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# Study of Automatic Fiber Placement Manipulator's Robotic Kinematics Manipulability Based on Volume Element

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**Abstract:** The method is proposed based on volume element in order to measure the manipulator's robotic kinematics manipulability. Then studied the series redundant automatic fiber placement robotic manipulator's operation space, draw the conclusion that the greater of the robotic manipulator's operation space volume, the better of the robotic manipulator's manipulability, volume element based on redundant robotic manipulator's kinematics is proposed as an operational performance index. *n*-DOF serial robotic manipulator's operation space is *n*-dimensional Riemannian manifold, the *n*-dimensional Riemannian manifold volume is calculated using the moving coordinate system and the exterior product definition in differential geometry and get the robotic manipulator's operation space volume then compared the obtained results with the operation space volume using inner product determinant in the literature, it shows that the volume element as a kinematics operational performance index is feasible.

**Keywords:** Exterior product, manipulability, riemannian manifold, volume element

## INTRODUCTION

Robotic manipulator's manipulability is a comprehensive measure of robotic manipulator's manipulability in all directions, is one of key performance indices of redundant robotic manipulator's overall dexterity. Therefore, study on manipulability indices have been one of the main researches in studying the redundant robotic manipulator's kinematics, Yoshikawa (1984) proposed manipulability as an overall dexterity index of the redundant robotic manipulators, definite that $w = [\det (JJ^T)]^{1/2}$, and drew the conclusion that $w$ is the greater and the redundant robotic manipulator's dexterity is the better; Salisbury proposed condition number (Salisbury and Craig, 1982); Angel proposed minimum condition number (Angeles and Rojas, 1987); Klein proposed minimum singular value (Klein and Blaho, 1987). They took condition number, minimum condition number and minimum singular value as robotic manipulator's manipulability indices and used to the redundant robotic manipulator's design and control.

But the manipulability indices above are based on the characteristic of Jacobin matrix and some indices dependent on the Euclidean measures of $R^6 \cong se(3)$, However, the Euclidean measure is changeable with the coordinate system, that is to the different coordinate systems connected end-effectors, the minimum singular value points of the corresponding Jacobin matrix is different, This shows that manipulability index based on the minimum singular value of Jacobin matrix is sick. For the condition number and minimum condition number there have similar conclusions (Richard *et al*., 1998). The index of the measuring the manipulator's robotic kinematics manipulability changes as the coordinate system changes in the above literature, so the manipulator's robotic kinematics manipulability index proposed by these references only adapt to a particular manipulator's robot and no reference for the other manipulator's robot.

Manipulability index based on differential geometry volume element is proposed in this study and used in the 7-DOF automatic fiber placement manipulator's robot, the manipulability index is suitable for measuring the general manipulator's robotic kinematics manipulability, by comparison with other literature and can be used as a manipulator's robotic kinematics manipulability index.

## VOLUME ELEMENT

Wherever Volume element is essentially a geometric description of the workspace and reflects the motion density of robotic manipulator's end-effectors. Quantitative workspace of robotic manipulator can be obtained through its integral and it reflected the manipulability of robotic manipulator's end-effectors (Zhang, 2004). Assume the $i^{th}$ joint space of $n$ degree of freedom series redundant robotic manipulator is $C_i$, so the configuration space $C$ can be expressed as $n$ independent $C_i$'s product space, that is:

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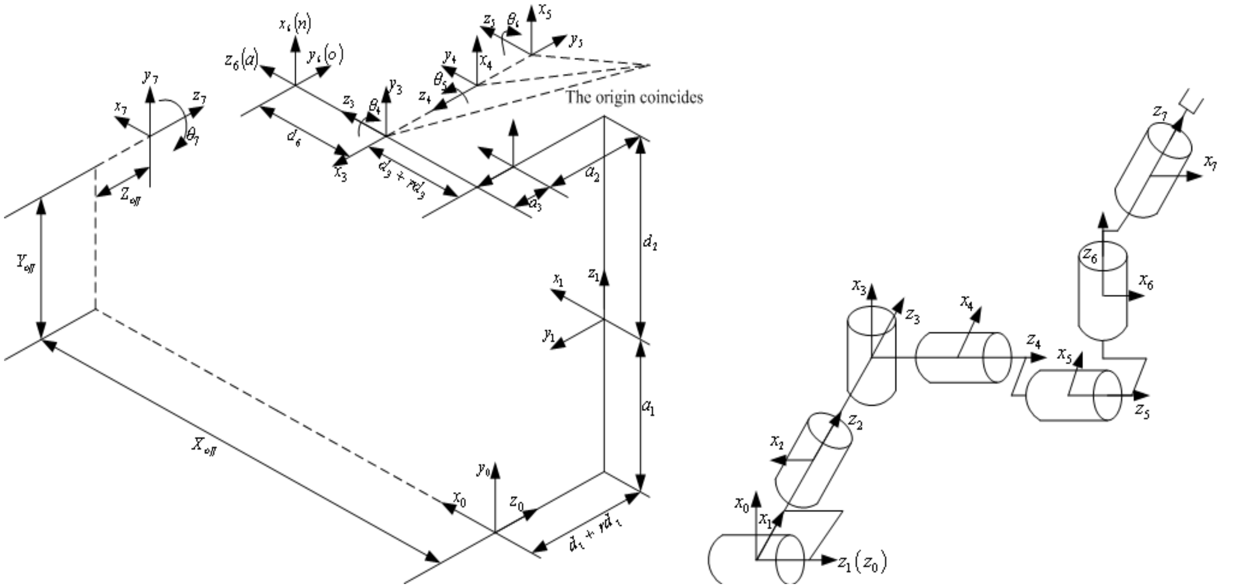

Fig. 1: The automated fiber placement robotic manipulator's structure and topology

$$C = C_1 \times C_2 \times \ldots \times C_n$$

The role of the robotic joints is equivalent to the role of mathematical functions, It maps the motion of the joint to the motion of end-effector (Zhao *et al*., 2007). It can be proved that the robotic manipulator's joint configuration space formed *n* dimensional Riemannian manifold.

Assume ($M$, $g$) is $n$ dimensional Riemannian manifold, ($U$; $u^i$) is the local coordinate system of $M$, so $*$ (Hodge star operator) (Chen, 2006) has the following corresponding:

$$*(\mathrm{d}u^{i_1} \wedge \cdots \wedge \mathrm{d}u^{i_r}) = \frac{\sqrt{G}}{(n-r)!} \sum_{k_1 \cdots k_n} g^{i_1 k_1} \cdots g^{i_r k_r} \delta^{1 \cdots n}_{k_1 \cdots k_n} \mathrm{d}u^{k_{r+1}} \wedge \cdots \wedge \mathrm{d}u^{k_n} \quad (1)$$

There, $g^{ij}$ is the inverse matrix of $g_{ij}$ and $g_{ij} = g\left(\frac{\partial}{\partial u^i}, \frac{\partial}{\partial u^j}\right), G = \det(g_{ij})$.

So the volume element of ($M$, $g$) in the local coordinate system ($U$; $u^i$) can be expressed as:

$$\mathrm{d}\sigma = \sqrt{G}\,\mathrm{d}u^1 \wedge \cdots \wedge \mathrm{d}u^n \quad (2)$$

**Example:**

**The structure and parameters of the automated fiber placement robot:** The automated fiber placement robotic manipulator developed by Nanjing University of Aeronautics and Astronautics as an example in this study and the structure as shown in Fig. 1. From its structure, there is a 6-DOF fiber placement robotic manipulator and a rotational mandrel. The following equivalent transformation in getting the robotic manipulator workspace using volume element can be done: mandrel as stationary and the coordinate system fixed mandrel coincides with the base coordinate system, a virtual revolute joints linked the base of the fiber placement robotic manipulator and the mandrel together, The rotational motion of the mandrel is equivalent to the robotic manipulator's rotation around the mandrel. So the fiber placement robotic manipulator with 6-DOF and the mandrel with 1-DOF becomes a 7-DOF redundant robotic manipulator. The shoulder has a revolute joint, the elbow has three displacements joint, wrist has three revolute joint. The three revolute joint axes of the wrist intersect at one point, the automatic fiber placement robotic manipulator's topology after equivalent motion as shown in Fig. 1. Establishing D-H coordinate system and its structural parameters as shown in Table 1.

**The automated fiber placement robotic manipulator's volume element:** The template Although the fiber placement robotic manipulator can be seen as 7-DOF redundant robotic manipulator according to the equivalent motion law, the range of the virtual revolute is no limit after the equivalent motion, therefore, the not limited virtual revolute joint does not affect the size of the robotic manipulator's workspace,

Table 1: Links parameters of automated fiber placement robotic manipulator

| Link *i* | $a_{i-1}$ (mm) | $\alpha_{i-1}$ ($^{\circ}$) | $d_i$ (mm) | $\theta_i$ ($^{\circ}$) | Range of joint |
|---|---|---|---|---|---|
| 1 | $a_0$ | 0 | 0 | $\theta_1$ | -180~180 |
| 2 | 0 | 0 | $d_2$ | 0 | -150~150 |
| 3 | 0 | 90 | $d_3$ | -90 | -110~110 |
| 4 | $a_3$ | 90 | $d_4$ | 0 | -100~100 |
| 5 | 0 | 0 | $c$ | $\theta_5$ | -90~90 |
| 6 | 0 | 90 | 0 | $\theta_6$ | -110~110 |
| 7 | 0 | -90 | 0 | $\theta_7$ | -260~260 |

so in getting the robotic manipulator's workspace volume using the exterior differential, in essence, the DOF number of the fiber placement robotic manipulator is 6.

To study the robotic manipulator's kinematics manipulability, established the moving coordinate system in end-effector. The moving set of the moving coordinate system is the robotic manipulator's workspace with the moving of the robotic manipulator. The workspace is the differential manifold in differential geometry, the relative weight of the moving coordinate system constitutes the tangent space of the differential manifold. The volume element of robotic manipulator can be definite in tangent space, which is invariant with coordinate variant. Establish the moving coordinate system $\{r;\ e_1,\ e_2,\ e_3\}$ on the next joint at the end of link, $l_i$ denotes the length of $i^{\text{th}}$ link, $a_i$ denotes offset distance, $\alpha_{i,i+1}$ denotes twist angle which joint $i$+1 relative to joint $i$, the moving coordinate system recurrence formula as follows:

$$\begin{cases} \boldsymbol{r}_i = \boldsymbol{r}_{i-1} + \left[ l_i \cos\theta_i + a_{i+1} \sin\alpha_{i,i+1} \sin\theta_i \right] \boldsymbol{e}_1^{(i-1)} \\ \qquad + \left[ l_i \cos\theta_i - a_{i+1} \sin\alpha_{i,i+1} \cos\theta_i \right] \boldsymbol{e}_2^{(i-1)} + a_{i+1} \cos\alpha_{i,i+1} \boldsymbol{e}_3^{(i-1)} \\ \boldsymbol{e}_1^{(i)} = \cos\theta_i \boldsymbol{e}_1^{(i-1)} + \sin\theta_i \boldsymbol{e}_2^{(i-1)} \\ \boldsymbol{e}_2^{(i)} = -\cos\alpha_{i,i+1} \sin\theta_i \boldsymbol{e}_1^{(i-1)} + \cos\alpha_{i,i+1} \cos\theta_i \boldsymbol{e}_2^{(i-1)} + \sin\alpha_{i,i+1} \boldsymbol{e}_3^{(i-1)} \\ \boldsymbol{e}_3^{(i)} = \sin\alpha_{i,i+1} \sin\theta_i \boldsymbol{e}_1^{(i-1)} - \sin\alpha_{i,i+1} \cos\theta_i \boldsymbol{e}_2^{(i-1)} + \cos\alpha_{i,i+1} \boldsymbol{e}_3^{(i-1)} \end{cases} \tag{3}$$

Expressed in matrix form:
To revolute joint:

$${}^{i-1}\boldsymbol{T}_i = \begin{bmatrix} \mathrm{c}\theta_i & -\mathrm{c}\alpha_{i,i+1}\mathrm{s}\theta_i & \mathrm{s}\alpha_{i,i+1}\mathrm{s}\theta_i & l_i\,\mathrm{c}\theta_i + a_{i+1}\,\mathrm{s}\alpha_{i,i+1}\,\mathrm{s}\theta_i \\ \mathrm{s}\theta_i & \mathrm{c}\alpha_{i,i+1}\mathrm{c}\theta_i & -\mathrm{s}\alpha_{i,i+1}\mathrm{c}\theta_i & l_i\,\mathrm{s}\theta_i - a_{i+1}\,\mathrm{s}\alpha_{i,i+1}\,\mathrm{c}\theta_i \\ 0 & \mathrm{s}\alpha_{i,i+1} & \mathrm{c}\alpha_{i,i+1} & a_{i+1}\,\mathrm{c}\alpha_{i,i+1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

To displacement joint:

$${}^{i-1}\boldsymbol{T}_i = \begin{bmatrix} 1 & 0 & 0 & l_i \\ 0 & \mathrm{c}\alpha_{i,i+1} & -\mathrm{s}\alpha_{i,i+1} & -a_{i+1}\,\mathrm{s}\alpha_{i,i+1} \\ 0 & \mathrm{s}\alpha_{i,i+1} & \mathrm{c}\alpha_{i,i+1} & d_i + a_{i+1}\,\mathrm{c}\alpha_{i,i+1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

where, $\mathrm{c}\theta_\mathrm{i} = \cos\theta_i$, $\mathrm{s}\theta_\mathrm{i} = \sin\theta_i$ others so.

So the position and posture of the robotic manipulator's end-effector is:

$$\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3; \boldsymbol{r}\} = {}^0\boldsymbol{T}_1(\theta_1)\, {}^1\boldsymbol{T}_2(\theta_2)\, {}^2\boldsymbol{T}_3(\theta_3)\, {}^3\boldsymbol{T}_4(\theta_4)\, {}^4\boldsymbol{T}_5(\theta_5)\, {}^5\boldsymbol{T}_6(\theta_6) \tag{6}$$

Substituting the automatic fiber placement robotic manipulator's link parameters in Table 1 into formula (6), solving by MATLAB programming and there will be:

$${}^{i-1}\boldsymbol{T}_i \begin{cases} \boldsymbol{e}_1 = (-\sin\theta_5 \cos\theta_6)\boldsymbol{i} + (-\sin\theta_4 \cos\theta_5 \cos\theta_6 - \cos\theta_4 \sin\theta_6)\boldsymbol{j} \\ \quad + (\sin\theta_4 \sin\theta_6 - \cos\theta_4 \cos\theta_5 \cos\theta_6)\boldsymbol{k} \\ \boldsymbol{e}_2 = (\sin\theta_5 \sin\theta_6)\boldsymbol{i} + (\sin\theta_4 \cos\theta_5 \sin\theta_6 - \cos\theta_4 \cos\theta_6)\boldsymbol{j} \\ \quad + (\cos\theta_4 \cos\theta_5 \sin\theta_6 + \sin\theta_4 \cos\theta_6)\boldsymbol{k} \\ \boldsymbol{e}_3 = (-\cos\theta_5)\boldsymbol{i} + (\sin\theta_4 \sin\theta_5)\boldsymbol{j} + (\cos\theta_4 \sin\theta_5)\boldsymbol{k} \\ \boldsymbol{r} = (-c - d_3)\boldsymbol{i} + (-d_2)\boldsymbol{j} + (d_1 - a_2)\boldsymbol{k} \end{cases} \tag{7}$$

Assuming the robotic manipulator's end-effector generalized speed is:

$$\boldsymbol{V} = \begin{bmatrix} u_1 & u_2 & u_3 & u_{23} & u_{31} & u_{12} \end{bmatrix}^{\mathrm{T}} \tag{8}$$

According to the moving coordinate system's motional equations and characteristics of the circular vector function and there will be:

$$\begin{cases} u_i = \mathrm{d}\boldsymbol{r} \cdot \boldsymbol{e}_i \\ u_{ij} = \mathrm{d}\boldsymbol{e}_i \cdot \boldsymbol{e}_j \end{cases} \tag{9}$$

Substituting the automatic fiber placement robotic manipulator's link parameters in Table 1 into formula (3) and (9) and there will be:

$$\begin{cases} u_1 = \sin\theta_5 \cos\theta_6 \mathrm{d}d_3 + (\sin\theta_4 \cos\theta_5 \cos\theta_6 + \cos\theta_4 \sin\theta_6)\mathrm{d}d_2 \\ \quad + (\sin\theta_4 \sin\theta_6 - \cos\theta_4 \cos\theta_5 \cos\theta_6)\mathrm{d}d_1 \\ u_2 = -\sin\theta_5 \sin\theta_6 \mathrm{d}d_3 - (\sin\theta_4 \cos\theta_5 \sin\theta_6 - \cos\theta_4 \cos\theta_6)\mathrm{d}d_2 \\ \quad + (\cos\theta_4 \cos\theta_5 \sin\theta_6 + \sin\theta_4 \cos\theta_6)\mathrm{d}d_1 \\ u_3 = \cos\theta_5 \mathrm{d}d_3 - \sin\theta_4 \sin\theta_5 \mathrm{d}d_2 + \cos\theta_4 \sin\theta_5 \mathrm{d}d_1 \\ u_{23} = \sin\theta_5 \cos\theta_6 \mathrm{d}\theta_4 - \sin\theta_6 \mathrm{d}\theta_5 \\ u_{31} = \sin\theta_5 \sin\theta_6 \mathrm{d}\theta_4 - \cos\theta_6 \mathrm{d}\theta_5 \\ u_{12} = \cos\theta_5 \mathrm{d}\theta_4 + \mathrm{d}\theta_6 \end{cases} \tag{10}$$

$u_1$, $u_2$, $u_3$, $u_{23}$, $u_{31}$, $u_{12}$ are the relative weight of the moving coordinate system connected on end-effector, the exterior product of the relatives constitutes the element volume $dV$ of robotic manipulator, that:

$$\mathrm{d}V = u_1 \wedge u_2 \wedge u_3 \wedge u_{23} \wedge u_{31} \wedge u_{12}$$

The displacement volume is:

$$\mathrm{d}T = u_1 \wedge u_2 \wedge u_3$$

The rotational volume is:

$$\mathrm{d}R = u_{23} \wedge u_{31} \wedge u_{12}$$

The total volume of robotic manipulator is:

$$V = \int \left|\mathrm{d}T \wedge \mathrm{d}R\right| = 8\pi^2 \left|(-c - d_3)(-d_2)(d_1 - a_2)\right|$$

The same results obtained in Udai and Duffy (1993). The robotic manipulator's motion is expressed by Euler angles, used the method of the inner product of determinant and got result after a complex operating in Udai and Duffy (1993). The same results obtained using volume element based on the moving coordinate system in this study, but the process is simple.

## CONCLUSION

The robotic manipulator's kinematics manipulability index-volume element is proposed based on combining the moving coordinate system and exterior product, giving the geometric meaning of the volume element. The automated fiber placement robotic manipulator developed by Nanjing University of Aeronautics and Astronautics as an example in this study, giving the detailed process of calculating the volume element and showing the volume element which used as measuring the kinematics manipulability index is feasible compared the known results in [12]. And using exterior product forms, the geometric meaning is clear, the process is simple and the computation is small.

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