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SOUND RADIATION FROM A BURIED NOZZLE WITH JET AND BYPASS FLOW

A. Demir, S.W. Rienstra

Department of Mathematics & Computer Science, Eindhoven University of Technology,
P.O. Box 513, 5600 MB Eindhoven, The Netherlands.

a.demir@tue.nl
s.w.rienstra@tue.nl

Abstract

Radiation of sound from a simplified model of a buried nozzle with bypass flow is studied. More precisely, a semi-infinite duct (the inner nozzle) is situated inside a larger semi-infinite duct. The exit plane of the larger duct either coincides with the exit plane of the smaller duct, or extends beyond it. Differences in the piece-wise subsonic mean flow velocity, density and temperature are taken into account. The inner nozzle issues the core flow inside the bypass jet flow. The bypass nozzle issues the bypass jet flow inside the ambient co-flow. Two vortex sheets, attached to the duct exits, separate the different flows from each other. These vortex sheets are unstable due to this mean velocity discontinuity. The application of the Kutta condition at the respective trailing edges guarantees shedding of vorticity which excites these instabilities. The system is set up to respond to an incident annular duct mode, but the analysis would be very similar for an inner duct mode.

To obtain an analytical solution a Wiener-Hopf approach with Idemen's method of "weak factorisation" is applied. Formulation of the boundary value problem following the classical approach leads to a couple of simultaneous Wiener-Hopf equations. These equations produce a matrix equation system, which is formally decoupled by the introduction of an infinite sum of poles with coefficients to be determined. The uncoupled scalar equations are solved independently by a standard application of analytical continuation. The unknown coefficients in the final solution are determined by solving an infinite linear algebraic system numerically. The contribution of the instability waves are separated from the rest of the solution.

The asymptotic far field is found by a standard application of the steepest descent method. Finally a series of practical examples are given.

1. INTRODUCTION

Accurate predictions of exhaust noise are required to guide the development of innovative noise reduction solutions. Much effort has already been spent on the intake noise radiation problem, but there is still a lack of research on the more complicated exhaust noise radiation problem, where sound propagates through the shear layers of high speed hot jets. In the European project

TURNEX (co-ordinator Brian Tester) this challenge is being faced by developing enhanced numerical prediction techniques. A very important aspect is the verification of the numerical solutions by detailed comparison with exact analytic solutions of idealized and geometrically simplified but not trivial cases. The model per se is to be validated too, but by other means.

The exploration of relevant problems that allow exact solutions was initiated by Munt [1], who developed a Wiener-Hopf solution [2] for the configuration of a jet from a hollow duct with co-flow. Based on this idea, a series of related configurations were developed by Gabard & Astley [3] of annular hard walled duct and Demir & Rienstra [4] of annular duct with full and partial lining of the centerbody.

These solutions have in common that the centerbody is doubly infinite. This is obviously a simplification which may be acceptable for a finite but relatively long, protruding afterbody, but not for a centerbody exit that is co-planar with or even buried inside the outer duct, as can be found in certain turbofan engines [5].

This is the case that will be presented here. Mathematically, its solution is more subtle than the classical Wiener-Hopf solutions of the doubly infinite geometry. The present solution is based on a so-called weak factorisation, originally due to Idemen [6]. Just like the classical Munt solution [1] and its variants [3, 7] it includes the effects of vortex shedding and Helmholtz instabilities of the inner and outer jet, while the mean flow is now triple piecewise constant in velocity, sound speed and density along the inner jet, the bypass jet and the outer flow.

The first who considered this same buried nozzle problem were Taylor, Crighton & Cargill [5]. However, they assumed low Mach and Strouhal numbers and a long distance between exit planes. Here we will consider the full problem without further approximations.

2. ANALYSIS

2.1. Formulation of the Problem

Consider the geometry which consists of a semi-infinite outer duct and a semi-infinite inner duct. Duct walls are assumed to be infinitely thin and they occupy the regions like depicted in the figures 1(a), 1(b). All quantities are made dimensionless by using outer duct radius and ambient flow properties as reference as $r, z \sim R_2, U \sim c_0, \rho \sim \rho_0, t \sim R_2/c_0$. The velocity

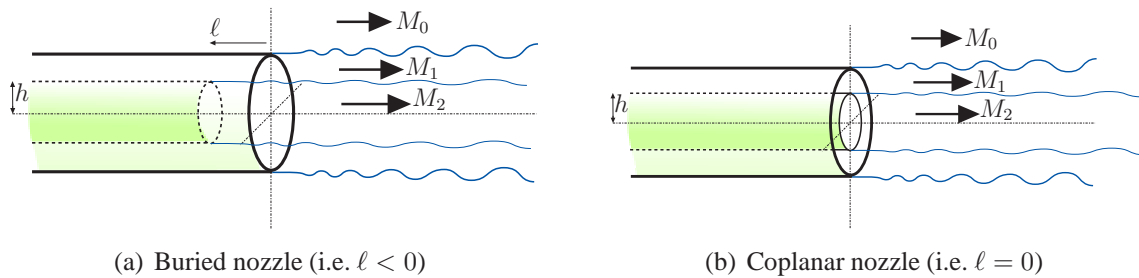


Figure 1.

potential ϕ will be used to obtain the acoustic pressure p , velocity \mathbf{v} and density ρ via the following equations [4]

$$p = -(\phi_t + M_0\phi_z), \quad \mathbf{v} = \nabla\phi, \quad \rho = p, \quad 1 < r \quad (1)$$

$$p = -D_1(\phi_t + M_1\phi_z), \quad \mathbf{v} = \nabla\phi, \quad \rho = pC_1^2, \quad h < r < 1 \quad (2)$$

$$p = -D_2(\phi_t + M_2\phi_z), \quad \mathbf{v} = \nabla\phi, \quad \rho = pC_2^2, \quad 0 \leq r < h \quad (3)$$

where $M_0 = U_0/c_0$, $M_1 = U_j/c_0$, $M_2 = U_c/c_0$, $C_1 = c_0/c_j$, $C_2 = c_0/c_c$, $D_1 = \rho_j/\rho_0$, $D_2 = \rho_c/\rho_j$. All mean flows are subsonic. Note that M_1 and M_2 are not the local Mach numbers.

From the symmetry of the geometry and the incident wave, the diffracted field will remain with the same azimuthal and time dependencies as the incident wave. The total field is written in different regions as:

$$\phi(r, z, \theta, t) = \psi^T(r, z) \exp(i\omega t - im\theta) \quad (4)$$

$$\psi^T(r, z) = \begin{cases} \psi_1(r, z), & 1 < r < \infty, \\ \psi_2(r, z) + \psi^i(r, z), & h < r < 1 \\ \psi_3(r, z), & 0 \leq r < h, \end{cases} \quad (5)$$

$$\psi^i(r, z) = \Psi_{mn}(r) \exp(-i\omega\mu_{mn}^+ z) \quad (6)$$

$$\Psi_{mn}(r) = Y'_m(\alpha_{mn}h)J_m(\alpha_{mn}r) - J'_m(\alpha_{mn}h)Y_m(\alpha_{mn}r) \quad (7)$$

where $\omega > 0$ is the angular frequency and $m \in \mathbb{Z}$ is the circumferential order. ψ^i denotes the incident field, which is here assumed to come from the bypass duct, but the analysis would have been similar for the core duct. Radial wave numbers α_{mn} , defined by the eigenvalue equation $Y'_m(\alpha_{mn}h)J'_m(\alpha_{mn}) - J'_m(\alpha_{mn}h)Y'_m(\alpha_{mn}) = 0$, yield the axial wave numbers $\omega\mu_{mn}^\pm$, where

$$\mu_{mn}^\pm = \frac{\pm \sqrt{C_1^2 - (1 - M_1^2 C_1^2)\alpha_{mn}^2/\omega^2 - M_1 C_1^2}}{1 - M_1^2 C_1^2} \quad (8)$$

and $\mu_{mn}^\pm = \mp i\sqrt{\dots}$ for negative arguments of the square root. The signs (+) and (−) show right and left running modes, respectively. For the special case of a plane wave we have $\Psi_{00}(r) = 1$ and $\mu_{00}^\pm = C_1/(M_1 C_1 \pm 1)$. Time dependence $e^{i\omega t}$ and azimuthal dependency $e^{-im\theta}$ are suppressed throughout this paper.

2.2. Derivation of the Wiener-Hopf System

The previously introduced velocity potentials $\psi_1(r, z)$, $\psi_2(r, z)$ and $\psi_3(r, z)$ satisfy the Helmholtz equations

$$\left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial z^2} - \frac{m^2}{r^2} - (i\omega + M_0 \frac{\partial}{\partial z})^2 \right] \psi_1(r, z) = 0, \quad (9)$$

$$\left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial z^2} - \frac{m^2}{r^2} - C_1^2 (i\omega + M_1 \frac{\partial}{\partial z})^2 \right] \psi_2(r, z) = 0, \quad (10)$$

$$\left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial z^2} - \frac{m^2}{r^2} - C_2^2 (i\omega + M_2 \frac{\partial}{\partial z})^2 \right] \psi_3(r, z) = 0. \quad (11)$$

These can be solved in the usual way by Fourier transformation to z , leading formally to

$$\psi_1(r, z) = \frac{\omega}{2\pi} \int_L A(u) H_m^{(2)}(\lambda_0 \omega r) e^{-i\omega u z} du \quad (12)$$

$$\psi_2(r, z) = \frac{\omega}{2\pi} \int_L [B(u) J_m(\lambda_1 \omega r) + C(u) Y_m(\lambda_1 \omega r)] e^{-i\omega u z} du \quad (13)$$

$$\psi_3(r, z) = \frac{\omega}{2\pi} \int_L D(u) J_m(\lambda_2 \omega r) e^{-i\omega u z} du \quad (14)$$

where L is a suitable integration contour along the real axis in complex u -domain (see [4]). The complex square roots λ_0 , λ_1 and λ_2 are defined as $\lambda_0(u) = \sqrt{(1 - uM_0)^2 - u^2}$, $\lambda_1(u) = \sqrt{C_1^2(1 - uM_1)^2 - u^2}$, $\lambda_2(u) = \sqrt{C_2^2(1 - uM_2)^2 - u^2}$, where $\text{Im}(\lambda) \leq 0$. The potentials sat-

isfy the usual condition of continuity of particle displacement along $r = 1$ and $r = h$. When $r = 1 + \xi(z) \exp(i\omega t - im\theta)$ and $r = h + \eta(z) \exp(i\omega t - im\theta)$ denotes the complex radial displacement of vortex sheets at $r = 1$ and $r = h$, the full Kutta condition applied at either trailing edge implies, respectively, $\xi(z) = \mathcal{O}(z^{3/2})$, and $\eta(z) = \mathcal{O}(z^{3/2})$. Application of the boundary conditions on $r = h$ yields expressions for $\frac{\partial}{\partial r}\psi_2(h, z)$ and $\frac{\partial}{\partial r}\psi_3(h, z)$, which gives

$$D(u)\lambda_2 J'_m(\lambda_2\omega h) = i(1 - uM_2) e^{i\omega u\ell} \Phi_1^+(u), \quad (15)$$

$$B(u)\lambda_1 J'_m(\lambda_1\omega h) + C(u)\lambda_1 Y'_m(\lambda_1\omega h) = i(1 - uM_1) e^{i\omega u\ell} \Phi_1^+(u), \quad (16)$$

where $\Phi_1^+(u) = \int_{\ell}^{\infty} \eta(z) e^{i\omega u(z-\ell)} dz$. Continuity of pressure at $r = h$ gives us

$$\begin{aligned} -i\omega(1 - uM_1)J_m(\lambda_1\omega h)B(u) - i\omega(1 - uM_1)Y_m(\lambda_1\omega h)C(u) &= e^{i\omega u\ell} \Phi_1^-(u) \\ - (1 - \mu_{mn}^+ M_1)\Psi_{mn}(h) \frac{e^{i\omega(u-\mu_{mn}^+)\ell}}{u - \mu_{mn}^+} + \omega D_2(1 - uM_2)^2 \frac{J_m(\lambda_2\omega h)}{\lambda_2 J'_m(\lambda_2\omega h)} e^{i\omega u\ell} &\Phi_1^+(u) \end{aligned} \quad (17)$$

and the matrix equation

$$\begin{bmatrix} \lambda_1 J'_m(\lambda_1\omega h) & \lambda_1 Y'_m(\lambda_1\omega h) \\ -i\omega(1 - uM_1)J_m(\lambda_1\omega h) & -i\omega(1 - uM_1)Y_m(\lambda_1\omega h) \end{bmatrix} \begin{bmatrix} B(u) \\ C(u) \end{bmatrix} = \begin{bmatrix} i(1 - uM_1) e^{i\omega u\ell} \Phi_1^+(u) \\ e^{i\omega u\ell} \Phi_1^-(u) - (1 - \mu_{mn}^+ M_1)\Psi_{mn}(h) \frac{e^{i\omega(u-\mu_{mn}^+)\ell}}{u - \mu_{mn}^+} + \omega D_2(1 - uM_2)^2 \frac{J_m(\lambda_2\omega h)}{\lambda_2 J'_m(\lambda_2\omega h)} e^{i\omega u\ell} \Phi_1^+(u) \end{bmatrix} \quad (18)$$

where $\Phi_1^-(u) = \int_{-\infty}^{\ell} [D_2(i\omega + M_2 \frac{\partial}{\partial z})\psi_3(h, z) - (i\omega + M_1 \frac{\partial}{\partial z})\psi_2(h, z)] e^{i\omega u(z-\ell)} dz$. The coefficients $B(u)$ and $C(u)$ may now be expressed explicitly in $\Phi_1^+(u)$ and $\Phi_1^-(u)$.

The boundary condition along $r = 1$ results into

$$A(u)\lambda_0 H_m^{(2)'}(\lambda_0\omega) = i(1 - uM_0)\Phi_2^+(u), \quad (19)$$

$$B(u)\lambda_1 J'_m(\lambda_1\omega) + C(u)\lambda_1 Y'_m(\lambda_1\omega) = i(1 - uM_1)\Phi_2^+(u), \quad (20)$$

where $\Phi_2^+(u) = \int_0^{\infty} \xi(z) e^{i\omega uz} dz$. After eliminating A , B , and C , using continuity of pressure at $r = 1$, and introducing $\Phi_2^-(u) = \int_{-\infty}^0 [(i\omega + M_0 \frac{\partial}{\partial z})\psi_1(1, z) - D_1(i\omega + M_1 \frac{\partial}{\partial z})\psi_2(1, z)] e^{i\omega uz} dz$, we obtain a couple of Wiener-Hopf equations, which are to be solved:

$$\begin{aligned} \Phi_1^+(u) - \frac{\lambda_1\lambda_2}{\omega N(u)} \left[\Phi_1^-(u) - (1 - \mu_{mn}^+ M_1)\Psi_{mn}(h) \frac{e^{-i\omega\mu_{mn}^+\ell}}{u - \mu_{mn}^+} \right] \\ = -\frac{2}{\pi\omega h} (1 - uM_1)^2 \frac{e^{-i\omega u\ell} \Phi_2^+(u)}{\chi_1(u)}, \end{aligned} \quad (21)$$

$$\begin{aligned} \omega\Phi_2^+(u)K(u) - \frac{2}{\pi\omega} \frac{D_1(1-uM_1)^2}{\chi_1(u)} \left[e^{i\omega u\ell} \Phi_1^-(u) - (1 - \mu_{mn}^+ M_1)\Psi_{mn}(h) \frac{e^{i\omega(u-\mu_{mn}^+)\ell}}{u - \mu_{mn}^+} \right] \\ = \Phi_2^-(u) - D_1\Psi_{mn}(1) \frac{1 - \mu_{mn}^+ M_1}{u - \mu_{mn}^+} \end{aligned} \quad (22)$$

with kernels defined by

$$N(u) = \lambda_2(1 - uM_1)^2 \frac{Y_m(\lambda_1\omega h)J'_m(\lambda_1\omega) - J_m(\lambda_1\omega h)Y'_m(\lambda_1\omega)}{Y'_m(\lambda_1\omega h)J'_m(\lambda_1\omega) - J'_m(\lambda_1\omega h)Y'_m(\lambda_1\omega)} - \lambda_1 D_2(1 - uM_2)^2 \frac{J_m(\lambda_2\omega h)}{J'_m(\lambda_2\omega h)} \quad (23)$$

$$K(u) = D_1(1 - uM_1)^2 \chi_2(u)/\chi_1(u) - (1 - uM_0)^2 H_m^{(2)}(\lambda_0\omega)/\lambda_0 H_m^{(2)'}(\lambda_0\omega) \quad (24)$$

$$\chi_1(u) = (1 - uM_1)^2 \lambda_1 [Y_m(\lambda_1\omega h)J'_m(\lambda_1\omega) - J_m(\lambda_1\omega h)Y'_m(\lambda_1\omega)] - D_2(1 - uM_2)^2 \frac{\lambda_1^2 J_m(\lambda_2\omega h)}{\lambda_2 J'_m(\lambda_2\omega h)} [Y'_m(\lambda_1\omega h)J'_m(\lambda_1\omega) - J'_m(\lambda_1\omega h)Y'_m(\lambda_1\omega)] \quad (25)$$

$$\chi_2(u) = (1 - uM_1)^2 [Y_m(\lambda_1\omega h)J_m(\lambda_1\omega) - J_m(\lambda_1\omega h)Y_m(\lambda_1\omega)] - D_2(1 - uM_2)^2 \frac{\lambda_1 J_m(\lambda_2\omega h)}{\lambda_2 J'_m(\lambda_2\omega h)} [Y'_m(\lambda_1\omega h)J_m(\lambda_1\omega) - J'_m(\lambda_1\omega h)Y_m(\lambda_1\omega)] \quad (26)$$

2.3. Solution

Due to the space limitations for the present conference paper it is impossible for us to give the solution procedure in detail here. In short, it is based on the usual Wiener-Hopf splitting of the kernels $N(u) = N_+(u)/N_-(u)$ and $K(u) = K_+(u)/K_-(u)$ in functions that are analytic in lower, resp. upper complex u -space. The equations may then be rewritten in a lhs/rhs, analytic in the lower/upper halfspace, except for isolated poles. These poles may then be removed at the expense of a linear system to be solved [6, 8]. For further details on the Kutta condition, vortex sheet instabilities, and numerical calculation of the split function we refer to [4] and the forthcoming paper. The resulting solution for Φ_2 takes the form

$$\Phi_2^+(u) = \frac{1}{\omega K_+(u)} \sum_{p=1}^{\infty} \frac{b_{mp}}{\beta_{mp}^+ - u_1} \left[\frac{u - u_1}{u - \beta_{mp}^+} - \Gamma_1 \right] - \delta_{mn} \frac{D_1(1 - \mu_{00}^+ M_1)}{\omega(u - \mu_{00}^+) K_+(u)} K_-(\mu_{00}^+) \left\{ \frac{2}{\pi\omega} \frac{(1 - \mu_{00}^+ M_1)^2}{\chi_1(\mu_{00}^+)} + 1 \right\} \quad (27)$$

where β_{mp}^+ denotes χ_1 's p -th lower half plane zero; $\delta_{00} = 1$ and $\delta_{mn} = 0$ otherwise; u_1 is related to the instability pole of the outer vortex sheet; $\Gamma_1 = 1, = 0$ corresponds to full, no Kutta condition at the outer trailing edge. At the inner trailing edge a Kutta condition is always assumed. The instability pole of the inner vortex sheet is included in the kernel N .

The far field $\bar{R} \rightarrow \infty$ may be expressed by a usual steepest descent approximation as

$$\psi_1 \simeq D_\psi(\bar{\theta}) \exp(i\omega \bar{R} [M_0 \cos \bar{\theta} - (1 - M_0^2 \sin^2 \bar{\theta})^{1/2}] / (1 - M_0^2) + i \frac{m+1}{2} \pi) / \bar{R}, \quad (28)$$

where $r = R \sin \theta (1 - M_0^2)^{1/2} = \bar{R} \sin \bar{\theta}$, $z = R \cos \theta (1 - M_0^2) = \bar{R} \cos \bar{\theta}$, and directivities for potential and pressure

$$D_\psi(\bar{\theta}) = \frac{i(1 - u' M_0) \Phi_2^+(u')}{\pi \sin \bar{\theta} H_m^{(2)'}(\lambda'_0 \omega)}, \quad D_P(\bar{\theta}) = \frac{\omega(1 - u' M_0)^2 \Phi_2^+(u')}{\pi \sin \bar{\theta} H_m^{(2)'}(\lambda'_0 \omega)}, \quad (29)$$

with $u' = (\cos \bar{\theta} (1 - M_0^2 \sin^2 \bar{\theta})^{-1/2} - M_0) / (1 - M_0^2)$, $\lambda'_0 = \sin \bar{\theta} / (1 - M_0^2 \sin^2 \bar{\theta})^{1/2}$.

3. NUMERICAL EXAMPLES

A series of examples are numerically evaluated to see the effect of the buried versus co-planar nozzle, with mean flow and Kutta condition. These examples are not meant to be more than an illustration of the found solutions, as it is not possible here to make an exhaustive investigation of all possible combinations of problem parameters nor scan all the physical consequences. The problem parameters that were used are:

Table 1. Problem parameters for far field examples.

| parameter | definition | | value |
|-----------|-----------------------|--------------------------------|------------|
| ω | $2\pi f R_2/c_0$ | $2\pi \cdot 876 \cdot 1.2/330$ | 20.0 |
| m | | | 1 and 10 |
| M_0 | U_0/c_0 | 99/330 | 0.30 |
| M_1 | U_j/c_0 | 210/330 | 0.64 |
| M_2 | U_c/c_0 | 265/330 | 0.80 |
| C_1 | c_0/c_j | 330/350 | 0.94 |
| C_2 | c_0/c_c | 330/530 | 0.62 |
| D_1 | ρ_j/ρ_0 | 1.158/1.3026 | 0.89 |
| D_2 | ρ_c/ρ_j | 0.505/1.158 | 0.44 |
| h | R_1/R_2 | 0.8/1.2 | 0.67 |
| ℓ | inner duct exit plane | 0 and $-1.2/R_2$ | 0 and -1 |

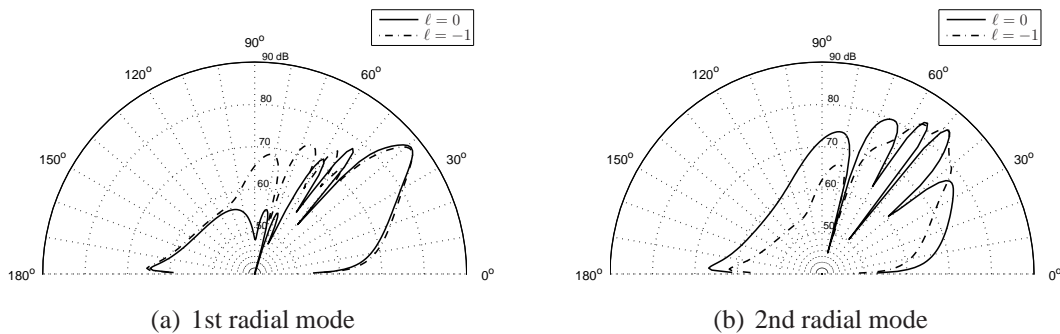


Figure 2. Comparing co-planar and buried exits with $m = 1$, full Kutta condition

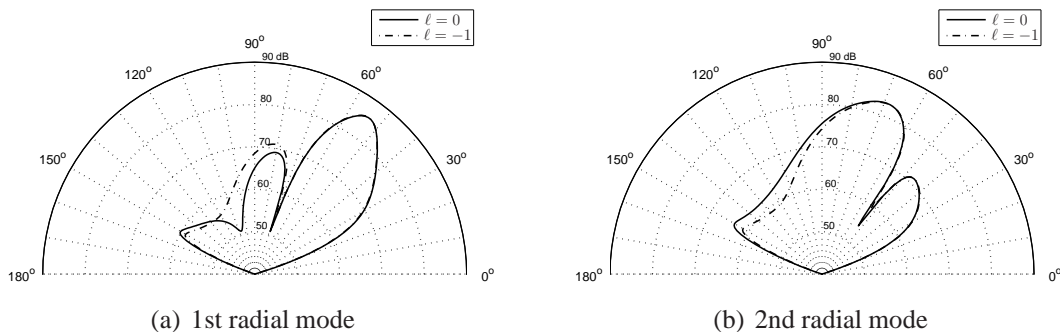


Figure 3. Comparing co-planar and buried exits with $m = 10$, full Kutta condition

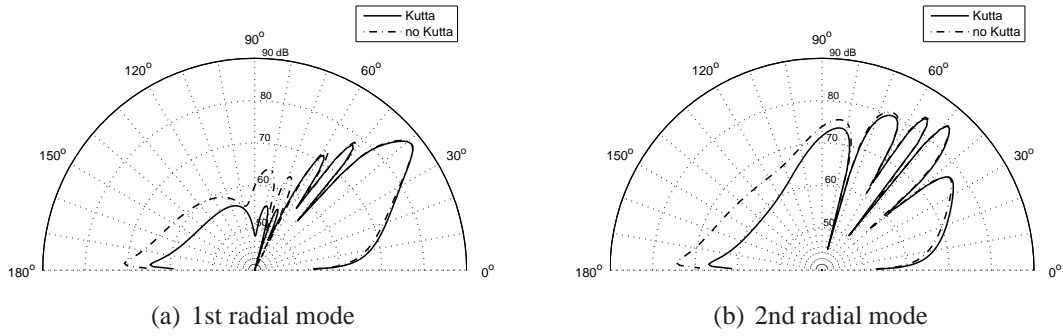


Figure 4. Co-planar exit with $m = 1$, comparing with and without outer edge Kutta condition

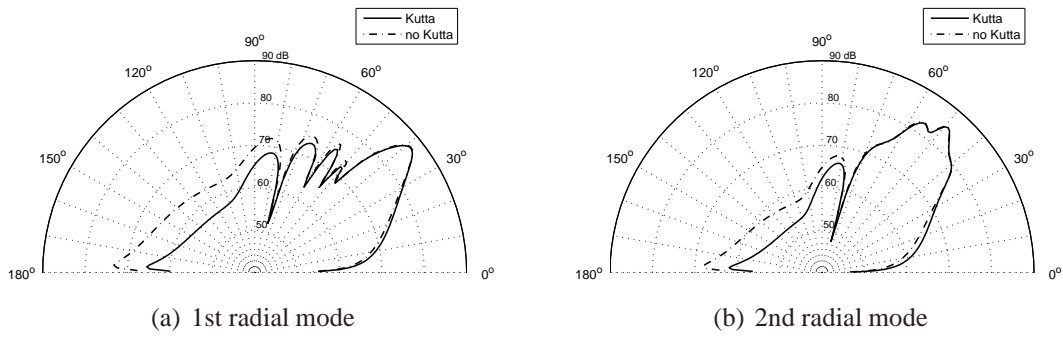


Figure 5. Buried exit with $m = 1$, comparing with and without outer edge Kutta condition

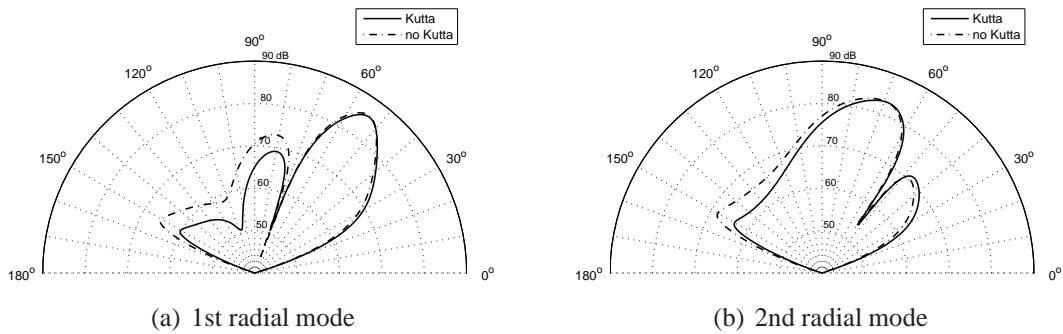


Figure 6. Co-planar exit with $m = 10$, comparing with and without outer edge Kutta condition

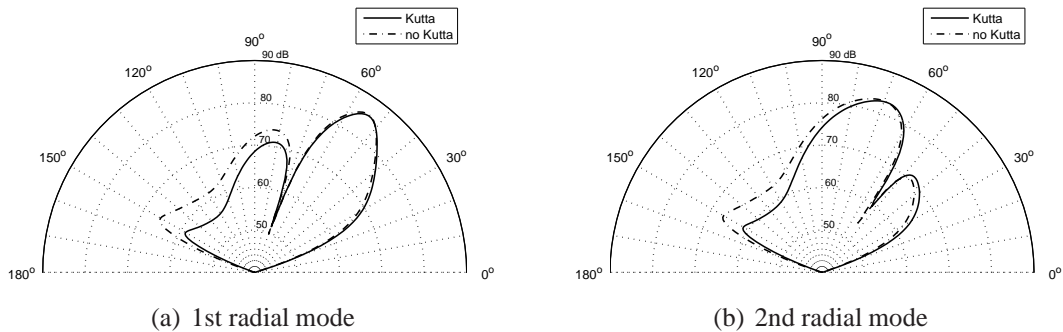


Figure 7. Buried exit with $m = 10$, comparing with and without outer edge Kutta condition

The 1st and 2nd radial mode far fields are plotted dimensionally, at 50.0 m away from the exhaust plane. Amplitudes are such that the cross-wise averaged intensity at $z = \ell$ is 1 W/m^2 .

In the figures 2,3 the difference is studied of a buried against co-planar nozzle. For high m the practically vanishing inner part of the modal radial profile makes the inner exit unimportant. Otherwise we see strong interference between the coplanar inner and outer exit (figure 2b).

In the figures 4,5, 6, 7 the effect is studied of the outer edge Kutta condition. In all cases the effect is relatively small for downstream angles. This is due to the prevailing Strouhal numbers being high, which curtails the regions of significant vorticity-trailing edge interaction.

4. CONCLUSIONS

Analytically exact solutions are constructed and numerically implemented for the relatively complicated model problem of sound radiation from an aircraft engine exit with piecewise uniform mean flow of buried or co-planar hot core jet, colder annular by-pass jet and cold mean co-flow. The solution includes the effects of Kutta condition and unstable vortex sheets. The solution method is based on a Wiener-Hopf approach, supplemented by weak factorisation. This is only possible for the buried jet configuration. For a protruding inner duct geometry other courses should be pursued.

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