Ensuring Pareto Optimality by Referendum Voting*

İpek Özkal-Sanver† and M. Remzi Sanver

Department of Economics, İstanbul Bilgi University

December 11, 2003

Abstract

We consider a society confronting the decision of accepting or rejecting a list of (at least two) proposals. Assuming separability of preferences, we show the impossibility of guaranteeing Pareto optimal outcomes through anonymous referendum voting, except for the case of an odd number of voters confronting precisely two proposals. In this special case, majority voting is the only anonymous social choice rule which guarantees Pareto optimal referendum outcomes.

Key Words: Referendum Voting, Majority Rule, Paradox of Multiple Elections, Separable Preferences

JEL Classification Number: D 71

---

*We thank Steven J. Brams for his valuable comments.

†Corresponding Author: İstanbul Bilgi University, Kuştepe, 80310, İstanbul, Turkey. e-mail: isanver@bilgi.edu.tr

1 Introduction

Given a society confronting the decision of accepting or rejecting a list of proposals, a social choice outcome is a list which indicates whether each proposal is accepted or rejected. To be more concrete, given m proposals, assuming that each proposal will either be accepted or rejected, there are 2^m possible outcomes. It is of course possible to model the problem in a standard social choice framework where the basic information is voters' preferences on outcomes. On the other hand this may cause practical problems as the number of outcomes can explode (e.g., there would be 131072 possible outcomes when there are only 17 proposals) and voters would confront difficulties in ranking such a high number of outcomes.

A typical solution to this problem is to decide over proposals separately, an approach which is called referendum voting. Under referendum voting, the preference of voters about the acceptance/rejection of every proposal is aggregated into a social decision regarding that particular proposal- usually but not necessarily through majority voting.

Referendum voting has less informational requirements. In fact, it suffices that every voter indicates for every proposal whether he/she wants that proposal to be accepted or rejected. So referendum voting uses as input the best outcome of every voter, without requiring a further ordering. Nevertheless, these orderings are necessary to have a finer description of the social choice outcome. A typical approach is connecting the best outcome of a voter with one or more admissible orderings over outcomes through certain axioms. A standard axiom in this context is separability. A voter is said to have separable preferences over outcomes if for every proposal the voter either always prefers that proposal to be accepted or always prefers it to be rejected, independent of what happens with the remaining proposals.[1]

Although referendum voting is not the only way of handling the complications due to the size of the social choice problem,[2] it is a very popular one. That is why we concentrate on referendum voting. We ask whether

[1]Kilgour (1997) gives a formal treatment of this property. Brams et al.(1997) analyze referendum voting under nonseparable preferences.

[2]For example one can use "Yes-No voting" proposed by Brams and Fishburn (1993) whereby a voter can indicate multiple packages of proposals he/she supports or "fallback bargaining with unanimity" proposed by Brams et al. (2003) which finds an agreement on multilateral treaties which is a compromise minimizing the maximum distance between it and the top preferences of all players.

Pareto optimality can be ensured by the informational input it requires. We consider a social choice problem with at least two proposals and two voters. Each voter indicates for each proposal whether he/she wants it to be accepted or rejected. A fixed social choice rule is applied to each proposal separately to decide whether it should be accepted or rejected, hence leading to the social outcome. While the Pareto optimality of this outcome depends on how voters order outcomes, the only information we have on hand is their first best. To be able to analyze the efficiency of the social choice, we assume that voters have separable preferences over outcomes. In other words, once we know the first best outcome of a voter, we allow him/her to have any separable ordering over outcomes. As a result any list of individual opinions of voters regarding the acceptance/rejection of proposals leads to a set of admissible preference profiles over outcomes obtained through the separability axiom. We say that a referendum voting rule is Pareto ensuring if and only if given any list of individual opinions regarding the proposals, it picks an outcome which is Pareto optimal according to every admissible preference profile over outcomes.

We illustrate these concepts through an example. Consider three voters who have to decide over three proposals. Let the best outcome of the first voter be (Y, Y, N). In other words, he/she wishes the first two proposals to be accepted and the third one to be rejected. Similarly, let the second and third voters opinion be (Y, N, Y) and (N, Y, Y) respectively. Assume we use the majority rule on each issue. The first proposal receives two approvals- by the first and second voters- hence it is accepted. In the same manner, the second and third proposals also receive two approvals each. Thus the referendum outcome via the majority rule is (Y, Y, Y), i.e., every proposal is accepted. However, every voter may prefer the rejection of all proposals, i.e. the outcome (N, N, N) to (Y, Y, Y). This is perfectly compatible with separability and when this is the case, the referendum voting outcome is not Pareto optimal.

This small example shows that referendum voting via the majority rule is not Pareto ensuring with three voters and three proposals. We question the generality of this result and ask whether it depends on the size of the social choice problem or the social choice rule via which the referendum is made. Interestingly, the class of Pareto ensuring referendum voting rules can be characterized in terms of the escape from a paradox for multiple elections introduced by Brams, Kilgour and Zwicker (1998). This paradox -to which we refer as the BKZ-paradox- is about referendum voting rules which pick an outcome which is the best for (or voted by) no voter. We show that a referendum voting rule is Pareto ensuring if and only if it escapes the BKZ-paradox. Moreover, except for a particular case of two proposals and an odd number of voters, no anonymous voting rule can escape the BKZ-paradox, hence be Pareto ensuring. So our main result is the impossibility of guaranteeing Pareto optimality by referendum voting.[3]

Our paper proceeds as follows: In Section 2 we give the basic notions. Section 3 contains the main results and Section 4 makes some concluding remarks.

2 Basic Notions

Picking any two integers $m, n \geq 2$, we consider a society $N = \{1, ..., n\}$ confronting a set of proposals $\pi = \{\pi^1, ..., \pi^m\}$. Let $M = \{1, ..., m\}$ be the set of indices of the proposals. The *vote* of a voter $i \in N$ is a m-tuple $v_i \in \{-1, 1\}^m$ where for each $j \in M$, the j'th entry $v_i^j \in \{-1, 1\}$ reflects his opinion over the proposal π^j in the following manner: If $v_i^j = 1$ (resp., $v_i^j = -1$), then voter i wants proposal π^j to be accepted (resp., rejected). We write $v = \{v_i\}_{i \in N} \in V$ for a *vote profile* of the society where $V = \{-1, 1\}^{m.n}$ is the set of all possible vote profiles. An outcome is an m-tuple $x \in \{-1, 1\}^m$ where for each $j \in M$, the j'th entry $x^j \in \{-1, 1\}$ reflects the social decision about proposal π^j. Once again $x^j = 1$ (resp., $x^j = -1$) means that π^j is accepted (resp., rejected).[4] Let $A = \{-1, 1\}^m$ stand for the set of all possible outcomes and $\Re$ be the set of all complete and transitive binary relations over A. Every voter $i \in N$ has a preference $R_i \in \Re$ on A. For all $x, y \in A$, $x \; R_i \; y$ means that voter i finds outcome x at least as good as outcome y. We write $x \; P_i \; y$ whenever the preference relation is strict, ie., $x \; R_i \; y$ but not $y \; R_i \; x$. Similarly, $x \; I_i \; y$ stands for the indifference counterpart of R_i, i.e., we have $x \; I_i \; y$ whenever $x \; R_i \; y$ and $y \; R_i \; x$ both hold. An n-tuple $R = (R_1, ..., R_n) \in \Re^n$ of these binary relations reflects a preference profile of the society over the possible outcomes.

[3]A related result is due to Lacy and Niou (2000) who show that nonseparable preferences can lead to the social choice of an outcome which is a Condorcet loser or even Pareto dominated. A similar analysis is made by Benoit and Kornhauser (1994) who examine the efficiency properties of voting systems electing assemblies as a function of the preferences of voters over individual candidates.

[4]Note that we do not allow for indifferences in neither individual nor social preference. This is a matter which we will analyze at the end of Section 3.

We do not have strategic considerations. So the vote of a voter is also the outcome he/she prefers most. We assume that for every voter $i \in N$, his/her vote v_i and his/her preference R_i on A are related. This relation is established through a binary relation $B(v_i)$ over A which is defined for any two distinct outcomes $x, y \in A$, as follows: $x\ B(v_i)\ y$ if and only if for every $j \in M$, we have

$x^j \geq y^j$ whenever $v_i^j = 1$,

and

$x^j \leq y^j$ whenever $v_i^j = -1$.

So given any voter $i \in N$ with a vote $v_i \in \{-1, 1\}^m$, the outcome x beats the outcome y through $B(v_i)$ if and only if for every separate proposal, voter $i \in N$ finds the decision according to x at least as good as the decision according to y. Given any $i \in N$, a preference $R_i \in \Re$ is said to be *separable* with respect to $v_i \in \{-1, 1\}^m$ if and only if for all $x, y \in A$, with $x\ B(v_i)\ y$ we have $x\ P_i\ y$. We say that a preference profile $R \in \Re^n$ is separable with respect to a vote profile $v \in V$ whenever R_i is separable with respect to v_i for all $i \in N$. Given any $v \in V$, we write $\Sigma(v) \subset \Re^n$ for the set of preference profiles over A which are separable with respect to v.

A *voting rule* is a function $F : V \to A$ which assigns an outcome $F(v) \in A$ to every vote profile $v \in V$.

An outcome $x \in A$ is said to be *Pareto optimal at* $R \in \Re^n$ whenever there exists no $y \in A$ such that $y\ R_i\ x$ for all $i \in N$, and $y\ P_j\ x$ for some $j \in N$. A *voting rule* is *Pareto ensuring* if and only if given any vote profile $v \in V$, the outcome $F(v)$ is Pareto optimal at every $R \in \Sigma(v)$.

3 Results

We quote a version of a paradox introduced by Brams, et al. (1998). A voting rule $F : V \to A$ is said to escape the *paradox of multiple elections* (to which we refer as the *BKZ-paradox*) if and only if for all $v \in V$,there exists $i \in N$ such that $F(v) = v_i$. So escaping the BKZ-paradox means that at every vote profile the voting rule picks an outcome which is voted by (or which is the best for) at least one voter. In other words, a voting rule exhibits the BKZ-paradox if and only if the outcome it picks at some vote profile is voted by nobody.

Interestingly the class of Pareto ensuring voting rules can be characterized through the escape of the BKZ-paradox, as the following theorem states:

Theorem 3.1 *A voting rule $F : V \to A$ is Pareto ensuring if and only if F escapes the BKZ-paradox.*

Proof. To show the "if" part, consider a voting rule $F : V \to A$ which escapes the BKZ-paradox. Take any vote profile $v \in V$. Let $x = F(v)$. As F escapes the BKZ-paradox, there exists some $i \in N$ such that $x = v_i$.Now take any $y \in A \backslash \{x\}$. Clearly $x\ B(v_i)\ y$. Hence at every $R \in \Sigma(v)$, we have $x\ P_i\ y$. Thus $x = F(v)$ is Pareto optimal at every $R \in \Sigma(v)$, showing that F is Pareto ensuring.

To show the "only if" part, suppose F exhibits the BKZ-paradox. So there exists some $v \in V$ such that $F(v) \neq v_i$ for all $i \in N$. We claim that there exists some $R \in \Sigma(v)$ such that $-F(v)\ P_i\ F(v)$ for all $i \in N$. Showing our claim completes the proof as it establishes the existence of some $R \in \Sigma(v)$ according to which $F(v)$ is not Pareto optimal. We show our claim by proving that $F(v)\ B(v_i) - F(v)$ holds for no $i \in N$. Write $F(v) = x$ and $-F(v) = -x$. Take any $i \in N$. There exists some $j \in M$ such that $x^j \neq v_i^j$. Note that $v_i^j = -x^j$. Hence, $F(v)\ B(v_i) - F(v)$ does not hold, completing the proof. ■

Remark 3.1 *The equivalence established by Theorem 3.1 could be stated under any set of admissible preferences over outcomes containing the set $\Sigma(v)$ of separable preferences. In particular, Theorem 3.1 would hold without assuming separable preferences and allowing voters to have any ordering over outcomes, independent of the vote they cast.*

We now show a basic impossibility of escaping the BKZ-paradox -hence ensuring Pareto optimal outcomes- through anonymous referendum voting when there are an even number of voters or at least three proposals. First we give the necessary definitions. A voting rule $F : V \to A$ is said to be *simple* whenever the number of proposals $m = 1$. Now for every $j \in M$, let $v^j = (v_1^j, ..., v_n^j)$ the list of opinions of the voters for proposal π^j. We call a voting rule F *referendum voting* if and only if there exists a simple voting rule f such that $F(v) = (f(v^1), ..., f(v^m))$. We refer to f as the corresponding simple rule of the referendum voting rule F. So a referendum voting rule is one where you apply a given simple voting rule to all proposals separately. A voting rule $F : V \rightarrow A$ is said to be *anonymous* if and only if given any vote profile $v = (v_1, ..., v_n)$ and any permutation $\tau : N \longleftrightarrow N$ of voters, we have $F(v_1, ..., v_n) = F(v_{\tau(1)}, ..., v_{\tau n)})$. Note that anonymity of a referendum voting rule F implies the anonymity of its corresponding simple voting rule f.

Theorem 3.2 *Let $m \geq 3$ or n be even. There exists no anonymous referendum voting rule $F : V \rightarrow A$ which escapes the BKZ-paradox.*

Proof. Let $F : V \rightarrow A$ be any anonymous referendum voting rule. So given any vote profile $v \in V$, writing $v^j = (v_1^j, ..., v_n^j)$ for the list of opinions of the voters for proposal $j \in M$, we have $F(v) = (f(v^1), ..., f(v^m))$ for some simple voting rule f.

First let n be even and consider the following vote profile $v \in V$ where for every $j \in M \backslash \{m\}$ we have $v_i^j = 1$ for all $i \in \{1, ..., n/2\}$ and $v_i^j = -1$ for all $i \in \{n/2+1, ..., n\}$. On the other hand, $v_i^m = -1$ for all $i \in \{1, ..., n/2\}$ and $v_i^m = 1$ for all $i \in \{n/2+1, ..., n\}$. Note that by the anonimity of f, we have $f(v^i) = f(v^j)$ for all $i, j \in M$. Hence $F(v) \in \{(-1, -1, ... -1), (1, 1..., 1)\}$. However, there exists no $i \in N$ for whom. $v_i . \in \{(-1, -1, ... -1), (1, 1..., 1)\}$. Thus F exhibits the BKZ-paradox.

Now let $m \geq 3$ and n be odd. Writing n^* for the lowest integer no less than $n/2$, consider the following vote profile $v \in V$ where $v_i^1 = 1$ for all $i \in \{1, ..., n^*\}$ and $v_i^1 = -1$ for all $i \in \{n^*+1, ..., n\}$. On the other hand $v_i^2 = -1$ for all $i \in \{1, ..., n^*-1\}$ and $v_i^2 = 1$ for all $i \in \{n^*, ..., n\}$. Finally let $v_i^3 = 1$ for all $i \in \{1, ..., n^*-1, n^*+1\}$ and $v_i^3 = -1$ for all $i \in \{n^*, n^*+2, ..., n\}$. In case $m > 3$, let $v^j = v^1$ for all $j \in \{4, ..., m\}$. Note that by the anonimity of f, we have $f(v^i) = f(v^j)$ for all $i, j \in M$. Hence $F(v) \in \{(-1, -1, ... -1), (1, 1..., 1)\}$. However, there exists no $i \in N$ for whom $v_i \in \{(-1, -1, ... -1), (1, 1..., 1)\}$. Thus F exhibits the BKZ-paradox, completing the proof. ■

Theorems 3.1 and 3.2 lead to the following theorem as a corollary:

Theorem 3.3 *Let $m \geq 3$ or n be even. There exists no Pareto ensuring anonymous referendum voting rule $F : V \rightarrow A$.*

Our impossibility results do not cover the case where there are two proposals and an odd number of voters. In fact, for this particular case, we do have a unique Pareto ensuring and anonymous referendum voting rule which uses the well-known majority rule as its corresponding simple voting rule. To be sure, a simple voting rule $f : V \to A$ is the majority rule if and only if $f(v) = sgn(\sum\limits_{i \in N} v_i)$.[5]

Theorem 3.4 *Let $m = 2$ and n be odd. An anonymous referendum voting rule is Pareto ensuring if and only if its corresponding simple voting rule is the majority rule.*

Proof. Let $m = 2$ and n be odd. To show the "if" part, consider the referendum voting rule $F : V \to A$ with its corresponding simple voting rule f being the majority rule. For every vote profile $v \in V$, write $F(v) = (f(v^1), f(v^2))$. By definition of f, we have $\#\{i \in N : v_i^1 = f(v^1)\} > n/2$ and $\#\{i \in N : v_i^2 = f(v^2)\} > n/2$. So there exists at least one agent $i \in N$ with $v_i = (v_i^1, v_i^2) = (f(v^1),\ f(v^2)) = F(v)$. Thus F escapes the BKZ-Paradox. Hence, by Theorem 3.1, F is Pareto ensuring.

To show the "only if" part, consider any anonymous referendum voting rule $F : V \to A$ having a corresponding simple voting rule f which is not the majority rule. Again for every vote profile $v \in V$, write $F(v) = (f(v^1), f(v^2))$. As f is not the majority rule, there exists some $v \in V$ and $j \in \{1, 2\}$ such that $\#\{i \in N : v_i^j = f(v^j)\} = r < n - r$. Let $j = 1$ without loss of generality. Now define a vote profile $u \in V$ as follows: $u_i^1 = f(v^1)$ for all $i \in \{1, ..., r\}$, $u_i^1 = -f(v^1)$ for all $i \in \{r + 1, ..., n\}$, $u_i^2 = -f(v^1)$ for all $i \in \{1, ..., n - r\}$ and $u_i^2 = f(v^1)$ for all $i \in \{n - r + 1, ..., n\}$. By anonymity of f, we have $f(u^1) = f(u^2) = f(v^1)$, i.e., $F(u) = (f(v^1),\ f(v^1))$. However, there exists no $i \in N$ for whom $u_i = (f(v^1),\ f(v^1))$, showing that F exhibits the BKZ-paradox, which, again by Theorem 3.1, proves that F is not Pareto ensuring. ■

Note that the majority rule is the only simple voting rule which can induce a Pareto ensuring referendum voting rule - at least for some social choice problems with two proposals and an odd number of voters. We state this in the following corollary as a new characterization of the majority rule:

Corollary 3.1 *A simple voting rule is the majority rule if and only if it is anonymous and it can induce a Pareto ensuring referendum voting rule at some size of the social choice problem.*

[5]Given any real number r, sgn(r) respectively equals 1, 0 and -1 when r >0, r=0, r<0. Note that when n is odd, $\sum\limits_{i \in N} v_i \neq 0$.

In spite of the restricted positive result announced by Theorem 3.4, what we establish is a basic impossibility in ensuring Pareto optimal outcomes through referendum voting. We now ask whether a possibility result could be obtained for a larger class of voting rules which allow for social indifference in outcomes. In this more general world, an outcome is an m-tuple $x \in \{-1, 0, 1\}^m$ where $x^j = 0$ means social indifference for proposal $j \in M$. We write $\overline{A} = \{-1, 0, 1\}^m$ for the set of all possible outcomes. So a voting rule is a mapping $F : V \to \overline{A}$. A voting rule $F : V \to \overline{A}$ is said to be *decisive* if and only if given any $v \in V$, writing $x = F(v)$, we have $x_j = 0$ for no $j \in M$. So our results upto now are for decisive voting rules. On the other hand, it is not possible to escape the established impossibilities by allowing social indifference in outcomes, as decisiveness of a referendum voting rule is a necessary condition for its being Pareto ensuring. We state this in the following theorem:

Theorem 3.5 *A referendum voting rule $F : V \to \overline{A}$ is Pareto ensuring only if F is decisive.*

Proof. Take any anonymous voting rule $F : V \to \overline{A}$ which is not decisive. So there exists some $v \in V$ where, writing $x = F(v)$, we have $x^k = 0$ for some $k \in M$. Fix that particular $k \in M$ as well as the list v^k of individual opinions over k. Let f be the corresponding simple voting rule of F. Note that $f(v^k) = 0$.

First, consider the case where $v_i^k = v_j^k$ for all $i, j \in N$. Let $y \in \overline{A}$ be defined as $y^r = x^r$ for all $r \in M \backslash \{k\}$ and $y^k = v_i^k$ for some $i \in N$. Clearly y $B(v_i)$ x for all $i \in N$. So given any $R \in \Sigma(v)$, we have y P_i x for all $i \in N$. Thus F is not Pareto ensuring.

Next, consider the case where there exists $i, j \in N$ such that $v_i^k = 1$ and $v_j^k = -1$. Take a vote profile $u \in V$ where $u^r = v^k$ for all $r \in M$. So we have $F(u) = (f(u^1), ..., f(u^m)) = (0, ..., 0)$. Let $y \in \overline{A}$ be defined as $y^1 = 1, y^2 = -1$ and $y^r = 0$ for all $r \in M \backslash \{1, 2\}$. It is straightforward to check that $F(u)$ $B(u_i)$ y holds for no $i \in N$. Hence there exists some $R \in \Sigma(u)$ such that y P_i $F(u)$ for all $i \in N$, i.e., there exists some $R \in \Sigma(u)$ according to which $F(u)$ is not Pareto optimal, showing that F is not Pareto ensuring, thus completing the proof. ■

4 Final Remarks

We state an impossibility result about the non-existence of anonymous Pareto ensuring referendum voting rules - except for a particular case of two proposals and an odd number of voters. This result is established through the equivalence of Pareto ensurance and the escape of the paradox of multiple elections intorduced by Brams et al. (1998).

In the particular case of two proposals with an odd number of voters, Pareto optimality of the resulting referendum outcome can be guaranteed if and only if we use majority voting to decide on separate proposals. This restricted but positive result can be interpreted as another characterization of the majority rule in the context of referendum voting.[6]

We mainly consider a world where indifferences are ruled out both in individual preferences and in the final outcome reflecting the social preference. Our main results being negative, expanding the domains of voting rules through allowing indifferences in individual preferences can do no better. Nevertheless, it makes sense to ask whether it is possible to escape the impossibility in ensuring Pareto optimal outcomes by extending the range of voting rules by allowing social indifferences in outcomes. Theorem 3.4 is a strong negative answer to this question: Being decisive, i.e., not allowing for social indifferences in outcomes, is a necessary condition for a voting rule to be Pareto ensuring. Hence the positive but restricted result annonced by Theorem 3.3 is the best one can achieve regarding Pareto optimality in the context of anonymous referendum voting.

These negative results are valid when all separable orderings over outcomes are admissible. Of course enlarging the set of admissible orderings by allowing for additional non-separable preferences will strenghten the impossibility. On the other hand, it is easier to ensure Pareto optimality by restricting the set of admissible orderings. In fact, there exists strong but reasonable restrictions where one can escape the impossibility we establish. To see this, assume voters give equal importance to all proposals and rank the outcomes according to the number of components on which they differ from their best outcome.[7] This extension method is called the "Hamming

[6]The majority rule is first characterized by May (1952). For more recent characterizations one can refer to Aşan and Sanver (2002) as well as Woeginger (2003).

[7]For example, a voter whose best outcome is (1, -1) will be indifferent between (1, 1) and (-1, -1) which both disagree in one component from their top outcome. The worst outcome of this voter will be (-1, 1) where there is disagreement in both components.

distance" and for any best outcome, it leads to a unique separable ordering of outcomes. Brams et al. (2003) show that referendum voting via the majority rule leads to an outcome which minimizes the sum of the Hamming distances to the top preferences of all players. Hence it is Pareto ensuring when the set of admissible orderings over outcomes is (severely) restricted by assuming the Hamming distance.[8]

To sum up, referendum voting has the important merit of being simple but at the cost of a possible loss of efficiency - in case all separable orderings over outcomes are allowed. The heaviness of his cost depends on how often Pareto optimality is violated as a function of the number of proposals, a question which is subject to further research.

We wish to close by noting that referendum voting is defined through the use of the same simple voting rule for every proposal. This could be generalized by allowing to use different simple voting rules for different proposals. The effect of this generalization to our impossibility results remains as an open question to be investigated.

---

[8]In this regard, our negative result is weaker than the one shown by Kaymak and Sanver (2003) about the impossibility of ensuring that sets are Condorcet winners - a negative result which turns out to prevail almost independent of how one restricts the extensions of orderings over alternatives to sets.

References

Aşan, G. and M. R. Sanver (2002), Another Charazterization of the Majority Rule, Economics Letters, 409-413.

Benoit, J.P. and L. A. Kornhauser (1994), Social Choice in a Representative Democracy, American Political Science Review, 88, 185-192.

Brams, S.J. and P.C. Fishburn (1993), Yes-No Voting, Social Choice and Welfare, 10, 35-50.

Brams, S.J., Kilgour, D.M. and W.S. Zwicker (1997), Voting on Referenda: The Separability Problem and Possible Solutions, Electoral Studies, 16 (3), 359-377.

Brams, S.J., Kilgour, D.M. and W.S. Zwicker (1998), The Paradox of Multiple Elections, Social Choice and Welfare, 15, 211-236.

Brams, S.J., Kilgour, D.M. and M.R. Sanver (2003), A Minimax Procedure for Negotiating Multilateral Treaties, Preprint.

Kaymak, B. and M. R. Sanver (2003),. Sets of Alternatives as Condorcet Winners, Social Choice and Welfare, 20 (3), 477-494.

Kilgour, D.M. (1997), Separable and Nonseparable Preferences in Multiple Referenda, Preprint, Department of Mathematics, Wilfrid Laurier University.

Lacy, D. and E. M. S. Niou (2000),. A Problem with Referendums, Journal of Theoretical Politics,12 (1), 5-31.

May, K. (1952), A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision, Econometrica, 20, 680-684.

Woeginger, G. (2003), A New Characterization of the Majority Rule, Economics Letters, 81, 89-94.