climate, and therefore, the rainfall uncertainty is addressed with steady state probabilities in the SDP model, whereas, the inflow uncertainty is modeled with a one step Markov chain. The steady state operating policy specifies optimal end-of-the-period reservoir storage for given values of initial reservoir storage, inflow, rainfall in the command area, and the initial soil moistures of the crops in the period. The steady state policy is applied to the case study through simulation. Table 1 gives the sample simulation results for a rabi season (November–March) of an intermediate year in simulation. From the simulation based on the optimal allocations to individual crops in intra-seasonal periods, the relative yield for each crop in each year of simulation is determined, using a dated production function. Historical rainfall data of 88 years is used along with a corresponding generated sequence of inflows. The inflows are generated with a non-stationary Markov model. With 88 years of simulation using the optimal operating policy which includes optimal crop water allocations, there are 88 values of yield realised for each crop.

The performance indicators, reliability α and resiliency γ, are determined from the simulation by relative frequency approach. Reliability of the system under a given operating policy is defined as the probability that the system output is satisfactory (Hashimoto et al., 1982). The system output in this study is defined to be satisfactory in a given period t if the water available for irrigation, $X_t = \beta R_t$ (where R_t is the release from the reservoir in period t and β is the canal conveyance factor accounting for all losses from reservoir upto the field), and is at least equal to the total irrigation requirement of all the crops in that period. Resiliency gives the likelihood of the system recovery from a failure, once a failure occurs. Reliability and resiliency for the case study from simulation is found to be 0.87 and 0.38. Note that even with a high reliability and resiliency, the effect of failures can be quite significant on the crop yield, if such failures occur in initial growth periods and or if the extent of failure (deficit) is so large as to cause permanent damage to the crop. The performance of the system must also, therefore, be measured with respect to the crop yield resulting from the policy. The fuzzy risk provides one such measure.

To determine fuzzy risk from simulation, the yield of a crop is divided into 10 discrete intervals. For each crop, the frequency of occurrence of the yield of each discrete state is determined. The probability of occurrence of each discrete state is estimated by the frequency of that discrete state divided by the total number of simulated yield values (88) of a crop. Table 2 gives probability of occurrence of the yield of the crops for different discrete states. Since the acceptable value of the yield of each crop is different, fuzzy risk of low yield is calculated individually for each crop.

3.2. Membership function for low yield of a crop

In order to construct membership function for low yield of a crop, the values of the minimum desired yield, R_c^m and acceptable yield, R_c^a for each crop must be specified. A field survey of the farmers in the command area was conducted for the purpose. Farmers in the command area, growing different crops were randomly selected and their input obtained against a questionnaire [Appendix A]. With the feed back obtained from the field surveys, the

Table 1
Sample simulation results of an intermediate year for rabi season (November–March)

t	IS ($M\,m^3$)	Inflow ($M\,m^3$)	RF (mm)	Initial soil moisture (mm/cm)					Rel. ($M\,m^3$)	Evap. ($M\,m^3$)	FS$^+$ ($M\,m^3$)
				Cr1**	Cr2	Cr3	Cr4	Cr5			
16	870.00	30.60	1.30	2.12	3.50	3.50	3.50	3.50	18.52	5.63	870.00
17	870.00	49.04	0	1.92	2.19	2.45	2.45	1.96	14.75	5.60	870.00
18	870.00	26.71	1.30	1.85	2.50	2.12	2.21	1.43	85.34	5.49	809.86
19	809.86	23.58	0	1.62	1.83	2.00	1.91	1.21	113.42	4.38	715.64
20	715.64	14.11	1.30	1.42	1.69	1.93	1.71	1.42	176.24	3.61	549.90
21	549.90	5.84	0	1.11	1.60	1.79	1.59	1.89	152.33	3.01	400.40
22	400.40	1.27	0	1.11	1.32	1.90	1.87	2.23	82.42	2.57	316.68
23	316.68	0	0	1.76	1.91	1.90	2.40	1.90	71.12	2.30	243.26
24	246.26	0	0	2.42	2.30	2.35	2.29	2.48	79.76	1.98	164.52
25	164.52	0.17	0	2.00	2.30	2.35	2.29	2.52	38.24	1.85	124.60
26	124.60	0.40	0	1.64	2.30	2.35	2.29	2.71	0	1.82	123.18
27	123.18	2.04	0	1.64	2.30	2.35	*	2.71	0	1.82	123.40
28	123.40	0.97	0	1.56	2.30	*	*	2.64	0	2.35	122.02
29	122.02	0.94	21.8	1.56	*	*	*	2.64	0	2.57	120.38
30	120.38	1.51	13.4	1.59	*	*	*	*	0	2.55	119.35
31	119.35	1.01	1.40	1.58	*	*	*	*	0	2.57	117.78
32	117.35	9.83	29.60	1.58	*	*	*	*	0	2.61	125.00

	Allocations ($M\,m^3$)					Final soil moisture (mm/cm)				
	Cr1	Cr2	Cr3	Cr4	Cr5	Cr1	Cr2	Cr3	Cr4	Cr5
16	6.17	0	0	0	0	2.63	2.19	2.45	2.45	2.64
17	4.91	0	0	0	0	2.40	2.50	2.76	2.53	2.65
18	5.90	5.64	1.68	5.64	9.56	2.20	2.50	2.84	2.57	2.65
19	11.81	4.72	9.45	4.72	7.08	2.07	2.53	2.82	2.62	2.68
20	18.35	6.45	14.68	8.23	11.06	1.98	2.57	2.70	2.59	2.71
21	31.18	2.32	1.98	3.21	12.07	1.89	2.52	2.58	2.50	2.72
22	19.05	1.02	0.98	1.43	4.98	1.76	2.42	2.47	2.40	2.74
23	17.24	0.89	1.34	1.85	2.37	1.64	2.30	2.35	2.29	2.71
24	24.38	0	0	0	2.20	1.64	2.30	2.35	2.29	2.71
25	6.37	0	0	0	6.37	1.64	2.30	2.35	2.29	2.71
26	0	0	0	0	0	1.64	2.30	2.35	*	2.71
27	0	0	0	0	0	1.56	2.30	*	*	2.64
28	0	0	0	0	0	1.56	*	*	*	2.64
29	0	0	0	0	0	1.59	*	*	*	*
30	0	0	0	0	0	1.58	*	*	*	*
31	0	0	0	0	0	1.58	*	*	*	*
32	0	0	0	0	0	*	*	*	*	*

(**): [Cr1, cotton; Cr2, sorghum; Cr3, pulses; Cr4, wheat; Cr5, safflower]-rabi. FS$^+$ final storage; *, no crop; IS, initial storage; RF, rainfall; *t*, time period; Rel.: water released to the command area; Evap.: evaporation loss from the reservoir.

expenditure incurred and the income expected was worked out. This forms the basis for the minimum desired yield, R_c^m. Based on the input from agricultural economists (Profs. Reddy and Gurappa, personal communication, 2001) from the University of Agricultural Sciences, Bangalore, India, the acceptable value of the yield R_c^a was fixed for all the crops. Table 2

Table 2
Acceptable and minimum desired crop yields all crops

Sl.	Crop	Minimum desired value of yield (R_c^m)	Acceptable value of yield (R_c^a)
1	Maize	0.40	0.80
2	Pulses	0.35	0.80
3	Sorghum	0.35	0.80
4	Cotton	0.35	0.85
5	Wheat	0.35	0.80
6	Safflower	0.35	0.80

gives the acceptable yield R_c^a and minimum desired yield R_c^m for the crops. Knowing the acceptable and minimum values, R_c^a and R_c^m, of yield of crop c, simple linear membership functions were constructed. Since the acceptable value of the yield of a crop R_c^a varies from crop to crop, membership functions were also different for different crops. A typical membership function for the low yield of a crop is shown in Fig. 3.

3.3. *Fuzzy risk*

The degree of membership function for different states of low yield of a crop is known from the membership function. The probability of occurrence of each discrete state of low yield of a crop was determined from simulation (Table 3) . The fuzzy risk of low yield of a crop was determined using the Eq. (9). In a command area, irrespective of the number of crops present, the crop yield is referred generally with respect to the entire command area rather than for an individual crop. In the present case, the fuzzy risk of low yield for the entire command area was calculated by taking the weighted average fuzzy risk of low yield (average of fuzzy risks of crops, weighted with crop areas) of all crops. The weighted average fuzzy risk was found to be 8.75%.

The crisp risk (Eq. (11)) was also determined for the case study. The output from the simulation was the actual evapotranspiration of a crop ET_{ac}^t in time period t. Using a multiplicative dated production function (Doorenboss and Kassam, 1979), the yield of a crop Ψ_c was determined. The yield of a crop Ψ_c was compared with the acceptable value of the yield R_c^a. The number of the yield values, which were less than the acceptable

Table 3
Probability of occurrence of crop yields corresponding to different discrete states

Interval no.	Class interval	Mid point	Cr1	Cr2	Cr3	Cr4	Cr5	Cr6	Cr7	Cr8
6	0.5–0.6	0.55	0	0	0	0.13	0	0	0	0
7	0.6–0.7	0.65	0	0	0	0.20	0.15	0.05	0.19	0.18
8	0.7–0.8	0.75	0.17	0.17	0.31	0.26	0.28	0.31	0.33	0.24
9	0.8–0.9	0.85	0.63	0.60	0.43	0.28	0.38	0.41	0.33	0.33
10	0.9–1.0	0.95	0.20	0.23	0.26	0.13	0.19	0.23	0.15	0.25

Note: frequencies of class intervals from 1 to 5 are zero. Cr1, maize; Cr2, pulses; Cr3, sorghum (kharif); Cr4, cotton; Cr5, sorghum (rabi); Cr6, pulses (rabi); Cr7, wheat; Cr8, safflower.

value of the yield of a crop divided by the total number of years of simulation, gave the crisp risk.

3.4. Standard operating policy (SOP)

Crisp risk and fuzzy risk of low yield of a crop were also computed for the standard operating policy. The standard operating policy (SOP) was to release an amount of water equal to the total demand in period t, if possible. When it was not possible to meet the total demand, all the water in storage was released. In case of SOP, the demand from a crop in each period DM_c^t was given by,

$$\mathrm{DM}_c^t = (ET_{pc}^t - ER_t)A_c \tag{12}$$

where ER_t is the effective rainfall in mm and was taken as 0.65 times the actual rainfall for black cotton soils, ET_{pc}^t was the potential evapotranspiration of a crop c in period t and A_c is the area of crop c in hectares. The allocations to individual crops was obtained as:

$$x_c^t = \frac{(X_t A_c)}{\sum A_c} \tag{13}$$

where X_t is the known amount of water available at the field for irrigation in period t, x_c^t is the allocation to crop c in period t and A_c is the area of the crop c.

Table 4 shows the results of the fuzzy risk values and crisp risk values for all the eight crops in the command area for optimal as well as standard operating policies. From the table it is observed that the crisp risk values are greater than the fuzzy risk values. In case of crisp risk, any value of yield of a crop less than acceptable value of the yield represents a non-satisfactory state with a degree of membership equal to 1. In case of fuzzy risk, yield of a crop greater than or equal to the acceptable value R_c^a represents a non-satisfactory state with a degree of membership equal to 0. Yield values less than or equal to the minimum desired value R_c^m correspond to a non-satisfactory state with a degree of membership function equal to 1. Fuzzy risk considers the full range of the yield of a crop (i.e., from 0 to 1) and its

Table 4
Comparison of fuzzy risk and crisp risk values between optimal and standard operating policies

Crop	Optimal operating policy		Standard operating policy	
	Fuzzy risk	Crisp risk	Fuzzy risk	Crisp risk
Maize	0.02	0.17	0.35	0.58
Pulses	0.02	0.15	0.29	0.57
Sorghum	0.03	0.31	0.32	0.68
Cotton	0.21	0.59	0.46	0.76
Sorghum R	0.08	0.43	0.39	0.70
Pulses R	0.05	0.36	0.37	0.66
Wheat	0.10	0.52	0.44	0.71
Safflower	0.09	0.57	0.50	0.78
W. average	0.08	0.41	0.42	0.74

W. average, weighted average.

associated probability of occurrence. On the other hand, crisp risk obtained from probability considerations takes into account only the yield values less than the acceptable value and not the full range of yield values. In case of fuzzy risk, the probabilities are multiplied by the membership values (which are less than 1), and thus the fuzzy risk values are less than the crisp risk values. It must be noted here that while the crisp risk gives *probability of failure*, the fuzzy risk measures the *expected degree of failure*.

From Table 4, it is also observed that the fuzzy risk and crisp risk from optimal operating policy are much less than that of standard operating policy. This is because in case of standard operating policy, in calculating the crop water demand, the effect of soil moisture is not considered. It is simply calculated based on the potential evapotranspiration and the effective rainfall in a period resulting in higher demands and implying lower water availability. In many time periods the available water at the field level is less than the requirement, and the crops compete for the available water. Also, the allocations to a crop is based on the area in which it is grown, without any consideration for sensitivity to water deficit. Also, an optimal operating policy is a long term policy, which takes into account the soil moisture state and sensitivity of the crop to water deficit while allocating the water to a crop. It also considers the variation of inflows and rainfall in the command area in the form of inflow transition probabilities and steady state probabilities of rainfall. The optimal operating policy thus considers a long term system performance. Due to these reasons, the fuzzy risk values resulting from the optimal operating policy are less than those obtained from the standard operating policy. The weighted average fuzzy and crisp risk for all crops in case of the optimal operating policy worked out to be 8 and 41%, whereas, in case of the standard operating policy, these values are 42 and 74%, respectively. The fuzzy risk computed in the present analysis may be used as measure to measure the performance of a system with respect to the yield of a crop.

4. Conclusions

A model for fuzzy risk of low yield of a crop, as a performance indicator for a reservoir operating policy, was developed in this paper. To study this indicator, the system was simulated over a number of years with a known operating policy. The yield of a crop is determined using a dated production function with resulting actual evapotranspiration of crops. The allowable values of yield for all the crops are fixed based on agricultural economics experts input. The minimum values of yield of all crops are determined based on the feed back obtained from the farmers. Fuzzy risk for low yield was defined, based on Zadeh's definition of probability of a fuzzy event. The model was applied to a case study of Malaprabha reservoir in Karnataka. Fuzzy risk and crisp risk were calculated. These values were compared with the values obtained from standard operating policy.

Fuzzy risk represents the expected degree of failure rather than merely the probability of failure. It thus contains more information than the crisp risk. The crisp risk deals with a threshold value of failure and in this scenario, a particular value of crop yield is either a failure or a non-failure. Fuzzy risk, on the other hand, deals with low crop yield as a fuzzy event and treats all crop yields as failures of various degrees. To account for the uncertainty in determining a failure, occurrence of failure itself was treated as a fuzzy event. The fuzzy definition of low crop yield ensures that there is no single threshold value of crop yield which defines a failed state. All discrete crop yields are treated as failures of different degrees. The fuzzy set of low crop yield maps all crop yields to 'low crop yield' and its membership function denotes the degree to which a particular crop yield is low. A high crop yield, for example, will have a membership value of zero in the fuzzy set of low crop yield. The information content in the fuzzy risk is much more than that in the crisp risk. The crisp risk is a special case of the general fuzzy risk, and therefore, the fuzzy risk provides a wider and more useful measure of performance of irrigation systems.

Appendix A. Questionnaire used in field surveys to construct fuzzy membership functions

Name of the farmer: . . . .
Address:
Village: . . . .
Taluk: . . . .
District: . . . .
Crop: . . . . Variety: . . . .
Area (ha/acres): . . . .
Season: . . . .
Soil type: . . . .

1. **Type of Seedlings used**:

   a. Hybrid
   b. High yielding variety
   c. Locally improved

2. **Type of land preparation**:

   a. By wooden ploughs
   b. By tractors

3. **Density index**:

   a. Spacing between the rows: . . . .
   b. Spacing between plant to plant: . . . .
   c. Depth of sowing: . . . .
   d. Random
   e. Quantity of seeds used: . . . . kg/acre.

4. **Type and amount of fertilisers used**:

   a. Bio-fertilisers (Farm yard Manure): . . . . t/acre.
   b. Fertilisers:

   4.1. Basal dose
   4.2. Top dress

4.3. Any other dose

5. **Type of weedicides**:

5.1. Weeder
5.2. Manual weeding
5.3. Chemical weeding

6. **Plant protection**:

6.1. Cultural
6.2. Mechanical
6.3. Biological
6.4. Chemical
6.5. Physical

7. **Type of irrigation**:

7.1. Basin flooding
7.2. Furrow method
7.3. Any other method

8. **Crop rotation**:

a. Last crop grown: . . . .
b. Next crop: . . . .

9. **Yield expected**: . . . . t/ha or . . . . quintals/acre
10. **Farmer's experience**: . . . .

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