#### Novel Analytical Calculation Method for the Non-Linear Ψ-i-Characteristic of Switched-Reluctance-Machines in Arbitrary Rotor Positions

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«Switched reluctance drive», «Modelling»

## Abstract

The non-linear  $\Psi$ -i-characteristic is crucial for the design of switched reluctance machines. Known analytical calculations are based on complex models of the magnetic circuit or on functions needing a fitting procedure (using measured or FEM-calculated data). In this paper, a method is presented that requires only very few input data, which can be deduced easily from the geometry of the machine. Comparisons with measured data show an acceptable correlation for arbitrary rotor positions, qualifying this method to be used in the design stage of new drives.

## Introduction

The non-linear  $\Psi$ -i-characteristic (flux linkage versus phase current) is essential, if the performance of switched reluctance machines has to be calculated. It is desirable to have an analytical calculation procedure, but up to now either fitting data [1-3] or complex models [4,5] are required. In [6] a calculation method for the aligned rotor position has been described that is quite simple and uses only very few input data.

In this paper, the analytical approach for calculating the non-linear  $\Psi$ -i-characteristic in the aligned position [6] will be expanded to arbitrary rotor positions.

## Characteristic for low phase current

If the slope of the  $\Psi$ -i-characteristic for low currents is known ( $\Psi(i) = A \cdot i$  for the aligned position and  $\Psi(i) = D \cdot i$  for the unaligned position) the following assumptions are made for parameter "A" as a function of rotor position:

- 1. A rotational shift from the aligned position (i.e.  $0^{\circ}$ ) does not affect the slope of the  $\Psi$ -icharacteristic, as long as this shift is less than half of the difference between stator tooth angle and rotor tooth angle (in the following example:  $0.5^{\circ}$ ). For details see Fig. 1.
- 2. When the rotor tooth and the stator tooth are positioned edge-on-edge (in the following example: 18°), the slope in this position relating to the slope in the aligned position is inverse proportional to the effective length of the medium field line. As the main effect in reducing the flux is the reduced area of the small gap between stator and rotor, this reduction is assumed being linear with the rotational angle. For details see Fig 2.
- 3. A further rotation from the position described in 2. to the unaligned position (i.e. 30° for the regarded 8/6 machine) reduces the flux inversely proportional to the effective length of the medium field line. If the slope "D" is reached, this is fixed for the rest of the rotation.

In addition the parameters "B" and "C" (please refer to [6]) are assumed to vary linearly with the rotor position towards "D" and zero, respectively.



Fig. 1: Left hand side: aligned position, right hand side the position of stator and rotor after marginal rotational shift. Grey lines between stator and rotor indicate magnetic flux lines.  $\gamma_1 = 0.5^\circ$  for the regarded machine.



Fig. 2: Rotor tooth and stator tooth positioned edge to edge. Grey lines between rotor and stator: magnetical flux.  $\gamma_2 = 18^{\circ}$  for the regarded 8/6 machine.

In the following it is assumed that "A" at aligned position and "D" at unaligned position are known. The radius of the medium field line at  $\gamma = \gamma_2$  (see Fig. 2) can be calculated as follows:

$$r = \frac{0.5 \cdot (\gamma_2 - \gamma_1)}{360^\circ} \cdot 2 \cdot \pi \cdot R_1 \cdot \frac{1}{2} \tag{1}$$

 $R_1$  ist the radius of the rotor. Now we get the ratio "X" between the field line lengths of the aligned position and the rotor position at  $\gamma = \gamma_2$ :

$$X = \frac{\delta}{\pi \cdot \frac{r}{2} + \delta}$$
(2)

(4)

With

$$Y = D + X \cdot (A - D) \tag{3}$$

we get for the slope  $A_{var}$  which is dependent on the rotor position  $\gamma$ : If  $\gamma \le \gamma_1$ :  $A_{var} = A$ 

$$\gamma_1 < \gamma \le \gamma_2$$
:  $A_{\text{var}} = A + (Y + A) \cdot \frac{\gamma - \gamma_1}{\gamma_2 - \gamma_1}$  (5)

$$\gamma > \gamma_2: \qquad \qquad A_{\text{var}} = Y \cdot \frac{\pi \cdot \frac{r}{2} + \delta}{\pi \cdot \frac{0.5 \cdot (\gamma_2 - \gamma_1) + (\gamma - \gamma_2)}{2 \cdot 2 \cdot 360^\circ} \cdot 2 \cdot \pi \cdot R_1 + \delta}$$
(6)

If the equation  $A_{\text{var}} \ge D$  is false it is defined that  $A_{\text{var}} = D$ .

A comparison of measured data and the described calculation model is presented in Fig. 3. The difference between calculation and measurement in the unaligned position comes from the fact that for the measurements the slope for low current and for the analytical model the slope for high current are given (which are slightly different because the magnetization curve of iron is not completely linear at the beginning).



Fig. 3: Slope of the  $\Psi$ -i-characteristic for low phase current and different rotor positions (red: measurements; blue: calculation).

## **Arbitrary Rotor Positions and Current Levels**

Using the analytical calculation procedure given in [6], the  $\Psi$ -i-characteristic can now be calculated for arbitrary rotor positions and arbitrary current levels. The results are given in Fig. 4a and 4b :



phase current (A)

Fig. 4a:  $\Psi$ -i-characteristics, rotor angle: 0°, 10°, 20°, and 30° (red: measurements; blue: calculations)



Fig. 4b:  $\Psi$ -i-characteristics, rotor angle: 5°, 15°, and 25° (red: measurements; blue: calculations)

# Inductivity

Another important characteristic is the inductivity versus phase current for different rotor positions. The inductivity is calculated from  $L = \frac{\partial \Psi}{\partial i}$  and shown in the following figures:







Fig. 5b: L-i-characteristics rotor angle: 5°, 15°, and 25° (red: measurements; blue: calculations).

#### Conclusion

The presented method for calculating the non-linear  $\Psi$ -i-characteristic is based on the knowledge of the linear branches of the  $\Psi$ -i-characteristic. These parameters can be deduced from the geometry of the machine quite easily. Consequently, complex models or functions needing fitting procedures are avoided. Comparisons with measured data show an acceptable preciseness for any rotor position. Therefore, this method is qualified to be used in the design procedure of new switched reluctance machines as a first step, e.g. for very fast torque calculation or estimation of the material utilization.

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