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A NEW APPROACH ON SOLVING INTUITIONISTIC FUZZY LINEAR PROGRAMMING PROBLEM

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ABSTRACT

In this paper, we propose a new approach for solving Intuitionistic Fuzzy Linear Programming Problems (IFLPP) involving triangular intuitionistic fuzzy numbers (TIFN). We introduce a new algorithm for the solution of an Intuitionistic Fuzzy Linear Programming Problem without converting in to one or more classical Linear Programming Problems. Numerical examples are provided to show the efficiency of the proposed algorithm.

Keywords: intuitionistic fuzzy set, fuzzy number, triangular intuitionistic fuzzy number, fuzzy linear programming problem.

INTRODUCTION

Modelling of real life problems involving optimization process. It is often difficult to get crisp and exact information for various parameters affecting the process and it involves high information cost. Furthermore the optimal solution of the problem depends on a limited number of constraints or conditions and thus some of the information collected is not useful. Under such situations it is highly impossible to formulate the mathematical model through the classical traditional methods. Hence in order to reduce information costs and also to construct a real model, the use of intuitionistic fuzzy number is more appropriate. Fuzzy sets are an efficient and reliable tool that allows us to handle such systems having imprecise parameters effectively. Atanossov [6] extended the fuzzy sets to the theory intuitionistic fuzzy sets. His studies emphasized that in view of handling imprecision, vagueness or uncertainty in information both the degree of belonging and degree of non-belonging should be considered as two independent properties as these are not complement of each other.

Bellmann and Zadeh [7] proposed the concept of decision making in fuzzy environment. Angelo [2] extended the Bellman and Zadeh [7] approach of maximizing the degree of (membership function) acceptance of the objective functions and constraints to maximizing the degree of acceptance and minimizing the degree of rejection of objective functions and constraints. Many authors such as Mahapatra *et al*. [15], Nachammai [18] and Nagoorgani [21], Tanaka[26] , Luhandjula [12], Sakawa [25] etc have also studied linear programming problems under intuitionistic fuzzy environment. Jana and Roy [11] studied the multi objective intuitionistic fuzzy linear programming problem and applied it to transportation problem. Luo [13] applied the inclusion degree of intuitionistic fuzzy set to multi criteria decision making problem. Recently Dubey *et al* [8] have studied fuzzy linear programming with intuitionistic fuzzy numbers. Parvathi and Malathi [23], [23] developed intuitionistic fuzzy simplex method. Nagoor Gani and Abbas [20] proposed a method for solving intuitionistic Transportation Problem. By using a new type of arithmetic operations and a new ranking method, we propose a simple method for the solution for the fuzzy linear programming with intuitionistic fuzzy numbers and without converting to crisp linear programming problems.

The rest of the paper is arranged as follows: Section 2 is preliminaries to intuitionistic fuzzy set and intuitionistic fuzzy numbers needed for consequent sections. Section 3 comprise of modelling of an intuitionistic fuzzy optimization problem and its solution algorithm. Section 4 illustrates the implementation of the theory developed in section 3 to a linear programming problem. Last section presents the results of the undertaken problem and provides a brief discussion on the developed method. Numerical example is also provided to illustrate the efficiency of the developed method.

PRELIMINARIES

The aim of this section is to present some notations, notions and results which are of useful in our further study.

Definition 1

Let X be a universe of discourse, then an Intuitionistic Fuzzy Set (IFS) A in X is given by $\tilde{A}^I = \{ (x, \mu_A(x), \gamma_A(x)) / x \in X \}$ where the functions $\mu_A(x) : X \to [0,1]$ and $\gamma_A(x) : X \to [0,1]$ determine the degree of membership and degree of non membership of the element $x \in X$, respectively, and for every $x \in X$, $0 \le \mu_A(x) + \gamma_A(x) \le 1$.

Note: Throughout this paper μ represents membership values and γ represents non membership values.

Definition 2

For every common fuzzy subset A on X, Intuitionistic Fuzzy Index of x in A is defined as

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$\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$. It is also known as degree of hesitancy or degree of uncertainty of the element x in A. Obviously, for every $x \in X, 0 \le \pi_A(x) \le 1$.

Definition 3

An Intuitionistic Fuzzy Number (IFN) $\tilde{A}^I$ is

(i) an Intuitionistic fuzzy subset of the real line,

(ii) normal, that is there is any $x_0 \in R$, such that $\mu_{\tilde{A}^I}(x_0) = 1, \gamma_{\tilde{A}^I}(x_0) = 0$.

(iii) convex for the membership function $\mu_{\tilde{A}^I}(x)$, that is, $\mu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \ge \min(\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2))$ for every $x_1, x_2 \in R, \lambda \in [0,1]$.

(iv) concave for the non-membership function $\gamma_{\tilde{A}^I}(x)$, i.e. $\gamma_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \le \max(\gamma_{\tilde{A}^I}(x_1), \gamma_{\tilde{A}^I}(x_2))$ for every $x_1, x_2 \in R, \lambda \in [0,1]$.

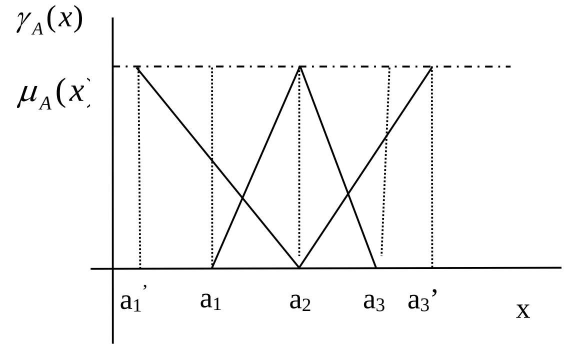
Definition 4

$\tilde{A}^I$ is Triangular Intuitionistic Fuzzy Number (TIFN) with parameters $a_1' \le a_1 \le a_2 \le a_3 \le a_3'$ and denoted by $\tilde{A}^I = (a_1, a_2, a_3; a_1', a_2, a_3')$ having the membership function and non-membership function as follows:

$$\mu_{\tilde{A}^I}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \le x \le a_2 \\ 1 & \text{for } x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \le x \le a_3 \\ 0 & \text{for } x > a_3 \end{cases}$$

and

$$\gamma_{\tilde{A}^I}(x) = \begin{cases} 1 & \text{for } x < a_1' \\ \dfrac{a_2 - x}{a_2 - a_1'} & \text{for } a_1' \le x \le a_2 \\ 0 & \text{for } x = a_2 \\ \dfrac{x - a_2}{a_3' - a_2} & \text{for } a_2 \le x \le a_3' \\ 1 & \text{for } x > a_3' \end{cases}$$



Figuer-1. Membership and non-membership functions of a Triangular intuitionistic fuzzy number.

Note: Here $\mu_{\tilde{A}^I}(x)$ increases with constant rate for $x \in [a_1, a_2]$ and decreases with constant rate for $x \in [a_2, a_3]$, but $\gamma_{\tilde{A}^I}(x)$ decreases with constant rate for $x \in [a_1', a_2]$ and increases with constant rate for $x \in [a_2, a_3']$.

Particular case

Let $\tilde{a}^I = (a_1, a_2, a_3; a_1', a_2, a_3')$ be a Triangular Intuitionistic fuzzy number then the following cases arises

Case: 1 If $a_1' = a_1$, $a_3' = a_3$ then $\tilde{A}^I$ represent Triangular Fuzzy number (TFN).

Case: 2 If $a_1' = a_1 = a_3' = a_3 = m$ then $\tilde{A}^I$ represent a real number m.

We denote this triangular Intuitionistic fuzzy number by $\tilde{a}^I = (a_1, a_2, a_3; a_1', a_2, a_3')$. We use $F(R)$ to denote the set of all intuitionistic fuzzy numbers.

Also if $m = a_2$ represents the modal value (or) midpoint, $\alpha_1 = (a_2 - a_1)$ represents the left spread and $\beta_1 = (a_3 - a_2)$ right spread of membership function and $\alpha_2 = (a_2 - a_1')$ represents the left spread and $\beta_2 = (a_3' - a_2)$ right spread of non- membership function.

Definition 5

Triangular Intuitionistic fuzzy numbers $\tilde{a}^I \in F(R)$ can also be represented as a pair $\tilde{a}^I = (\underline{a}, \overline{a}; \underline{a}', \overline{a}')$ of functions $\underline{a}(r), \overline{a}(r), \underline{a}'(r)$ and $\overline{a}'(r)$ for $0 \le r \le 1$ which satisfies the following requirements:

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(i) $\underline{a}(r)$ is a bounded monotonic increasing left continuous function for membership function
(ii) $\overline{a}(r)$ is a bounded monotonic decreasing left continuous function for membership function.
(iii) $\underline{a}(r) \le \overline{a}(r)$, $0 \le r \le 1$
(iv) $\underline{a}'(r)$ is a bounded monotonic decreasing left continuous function for non- membership function.
(v) $\overline{a}'(r)$ is a bounded monotonic increasing left continuous function for non-membership function.
(vi) $\underline{a}'(r) \le \overline{a}'(r)$, $0 \le r \le 1$.

Definition 6

For an arbitrary Intuitionistic triangular fuzzy number $\tilde{a}^I = \left(\underline{a}, \overline{a}; \underline{a}', \overline{a}'\right)$, the number $a_0 = \left(\frac{\underline{a}(1)+\overline{a}(1)}{2}\right)$ or $a_0 = \left(\frac{\underline{a}'(1)+\overline{a}'(1)}{2}\right)$ are said to be a location index number of membership and non-membership functions. The non-decreasing left continuous functions $a_* = (a_0 - \underline{a})$, $a^* = (\overline{a} - a_0)$ are called the left fuzziness index function and the right fuzziness index function for membership function and the non-decreasing left continuous functions $a'_* = (a_0 - \underline{a}')$, $a'^* = (\overline{a}' - a_0)$ are called the left fuzziness index function and the right fuzziness index function for non-membership function respectively. Hence every triangular Intuitionistic fuzzy number $\tilde{a}^I = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ can also be represented by $\tilde{a}^I = \left(a_0, a_*, a^*; a_0, a'_*, a'^*\right)$.

Arithmetic operation on triangular intuitionistic fuzzy numbers

Ming Ma *et al.*, [17] have proposed a new fuzzy arithmetic based upon both location index and fuzziness index functions.

$$Mag(\tilde{a}^I) = \frac{1}{2}\left(\int_0^1 (\underline{a} + \overline{a} + 2a_0 + \underline{a}' + \overline{a}')\ f(r)\ dr\right)$$
$$= \frac{1}{2}\left(\int_0^1 (a^* + a^{*'} + 8a_0 - a_* - a'_*)\ f(r)\ dr\right).$$

The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice L. That is for a, b∈L we define $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$.

For arbitrary triangular intuitionistic fuzzy numbers $\tilde{a}^I = \left(a_0, a_*, a^*; a_0, a'_*, a'^*\right)$ and $\tilde{b}^I = \left(b_0, b_*, b^*; b_0, b'_*, b'^*\right)$ and $* = \{+, -, \times, \div\}$, the arithmetic operations on the triangular intuitionistic fuzzy numbers are defined by

$$\tilde{a}^I * \tilde{b}^I = \left(a_0 * b_0,\ a_* \vee b_*,\ a^* \vee b^*; a_0 * b_0,\ a'_* \vee b'_*,\ a'^* \vee b'^*\right).$$

In particular for any two triangular intuitionistic fuzzy numbers $\tilde{a}^I = \left(a_0, a_*, a^*; a_0, a'_*, a'^*\right)$ and $\tilde{b}^I = \left(b_0, b_*, b^*; b_0, b'_*, b'^*\right)$, we define

Addition

$$\tilde{a}^I + \tilde{b}^I = \left(a_0, a_*, a^*; a_0, a'_*, a'^*\right) + \left(b_0, b_*, b^*; b_0, b'_*, b'^*\right)$$
$$= \left(a_0 + b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\};\right.$$
$$\left. a_0 + b_0, \max\{a'_*, b'_*\}, \max\{a'^*, b'^*\}\right)$$

Subtraction

$$\tilde{a}^I - \tilde{b}^I = \left(a_0, a_*, a^*; a_0, a'_*, a'_*\right) - \left(b_0, b_*, b^*; b_0, b'_*, b'^*\right)$$
$$= \left(a_0 - b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\};\right)$$
$$\left. a_0 - b_0, \max\{a'_*, b'_*\}, \max\{a'^*, b'^*\}\right)$$

Ranking of triangular intuitionistic fuzzy number

Many authors such as Tanaka [26], Grzegorzewski [10], Parvathi and Malathi [23] have developed the ranking methods for intuitionistic fuzzy numbers. Abbasbandy and Hajjari 0 proposed a new ranking method based on the left and the right spreads at some α -levels of fuzzy numbers

For an arbitrary triangular fuzzy number $\tilde{a}^I = \left(a_0, a_*, a^*; a_0, a'_*, a'^*\right)$ with parametric form $\tilde{a}^I = \left(\underline{a}, \overline{a}; \underline{a}', \overline{a}'\right)$, we define the magnitude of the triangular fuzzy number $\tilde{a}^I$ by

where the function $f(r)$ is a non-negative and increasing function on $[0,1]$ with $f(0)=0$, $f(1)=1$ and $\int_0^1 f(r)\,dr = \frac{1}{2}$. The function $f(r)$ can be considered as a weighting function. In real life applications, $f(r)$ can be chosen by the decision maker according to the situation. In this paper, for convenience we use $f(r) = r$.

Hence

$$Mag(\tilde{a}^I) = \left(\frac{a^* + a^{*'} + 8a_0 - a_* - a'_*}{4}\right) = \left(\frac{\underline{a} + \overline{a} + 2a_0 + \underline{a}' + \overline{a}'}{4}\right).$$

The magnitude of a triangular intuitionistic fuzzy number $\tilde{a}^I$ synthetically reflects the information on every membership degree, and meaning of this magnitude is

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visual and natural. $Mag(\tilde{a}^I)$ is used to rank fuzzy numbers. The larger $Mag(\tilde{a}^I)$ is larger fuzzy number.

For any two triangular intuitionistic fuzzy numbers $\tilde{a}^I = (a_0, a_*, a^*; a_0, a'_*, a'^*)$ and $\tilde{b}^I = (b_0, b_*, b^*; b_0, b'_*, b'^*)$ in F(R), we define the ranking of $\tilde{a}^I$ and $\tilde{b}^I$ by comparing the $Mag(\tilde{a}^I)$ and $Mag(\tilde{b}^I)$ on R as follows:

(i) $\tilde{a}^I \succeq \tilde{b}^I$ if and only if $Mag(\tilde{a}^I) \geq Mag(\tilde{b}^I)$

(ii) $\tilde{a}^I \preceq \tilde{b}^I$ if and only if $Mag(\tilde{a}^I) \leq Mag(\tilde{b}^I)$

(iii) $\tilde{a}^I \approx \tilde{b}^I$ if and only if $Mag(\tilde{a}^I) = Mag(\tilde{b}^I)$

FUZZY INTUITIONISTIC LINEAR PROGRAMMING PROBLEM

Intuitionistic Linear programming problem with triangular intuitionistic fuzzy variables is defined as follows:

$$\max \tilde{Z}^I \approx \sum_{j=1}^{n} \tilde{c}_j^{\,I} \ \tilde{x}_j^{\,I}$$

$$\text{subject to } \sum_{j=1}^{n} \tilde{a}_{ij}^{\,I} \ \tilde{x}_j^{\,I} \preceq \tilde{b}_i^{\,I} \text{ for all } i=1,2,...,m.$$

where $\tilde{A}^I = (\tilde{a}_{ij}^{\,I})$, and $\tilde{c}_j^{\,I}$, $\tilde{x}_j^{\,I}$ and $\tilde{b}_i^{\,I}$ are intuitionistic fuzzy numbers and $\tilde{x}_j^I \succeq \tilde{0}$, $j = 1,2,............n$.

Then the above problem can be rewritten as follows:

$$\max \tilde{Z}^I \approx \sum_{j=1}^{n} \left\langle (c_j)_0, (c_j)_*, (c_j)^*; (c_j)_0, (c_j)'_*, (c_j)'^* \right\rangle$$

$$\times \left\langle (x_j)_0, (x_j)_*, (x_j)^*; (x_j)_0, (x_j)'_*, (x_j)'^* \right\rangle$$

Subject to

$$\sum_{j=1}^{n} \left\langle (a_{ij})_0, (a_{ij})_*, (a_{ij})^*; (a_{ij})_0, (a_{ij})'_*, (a_{ij})'^* \right\rangle$$
$$\times \left\langle (x_j)_0, (x_j)_*, (x_j)^*; (x_j)_0, (x_j)'_*, (x_j)'^* \right\rangle$$
$$\preceq \left\langle (b_j)_0, (b_j)_*, (b_j)^*; (b_j)_0, (b_j)'_*, (b_j)'^* \right\rangle$$

for all $i = 1, 2, ..., m_0$

$$\sum_{j=1}^{n} \left\langle (a_{ij})_0, (a_{ij})_*, (a_{ij})^*; (a_{ij})_0, (a_{ij})'_*, (a_{ij})'^* \right\rangle$$

$$\times \left\langle (x_j)_0, (x_j)_*, (x_j)^*; (x_j)_0, (x_j)'_*, (x_j)'^* \right\rangle$$

$$\succeq \left\langle (b_j)_0, (b_j)_*, (b_j)^*; (b_j)_0, (b_j)'_*, (b_j)'^* \right\rangle$$

for all $i = m_0 + 1, m_0 + 2, ...m$

$$\left\langle (x_j)_0, (x_j)_*, (x_j)^*; (x_j)_0, (x_j)'_*, (x_j)'^* \right\rangle \succeq \tilde{0}$$

for all $j = 1, 2, ...n.$

Definition 7

Any vector $\tilde{x} = \left(\tilde{x}_1^{\,I}, \tilde{x}_2^{\,I}, ..., \tilde{x}_n^{\,I}\right)^T$ in $\left(F(R)\right)^n$, i.e.

$$\tilde{x}^I = \left\langle (x_1)_0, (x_1)_*, (x_1)^*; (x_1)_0, (x_1)'_*, (x_1)'^* \right\rangle,$$
$$\left\langle (x_2)_0, (x_2)_*, (x_2)^*; (x_2)_0, (x_2)'_*, (x_2)'^* \right\rangle, ...,$$
$$\left\langle (x_n)_0, (x_n)_*, (x_n)^*; (x_n)_0, (x_n)'_*, (x_n)'^* \right\rangle \in \left(F(R)\right)^n$$

is said to be a Intuitionistic fuzzy feasible solution, if for each

$$\tilde{x}_j = \left\langle (x_j)_0, (x_j)_*, (x_j)^*; (x_j)_0, (x_j)'_*, (x_j)'^* \right\rangle \in \left(F(R)\right)^n$$

satisfies the constraints and non-negativity restrictions .

Definition 8

An Intuitionistic fuzzy feasible solution is said to be a Intuitionistic fuzzy optimal solution if it optimizes the objective function of the Intuitionistic fuzzy linear programming problem.

Standard form

A standard form of Intuitionistic fuzzy linear programming problem is defined as follows:

$$\max \tilde{Z}^I \approx \tilde{C}^I \tilde{x}^I$$

subject to $\tilde{A}^I \tilde{x}^I \approx \tilde{b}^I$

and $\tilde{x}^I \succeq \tilde{0}$

where $\tilde{C}^I, \tilde{x}^I \in \left(F(R)\right)^n$ and $\tilde{b}^I \in \left(F(R)\right)^m$

NUMERICAL EXAMPLES

Example 1: Consider an example discussed by Nagoorgani and Ponnalagu [21].

$$\max \ \tilde{z}^I = (4, 5, 6; 4, 5, 6.1)\tilde{x}_1^{\,I} + (2.5, 3, 3.2; 2, 3, 3.5)\tilde{x}_2^{\,I}$$

$$\text{subject to } (3.5, 4, 4.1; 3, 4, 5)\tilde{x}_1^{\,I} + (2.5, 3, 3.5; 2.4, 3, 3.6)\tilde{x}_2^{\,I} \preceq (11,12,13;11,12,14)$$

$$(0.8, 1, 2; 0.5, 1, 2.1)\tilde{x}_1^{\,I} + (2.8, 3, 3.2; 2.5, 3, 3.2)\tilde{x}_2^{\,I} \preceq (5.6, 6, 7.5; 5, 6, 8.1)$$

and $\tilde{x}_1^{\,I}, \tilde{x}_2^{\,I} \succeq \tilde{0}$.

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Solution

Representing the intuitionistic triangular fuzzy numbers in terms of left and right index function, we have

$$\max \tilde{z}^I = \left(5,1-1r,1-1r;5,1-1r,1.1-1.1r\right)\tilde{x}_1^{\,I} + \left(3,0.5-0.5r,0.7-0.7r;3,0.7-0.7r,0.5-0.5r\right)\tilde{x}_2^{\,I}$$

subject to the constraints

$$\left(4,0.5-0.5r,0.1-0.1r;4,1-1r,1-1r\right)\tilde{x}_1^{\,I} + \left(3,0.5-0.5r,0.5-0.5r;3,0.6-0.6r,0.6-0.6r\right)\tilde{x}_2^{\,I} \preceq \left(12,1-1r,1-1r;12,1-1r,2-2r\right)$$

$$\left(1,0.2-0.2r,1-1r;1,0.5-0.5r,1.1-1.1r\right)\tilde{x}_1^{\,I} + \left(3,0.2-0.2r,0.2-0.2r;3,0.5-0.5r,0.2-0.2r\right)\tilde{x}_2^{\,I} \preceq \left(6,0.5-0.5r,1.5-1.5r;6,1-1r,2.1-2.1r\right)$$

and the triangular intuitionistic fuzzy numbers $\tilde{x}_1^{\,I}, \tilde{x}_2^{\,I} \succeq \tilde{0}$

By introducing non negative slack variables, the standard form of the Intuitionistic Linear programming problem with triangular intuitionistic fuzzy variables is

$$\max \tilde{z}^I = \left(5,1-1r,1-1r;5,1-1r,1.1-1.1r\right)\tilde{x}_1^{\,I} + \left(3,0.5-0.5r,0.7-0.7r;3,0.7-0.7r,0.5-0.5r\right)\tilde{x}_2^{\,I} + \tilde{0}\,\tilde{s}_1^I + \tilde{0}\,\tilde{s}_2^I$$

subject to the constraints

$$\left(4,0.5-0.5r,0.1-0.1r;4,1-1r,1-1r\right)\tilde{x}_1^{\,I} + \left(3,0.5-0.5r,0.5-0.5r;3,0.6-0.6r,0.6-0.6r\right)\tilde{x}_2^{\,I} + \tilde{s}_1^I \approx \left(12,1-1r,1-1r;12,1-1r,2-2r\right)$$

$$\left(1,0.2-0.2r,1-1r;1,0.5-0.5r,1.1-1.1r\right)\tilde{x}_1^{\,I} + \left(3,0.2-0.2r,0.2-0.2r;3,0.5-0.5r,0.2-0.2r\right)\tilde{x}_2^{\,I} + \tilde{s}_2^I \approx \left(6,0.5-0.5r,1.5-1.5r;6,1-1r,2.1-2.1r\right)$$

and $\tilde{x}_1^{\,I}, \tilde{x}_2^{\,I}, \tilde{s}_1^I, \tilde{s}_2^I \succeq \tilde{0}$.

The initial basic feasible intuitionistic fuzzy solution is given by

$$\tilde{x}_1^{\,I} \approx \tilde{0}, \tilde{x}_2^{\,I} \approx \tilde{0}, \tilde{s}_1^I \approx \left(12,1-1r,1-1r;12,1-1r,2-2r\right),$$
$$\tilde{s}_2^I \approx \left(6,0.5-0.5r,1.5-1.5r;6,1-1r,2.1-2.1r\right).$$

Table-1: Initial iteration.

$\tilde{C}_B^{\,I}$	$\tilde{y}_B^{\,I}$	$\tilde{x}_B^{\,I}$	$\tilde{x}_1^{\,I}$	$\tilde{x}_2^{\,I}$	$\tilde{s}_1^{\,I}$	$\tilde{s}_2^{\,I}$	θ
$\tilde{0}^I$	$\tilde{s}_1^{\,I}$	$(12,1-1r,1-1r;\ 12,1-1r,1-1r)$	$(4,0.5-0.5r,0.1-0.1r;\ 4,1-1r,1-1r)$	$(3,0.5-0.5r,0.5-0.5r;\ 3,0.6-0.6r,0.6-0.6r)$	$\tilde{1}^I$	$\tilde{0}^I$	$(3,1-1r,1-1r;\ 3,1-1r,1-1r)$
$\tilde{0}^I$	$\tilde{s}_2^{\,I}$	$(6,0.5-0.5r,1.5-1.5r;\ 6,1-1r,2.1-2.1r)$	$(1,0.2-0.2r,1-1r;\ 1,0.5-0.5r,1.1-1.1r)$	$(3,0.2-0.2r,0.2-0.2r;\ 3,0.5-0.5r,0.2-0.2r)$	$\tilde{0}^I$	$\tilde{1}^I$	$(6,0.5-0.5r,1.5-1.5r;\ 6,1-1r,2.1-2.1r)$
$\tilde{Z}_j^{\,I}-\tilde{C}_j^{\,I}$		$\tilde{0}^I$	$(-5,1-1r,1-1r;\ -5,1.1-1.1r,1-1r)$	$(-3,0.7-0.7r,0.5-0.5r;\ -3,0.5-05r,0.7-0.7r)$	$\tilde{0}^I$	$\tilde{0}^I$	

Here $\left(\tilde{Z}_j^{\,I}-\tilde{C}_j^{\,I}\right) \preceq \tilde{0}^I$, so $\tilde{x}_1^{\,I}$ enter the basis and $\tilde{s}_1^{\,I}$ leaves the basis.

Table-2: First iteration.

$\tilde{C}_B^{\,I}$	$\tilde{y}_B^{\,I}$	$\tilde{x}_B^{\,I}$	$\tilde{x}_1^{\,I}$	$\tilde{x}_2^{\,I}$	$\tilde{s}_1^{\,I}$	$\tilde{s}_2^{\,I}$
$(5,1-1r,1-1r;\ 5,1.1-1.1r,1-1r)$	$\tilde{x}_1^{\,I}$	$(3,1-1r,1-1r;\ 3,1-1r,1-1r)$	$(1,0.5-0.5r,0.1-0.1r;\ 1,1-1r,1-1r)$	$\left(\frac{3}{4},0.5-0.5r,0.5-0.5r;\ \frac{3}{4},1-1r,1-1r\right)$	$\left(\frac{1}{4},0.5-0.5r,0.5-0.5r;\ \frac{1}{4},1-1r,1-1r\right)$	$\tilde{0}^I$
$\tilde{0}$	$\tilde{s}_2^{\,I}$	$(3,1-1r,1-1r;\ 3,1-1r,2.1-2.1r)$	$\tilde{0}^I$	$\left(\frac{9}{4},0.5-0.5r,1-1r;\ \frac{9}{4},1-1r,1.1-1.1r\right)$	$\left(\frac{-1}{4},0.5-0.5r,1-1r;\ \frac{-1}{4},1-1r,1.1-1.1r\right)$	$\tilde{1}^I$

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$\tilde{Z}_j^I - \tilde{C}_j^I$	$(15,1-r,1-r;$ $15,1-1r,2.1-2.1r)$	$\tilde{0}^I$	$\left(\frac{3}{4},1-1r,1-1r;\right.$ $\left.\frac{3}{4},1-1r,1.1-1.1r\right)$	$\left(\frac{5}{4},1-1r,1-1r;\right.$ $\left.\frac{5}{4},1-1r,1.1-1.1r\right)$	$\tilde{0}^I$

Since all $\left(\tilde{Z}_j^I - \tilde{C}_j^I\right) \succeq \tilde{0}^I$, the current intuitionistic fuzzy basic feasible solution is intuitionistic fuzzy optimal. The intuitionistic fuzzy optimal solution is $\tilde{x}_1^I = (3,1-1r,1-1r;3,1-1r,1-1r)$, and $\tilde{x}_2^I = \tilde{0}^I$, $\tilde{Z}^I = (15,1-r,1-r;15,1-1r,2.1-2.1r)$.

For $r=0$, we have the intuitionistic fuzzy optimal solution in terms of location index and fuzziness index as $\tilde{x}_1^I = (3,1,1;3,1,2)$, $\tilde{x}_2^I = 0$, $\tilde{Z}^I = (15,1,1;15,1,2)$.

Hence the fuzzy optimal solution in the general form, in terms of $\tilde{a}^I = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ for $r=0$ is $\tilde{x}_1^I = (2,3,4;2,3,5)$, $\tilde{x}_2^I = 0$, $\tilde{Z}^I = (14,15,16;14,15,17)$.

Table-3. Comparison of intuitionistic fuzzy optimal solution obtained by our method and by Nagoorgani and Ponnalagu [21] method.

	Our method	**Nagoorgani and Ponnalagu [21]method**
Fuzzy optimal solution	$\max\ \tilde{z}^I = (4,5,6;4,5,6.1)\tilde{x}_1^I + (2.5,3,3.2;2,3,3.5)\tilde{x}_2^I$	$\max\ \tilde{z}^I = (4,5,6;4,5,6.1)\tilde{x}_1^I + (2.5,3,3.2;2,3,3.5)\tilde{x}_2^I$
Fuzzy optimal solution in terms of left and right index function	$\max \tilde{z}^I = (5,1-1r,1-1r;5,1-1r,1.1-1.1r)\tilde{x}_1^I + (3,0.5-0.5r,0.7-0.7r;3,0.7-0.7r,0.5-0.5r)\tilde{x}_2^I$	$\max \tilde{Z}^I = (10.72,15,22.26;8.8,15,28.48)$. (With out using "r" or without any flexibility)
For $r=0$	$\max \tilde{Z}^I = (14,15,16;14,15,17)$.	
For $r=0.25$	$\max \tilde{Z}^I = (14.25,15,15.75;14.25,15,16.5)$.	
For $r=0.5$	$\max \tilde{Z}^I = (14.5,15,15.5;14.5,15,16)$.	
For $r=0.75$	$\max \tilde{Z}^I = (14.75,15,15.25;14.75,15,15.5)$.	
For $r=1$	$\max \tilde{Z}^I = 15$	

It is to be noted that the Decision Maker have the flexibility of choosing $r \in [0,1]$ depending upon the situation and his wish by applying the proposed method in this paper whereas it is not possible by applying Nagoorgani and Ponnalagu [21] method.

Example 2: Consider an example discussed by Nachammai and Thangaraj [18].

$$\max\ \tilde{z}^I = (6,9,11;5,9,14)\tilde{x}_1^I + (9,10,15;7,10,17)\tilde{x}_2^I$$
$$\text{subject to } (2,3,6;1,3,8)\tilde{x}_1^I + (4,6,9;2,6,15)\tilde{x}_2^I \preceq (15,18,24;11,18,27)$$
$$(4,8,9;2,8,14)\tilde{x}_1^I + (6,7,10;5,7,13)\tilde{x}_2^I \preceq (14,20,26;10,20,29)$$
and triangular intuitionistic fuzzy numbers $\tilde{x}_1^I, \tilde{x}_2^I \succeq \tilde{0}$

Solution

Representing the intuitionistic triangular fuzzy numbers in terms of left and right index function, we have

$$\max \tilde{z}^I = (9,3-3r,2-2r;9,4-4r,5-5r)\tilde{x}_1^I + (10,1-1r,5-5r;10,3-3r,7-7r)\tilde{x}_2^I$$
subject to constraints
$$(3,1-1r,3-3r;3,2-2r,5-5r)\tilde{x}_1^I + (6,2-2r,3-3r;6,4-4r,9-9r)\tilde{x}_2^I \preceq (18,3-3r,6-6r;18,7-7r,9-9r)$$
$$(8,4-4r,1-1r;8,6-6r,6-6r)\tilde{x}_1^I + (7,1-1r,3-3r;7,2-2r,6-6r)\tilde{x}_2^I \preceq (20,6-6r,6-6r;20,10-10r,9-9r)$$
and triangular intuitionistic fuzzy numbers $\tilde{x}_1^I, \tilde{x}_2^I \succeq \tilde{0}$

By introducing non negative slack variables, the standard form of the given problem is

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$$\max \tilde{z}^I = (9,3-3r,2-2r;9,4-4r,5-5r)\,\tilde{x}_1{}^I + (10,1-1r,5-5r;10,3-3r,7-7r)\,\tilde{x}_2{}^I + \tilde{0}\,\tilde{s}_1^I + \tilde{0}\,\tilde{s}_2^I$$

subject to the constraints

$$(3,1-1r,3-3r;3,2-2r,5-5r)\,\tilde{x}_1{}^I + (6,2-2r,3-3r;6,4-4r,9-9r)\,\tilde{x}_2{}^I + \tilde{s}_1^I \approx (18,3-3r,6-6r;18,7-7r,9-9r)$$

$$(8,4-4r,1-1r;8,6-6r,6-6r)\,\tilde{x}_1{}^I + (7,1-1r,3-3r;7,2-2r,6-6r)\,\tilde{x}_2{}^I + \tilde{s}_2^I \approx (20,6-6r,6-6r;20,10-10r,9-9r)$$

and triangular intuitionistic fuzzy numbers $\tilde{x}_1{}^I, \tilde{x}_2{}^I, \tilde{s}_1^I, \tilde{s}_2^I \succeq \tilde{0}$

The initial basic feasible intuitionistic fuzzy solution is given by

$$\tilde{x}_1{}^I \approx \tilde{0}, \tilde{x}_2{}^I \approx \tilde{0}, \tilde{s}_1^I \approx (18,3-3r,6-6r;18,7-7r,9-9r),\ \tilde{s}_2^I \approx (20,6-6r,6-6r;20,10-10r,9-9r).$$

Table-4: Initial iteration.

$\tilde{C}_B{}^I$	$\tilde{y}_B{}^I$	$\tilde{x}_B{}^I$	$\tilde{x}_1{}^I$	$\tilde{x}_2{}^I$	$\tilde{s}_1{}^I$	$\tilde{s}_2{}^I$	θ
$\tilde{0}^I$	$\tilde{s}_1{}^I$	$(18,3-3r,6-6r;\ 18,7-7r,9-9r)$	$(3,1-1r,3-3r;\ 3,2-2r,5-5r)$	$(6,2-2r,3-3r;\ 6,4-4r,9-9r)$	$\tilde{1}^I$	$\tilde{0}^I$	$(3,3-3r,6-6r;\ 3,7-7r,9-9r)$
$\tilde{0}^I$	$\tilde{s}_2{}^I$	$(20,6-6r,6-6r;\ 20,10-10r,9-9r)$	$(8,4-4r,1-1r;\ 8,6-6r,6-6r)$	$(7,1-1r,3-3r;\ 7,2-2r,6-6r)$	$\tilde{0}^I$	$\tilde{1}^I$	$(20/7,6-6r,6-6r;\ 20/7,10-10r,9-9r)$
$\tilde{Z}_j{}^I - \tilde{C}_j{}^I$		$\tilde{0}^I$	$(-9,3-3r,2-2r;\ -9,4-4r,5-5r)$	$(-10,1-1r,5-5r;\ -10,3-3r,7-7r)$	$\tilde{0}^I$	$\tilde{0}^I$	

Here $\left(\tilde{Z}_j{}^I - \tilde{C}_j{}^I\right) \preceq \tilde{0},^I$ so $\tilde{x}_2{}^I$ enter the basis and $\tilde{s}_2{}^I$ leaves the basis

Table-5: First iteration.

$\tilde{C}_B{}^I$	$\tilde{y}_B{}^I$	$\tilde{x}_B{}^I$	$\tilde{x}_1{}^I$	$\tilde{x}_2{}^I$	$\tilde{s}_1{}^I$	$\tilde{s}_2{}^I$
$\tilde{0}$	$\tilde{s}_1{}^I$	$(6/7,6-6r,6-6r;\ 6/7,7-7r,9-9r)$	$(-27/7,4-4r,3-3r;\ -27/7,6-6r,9-9r)$	$\tilde{0}^I$	$\tilde{1}^I$	$(-6/7,2-2r,3-3r;\ -6/7,4-4r,9-9r)$
$(10,1-1r,5-5r;\ 10,3-3r,7-7r)$	$\tilde{x}_2{}^I$	$(20/7,6-6r,6-6r;\ 20/7,10-10r,9-9r)$	$(8/7,4-4r,3-3r;\ 8/7,6-6r,6-6r)$	$\tilde{1}^I$	$\tilde{0}^I$	$(1/7,1-1r,3-3r;\ 1/7,2-2r,6-6r)$
$\tilde{Z}_j{}^I - \tilde{C}_j{}^I$		$(200/7,6-6r,6-6r;\ 200/7,10-10r,10-10r)$	$(17/7,4-4r,5-5r;\ 17/7,6-6r,7-7r)$	$\tilde{0}^I$	$\tilde{0}^I$	$(10,1-1r,5-5r;\ 10,3-3r,7-7r)$

Since all $\left(\tilde{Z}_j{}^I - \tilde{C}_j{}^I\right) \succeq \tilde{0},^I$ the current intuitionistic fuzzy basic feasible solution is intuitionistic fuzzy optimal. The intuitionistic fuzzy optimal solution is $\tilde{x}_1{}^I = \tilde{0}^I$ and $\tilde{x}_2{}^I = (20/7,6-6r,6-6r;20/7,10-10r,10-10r)$,

$\tilde{Z}^I = (200/7,6-6r,6-6r;200/7,10-10r,10-10r)$.

For $r = 0$, we have the intuitionistic fuzzy optimal solution in terms of location index and fuzziness index as $\tilde{x}_1{}^I = \tilde{0}^I$, $\tilde{x}_2{}^I = (20/7,6,6;20/7,10,9)$; $\tilde{Z}^I = (200/7,6,6;200/7,10,10)$.

Hence the fuzzy optimal solution in the general form, in terms of $\tilde{a}^I = (a_1, a_2, a_3; a_1', a_2, a_3')$ for $r = 0$ is

$\tilde{x}_1{}^I = \tilde{0}^I$, $\tilde{x}_2{}^I = (-3.14, 2.85, 8.85; -7.15, 2.85, 12.85)$

$\tilde{Z}^I = (22.57, 28.57, 34.57; 18.57, 28.57, 38.57)$.

Table-6. Comparison of intuitionistic Fuzzy Optimal Solution obtained by our method and by Nachammai and Thangaraj [18] method.

	Our method	**Nachammai and Thangaraj method[18]**
Fuzzy optimal solution	$\max\ \tilde{z}^I = (6,9,11;5,9,14)\,\tilde{x}_1{}^I + (9,10,15;7,10,17)\,\tilde{x}_2{}^I$	$\max\ \tilde{z}^I = (6,9,11;5,9,14)\,\tilde{x}_1{}^I + (9,10,15;7,10,17)\,\tilde{x}_2{}^I$
Fuzzy optimal solution in terms of left and right index function	$\max \tilde{z}^I = (9, 3-3r, 2-2r; 9, 4-4r, 5-5r)\,\tilde{x}_1{}^I + (10, 1-1r, 5-5r; 10, 3-3r, 7-7r)\,\tilde{x}_2{}^I$	$\max\ \tilde{Z}^I = (12.6, 29, 64.65;\ \ 5.6, 29, 98.6)$ (With out using "r" or without any flexibility)
For $r = 0$	$\max\ \tilde{Z}^I = (22.57, 28.57, 34.57; 18.57, 28.57, 38.57)$	
For $r = 0.25$	$\max \tilde{Z}^I = (24.07, 28.57, 33.07;\ 21.07, 28.57, 36.07)$	
For $r = 0.5$	$\max \tilde{Z}^I = (25.57, 28.57, 31.57;\ 23.57, 28.57, 33.57)$	
For $r = 0.75$	$\max \tilde{Z}^I = (27.07, 28.57, 30.07;\ 26.07, 28.57, 31.07)$	
For $r = 1$	$\max \tilde{Z}^I = 28.57$	

It is to be noted that the Decision Maker have the flexibility of choosing $r \in [0,1]$ depending upon the situation and his wish by applying the proposed method in this paper whereas it is not possible by applying Nachammai and Thangaraj [18] method.

CONCLUSIONS

We have proposed a new method by introducing a new arithmetic operations and a ranking method for the solution of Intuitionistic Fuzzy Linear Programming Problems without converting it to an equivalent crisp problem. The optimal solution obtained by the proposed method is sharper than the solutions obtained by other methods. Moreover the proposed method is more flexible to the Decision maker to choose $r \in [0,1]$ depending upon the situation and his wish.

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