Portfolio optimization tool with risk calculations

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***Abstract*—The electricity exchange include the uncertainty of many aspects, e.g. energy demand. The techniques to risk management used widely in financial markets must be adopted, because of differences between electricity and other products.**

The uncertainty in presented approach is modelled by random variables, described by probability density function or probability distribution. Risk management involve the measurement of many instruments, such as the uncertainty of energy consumption, energy price, CO2 certificates price etc. The model included most of technical and economical aspects is created and is resolved numerically by Monte Carlo simulations.

The changes in time are modelled by the stochastic processes, the calculations includes the prediction of the time values using the auto-regressive models and/or the memory based methods and figure the linear statistical dependencies (correlations). The system provides portfolio management methods, including optimisation of the portfolio, given technical and economical restrictions.

The genetic algorithm is used to find the global optimum, then the hill climbing (local optimisation) improves the result.

I. Introduction

The electricity exchange include the uncertainty of many aspects, e.g. energy demand [1], [2], [3]. The techniques to risk management used widely in financial markets must be adopted, because of differences between electricity and other products [4], [5]. The main difference is connected with equality between demand and supply. The power, which is produced by suppliers (power plants), must be used immediately by consumers because the frequency in grid must be stable. This simple dependency from physics field causes a lot of problems from financial modeling point of view, especially in short-term period of time. Due to lack of efficient methods of energy storage all ISOs (Independent System Operator) have implemented additional mechanisms of refunding the system stabilization – so called balancing market. Additionally, the ISOs costs have influence on market prices and gives some indicators to risk managing. The market player simple behavior could attach only selling (or buying) electricity to (from) balancing market. Trading strategy is simple when the price is known at transaction closing, but in vast majority of cases this knowledge is unknown before committing the transaction. The price could be found using price forecasting models, but usually the balancing market price is the worst on mature markets. It means the better prices from purchaser and supplier points of view could be achieved on different markets: bilateral (individual agreements between consumer and supplier), exchange (using power exchange transactions), or free market turnover (using energy trading Internet platforms). And the point is: the better portfolio you are able to construct the higher figures you are able to achieve. The portfolio ingredients include the risk management.

Presented approach to risk managing on energy markets include coincidence between risk indicators on power and financial markets. At the first sight the buyer is a source of risk – especially when we are considering the cash flow and payment risk. Then to avoid bad debts and payment problems typical financial rules of insurance are used: deposits, insurance guarantees etc. Moreover, the maximum of the financial volume in transactions is fixed and restricted – nobody can pass it without special permission. It means that all income from the transactions is determined and had a upper limit. That approach cause the reduction of risk of customer to minimum – i.e. set the proper payment period so that when problem occurs the supplier can used the guaranties to cover actual charges. And that kind of practices are widely used in financial risk management. When the customer risk is minimized (including payment problems) the larger game of profit maximization is starting. This is the every business basics: buy cheap to sell dear. This rules is adopted towards any markets – including energy. And when we are looking more profound into transaction details then that simple rule emerges. Typical situation form the trader point of view starts from tender (call, email or some other communication medium) concerns a few numbers: how much energy is in scope and what is the price. In general of course – because energy offers are little bit complicated and includes terms of delivery, time of delivery and periods (peek, off peek etc.). On mature wholesale markets some basic products are fixed – and energy is exchanging using it. But all offers has to be split into single period of time specific for energy market (typical single period of time is equal 15 minutes in some markets or 1 hour) despite the fact that tender either concerns 1 trading day or 1 trading year.

So when somebody is buying 10 MW base stripe European peek in working days trough 1 month (let say June 2011) in Poland it means per each period from Monday till Friday for every single hour from 8.00AM to 8.00PM volume 10 MWh energy is buying. And the total amount of energy is 2640 MWh (22 days in June 2011 x 12 hours per each day x 10MWh per each hour). The same contract is perceived in completely different ways. It depends on participants role on energy market. Thus he is interesting in minimization of the average price – then it could be said that form the buyers perspective he manages the price risk. On the other hand somebody who is in charge of production required amount of energy is the owner of volume risk in vast majority of cases. Because the production cost is known (simplifying assumption – the cost curve of generation is known) then to maximize income (and profit) the supplier should produce as much energy as he can. Thus the statement could be used form the suppliers perspective he manages the volume risk.

Finally on energy markets a lot of trading companies are existing. And they are owners of both types of risk. They interests are simple as business is – maximization of profit which is some derivative from turnover. Thus natural method of risk managing is speculation. Using availability of: markets, energy products, time zones shift traders are trying to find the best offers to maximize the portfolio profit. When the portfolios risk of volume is not exist (we are buying the same amount of energy than were selling) and portfolios profit is higher than some minimum, assumed profit then we won the game. So much for the theory says.

In reality it could be very hard to find in the same tame sufficient number of ”long” and ”short” transactions with sufficient price spread. If yes – then we are the lucky winners, if not – then risk factor emerges. To describe the possible scenarios well use our upper example: somebody wants to buy 10 MW base stripe European peek trough June 2011. And the average price is 49 €/MWh. But today it is 22nd February and we do not have what will the price be on June. Our possible answers are presented below:

- when we are able to find short position of forward contract on June 2011 on exact amount of energy when the price is lower than 49 €/MWh we are commit the both transactions and earning on price' spread with avoiding the risk,
- when we could nott find the right contract then our answer depends on our strategy:
  - the ”long” position is rejected - so we do nothing with our portfolios profit but avoid the risk,
  - the ”long” position is accepted (no matters of reason why the offer was accepted) but there are lack of ”short” positions and portfolio is under risk.

This example focus attention on meaning of word risk. Word has some negative background and associates with money loss but it should be perceived as some probability of earning or losing. In examples context unbalancing the portfolio gives us a promise of more profits (on the contract in question), when the price decrease or some loss when the price increase above 44 €/MWh.

Our software is trying to find good opportunities to earning when the risk is unavoidable. Our idea is – the portfolio should be described by at least 2 factor: minimal profit from portfolio and maximum risk factor for it. This approach creates space for open trading which makes peoples behavior more creative in daily transactions. Nowadays many traders have to avoid any risk in transactions because they have to be incline with company risk strategy. In our approach they could have opportunities to earn more money under controllable risk.

The energy volume is known but the question is: what is the price of single MWh.

The risk management was divided in short-term (e.g. daily), middle-term (e.g. monthly) and long-term (e.g. annually) models, where the values at higher level became the limits for the lower level planning or calculations. The final results are presented using typical risk measures e.g. value at risk.

II. Risk calculations

The computer system with risk modeling was developed using the models presented below [6], [7]. The uncertainty in presented approach is modeled by random variables, described by probability density function and/or cumulative distribution function.

A. Random variables

The four types of continuous probability values are available:

- the degenerate distribution, i.e. distribution with cumulative distribution function

$$d(x) = \begin{cases} 0 & \text{for } x \leq x_0 \\ 1 & \text{for } x > x_0 \end{cases}$$

- the normal distribution, parametrized by μ (mean) and σ (standard deviation)
- the continuous uniform distribution, parametrized by x_{min} and x_{max}, with probability density function $u(x)$ given below.

$$u(x) = \begin{cases} \frac{1}{x_{max}-x_{min}} & \text{for } x_{min} \leq x < x_{max} \\ 0 & \text{for } x < x_{min} \text{ or } x \geq x_{max} \end{cases}$$

- general continuous distribution. This distribution is represented by segments, set of n disjoint uniform distributions. The example of probability density function $h(x)$ is shown in Fig. 1.

$$h(x) = \begin{cases} p^1 & \text{for } x^1_{min} \leq x < x^1_{max} \\ p^2 & \text{for } x^2_{min} \leq x < x^2_{max} \\ \ldots & \\ p^n & \text{for } x^n_{min} \leq x < x^n_{max} \\ 0 & \text{for } x < x^1_{min} \text{ or } x \geq x^n_{max} \end{cases}$$

  The general continuous distribution is given by user or created automatically e.g. from histogram generated by Monte Carlo calculations. The p^i factors are normalized.

$$\int_{-\infty}^{+\infty} h(x)\,\mathrm{d}x = \sum_{i=1}^{n} p^i (x^i_{max} - x^i_{min}) = 1$$

The expressions using random variables are created to model technical and/or economical aspects. For example the contract may contain the volume described by random variable of given probability density, to model the uncertainty of the amount and/or the price per MWh – to describe the uncertainty of the price.

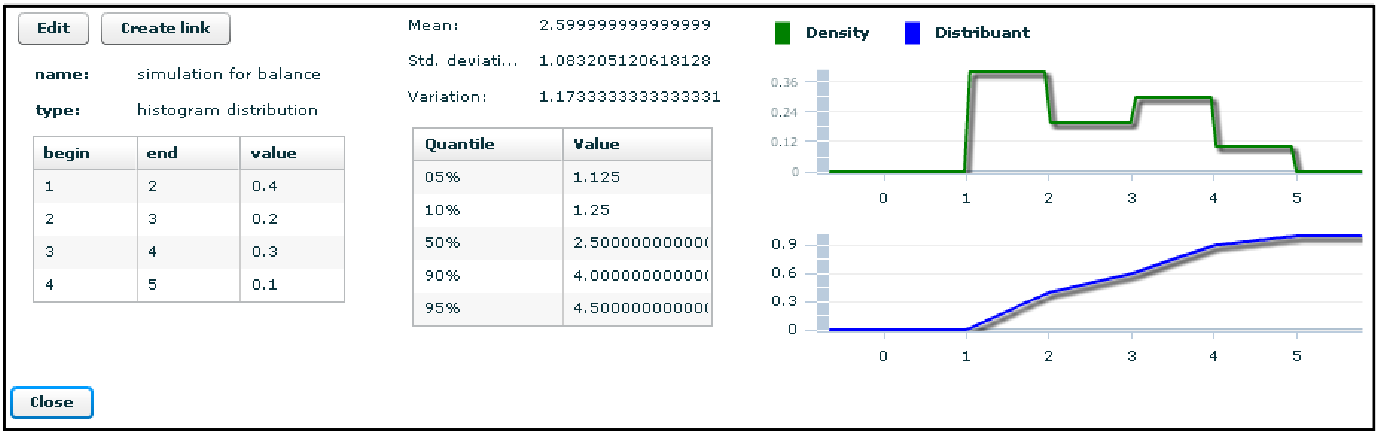

Fig. 1. The general continuous distribution, screen-shoot from presented computer program

B. Monte Carlos simulations

The expressions using random variables are resolved numerically by Monte Carlo simulations [8]. The simulation is parametrized by the accuracy i.e. number of steps when Monte Carlo is performed. The step includes the calculation of value of each random variable (draw) and the calculation of value of expression. Result of Monte Carlo simulation is the general continuous distribution, where the length of each segments is the same. The length of segment ϵ is approximated by (1),

$$\epsilon = \frac{\sigma}{\sqrt{n} * \mathcal{N}_{0,1}(3)} \tag{1}$$

where:
- $\mathcal{N}_{0,1}$ is the pdf of normal distribution
- n is the number of steps in Monte Carlo
- σ is standard deviation of result distribution.

The standard deviation of result distribution for expression v is approximated by (2), given the approximated parameters (μ and σ) of sub-expression.

$$\sigma^2 = \begin{cases} \sigma_a^2 + \sigma_b^2 & \text{where } v = v_a + v_b \\ \mu_a^2 * \sigma_b^2 + \mu_b^2 * \sigma_a^2 + \sigma_a * \sigma_b & \text{where } v = v_a * v_b \end{cases} \tag{2}$$

The expression to calculate the mean and standard deviation for supported variable types are given in Tab. I.

C. Time-series predictions

The transactions in energy markets are made on future periods of time. The prices in future are not known, this is also the risk factor in described transactions. To estimate of energy prices and/or energy amounts the set of algorithms are proposed. These algorithms, supplied by presented system, base on historical data, are described below.

The changes in time are modeled by the stochastic processes, represented by time series, a collection of couples (time stamp, value). During calculations time series is sampled to obtain samples at equal intervals, using linear approximation of the two closest elements, as showed in Fig. 2.

The two prediction algorithms are used: using the autoregressive models and/or the memory based method. Autoregressive model AR of order p (called AR(p)) calculates the prediction based on the history, using (3), where $v(t)$ is the value at time t, α_i are parameters of the model.

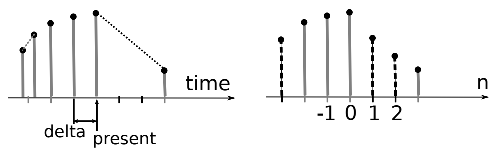

Fig. 2. Linear approximation of time series to obtain samples at equal intervals. The interval length is denoted delta, the present time-stamp is represented by 0, the past by negative integrals, the prediction by positive integrals.

$$v(t) = \sum_{i=1}^{p} \alpha_i v(t-i) \tag{3}$$

The memory based algorithm finds the closest (to the reference) sequence in past and then calculates the prediction. The algorithm is depicted in Fig. 3. In general case the k closest sample sequences are considered, the prediction is the average of them.

Presented algorithms are used to determine of unknown values based on historical data. This approach allows to find the trends and periodic changes of prices and/or amounts in energy market, therefore allows specify the risk of portfolio.

D. Portfolio optimization

The system provides portfolio management methods, including optimization of the portfolio, given technical and economical restrictions. The genetic algorithm is used to find the global optimum, then the hill climbing (local optimization) improves the result.

Optimization algorithms uses the Monte Carlo simulation to calculate the fitness function. Therefore the portfolio may contain the parameters modeled by random variables, e.g. uncertain prices and/or uncertain amounts.

The best solution is calculated at given value of risk (e.g. at 5% risk). The small values indicates the safer transactions are considered in portfolio. The greater values of given value of

name	parameters	μ	σ
degenerate distribution	x_0	x_0	0
normal distribution	μ, σ	μ	σ
uniform distribution	x_{min}, x_{max}	$\frac{1}{2}(x_{min} + x_{max})$	$\frac{1}{2\sqrt{3}}(x_{max} - x_{min})$
general distribution	$p^1, p^2, ..., p^n$, $x^1_{min} x^2_{min}, ..., x^n_{min}$, $x^1_{max}, x^2_{max}, ..., x^n_{max}$	$\frac{1}{2}\sum_{i=1}^{n} p^i(x^i_{min} + x^i_{max})$	$\frac{1}{\sqrt{3}}\sqrt{\sum_{i=1}^{n} p^i((x^i_{max} - \mu)^3 - (x^i_{min} - \mu)^3)}$

TABLE I
THE MEAN AND STANDARD DEVIATION FOR SUPPORTED RANDOM DISTRIBUTIONS

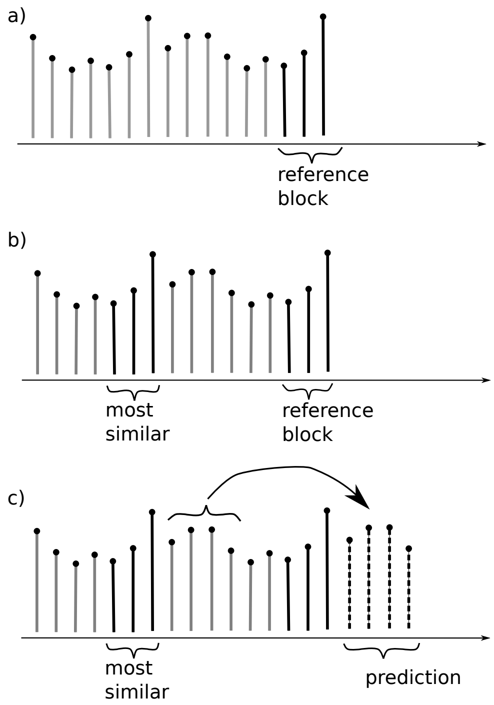

Fig. 3. Memory based prediction algorithm. For given reference block (a), find the closest sample sequence in the history (b). The prediction corresponds to the closest sample (c), the following samples are used.

risk for optimization – more profit (an more risky) portfolio is proposed.

III. IMPLEMENTATION

The comprehensive system of risk manager was developed in Web Application architecture, depicted in Fig. 4 and in [9]. The user require only Web Browser with Adobe Flash Player plug-in, the graphical user interface allow to manage the stored data about clients (contracts, energy consumption, profiles, limits), energy price, CO_2 certificates price, fuel prices, calendars and perform the calculations: predict the prices, optimize the contract portfolio, calculate the production costs, etc. The Fig. 5 shows the calendar manager containing the type of day (working days, free days, bridge days), used in energy consumption prediction algorithms.

The SCRUM framework was used for managing product development, the 3 weeks sprint cycle was established. The backlog containing about 400 items were used. The calculations i.e. optimization, simulation and prediction algorithms

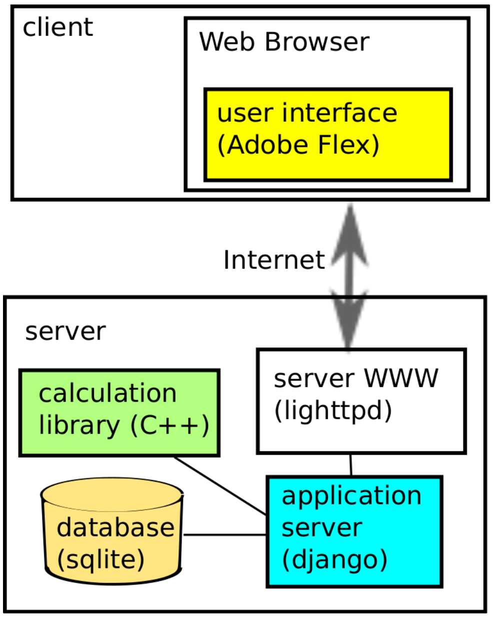

Fig. 4. The architecture of presented computer program. The server uses django framework, relational database and specialized multithreaded calculation library. The client uses Web browser with Adobe Flash Player.

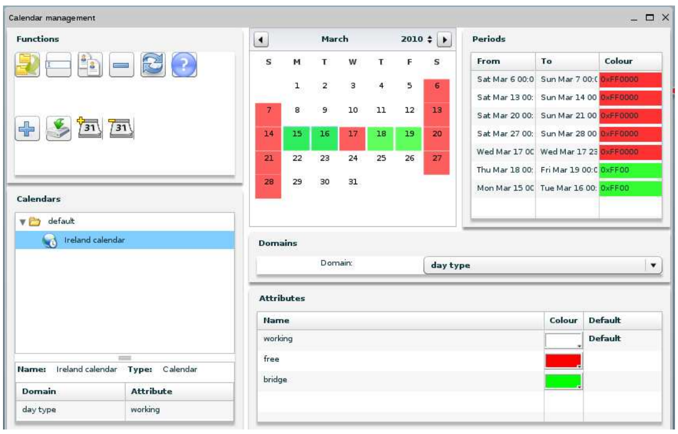

Fig. 5. The calendar manager with types of days, the example screen from application.

are implemented in shared library using C++. The server based on Django Web Framework provides all functionality and stores objects in relational database. The graphical user interface uses Adobe Flex software development kit. The size of the source code, reported by http://cloc.sourceforge.net, is 8k (calculation library), 32k (server), 32k(graphics user interface). Over 200 unit tests are created using boost test library, unittest and flexunit.

IV. Results

When the background is known then we would like to simulate assumptive energy market transactions and compare traditional and our approach to sign the transactions. The minimum limit of portfolios profit (comes from company policy) is known and equal 8000 €. Example: You are the trader in one of the biggest European trading company. And your first transaction was describe in introduction to this article. Our client wants to buy an energy in European peek in June 2011. Total volume of contract amounts 2640 MWh (22 days in June 2011 x 12 hours per each day x 10MWh per each hour), as depicted in Fig. 6.

Fig. 6. The contract representation in the application.

SCENARIO 1: Client is asking us about price.

1) To find a good price were starting checking the energy prices in short transactions on the market. Three offers are matching our need and we are finding the best and the worst just to creates the alternatives for bidding. And those values (as average price) start from 45 €to 50 €.
2) The higher price (50 €per MWh) is presented to the buyer
3) The final price from the customer (as the answer on our proposal) is 48 €per MWh and the transaction is finalized (Fig. 6).

The risk does not exists in this scenario, the gain as a function of risk is presented in Fig. 7.

SCENARIO 2: Client is asking about price and gives some ranges of MWh. The market offers are the same as in scenario 1. But from the trader point of view there some new risk factor is emerged, e.g. the prediction of prices on the market or the average prices from the same period in historical data. The profit as a function of risk is modeled as shown in Fig. 8.

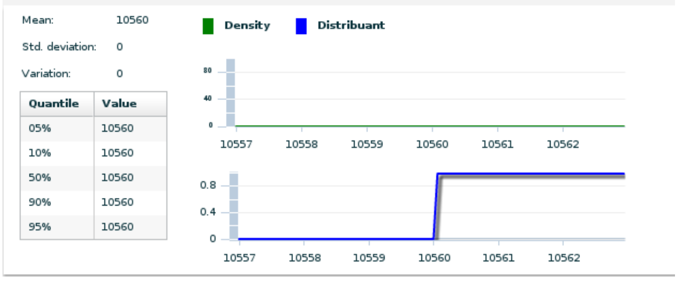

Fig. 7. The gain as function of risk for scenario 1.

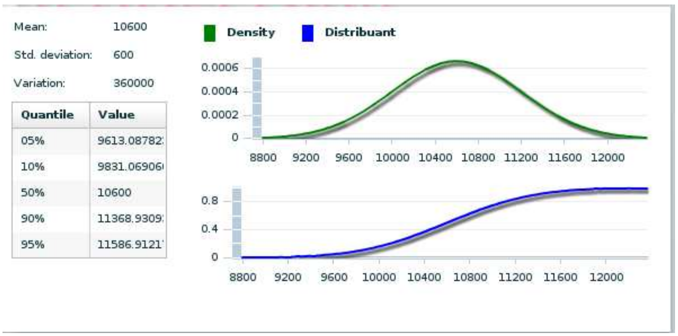

Fig. 8. The gain as function of risk for scenario 2, the uncertainty of price is considered.

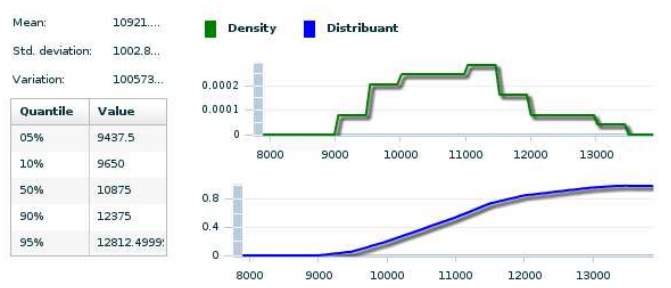

Fig. 9. The gain as function of risk for scenario 3. When no risk is considered the gain is lower than in scenario 1, but higher than minimum portfolio profit. With 11% of risk the scenario 3 gives the same profit as the scenario 1. On average the profit will be greater (we will gain about 300€more), in the most favorable situation the proposed scenario is 25% better.

SCENARIO 3: This is active point of work for trader. Using available market contracts he is in charge of building offers for customers. The proposed profit distribution is shown in Fig. 9,

SCENARIO 4

The choose of the best scenario. the optimal portfolio is proposed by optimization algorithm at given value of risk. The comparison of many scenarios, as shown in Fig. 10 is used.

V. Conclusions

The algorithms for risk management were adopted to electricity exchange. The computer program using these methods was implemented and tested on Windows XP and Debian Linux, the calculations on historical data from polish electricity market proved the usability of presented approach. The

Fig. 10. The profit as function of risk, comparison for scenario 1 and scenario 3. The trader choose the acceptable value of risk and application suggests the best scenario.

practical implementations in trading companies and energy plants are planned and partners able to test and develop presented system are searched.

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REFERENCES

[1] L. Clewlow and C. Strickland, *Energy derivatives: Pricing and risk management*. Lacima, London, 2000.

[2] A. Eydeland and K. Wolyniec, *Energy and power risk management: New developments in modeling, pricing, and hedging*. John Wiley & Sons Inc, 2002.

[3] H. Kaproń, "Teoria konkurencji producentów energii elektrycznej," *Rynek Energii*, vol. 6, pp. 13–16, 2007.

[4] R. Weron, "Energy price risk management," *Physica A: Statistical Mechanics and its Applications*, vol. 285, no. 1-2, pp. 127–134, 2000.

[5] K. Pawlak, "Risk management in the electricity market from the side of TPA client (end user case)," *Biuletyn Urzedu Regulacji Energetyki Selected full texts*, vol. 1, pp. 63–68, 2006.

[6] J.Arabas, R.Nowak, P.Kaczyski, B.Lipiski, H.Lipiski, and P.Stalewski, "Zasady funkcjonowania systemu zarzdzania ryzykiem 8decision," Politechnika Warszawska, grant celowy: Opracowanie, realizacja i wdroenie systemu wspomagania decyzji handlowych na rynkach energii, Tech. Rep., 2008.

[7] ——, "Model prognozowania szeregw czasowych oraz zagadnienia optymalizacji modeli w prognozowaniu cen i wolumenw," Politechnika Warszawska, grant celowy: Opracowanie, realizacja i wdroenie systemu wspomagania decyzji handlowych na rynkach energii, Tech. Rep., 2008.

[8] R. Wieczorkowski and R. Zieliński, "Komputerowe generatory liczb losowych," *WNT, Warszawa*, 1997.

[9] R. Biedrzycki and R. Nowak, "Internet application architecture for data fusion," Institute of Electronic Systems Warsaw University of Technology, Tech. Rep., 2010.