On the Performance of Interference-Aware Cognitive Ad-Hoc Networks

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***Abstract*—In this paper we analyze the effects of channel availability on channel access delay and service probability of cognitive networks using a modified IEEE 802.11 Media Access Control Protocol (MAC). For the designed cognitive network, cognitive communication is limited by the interference imposed on primary users. We determine the probability of accessing the channel under Rayleigh fading condition for this opportunistic network. We then use this probability to determine the embedded Markov model of the cognitive nodes. We use this Markov model to determine the average channel access delay, and service rate of cognitive nodes. Both simulation and analytical results are presented to access the system performance.**

***Index Terms*—Cognitive networks, IEEE 802.11, access delay, service rate.**

I. Introduction

MEDIA Access Control Protocol (MAC) layer plays an important role for cognitive communication. In [1], the authors survey advantages, design consideration, and challenges of proposed MAC protocols for cognitive networks. In the literature, IEEE 802.11-like MAC protocols have been proposed in [1], [2] and references there in. For instance in [2], the authors proposed distributed multi channel MAC protocol for cognitive networks. In [3] channel access delay for nodes is optimized over sensing time for cognitive networks.

Apart from the above mentioned studies, the authors in [4] and [5] used Markov model to determine performance metrics such as access delay, throughput, offered load for IEEE 802.11 MAC for both saturated and unsaturated traffic cases. However, for primary users' interference limited cognitive communication, the channel access delay and service rate is affected by the spectrum sensing time, contention delay, RTS (Request to send) and CTS (Clear to send) exchange period, and channel unavailability period due to primary users' interference limitations. To the best of our knowledge, the effect of primary users' interference constraint on the performance of cognitive ad-hoc networks has not been evaluated to date. From this point of view, in this paper our main contributions are, 1) We determine the channel access probability of Multiple-Input and Multiple-Output (MIMO) cognitive ad-hoc networks. 2) We model the transition of state in a node using an embedded Markov model. 3) This Markov model is used to determine the average channel access delay and service rate of nodes for interference limited communication.

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The rest of the paper is organized as follows. The system model and analysis for the access delay and service probability are presented in Section II. Simulation results are reported in Section III. Finally, conclusions are drawn in Section V.

II. System Model

We consider N pairs of cognitive ad-hoc nodes coexist with licensed primary users in the same geographical area. Cognitive and primary users access the adjacent channels but due to spill over energy [6], cognitive communication may cause interference on primary users. We assume all cognitive nodes are within the radio range of each other. Cognitive source-destination pairs use N_t transmit and N_r receive antennas and achieve multiplexing gain. On the other hand, for the sake of simplicity we limit our study to Single-Input and Single-Output (SISO) primary users. In the MAC sublayer of the data link layer, nodes use Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) protocol with RTS/CTS mechanism. During the distributed coordination function inter frame space (DIFS) period of the MAC protocol, nodes perform channel sensing to determine the transmission opportunity [2]. Following this, nodes move to the back-off state of the MAC protocol, if interference imposed on the primary users is below the specified threshold. Otherwise, cognitive nodes wait T_f amount of time before sensing the channel again. To model the transitions of these states for a packet in a node, a discrete-time Markov renewal process is established as illustrated in Fig. 1. The states in the figure can be divided into 4 categories: 1) channel access state ($F_i, i = 0, 1, 2, 3....., K$) 2) back-off state ($B_i, i = 0, 1, 2, ..., K$) 3) collision state ($C_i, i = 0, 1, 2, ..., K$) and 4) transmission state (T).

As illustrated in Fig. 1, nodes start back off process after the state F_i with channel access probability P_a. From the back-off state, the packet moves to the transmission state with probability P, if the request is successful, else moves to the collision state for unsuccessful requests. After each collision state, the packet is moved to higher level of channel access state or back-off state with probability $1 - P_a$ and P_a, respectively. This process continues until the packet is dropped after K retransmission or collision events.

Throughout the paper, boldface letters are used to represent vectors and matrices.

A. Probability of Channel Availability

To develop the mathematical model for the probability of channel availability, we define the interference signal y^i_{pl} at any primary user l due to spill over energy [6] by the cognitive communication using adjacent channels $i \in \hat{C}$ as,

$$y^i_{pl} = \boldsymbol{G}_i \boldsymbol{x} \tag{1}$$

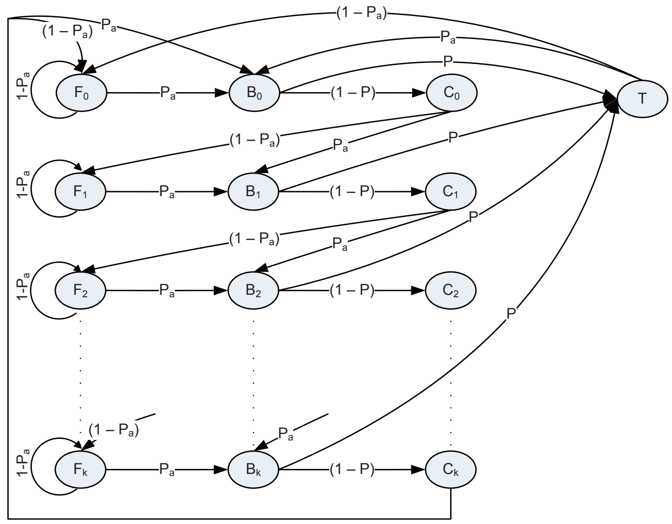

Fig. 1. Embedded Markov model for the state transition process in each node.

where $\boldsymbol{G}_i$ stands for an $1 \times N_t$ channel vector representing the corresponding channels between a primary user and cognitive node $i \in N$, $\boldsymbol{x}$ denotes $N_t \times 1$ cognitive user transmit symbol vector.

From (1), the instantaneous interference power at the l^{th} primary user can be written as

$$I_i^l = E((y_{pl}^i)^H y_{pl}^i) = \sigma \boldsymbol{G}_i.(\boldsymbol{G}_i)^H, \tag{2}$$

where $\sigma = E[\boldsymbol{x}^H \boldsymbol{x}]/N_t$. We also consider all cognitive users have uniform interference effect on primary users.

If Maximum Ratio Combining (MRC) is employed at primary nodes, from (1), we notice that the effective interference signal power at the primary user i is $\sigma \sum_{j=1}^{N_t} G_{ij}^2$. If we consider Rayleigh fading channel between cognitive and primary users, the effective interference power after combining is chi-square distributed with $2N_t$ degrees of freedom. That is, probability density function (pdf), [7] of the interference power can be written as,

$$p_{\sigma_{eff}}(u) = \frac{u^{N_t-1} \exp^{-u/\sigma}}{\sigma^{N_t}(N_t-1)!} \; u > 0. \tag{3}$$

For cognitive power σ and primary users interference threshold I_{th}, the probability of channel availability can be written as,

$$P_a = Probability(\sigma < I_{th}) = \int_o^{I_{th}} \frac{u^{N_t-1} \exp^{-u/\sigma}}{\sigma^{N_t}(N_t-1)!} du,$$
$$= 1 - \exp^{-\frac{I_{th}}{\sigma}} \left(\sum_{i=1}^{N_t} \frac{(I_{th}/\sigma)^{i-1}}{(i-1)!} \right). \tag{4}$$

B. Average Channel Access Delay

According to the system model described above, the transmission probability τ can be used to express the conditional collision probability ρ, and the probability of successful transmission P for a node as,

$$\rho = 1 - (1-\tau)^{N-1}, \tag{5}$$

$$P = (1-\tau)^{N-1}. \tag{6}$$

For short retransmission limit τ can be written in terms of the collision probability as [8],

$$\tau = \frac{2(1-2\rho)(1-\rho^{m+1})}{w(1-(2\rho)^{m+1})(1-\rho) + (1-2\rho)(1-\rho^{m+1}) + w2^m\rho^{m+1}(1-2\rho)(1-\rho^{K-m})}, \tag{7}$$

where K and m represent maximum number of retransmission events and maximum number of back-off states, respectively. w denotes the minimum value of contention window size. One can notice that the value of ρ and τ can be determined from (5) and (7) using numerical techniques.

Holding time in Fig. 1 in state T and in state C are fixed for MAC protocol and can be determined as, $T_s = t_{DIFS} + t_{RTS} + t_{SIFS} + t_{CTS} + t_{SIFS} + t_{Packet} + t_{SIFS} + t_{ACK}$, and $T_c = t_{DIFS} + t_{RTS} + t_{SIFS} + t_{EIFS}$, where t_{Packet} denotes the packet transmission time and the nominal values of other parameters in T_s and T_c for the IEEE 802.11 protocol are given in Table I. Holding time in the back-off state B depends on the time wasted due to packet collision, successful packet transmission by other nodes and waiting time of the back-off process for channel acquisition. The probability of successful transmission in the channel P_s, the collision probability in the channel P_c, and the probability of the channel being idle P_i can be expressed as, $P_s = N\tau(1-\tau)^{N-1}$, $P_c = 1 - (1-\tau)^N - N\tau(1-\tau)^{N-1}$, and $P_i = (1-\tau)^N$, respectively. Using these probabilities, the average time required for two successive back-off timer decrementing instants d is given by,

$$d = T_s P_s + T_c P_c + P_i t_{slot}, \tag{8}$$

where t_{slot} denotes the duration of a time slot. For short retry limit, the contention window size w_i is given by, $w_i = 2^i w$, if $0 \le i < m$ or $w_i = 2^m w$, if $m \le i \le K$ where i represents the number of retransmission events and w denotes the minimum value of contention window size. At each back-off state, the value of back-off timer is set uniformly between 0 and $w_i - 1$. Also, the average value of back-off counter is given by,

$$E\{w_i\} = \begin{cases} \frac{2^i w - 1}{2} & \text{if } 0 \le i < m, \\ \frac{2^m w - 1}{2} & \text{if } m \le i \le K. \end{cases} \tag{9}$$

Now, from (8) and (9), one can find the holding time Y_i in back-off state b_i for interference constraint as,

$$Y_i = w_i d + (1-P_a)w_i d + (1-P_a)^2 w_i d + ... = \frac{w_i d}{P_a}. \tag{10}$$

As w_i is not dependent on P_a and d, average holding time $E\{Y_i\}$ in back-off state b_i, can be written as,

$$E\{Y_i\} = \frac{E\{w_i\}d}{P_a}. \tag{11}$$

Average holding time G_f in the channel access state F_i for interference constraint is given by,

$$G_f = T_f + (1-P_a)T_f + (1-P_a)^2 T_f + ... = \frac{T_f}{P_a}. \tag{12}$$

The total channel access delay, D starts from state F_0 until the service completion in state T. It can happen through single stage as, $F_0 \rightarrow B_0 \rightarrow T$ or multiple stages as, $F_0 \rightarrow B_0 \rightarrow$

$C_0 \to F_1 \to B_1 \to C_1 \to F_2 \to B_2 \to T$ (Fig. 1). Access delay D_0 for stage $i = 0$ starts at F_0 to B_0 and ends at T with probability P as,

$$\begin{aligned} D_0 &= P_a(E\{Y_0\} + T_s) + (1 - P_a)(E\{Y_0\} + T_s + G_f) \\ &= E\{Y_0\} + T_s + \frac{1 - P_a}{P_a} T_f. \end{aligned} \tag{13}$$

In sequel, access delay at any stage D_i starts from state F_0 for $i = 0$ and after packet collision event C_{i-1} for $i = 1, .., K$ until the service completion in state T, given by

$$D_i = \begin{cases} \frac{1-P_a}{P_a} T_f + E\{Y_i\} + T_s, & \text{with prob. } P \\ \frac{1-P_a}{P_a} T_f + E\{Y_i\} + T_c + D_{i+1}. & \text{with prob. } 1 - P \end{cases} \tag{14}$$

It is worthwhile to note that the packet is dropped from the queue after the collision event at state $i = K$ and the node starts from state $i = 0$ with a new packet. Using (14) the average channel access delay for primary users' interference constraint can be determined as,

$$\begin{aligned} E(D) &= \frac{1 - P_a}{P_a} T_f + E\{Y_0\} + PT_s + (1 - P)(T_c + D_1) \\ &= \underbrace{(1 - (1 - P)^{K+1})T_s}_{\text{Packet transmission time}} + \underbrace{\frac{(1 - (1 - P)^K)(1 - P)}{P} T_c}_{\text{Collision time}} \\ &\quad + \underbrace{\sum_{i=0}^{K-1} E\{Y_i\}(1 - P)^i + (1 - P)^K P E\{Y_K\} +}_{\text{Back-off time}} \\ &\quad \underbrace{T_f \frac{1 - P_a}{P_a} \sum_{i=0}^{K-1} (1 - P)^i + T_f \frac{1 - P_a}{P_a} (1 - P)^K P}_{\text{Channel access time}} \end{aligned} \tag{15}$$

where $E(.)$ is the expectation operator.

III. The Service Rate

According to Fig. 1, steady state probabilities of the states are given by,

$$\pi_{C_i} = (1 - P)\pi_{B_i}, \quad \text{for } i = 0, 1, ..., K \tag{16}$$

$$\pi_{F_i} = \begin{cases} \frac{1-P_a}{P_a}(\pi_T + \pi_{C_K}) & \text{for } i = 0 \\ \frac{1-P_a}{P_a}\pi_{C_{i-1}}, & \text{for } i = 1, ..., K \end{cases} \tag{17}$$

and

$$\pi_{B_i} = \begin{cases} P_a(\pi_T + \pi_{C_K} + \pi_{F_i}) & \text{for } i = 0 \\ P_a(\pi_{F_i} + \pi_{C_{i-1}}), & \text{for } i = 1, ..., K. \end{cases} \tag{18}$$

Using (17), for $i = 1, ..., K$, (18) can be written as,

$$\pi_{B_i} = (1 - P_a)\pi_{C_{i-1}} + P_a \pi_{C_{i-1}} = \pi_{C_{i-1}} = (1 - P)\pi_{B_{i-1}}. \tag{19}$$

Accordingly, from (19), π_{B_K} and π_{C_K} can be written as,

$$\begin{aligned} \pi_{B_i} &= (1 - P)^i \pi_{B_0} \text{for } i = 1, ..., K \\ \pi_{C_K} &= (1 - P)^{K+1} \pi_{B_0}, \end{aligned} \tag{20}$$

TABLE I
Simulation Setting

Parameter	Value	Parameter	Value
MAC protocol	CSMA/CA	MAC header	272 bits
Packet Payload	8184 bits	Slot time	50 μs
PHY header	127 bits	DIFS	128 μs
ACK	112 bits+PHY header	SIFS	28 μs
RTS	160 bits+PHY header	Bit rate	2 Mb/s
CTS	112 bits+PHY header		

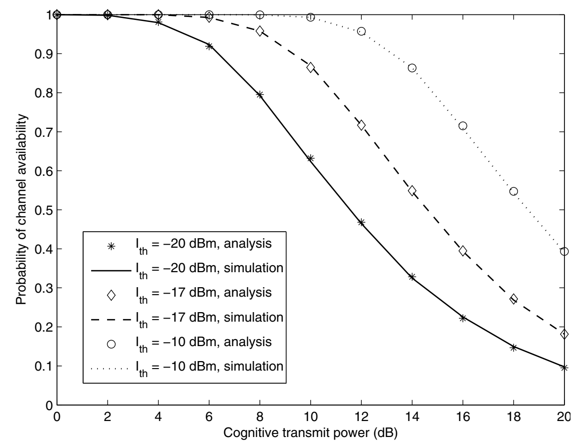

Fig. 2. Probability of channel availability as a function of interference constraint.

Now for $i = 0$, π_{B_0} can be determined using (17), (18) and (20) as,

$$\pi_{B_0} = \frac{P_a}{1 - P_a}\pi_{F_0} \tag{21}$$

$$\begin{aligned} &= \pi_{C_K} + \pi_T = (1 - P)^{K+1}\pi_{B_0} + \pi_T \\ &= \frac{1}{1 - (1 - P)^{K+1}}\pi_T. \end{aligned} \tag{22}$$

Using (20) to (22), and the analysis of limiting state probabilities of the Markov renewal process in [9], the service probability or service rate can be determined as,

$$\begin{aligned} \bar{\pi}_T = 1/\Bigg[&1 + \frac{\tau_f}{\tau_T}\left\{\frac{1 - P_a}{P_a P}\right\} + \frac{\tau_c}{\tau_T}\frac{1}{1 - (1 - P)^{K+1}} \\ &\left\{\frac{1 - (1 - P)^{K+2}}{P} - 1\right\} + \frac{\hat{d}}{P_a(1 - (1 - P)^{K+1})} \\ &\left\{\sum_{i=0}^{m}(1 - P)^i \frac{1 + w_i}{2} + \sum_{i=m+1}^{K}(1 - P)^i \frac{1 + w_m}{2}\right\}\Bigg], \end{aligned} \tag{23}$$

where τ_f, τ_c, τ_T and $\hat{d}$ denote holding time in states F, C, T and B expressed in terms of slot times, respectively. $\hat{d}$ is given by [5],

$$\hat{d} = \frac{1}{\tau_T} + (1 - P)\frac{\tau_C}{\tau_T} - \left(1 - \frac{\tau_C}{\tau_T}\right) P \log P. \tag{24}$$

IV. Simulation Results

For performance evaluation purposes, we consider 'cognitive-to-cognitive' communication is error free. We also assume nodes use blind channel estimation methods or primary users' pilot symbols to estimate the CSI of

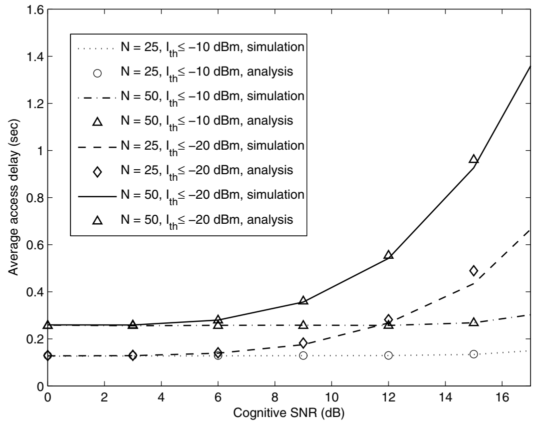

Fig. 3. Average channel access delay for $N_t = N_r = 1$.

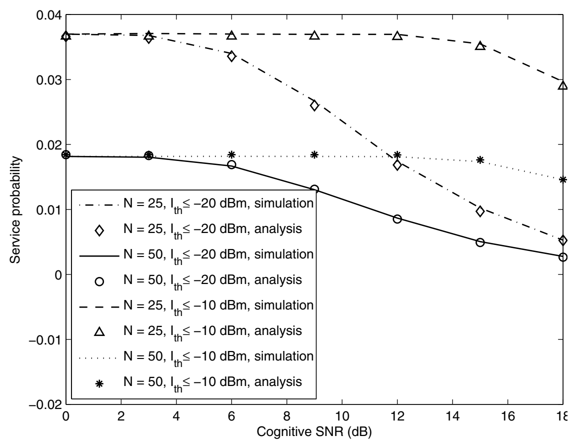

Fig. 4. Service probability of cognitive nodes for $N_t = N_r = 1$.

'primary-to-cognitive' channels. To model this spill over energy mentioned in the system model, we consider the elements of 'Cognitive-to-Primary' channel matrix G, as zero mean and 10^{-3} variance complex Gaussian variables. We use these settings and parameters [10] listed in Table I and build an IEEE 802.11 compatible ad-hoc network. We carry out an event driven simulation and record the access delay and service probability results for the above mentioned system. It is to be noted that in the following performance results, each data point represents an average over 10,000 events.

To validate the probability of channel availability model in (4), we compare the theoretical and simulation results in Fig. 2. For simulation, we consider a channel is available if the interference is below the specified threshold. We record the number of instants when the channel is available over 10,000 channel realizations, and determine the probability of channel availability results as indicated in the figure. The results demonstrate that the channel availability improves as primary users interference threshold increases. As the cognitive power increases, nodes are most likely fail to obey the primary users' zero interference rule. For this reason cognitive nodes need to halt their packet transmission. It is also clear from the results that the simulation and analytical results are very close which validates the model in (4).

It is evident form (15) that the average access delay is proportional to four contributing factors, namely: 1) channel unavailability time due to fading, 2) idle time due to the back-off period, 3) time wasted in packet collision events, and 4) packet transmission time. Also, from Fig. 2 it is understood that the channel unavailability time due to fading increases with the increase in cognitive SNRs, or decrease in the interference constraint. For that the access delay performance degrades (Fig. 3) with the increase in cognitive SNRs, or decrease in the interference constraint. Fig. 3 also indicates that the access delay increases with the number of nodes in the network. Other factors such as packet collision events and idle times are major parameters for this performance degradation.

From (24) one can notice that the service probability is inversely proportional to the 1) channel unavailability time due to fading, 2) idle time due to the back-off period, and 3) time wasted in packet collision events. As a result, the service probability degrades with the increase in SNR (Fig. 4). Similarly, the performance improves with the increase in the interference constraint. Fig. 4 also indicates that similar to the access delay analysis performance degradation is also noticed for higher number of nodes in the network. This happens due to the other contributing factors mentioned above.

V. Conclusions

We evaluated the average access delay, and service probability for interference limited cognitive networks. We also presented analytical results for channel availability of cognitive networks with respect to transmit power. Our analysis indicates that the network performance depends on transmit power, interference threshold and number of nodes in the network. For this reason, optimization techniques can be applied to achieve a desired performance goal for certain operating environment.

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