INTEREST GROUPS AND THE ELECTORAL CONTROL OF POLITICIANS¹

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Abstract

We develop a model of interest group influence in the presence of repeated electoral competition. In each period of the game, an interest group attempts to "buy" an incumbent's policy choice, and a voter chooses whether to replace the incumbent with an unknown challenger. The voter faces a tension between retaining good politician types and rewarding past performance. The model predicts that "above average" incumbents face little discipline, but others are disciplined increasingly—and re-elected at a higher rate—as the interest group becomes more extreme. Extensions of the model consider term limits, long-lived groups, and multiple groups.

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1. Introduction

Interest group politics is one of the most important topics in political economy and political science. However, while theorists have been analyzing formal models of interest group politics for more than thirty years, one aspect of the problem remains underdeveloped: How should strategic voters vote when they know that interest groups are trying to skew policies in ways the voters do not like? This issue has been overlooked because existing models focus on the calculations and strategic interactions of interest groups and politicians. As a result, these models treat voters as a black box, or at best as myopic actors that respond only to the short-run campaign promises of the current election.¹

One obvious place to turn is the work on political agency, which focuses on the calculations voters and politicians make in a principal-agent framework. That literature goes back approximately as far as the interest group literature – Barro (1973) vs. Stigler (1971) – and has produced important insights about the possibilities and limits of using elections to control the behavior of politicians and/or select "good" types of politicians.² None of the models in this literature, however, explicitly incorporate interest groups as a strategic actor.

This paper takes a step toward combining the two literatures – to our knowledge, the first step. We analyze an infinite horizon game in which a representative voter elects a single office-holder in each period. In each election, the voter chooses between an incumbent and a randomly-drawn challenger. The voter cares about policy outcomes, while politicians care about holding office. Each period, Nature also draws an interest group that can offer a contract to the incumbent for choosing particular policies.

A key parameter in our analysis is each challenger's value of office, which is drawn i.i.d. each period and cannot be revealed to the voter until she achieves office. We refer to this

¹See Peltzman (1976), Denzau and Munger (1986), Grossman and Helpman (1994, and 2001, Chapters 7-9), and Persson and Tabellini (2000, Chapter 7) for examples of the former, and Austen-Smith (1987), Baron (1994), and Grossman and Helpman (1996, and 2001, Chapter 10) for examples of the latter.

²A sampling of the literature includes Ferejohn (1986), Rogoff and Sibert (1988), Austen-Smith and Banks (1989), Rogoff (1990), Banks and Sundaram (1993, 1998), Harrington (1993), Fearon (1999), Barganza (2000), Hindriks and Belleflamme (2001), Le Borgne and Lockwood (2001, 2006), Smart and Sturm (2003, 2006), Besley (2006), Besley and Smart (2007).

value as the challenger's (or incumbent's) "type." The incumbent's type determines her "price" for an interest group's policy-buying efforts. A high-type incumbent who expects re-election will be relatively expensive for a group to buy, while a low-type incumbent who does not expect re-election will be relatively cheap. The voter thus has partially conflicting incentives. She can induce good performance through promises of re-election, but also prefers to retain only types who are less susceptible to interest group influence.

In addition to considering competition between voters and interest groups, the model can be viewed as a contest between two different kinds of interest groups. "Activist" groups such as the Sierra Club, National Association for the Advancement of Colored People, or American Association of Retired Persons have the attention of large numbers of voters in many constituencies, but relatively limited financial resources. On the other hand, groups such as Pharmaceutical Research and Manufacturers of America have impressive resources for lobbying, but relatively few voters. The model therefore captures some basic intuitions of competition between groups with heterogeneous resources.

We study two main variants of the model. In the first, the voter can commit to optimal stationary contracts for controlling the politician. In the second, we drop the commitment assumption and examine both stationary and simple (two-state) non-stationary equilibria. A non-stationary equilibrium would seem to demand an excessively high level of sophistication on the part of a voter. However, activist groups may in practice provide the link between desired punishment strategies and voter actions. By coordinating voting behavior through publications, advertisements, or endorsements, such groups can tune the responses of voters to incumbent behavior over multiple elections.

Our results reveal several important features of the tension between inducing performance and selecting types. In an environment where the voter can commit to re-election contracts, she will re-elect an incumbent only if the chosen policy is sufficiently close to her ideal point. The voter may allow policy to deviate from her ideal point somewhat, however, to prevent excessive interest group vote buying. Policies that are too far from the group's ideal will induce the group to "buy" its ideal policy instead, at a cost equal to the incumbent's expected lifetime payoff. An incumbent's price therefore depends on her anticipation of future re-elections.

For a given incumbent type, the voter thereby maximizes her policy utility by promising re-election in all future periods. However, the voter may not wish to induce maximum performance from every incumbent type. Since incumbents who value office more highly also command higher prices, voters have an incentive to remove "low-type" incumbents. Therefore, in the contracting equilibrium the voter will always keep sufficiently high-type incumbents and always remove sufficiently low-type incumbents. In between, incumbents may be retained if groups are extreme. This happens because even a temporary promise of re-election can improve policy performance. To a voter with a concave utility function, this performance is most valuable when a group is extreme relative to the voter. Our model thus makes the somewhat counterintuitive prediction that re-election rates should increase as policies diverge from the voter's ideal.

When we remove the assumption of re-election contracts, the results depend on the form of equilibrium assumed. In an equilibrium in stationary strategies, the voter's ability to induce performance is severely constrained. Since votes are cast after policies are chosen, the voter's type-selection incentives are too strong and all incumbent types produce the same policy results as the worst type.

When we additionally drop the strong stationarity restriction, however, the results can change dramatically. We show that by using simple non-stationary "trigger" strategies the contracting equilibrium can be restored. Note that this is not an obvious result, because we cannot use standard repeated-game punishment strategies, since the punishment instrument eliminates players.³

Finally, we examine a few extensions to this framework. When finite term limits are imposed, an incumbent's term of office acts much like her w. As an incumbent's experience increases, the price of her vote decreases. The voter therefore loses the ability to induce performance, and becomes less likely to re-elect incumbents as they become more experienced.

³See, e.g., Dutta (1995) for a more general treatment of such repeated dynamic games.

As a result, in both the contracting and non-stationary, non-contracting environments, voters cannot benefit from term limits. We also consider the effects of a long-lived interest group and of multiple groups. The former weakens a voter's electoral control by giving the group more "buying power," while the latter strengthens it when groups are on ideologically opposite sides of the voter.

Our model joins a number of recent models in examining the strategies of voters in the presence of interest groups. These models frequently focus on the interaction between interest group contributions and private information. Interest groups may provide resources for incumbents to advertise their private information (*e.g.*, Coate, 2004; Ashworth, 2006), or for signaling their own information about candidate characteristics (Prat, 2002). As in our work, voters are concerned with selecting candidate "types" as well as the possible nonconvergence between their preferences and policy.⁴ However, these models are not dynamic, and therefore do not present any incentives for the retrospective disciplining of incumbents.

Several recent papers in the literature on political agency argue that elections are mainly about "selecting good types" rather than "sanctioning poor performance" (*e.g.*, Fearon, 1999; Besley, 2006; Besley and Smart, 2007). The argument is that if politicians have policy preferences, and if the game is finite or if politicians are finitely-lived, then in any subgame perfect equilibrium voters will behave as if they care only about selecting politicians with "good" preferences. Fearon (1999, page 57) states it clearly: "although the electorate would like to commit to a retrospective voting rule to motivate self-interested politicians optimally, when it comes time to vote it makes sense for the electorate to focus *completely* on the question of type: which candidate is more likely to be principled and share the public's preferences?" If voters believe the incumbent's type is better than average, then they must re-elect her, and if they believe the incumbent's type is worse than average, then they must replace her.⁵ By contrast, our model shows that this argument does not necessary hold in

⁴An extensive literature also examines other causes of candidate non-convergence. See, *e.g.*, Alesina (1988), Bernhardt and Ingberman (1985), Calvert (1985), Ingberman and Villani (1993), Palfrey (1984), and Testa (2003).

⁵Besley (2006, pages 192-193) states the problem as follows: "With pure moral hazard, voters are indifferent (ex post) between voting for the incumbent and a randomly selected challenger. Hence, they can

a world where "policy preferences" are not intrinsic, but rather induced by interest group influence. As noted above, in our model the voter *can* both sanction and select, even with finitely-lived politicians. All that is required is that players use non-stationary strategies of a very simple form.

2. The Model

Our model is one of policy-making and elections in a single constituency over an infinite horizon. We denote periods with a subscript t. Players all discount future payoffs by a common factor $\delta \in (0, 1)$.

In each period t = 1, 2, ... there are four players; a politician (P), a median voter (M), an interest group (G_t), and an election challenger (C_t). Since the incumbent is endogenous, note that P (redundantly) denotes the incumbent politician of a given period, with C₀ representing the first incumbent. Thus if C₁ defeats C₀, then P "becomes" C₁ in period 2, and so on.

P and C_t care about holding office, while M and G_t care about policy. The policy in period t is an element x_t from the convex, compact set $X \subset \Re$. M and G_t can control P in different ways; the former by voting, and the latter by offering payments in return for policies chosen. It is assumed that G_t can credibly commit to these payments.

Player utilities are represented as follows. In each period t, M receives $u^M(x_t)$, where $u^M : X \to \Re$ is continuous and single-peaked and $m \equiv \arg \max u^M(\cdot) \in X$. For politicians, each period's challenger has a "type" w_t , where w is used to denote generic types. In each period t, w_t is drawn i.i.d. according to the probability density f_w with compact support $\Omega \subset \Re_+$. A politician who was first elected in period t' (*i.e.*, an incumbent who began as $C_{t'}$) then receives a fixed benefit of $w_{t'}$ per period upon each election. She does not have policy preferences, but has quasilinear utility over a non-negative transfer or "bribe"

pick a standard for incumbents to meet that creates the best possible incentives for incumbents to reduce their rent extraction. Voters cannot commit to this voting rule, however, when the incumbent could be a good type even though they would prefer to commit to the voting rule that is used under moral hazard. This finding suggests that the pure moral hazard model is rather fragile to a small variation in the model to include some good types of politicians. This is because the strict indifference rule that underpins incentives in that case allows the voters to commit. Once this strict indifference is broken, then there is actually a constraint on optimal voting strategies which can make things worse."

 $b_t \in B \equiv \{b : X \to \Re_+\}$ offered by G_t ; thus, in period t an incumbent initially elected in period $t' \leq t$ receives:

$$u^P(x_t, w_{t'}) = w_{t'} + b_t(x_t)$$

Finally, G_t may receive non-zero utility only in period t, when she receives:

$$u^{G_t}(x_t) - b_t(x_t),$$

where $u^{G_t}: X \to \Re_-$ is continuous and single-peaked. Let $g_t \equiv \arg \max_{x_t} u^{G_t}(\cdot) \in X$ denote G_t 's ideal policy. Each g_t is drawn i.i.d. according to the probability density f_g with compact, convex support $\Gamma \subseteq X$. We assume that for all G_t , $u^{G_t}(x_t) = v(x_t - g_t)$ with v(0) = 0, so that group utility functions are identical up to changes in ideal point.⁶

The sequence of moves in each period t is as follows. All actions are perfectly observable by all players unless otherwise noted.

Group Draw. Nature selects group G_t .

Vote Buying. G_t offers P a transfer schedule $b_t \in B$, unobserved by M.^{7,8}

Policy Choice. P chooses $x_t \in X$.

Challenger Draw. Nature selects challenger C_t , where w_t is unobserved by M.

Election Voting. M casts a vote $r_t \in \{0, 1\}$ over whether to re-elect P (1) or elect a challenger C_t (0). If C_t wins, w_t is revealed to M.

We will focus on subgame perfect equilibria in pure strategies. While we do not prove existence of such equilibria for the general game, they are readily derived for a wide range of parameters and functional forms. Strategies for each player are measurable mappings

⁶Alternatively, we could assume that all groups have the same ideal point g, but differ in the intensity of their preferences – *i.e.*, in their willingness to pay for favorable policy shifts. This yields qualitatively similar results.

⁷The unobservability of the transfer schedule simplifies the analysis but is not required for our results.

⁸This gives all of the bargaining/agenda-setting power to the group. However, our main results and intuitions hold under a variety of different assumptions about how the bargaining/agenda-setting power is divided between the politician and the group.

defined as follows. Let H_t denote the set of game histories prior to period t. Then G_t 's strategy $\beta_t : H_t \times \Omega \times \Gamma \to B$ maps the history through period t-1, the politician type, and her ideal point into a bribe schedule. P's strategy $\chi_t : H_t \times \Omega \times \Gamma \times B \to X$ maps histories, types, group ideal points, and bribe schedules into a policy choice. Finally, M's strategy $\rho_t : H_t \times \Omega \times \Gamma \times B \times X \to \{0, 1\}$ maps histories, types, group ideal points, bribes, and policy choices into a vote for the incumbent or challenger.

3. The Contracting Game

To establish a baseline for comparison, we first examine a case in which M may write "contracts." That is, in each period t, M commits to a vote based on actions from that period. Formally, this requires that the game be modified so that instead of choosing a vote after the policy choice, M announces ρ_t prior to G_t 's vote buying. Following convention, we restrict attention to stationary subgame perfect equilibria (SSPE). This requires that players act identically and optimally when faced with identical continuation games, and hence imply history-independent strategies. M's period t contract can thus condition only on g_t , x_t , and the incumbent's type. Additionally, we will focus on the set of optimal SSPE for M, which we label \mathcal{E}_c . Since there are multiple SSPE with similar characteristics, this restriction simplifies the analysis, but does not qualitatively change our results.

To derive the optimal voting contracts, we begin by supposing hypothetically that M promises re-election to P regardless of w and g_t . This strategy (which may not be Nash) may be considered a "pure" *ex post* monitoring solution, in that M's contract achieves the best policy possible in period t without any regard for the type of politician retained.

Consider G_t 's response to an arbitrary contract from M. In general, an optimal contract cannot be constant in x_t ; otherwise, G_t could achieve her ideal policy at negligible cost by offering the following contract:

$$b(x_t) = \begin{cases} \epsilon & \text{if } x_t = g_t \\ 0 & \text{otherwise.} \end{cases}$$
(1)

Hence, we may partition X into two disjoint, non-empty subsets; an "incumbent set" of

policies for which ρ promises re-election, and a "challenger set" of policies for which P is kicked out. G_t's optimal contract then must offer either:

(i) $\epsilon > 0$ for choosing $\tilde{p} = \arg \max_{\{x \in X | \rho(w, g_t, x) = 1\}} u^{G_t}(x)$ and 0 otherwise; or

(ii) $w/(1-\delta) + \epsilon$ ($\epsilon > 0$) for choosing $\tilde{c} = \arg \max_{\{x \in X | \rho(w, g_t, x) = 0\}} u^{G_t}(x)$ and zero otherwise.

 G_t offers the type (i) contract if the best policy (\tilde{p}) with re-election beats the best policy without re-election (\tilde{c}) plus the cost of buying out P, or $u^{G_t}(\tilde{p}) > u^{G_t}(\tilde{c}) - w$.

To derive M's voting contract, let $\mathcal{G}(y) = \{x \in X \mid u^{G_t}(x) \geq y\}$ denote the upper contour set of $u^{G_t}(\cdot)$. By the single-peakedness and continuity of $u^{G_t}(\cdot)$, $\mathcal{G}(y)$ is a closed interval containing g. There are two cases. First, if $m \in \mathcal{G}(-\frac{w}{1-\delta})$, then G_t is unwilling to pay the cost of a type (ii) contract to move P's policy away from m. M can then threaten to re-elect P if and only if m is chosen. Second, if $m \notin \mathcal{G}(-\frac{w}{1-\delta})$, then M cannot induce any policy choice outside of $\mathcal{G}(-\frac{w}{1-\delta})$, for otherwise G_t can use a type (ii) contract to achieve $x_t = g_t$. Letting $\overline{g}(y) = \max \mathcal{G}(y)$ and $\underline{g}(y) = \min \mathcal{G}(y)$, the best policy that M can induce by promising perpetual re-election is thus:

$$\tilde{x}(w,g_t) = \begin{cases} \underline{g}(-\frac{w}{1-\delta}) & \text{if } m < \underline{g}(-\frac{w}{1-\delta}) \\ m & \text{if } m \in [\underline{g}(-\frac{w}{1-\delta}), \overline{g}(-\frac{w}{1-\delta})] \\ \overline{g}(-\frac{w}{1-\delta}) & \text{if } m > \overline{g}(-\frac{w}{1-\delta}). \end{cases}$$
(2)

Note that as w decreases, $\mathcal{G}(-\frac{w}{1-\delta})$ shrinks. This reduces P's "price" for G_t , and in turn M's ability to discipline P. Additionally, when $m \neq g_t$, $\tilde{x}(w, g_t)$ is strictly better for M than g_t , as M is always able to pull policy some distance away from g_t .

One contract that achieves this result is:

$$\rho^*(w, g_t, x_t) = \begin{cases} 1 & \text{if } x_t = \tilde{x}(w, g_t) \\ 0 & \text{otherwise.} \end{cases}$$
(3)

 G_t 's optimal response is a null contract with zero payments (*i.e.*, a type (i) contract as $\epsilon \to 0$). P then chooses $\tilde{x}(w, g)$ and is re-elected.⁹ Figure 1 illustrates the equilibrium policy.

[Figure 1 here.]

⁹Here we briefly sketch what happens when all bargaining/agenda-setting power lies with P rather than G_t . Consider the case with $m < \underline{g}(-\frac{w}{1-\delta}) < g_t$. Suppose that there is a status quo or reversion policy

There are two simple cases in which this voting strategy can be sustained as an equilibrium. If w_t is constant over t, then M has no incentive to select politician types, and so it is optimal to retain an incumbent indefinitely. Similarly, if Γ is such that $m \in \mathcal{G}(-\frac{w_t}{1-\delta})$ for all g_t and w_t , then M can achieve her ideal policy for any type. In these cases, permanent retention obviously yields the optimal outcome for the voter.

In more complex cases, however, M will have an incentive to select politician types. This may reduce the performance that can be extracted from a given incumbent in equilibrium, since removing the promise of perpetual re-election will reduce P's expected lifetime payoff and hence her price to G_t . To describe the policies that M can induce, denote by $l(w; \{\rho(w, \cdot)\})$ a type-w incumbent's ex ante (i.e., prior to the draw of g_t) discounted expected payoff given a set of voting contracts $\{\rho(w, \cdot)\}$. We simply write l_w when the contracts in question are understood; thus, when M always re-elects a type-w incumbent, $l_w = w/(1-\delta)$. In each period t, M can then "myopically" induce the following policies:

$$\tilde{x}(w,g_t) = \begin{cases} \underline{g}_t(-w-\delta l_w) & \text{if } m < \underline{g}_t(-w-\delta l_w) \\ m & \text{if } m \in [\underline{g}_t(-w-\delta l_w), \overline{g}_t(-w-\delta l_w)] \\ \overline{g}_t(-w-\delta l_w) & \text{if } m > \overline{g}_t(-w-\delta l_w). \end{cases}$$
(4)

There are often many SSPE in the contracting game; however, their voter strategies share the same intuition. Generally, M must balance the short term performance that can be induced by a contract with the possibility of better long run performance from a challenger. By re-electing P, M can achieve policy $\tilde{x}(w, g_t)$, but if M does not re-elect P, then G can offer a contract similar to (1), resulting in policy g_t . Because a type-w incumbent's future

 $s \leq m$, which is enacted if P does not propose anything or if G_t rejects all of P's proposals. Suppose M offers a contract of the form: $\rho^*(w, g_t, x_t) = 1$ if $x_t \leq x_0$ and $\rho^*(w, g_t, x_t) = 0$ otherwise, with $x_0 < g_t$. Given the "threat point" s, P can extract at most $-u^G(s)$ in payments from G_t , by making G_t a take-it-or-leave-it offer of the form: $x_t = g_t$ and $b = -u^{G_t}(s)$ (which G_t will accept). This yields P a payoff of $-u^{G_t}(s)$, since P will not be re-elected under the proposed contract from M. Alternatively, P can choose a policy that allows her to be re-elected. The best such policy for P is x_0 itself, together with a take-it-or-leave-it offer to G_t of the form: $x_t = x_0$ and $b = u^{G_t}(x_0) - u^{G_t}(s)$ (which G_t will again accept). This yields P a payoff of $\frac{w}{1-\delta} + u^{G_t}(x_0) - u^{G_t}(s)$. The optimal contract for M then solves: minimize x_0 s.t. $\frac{w}{1-\delta} + u^{G_t}(x_0) - u^{G_t}(s) \geq -u^{G_t}(s)$, or minimize x_0 s.t. $u^{G_t}(x_0) \geq -\frac{w}{1-\delta}$. The solution is therefore $\tilde{x}(w, g_t) = \underline{g}(-\frac{w}{1-\delta})$. Thus the contract, and the equilibrium policy outcome, is exactly as in the game where G_t has all of the bargaining/agenda-setting power. Note also that this is true *regardless* of the reversion policy s (as long as $s < g(-\frac{w}{1-\delta})$).

election prospects may depend on future realizations of g_t , her expected payoff conditional upon re-election must satisfy $l_w \in [w, \frac{w}{1-\delta}]$. Hence the policy $\tilde{x}(w, g_t)$ can be no better for M than (2).

For each incumbent of type w, let $v_c(w) = \sum_{t=1}^{\infty} \delta^{t-1} E u^M(x_t^* \mid w_0 = w)$ represent M's discounted expected utility in the repeated contracting game, where x_t^* is P's optimal policy choice. As intuition would suggest, $v_c(w)$ is weakly increasing in w, since M can feasibly offer a high-w incumbent the same contracts as she does a low-w incumbent.¹⁰ Now M's condition for re-electing P can be written:

$$u^{M}(\tilde{x}(w,g_{t})) + \delta v_{c}(w) \ge u^{M}(g_{t}) + \delta \int_{\Omega} v_{c}(\tilde{w}) f_{w}(\tilde{w}) d\tilde{w}.$$
(5)

The integral in (5) represents the discounted expected value from electing challenger C_t , prior to the revelation of w_t .

The first result uses these observations to characterize the main policy and re-election implications of optimal equilibria. It establishes a basic monotonicity of re-election results according to type. Since it provides an important point of comparison, it will be convenient to define an "average" type $\hat{w} = \sup \{w \in \Omega \mid v_c(w) \leq \int_{\Omega} v_c(\tilde{w}) f_w(\tilde{w}) d\tilde{w}\}$. In most cases, this is the lowest type that is *ex ante* no better than a challenger. Since an optimal contract for M must retain a positive measure of politicians types with positive probability, it is clear that $\hat{w} > 0$ in an optimal equilibrium. The result establishes that all incumbents with types above \hat{w} , and possibly some lower types as well, are always re-elected. Thus within the set of sufficiently high types, M does no type selection. For these incumbents, M focuses on extracting the optimal level of policy performance. Interestingly, the prospect of perpetual election does not result in opportunistic behavior by the incumbent. This is a consequence of both the voter's ability to commit as well as the politician's lack of intrinsic policy preferences.

As incumbent types become worse, re-election will depend on group ideal points, as seen in (5). In the latter case, if u^M is concave, then somewhat counter-intuitively, incumbents

¹⁰This statement is proved in Lemma 1 in the appendix.

are increasingly re-elected when groups (and hence equilibrium policies) are farther from m. This is because electing a challenger results in a larger one-period loss when g_t is very distant from m than when it is close. The voter may then prefer to wait for a closer g_t before eliminating the incumbent. Finally, sufficiently low incumbent types are subject only to type selection, and are never given a policy choice that could result in re-election.

Proposition 1 In any equilibrium in \mathcal{E}_c , there exist w^1 and w^2 such that $0 < w^1 \le w^2 \le \hat{w}$ and re-election outcomes are:

$$r_t^* = \begin{cases} 0 & \text{if } w < w^1 \\ \eta(w, g_t) & \text{if } w \in [w^1, w^2] \\ 1 & \text{if } w > w^2, \end{cases}$$

for some $\eta : [w^1, w^2] \times \Gamma \to \{0, 1\}$. $\eta(w, g_t)$ is non-decreasing in w, and if u^M is concave and symmetric, then $\eta(w, g_t)$ is non-decreasing in $|m - g_t|$.

Proof. All proofs are in the Appendix.

While the result applies to \mathcal{E}_c , it also holds for many (but not necessarily all) SSPEs. It is also worth noting that the comparative statics on δ are ambiguous. High values can enlarge $\mathcal{G}_t(\cdot)$ and hence the range of group ideal points for which M can achieve her ideal policy, holding re-election strategies constant. However, they also give M a greater incentive to select good future politicians, thus expanding the set of politician types and groups for which incumbents are kicked out.

The exercise here establishes that in an environment in which voters may write "contracts" for politicians, an optimal strategy in a repeated game solves both shirking and type-selection problems. But because of the bluntness of the voter's contract instrument, both problems are solved in crude fashion. The voter may only select against the worst politician types (and only under certain circumstances), and may only move policy a limited amount when groups are extreme.

4. Equilibria in Stationary Strategies

We now return to the case in which M cannot write contracts, but instead votes in a sequentially rational manner. We again restrict attention to SSPE, and focus on the optimal such equilibria for M, which we label \mathcal{E}_s .

Analogously with the previous section, we define $v_s(w)$ to be M's expected payoff in an arbitrary SSPE, starting from a period with a type-w incumbent. At the voting stage, M's problem is to choose between a type-w incumbent and a random draw from among the challengers. By stationarity, she re-elects P if:

$$v_s(w) \ge \int_{\Omega} v_s(\tilde{w}) f_w(\tilde{w}) d\tilde{w}.$$
(6)

This implies that if P is "above average," then she is always retained. Likewise, if P is "below average," she is never re-elected. As a result, for all types such that (6) is not satisfied with equality, $\rho_t^*(\cdot)$ must be constant in x_t . In response, G_t can offer a contract of the form in (1), and thus receives her ideal policy in every such period. The following result uses this intuition to establish that all continuation values (up to a set of measure zero) must be equal.

Comment 1 In any SSPE, $v_s(w) = k$ almost everywhere for some k.

Thus, in equilibrium M's incentive to select types effectively prevents monitoring of performance. Whereas in the contracting case M could induce high-w types to choose "better" policies, she cannot credibly force different types to choose different actions in a stationary equilibrium.

There are many SSPEs. As a trivial example, consider the equilibrium set \mathcal{E}_0 , in which G_t offers a contract of the form in (1) in each period. P chooses g_t , and M elects C_t . Note that since G_t always receives her ideal policy, $v_s(w)$ is constant. Clearly, this is the worst equilibrium for M. A more plausible equilibrium might feature the optimal (credible) use of re-election incentives.

To characterize \mathcal{E}_s , we make use of Comment 1. Denote by $\underline{w} \equiv \min \Omega$ the "lowest" candidate type. If $w = \underline{w}$, then (similar to the discussion of the contracting case) with the appropriate voting strategy, M can induce any policy in $\mathcal{G}_t(-\frac{w}{1-\delta})$ in each period t. At best,

it can then achieve a policy of m if $m \in \mathcal{G}_t(-\frac{w}{1-\delta})$, and $\underline{g}_t(-\frac{w}{1-\delta})$ or $\overline{g}_t(-\frac{w}{1-\delta})$ otherwise. Thus, if \underline{w} were the *only* type, then P would be indifferent between all candidates and could thereby achieve the contracting result.

The following result establishes that while M can credibly induce any type of politician to take the same action as type- \underline{w} (hence creating a uniform expected value for all types), she cannot do any better. The reason for this is straightforward: if M could induce a better policy in expectation, then this policy must sometimes lie outside of $\mathcal{G}_t(-\frac{w}{1-\delta})$. In these cases, G_t would then be willing to "buy out" type- \underline{w} politicians by offering $\frac{w}{1-\delta} + \epsilon$ for a policy choice of g_t .

Proposition 2 In any equilibrium in \mathcal{E}_s , for any type-w incumbent and all t, g_t , and b_t :

$$\chi_t^*(w, g_t, b_t) = \begin{cases} \underline{g}_t(-\frac{\underline{w}}{1-\delta}) & \text{if } m < \underline{g}_t(-\frac{\underline{w}}{1-\delta}) \\ m & \text{if } m \in [\underline{g}_t(-\frac{\underline{w}}{1-\delta}), \overline{g}_t(-\frac{\underline{w}}{1-\delta})] \\ \overline{g}_t(-\frac{\underline{w}}{1-\delta}) & \text{if } m > \overline{g}_t(-\frac{\underline{w}}{1-\delta}). \end{cases}$$

As in the contracting case, M's strategy has elements of type selection and *ex post* monitoring. However, removing commitment to re-election schedules makes the selection of types irrelevant in equilibrium. This happens because of stationarity, as well as the fact that M's election choice occurs after P's policy choice. The incentive to retain high types (respectively, remove low types) is then so strong that if they existed, they would always be re-elected (respectively, removed). Under these conditions G_t can easily buy its ideal policy. Thus the best that M can do is to monitor all types uniformly, and in a credible fashion. This causes the performance of each type to sink to that of the "least common denominator," or \underline{w} , regardless of the distribution of candidate types.

5. Non-Stationary Strategies

The preceding results suggest that the ability to commit might play a central role in voter welfare. Here we demonstrate that this is not so. Instead, by dropping the requirement of stationary equilibria and focusing instead on simple, non-stationary equilibria, the voter can do just as well as she could in the contracting case. As noted in the introduction, the role of the voter may be filled in practice by large, high-membership interest groups. By coordinating publicity or donations, such groups may play a central role in enforcing the implementation of non-stationary strategies.

We begin by noting a major difference between our model and other dynamic games in which non-stationary solution concepts are applied. In the game examined here, the only plausible "punishment" device (*i.e.*, not re-electing P) removes the punished player. That is, we do not have a repeated game. Combined with complete information, this greatly simplifies a potential defector's optimization problem. It also complicates the potential punisher's problem, in that it limits the promise of future "cooperative" interaction.¹¹

There are many kinds of non-stationary equilibria. For example, the voter might punish a politician by not re-electing him, and "cooperate" with new politicians at a later date. It may also condition on the group's ideal point. However, our results require only that we examine the simplest such class of equilibria, in which P and G_t play a "trigger" strategy against M. Such equilibria are characterized by the triple $(\mathcal{E}, \mathcal{E}_0, n)$, the elements of which correspond to a *cooperative* phase, *punishment* phase, and punishment length, respectively. Play begins in the cooperative phase at t = 1, and upon any deviation by M from its prescribed voting strategy, play continues in the punishment phase for $n \geq 1$ periods. During this phase, G_t offers a contract of the form in (1), P chooses g_t in each period, and M never re-elects P. After the punishment phase, play reverts to the cooperative phase. Subgame perfection requires that the game play within the punishment phase itself be a subgame perfect Nash equilibrium, and that the strategies in the cooperative phase be consistent with the incentives posed by the punishment phase.

Cooperative phase strategies may take many forms, and for simplicity we focus on strategies in \mathcal{E} that meet two criteria. First, they must be stationary.¹² Second, they cannot result in the punishment phase along the equilibrium path (though the punishment phase may be

¹¹See Dutta (1995) for a more general treatment of equilibria in dynamic games.

¹²Note that the punishment phase strategies are stationary as well.

essential for inducing M to play according to \mathcal{E}). Clearly, then, the punishment phase cannot be reached in equilibrium if the cooperative phase strategies yield a policy at least as good as g_t in each period, and n is sufficiently large. We call strategies satisfying these requirements stationary trigger strategies.

Even with its restrictions, many stationary trigger equilibria exist. The simplest example might be a "grim trigger" equilibrium $(\mathcal{E}_s, \mathcal{E}_0, \infty)$, where play proceeds according to the stationary equilibrium of the previous section. Another, more plausible, example might be $(\mathcal{E}'_c, \mathcal{E}_0, \infty)$, where \mathcal{E}'_c specifies that M votes according to the "myopic" contracting game (2). M has no incentive to deviate from its specified voting strategy, because doing so will result in receiving g_t forever, a result which is weakly dominated by that under \mathcal{E} . Clearly, then, there is room for discrimination amongst types even in a very rudimentary non-stationary setup.

As both of these examples illustrate, M's minmax payoff is simply that implied by \mathcal{E}_0 ; *i.e.*, receiving a policy at g_t in every period. Moreover, M's vote is the last action in each period, and does not affect her payoffs in that period.¹³ However, M may still exploit P by replacing a "bad" incumbent after she chooses a policy in expectation of re-election. Thus, to establish a trigger equilibrium it will be important to check that M does not have sufficient incentive to do so.

The following result uses these observations to characterize the optimal equilibrium for M, which restores the contracting outcome \mathcal{E}_c as the predicted result.¹⁴

Proposition 3 For *n* sufficiently large, in the optimal stationary trigger strategy equilibrium, $\mathcal{E} = \mathcal{E}_c$.

As intuition might suggest, sufficiently long (and possibly infinite) punishment periods will ensure that M does not exploit weak incumbents. Because there is no noise in the observables, the punishment phase is never invoked in equilibrium. Thus, the simple non-

¹³The latter point implies that there is no analog to "suckering" one's opponent in the prisoner's dilemma. ¹⁴The proof is easily modified to accommodate non-optimal stationary trigger equilibria as well.

stationary equilibrium derived in Proposition 3 produces policies that are identical in every period to those of the contracting equilibrium of Proposition 1.

6. Extensions

6.1 Term Limits

We now extend our results to the case in which legislators may serve no more than T > 0 terms. The model is unchanged with the exception that if, in period t, an incumbent completes her T-th period of office, she is automatically replaced with the challenger C_t . Note that at $T = \infty$, the model is identical to that in the previous sections. We continue to assume that a legislator who leaves office cannot return as a candidate.

The main intuition of our results is that an incumbent's continuation value will depend not only on her value of holding office, but also on the number of possible terms remaining. Thus the extent to which M can control politicians will also depend on both variables. Let $\theta \in \{1, \ldots, T\}$ denote the term that the legislator is currently serving.

We begin with the optimal contracting equilibria, which we label \mathcal{E}_T . Let $v_T(w, \theta)$ represent the expected value to M of a type-w legislator with θ terms of experience. As in Section 3, M has two choices at the contracting stage. First, she may extract the optimal performance given P's expected lifetime and re-elect the incumbent, who will be of type $(w, \theta+1)$ in the subsequent period. Second, M may allow G_t to buy its ideal policy and elect a type- $(w_t, 1)$ challenger, where w_t is randomly drawn as before.

To reflect the possible dependence of strategies on θ , we extend the notation of Section 3 as follows. Let $l_{w,\theta}$ denote the discounted expected payoffs of a type- (w,θ) incumbent prior to the draw of g_t . Next, let $\tilde{x}(w,\theta,g_t)$ represent the optimal policy that M can induce through its re-election contract (of the form in (3)) in a given period:

$$\tilde{x}(w,\theta,g_t) = \begin{cases} \underline{g}_t(-w-\delta l_{w,\theta+1}) & \text{if } m < \underline{g}_t(-w-\delta l_{w,\theta+1}) \\ m & \text{if } m \in [\underline{g}_t(-w-\delta l_{w,\theta+1}), \overline{g}_t(-w-\delta l_{w,\theta+1})] \\ \overline{g}_t(-w-\delta l_{w,\theta+1}) & \text{if } m > \overline{g}_t(-w-\delta l_{w,\theta+1}). \end{cases}$$
(7)

Thus, the optimal contract re-elects P if:

$$u^{M}(\tilde{x}(w,\theta,g_{t})) + \delta v_{T}(w,\theta+1) \ge u^{M}(g_{t}) + \delta \int_{\Omega} v_{T}(\tilde{w},1) f_{w}(\tilde{w}) d\tilde{w}.$$
(8)

Finite term limits allow us to pin down the expected payoff from an incumbent in her final term ($\theta = T$). Since she cannot be re-elected, no voting contract can induce any performance. By offering a contract of the form in (1), G_t can then obtain its optimal policy (*i.e.*, $\tilde{x}(w, T, g_t) = g_t$). Thus,

$$v_T(w,T) = \int_{\Gamma} u^M(g) f_g(g) dg + \delta \int_{\Omega} v_T(\tilde{w},1) f_w(\tilde{w}) d\tilde{w}.$$
(9)

Expression (9) additionally implies that legislators in their penultimate terms will also be difficult to control. With a single possible period of office remaining at period t, M can extract some performance by promising a single re-election. However, doing so will result in a period t+1 policy of g_{t+1} , followed by the election of challenger C_{t+1} . M may therefore prefer C_t to P.

This logic suggests that M has a greater ability to use future elections to discipline all types of incumbents earlier in their careers. Loosely speaking, low values of θ have an effect similar to that of high values of w.¹⁵ A number of easily established results follow. Term limits increase M's desire for policy performance as θ increases. Analogously with Proposition 1, when $u^{M}(\cdot)$ is concave a higher θ requires a worse draw of g_t to keep P in office. Term limits also reduce the expected quality of policies from M's perspective. In an environment with interest groups and voting contracts, voters therefore cannot benefit from finite term limits.¹⁶

Since incumbents become less desirable with seniority, our model predicts that re-election rates should change with θ . Previous models of term limits that do not incorporate citizens' type-selection incentives might therefore underestimate the impact of term limits on incum-

¹⁵This statement is proved formally in the Lemma of the proof of Proposition 4.

¹⁶There may be a counter-vailing force in favor of term limits if "power" in office rises with seniority, as in McKelvey and Riezman (1992). Additionally, in the model of Smart and Sturm (2006), term limits can help voters, but no interest groups are present.

bent turnover rates. The next result shows that an incumbent's *ex ante* (*i.e.*, prior to the draw of G_t) re-election probability decreases weakly over time.¹⁷

Proposition 4 The ex ante re-election probability of any incumbent is non-increasing in θ .

Because type-(w, T) incumbents have an *ex ante* re-election probability of zero, Proposition 4 implies that unless incumbents are never re-elected (*e.g.*, if $w_t = 0$ or $g_t = m$ for all t, or T = 1), there exist incumbent types for which the *ex ante* re-election probability is strictly decreasing over time. The result also compares usefully with Proposition 1, which established that some types $(w > \hat{w})$ are always retained. With finite term limits, a type- (w, θ) incumbent might be retained with certainty for sufficiently high w and low θ , but her *ex ante* re-election probability declines to zero eventually.

Our final result eliminates the assumption of voting contracts and is therefore an analog to Proposition 3. We consider the same class of stationary trigger strategies as in the previous section. As with the non-term limited case, the result is that simple non-stationary strategies (such as $(\mathcal{E}_T, \mathcal{E}_0, \infty)$) are sufficient to support strategy profiles in \mathcal{E}_T .

Proposition 5 For T finite and n sufficiently large, in the optimal stationary trigger strategy equilibrium, $\mathcal{E} = \mathcal{E}_T$.

6.2 Infinitely-Lived Groups

We now consider the effect of a single group, G, that is present in all periods.¹⁸ Shortterm groups may be an appropriate assumption for environments such as transportation or agriculture in the U.S., for which policies are determined periodically through multi-year funding laws. In such settings, both the groups and the politicians they influence may not

¹⁷For a model of term-limited elections that predicts constant re-election rates, see Lopez (2002). Consistent with our prediction, Smart and Sturm (2003) estimate that U.S. governors running in states with term limits are less likely to be re-elected than governors in states without term limits.

¹⁸For a treatment of dynamic interaction of long-lived interests and politicians in the regulatory context, see Salant (1995), Martimort (1999), and Faure-Grimaud and Martimort (2003).

be present for a subsequent round of policy-making. They are less realistic for issues such as defense, for which spending is determined on an annual basis. Short-lived groups simplify the analysis because they do not care about incumbents' electoral fates. Under repeated interaction, however, the group also participates in the retention of incumbents.

To see how an infinitely-lived group may affect voting and bribing strategies, suppose that a voter is able to offer optimal contracts, as in \mathcal{E}_c . Under its (myopic) bribing strategy, G must accept a relatively unfavorable policy in all periods once M finds a sufficiently good politician type. G does better in periods when M chooses not to re-elect the incumbent, as the policy will be her ideal point, g. It is clear, then, that G has an incentive to "buy out" high incumbent types in order to induce a fresh draw of politicians. In turn, M will have an incentive to loosen its contract to prevent G from forcing such replacements. The optimal contract will then allow policies closer to G's ideal and hence lower M's expected payoff.

The next result illustrates this logic using a simplified version of the model. Suppose that in the basic game the group ideal points $g_t > 0$ are constant over time. We compare this against a world in which there is a single infinitely-lived group with the same ideal point, g. There are two candidate types; each candidate is of type \underline{w} with probability $p \in (0, 1)$ and type \overline{w} with probability 1 - p, where $g^2(1 - \delta) > \overline{w} > \underline{w}$. Additionally, let $u^M(x) = -x^2$ and $u^G(x) = -(g - x)^2$. Let $v_c^{\infty}(w)$ denote M's expected payoff under an infinitely-lived group, given an incumbent of type w. We can then show that, when only type- \overline{w} incumbents are retained in the equilibrium of both games, M does strictly worse under both incumbent types when interest group is infinitely-lived.

Proposition 6 If type- \underline{w} incumbents are removed and type- \overline{w} incumbents are retained in the optimal contracting equilibrium under both short-lived and infinitely-lived groups, then $v_c^{\infty}(w) < v_c(w)$ for all w.

The assumed retention rule captures the spirit of games with larger type spaces, where some incumbents are always eliminated. As intuition would suggest, it is the optimal equilibrium contract for M in both games whenever $\overline{w} - \underline{w}$ is sufficiently large. The restriction to this particular equilibrium retention rule simplifies the comparison of M's expected payoffs by inducing the same distribution of incumbent types in both games. Since a long-lived group produces a worse policy for M under a type- \overline{w} incumbent and the same policy under a type- \underline{w} incumbent, and M's retention rules are identical, M must then receive a strictly lower expected payoff under both types.

6.3 Two Interest Groups

The final extension considers what happens under pluralistic interest group competition. Suppose that two groups are drawn in each period, who simultaneously offer voting contracts to P. The groups are labeled G^i (i = 1, 2), with ideal points g^i $(g^1 < g^2)$, utility functions $u^{G^i}(\cdot)$, and upper contour sets $\mathcal{G}^i(\cdot)$. As in the basic model, each group "lives" for a single period. Now each group's bribes must take into account not only the cost of the politician's expected remaining payoff from office, but also any bribes from the other group as well. To simplify the exposition, we drop time subscripts throughout.

The effect of an additional group is again usefully illustrated by supposing that a voter offers P optimal contracts according to \mathcal{E}_c . In the presence of only G^1 and an incumbent of type w, this induces some policy x' between m and g^1 . But if $x' \notin \mathcal{G}^2(-\frac{w}{1-\delta})$, then G^2 will have an incentive to "buy out" P. This upsets the bribing strategy of G^1 , and raises the possibility that G^1 might in turn be willing to pay more to buy back P. M may also have an incentive to adjust its contract to prevent policy from being drawn too far away from m.

The fully-developed model of competitive vote-buying is quite complex, so the following result simply characterizes two general features. If both groups are sufficiently close to m, or if they are on opposite sides of m, then the voter can achieve her ideal policy with simple stationary voting contracts.

Proposition 7 In an optimal contracting equilibrium, $x^* = m$ and P is re-elected each period if for any g^1 , g^2 and all $w \in \Omega$, either:

(i) $m \in \mathcal{G}^1(-\frac{w}{1-\delta}) \cap \mathcal{G}^2(-\frac{w}{1-\delta})$, or (ii) $m \notin \mathcal{G}^1(-\frac{w}{1-\delta}) \cap \mathcal{G}^2(-\frac{w}{1-\delta})$, $m \in (g^1, g^2)$ and $u^{G^i}(\cdot) = u^G(\cdot)$ is concave and symmetric.

The result can be extended straightforwardly to show that under similar conditions, competition between groups would allow the voter to receive policy at m with term-limited politicians as well. Part (ii) of the proposition suggests that opposing groups can play a significant role in moderating policies and maintaining political careers. Compared to a world with a single interest group, the voter receives better policy, and also uses a less complex monitoring strategy. Note that this does not require that the groups' ideal points are distributed symmetrically about the voter – they just have to lie on opposite sides of the voter. Because of her strategic behavior, the voter can also benefit relative to a world in which interest groups can offer menu auctions to the politician (Grossman and Helpman, 1994). The effect on incumbent retention is even more pronounced. Even a low-w incumbent can remain in office forever if the groups' ideal points lie on opposite sides of m. Incumbent politicians should therefore have a preference for "pluralism." This may be one factor contributing to an apparent preference for procedural openness in many modern legislatures, since one way to promote a pluralistic environment is to ensure that a variety of groups, with differing views, are allowed to lobby the legislature.

Extreme policies may still result if both groups are on the same side of m, and at least one group is not close. It seems safe to conjecture that the outcome will never be more extreme than the most extreme group. However, since the most extreme group will tend to be more extreme as the number of groups increases, the overall effect of adding groups on the same side of m is unclear. We leave this for future work.

7. Discussion

The models developed here integrate strategic interest groups and strategic voters in a general framework of policy-making and elections. This combination introduces a tension in a voter's incentives, because there may be a strong short-run incentive to use electoral discipline to improve policy, and because a group can exploit a promise to retain a politician. As a result, the strategies of selecting good politician types and rewarding good performance are often incompatible.

The results illustrate the effects of this tension under a variety of assumptions. The models generally predict that incumbents who value office highly are difficult to discipline. Incumbents who value office less highly can be disciplined as the interest group becomes more extreme, because the short-term gain from policy performance is greater. This causes re-election rates to increase when adopted policies are more extreme. We also find that a long-lived interest group weakens a voter's electoral control, while multiple groups may strengthen it.

The results also highlight the need for constant vigilance by voters or activist groups in the presence of vote-buying interest groups. In a world in which politicians actually have policy preferences, voters can simply find a "good" politician and let her act according to her preferences. However, some theorists argue that, at least in the long run, office-motivated politicians will drive out policy-motivated politicians (*e.g.*, Calvert, 1986). An alternative rationale for assuming that politicians nonetheless act as if they are policy motivated is to assert that they have "induced" policy preferences – induced by some underlying contract with a set of interest groups. We show that there is a key difference between true policy preferences and preferences that are induced by payments from interest groups. In the latter environment, voters must *always* monitor politicians in order to prevent bad policy outcomes. By employing a simple, non-stationary voting strategy, the disciplining of incumbents can be accomplished alongside the selection of better politician types.

The extension to term-limited officials predicts that term limits have an effect similar to a reduction in the value of holding office. Thus, term limits reduce policy performance from a voter's perspective. In turn, they give voters a greater incentive to replace officials as their limit approaches. Our model cannot explain the apparently high level of support for term limits among citizens.¹⁹ While it is predicted that most voters should oppose them, in

¹⁹The model's predictions may be consistent with the general direction of support for term limits. Conservative and Republican voters are more supportive of term limits than liberal and Democratic voters. The policy preferences of conservative voters are probably more closely aligned with organized interest groups,

survey and initiative voting data voters typically support term limits by a 2 to 1 margin.

since most well-organized interest groups have a pro-business, conservative orientation (e.g., Schlozman and Tierney, 1986). Term limits are therefore likely to move policy in their direction.

Appendix

Proof of Proposition 1. For convenience we use w to denote the type of a generic incumbent. By stationarity, we also drop all time subscripts. Throughout the proof, we let $\omega(\{\cdot\})$ and $\gamma(\{\cdot\})$ denote the probability mass of sets of incumbent types and group ideal points associated with densities f_w and f_g , respectively.

We first define an extended value function $\tilde{v}_c : \Re_+ \to \Re$, which satisfies: (i) $\tilde{v}_c(w) = v_c(w)$ for any $w \in \Omega$, and (ii) $\tilde{v}_c(w) = \sum_{t=1}^{\infty} \delta^{t-1} \mathbb{E}[u^M(x^* \mid w_0 = w)]$ for any $w \notin \Omega$. That is, \tilde{v}_c coincides with v_c on Ω , and equals M's expected payoff in a fictional game with a type-wincumbent and challengers drawn according to density f_w otherwise. The following lemma will be useful for deriving the result.

Lemma 1 $\tilde{v}_c(w)$ is non-decreasing in w.

Proof. We show that for any two incumbent types, w' and w'', where w' < w'', there exists a voting strategy $\tilde{\rho}$ that implements the same policy and re-election outcome for both types.

For any optimal re-election strategy $\rho^*(w, g, x)$ we partition X into two disjoint (possibly empty) subsets; $\mathcal{P}(w, g) = \{x \in X \mid \rho^*(w, g, x) = 1\}$ and $\mathcal{C}(w, g) = \{x \in X \mid \rho^*(w, g, x) = 0\}$. It is then clear that G's optimal contract must be of one of two types: (i) offer ϵ for choosing $\tilde{p} = \arg \max_{x \in \mathcal{P}(w,g)} u^G(x)$ and 0 otherwise; or (ii) offer $w + \delta l_w + \epsilon$ for choosing $\tilde{c} = \arg \max_{x \in \mathcal{C}(w,g)} u^G(x)$ and 0 otherwise. G then offers the type (i) contract if:

$$u^G(\tilde{p}) > u^G(\tilde{c}) - w - \delta l_w.$$
⁽¹⁰⁾

Because P can assure herself of re-election whenever $\mathcal{P}(w,g) \neq \emptyset$, letting $\epsilon \to 0$ it is clear that $l_w = \frac{\xi w}{1-\xi\delta}$, where $\xi = \gamma(\{g \in \Gamma \mid \mathcal{P}(w,g) \neq \emptyset\})$.

Let $x_{w,g}^*$ denote the equilibrium policy with a type-w incumbent and group ideal point g. There are two cases. First, if w' and g are such that $x_{w',g}^* \in \mathcal{C}(w',g)$, then we claim $x_{w',g}^* = g$. To show this, note that $g \in \mathcal{C}(w',g)$ implies (by the definition of \tilde{c}) $x_{w',g}^* = g$ automatically. Now suppose $g \in \mathcal{P}(w',g)$. Then by (10), G should offer a type (i) contract: contradiction. To implement the same outcome with a type-w'' incumbent, M can choose $\tilde{\rho}(w'', g, x) = 0$ for all x. G then offers a contract promising ϵ for choosing policy g and 0 otherwise. Thus P chooses g and is not re-elected.

Second, if w' and g are such that $x_{w',g}^* \in \mathcal{P}(w',g)$, then to implement the desired outcome with a type-w'' incumbent, M can choose: $\tilde{\rho}(w'',g,x) = 1$ if $x = x_{w',g}^*$, and $\tilde{\rho}(w'',g,x) =$ 0 otherwise. To show that policy $x_{w',g}^*$ is chosen, note that $\tilde{\rho}(w'',g,x)$ implies that reelection is feasible for any group type in $\{g \in \Gamma \mid \mathcal{P}(w',g) \neq \emptyset\}$; hence $l(w''; \{\tilde{\rho}(w'',g,x)\}) >$ $l(w'; \{\rho^*(w',g,x)\})$. Further, $x_{w',g}^* \in \mathcal{P}(w',g)$ implies that (10) holds. This implies that for all $x \neq x_{w',g}^*$, $u^G(x_{w',g}^*) > u^G(x) - w - \delta l(w''; \{\tilde{\rho}(w'',g,x)\})$. Hence, G offers a type (i) contract $\beta^*(w',g)$ identical to that offered to a type-w' incumbent. P then receives $\epsilon + w'' + \delta l(w''; \{\tilde{\rho}(w'',g,x)\})$ for choosing policy $x_{w',g}^*$ and 0 otherwise. Thus she chooses $x_{w',g}^*$ and is re-elected.

Now consider any optimal stationary contracting equilibrium. Define the "average" type as: $\hat{w} = \sup \{ w \in \Omega \mid v_c(w) \leq \int_{\Omega} v_c(\tilde{w}) f_w(\tilde{w}) d\tilde{w} \}$. We use this notation to rewrite (5), which characterizes the re-election choice, as follows. Denote by $\tilde{x}(w,g)$ the optimal policy that M can induce from P given $v_c(w)$, following the approach in (4) (*i.e.*, the optimal policy in $\mathcal{G}(-w-\delta l_w)$). Then M re-elects P of type w if:

$$u^{M}(\tilde{x}(w,g)) + \delta v_{c}(w) \ge u^{M}(g) + \delta \int_{\Omega} v_{c}(\tilde{w}) f_{w}(\tilde{w}) d\tilde{w}.$$
(11)

By Lemma 1 and the fact that $u^M(\tilde{x}(w,g)) \ge u^M(g)$ holds trivially, (11) holds for all g if $w > \hat{w}$.

To characterize w^2 , rearranging (11) we have that P is not re-elected if:

$$u^{M}(\tilde{x}(w,g)) - u^{M}(g) < \delta\left(\int_{\Omega} v_{c}(\tilde{w}) f_{w}(\tilde{w}) d\tilde{w} - v_{c}(w)\right).$$

$$(12)$$

Denote by \mathcal{D}_w the (possibly empty) set of group ideal points satisfying (12). Let $w^2 = \sup\{w \in \Omega \mid \exists g \in \Gamma \cap \mathcal{D}_w\}$, and 0 otherwise. Note that for any $w > \hat{w}$, the right-hand side of (12) is non-positive and thus (12) cannot be satisfied. Thus $w^2 \leq \hat{w}$.

To characterize w^1 , note that given w P is never re-elected if (12) holds for all g. Thus let $w^1 = \sup\{w \in \Omega \mid \Gamma \subseteq \mathcal{D}_w\}$, and 0 otherwise. Because this condition is stricter than that for w^2 , it is clear that $w^1 \leq w^2$. To show that $w^1 > 0$, consider w = 0. Clearly, $l_0 = 0$ for any ρ ; hence, $\tilde{x}(w,g) = g$ and the left-hand side of (12) is zero. Additionally, it is trivial to establish that $\int_{\Omega} v_c(\tilde{w}) f_w(\tilde{w}) d\tilde{w} > \tilde{v}_c(0)$. Therefore the right-hand side is positive and (12) holds at w = 0. Putting all of the derived inequalities together, we have $\hat{w} \geq w^2 \geq w^1 > 0$.

To characterize the re-election function η for $w \in [w^1, w^2]$, the next lemma will be useful.

Lemma 2 $u^M(\tilde{x}(w,g))$ is non-decreasing in w.

Proof. Suppose otherwise; *i.e.*, $u^M(\tilde{x}(w',g)) > u^M(\tilde{x}(w'',g))$ for some g and types w' and w'', where w' < w''. By (4), this is possible only if:

$$w' + \delta l_{w'} > w'' + \delta l_{w''}.$$
(13)

We begin with two observations that follow from (13): (i) by (4), we have $u^{M}(\tilde{x}(w',g)) \geq u^{M}(\tilde{x}(w'',g))$ for all g; (ii) because w' < w'', we have $l_{w'} > l_{w''}$, and hence $\gamma(\Gamma_{w'}) > \gamma(\Gamma_{w''})$, where $\Gamma_{w} = \{g' \in \Gamma \mid (11) \text{ holds for type } w\}$ is the set of group ideal points such that a type-w incumbent is re-elected in equilibrium.

Partition Γ into $\Gamma^1 = \Gamma_{w'} \cap (\Gamma \setminus \Gamma_{w''})$ and $\Gamma^2 = \Gamma \setminus \Gamma^1$. Note that by observation (ii), $\gamma(\Gamma^1) > 0$. We now consider the behavior of $\tilde{v}_c(w)$ over Γ^1 and Γ^2 . Let $\hat{v}_c(w, g')$ represent M's discounted expected payoff conditional on a type-*w* incumbent and group ideal point g'.

First, for $g' \in \Gamma^1$, the definition of Γ^1 and (11) imply:

$$u^{M}(\tilde{x}(w',g')) + \delta v_{c}(w') \ge u^{M}(g') + \delta \int_{\Omega} v_{c}(\tilde{w}) f_{w}(\tilde{w}) d\tilde{w} > u^{M}(\tilde{x}(w'',g')) + \delta v_{c}(w'').$$
(14)

Noting that for any w, $\hat{v}_c(w, g') = \max\{u^M(\tilde{x}(w, g')) + \delta \tilde{v}_c(w), u^M(g') + \delta \int_{\Omega} v_c(\tilde{w}) f_w(\tilde{w}) d\tilde{w}\},\$ it is clear that:

$$v_c(w')_{|g'\in\Gamma^1} = \frac{\int_{\Gamma^1} \hat{v}_c(w',\tilde{g}) f_g(\tilde{g}) d\tilde{g}}{\gamma(\Gamma^1)} > v_c(w'')_{|g'\in\Gamma^1} = \frac{\int_{\Gamma^1} \hat{v}_c(w'',\tilde{g}) f_g(\tilde{g}) d\tilde{g}}{\gamma(\Gamma^1)}$$

Second, for $g' \in \Gamma^2$, we show that with a type-w' incumbent M can guarantee an outcome identical to that with a type-w'' incumbent. For any $g' \in \Gamma^2 \setminus \Gamma_{w''}$ (*i.e.*, where neither incumbent is re-elected), M may offer $\rho(w', g', x) = 0$ for all x. This contract clearly results in no re-election and x = g, which is also the outcome for a type-w'' incumbent. And finally for any $g' \in \Gamma_{w''}$, M may offer $\rho(w', g', x) = 1$ for $x = \tilde{x}(w'', g')$ and $\rho(w', g', x) = 0$ otherwise. To show that $\tilde{x}(w'', g')$ is chosen by P, note that $\tilde{x}(w'', g') = \arg \max_{x \mid \rho^*(w'', g', x) = 1} u^G(x)$. Thus, $u^G(\tilde{x}(w'', g')) > u^G(g') - w'' - \delta l_{w''}$. By (13), this implies $u^G(\tilde{x}(w'', g')) > u^G(g') - w' - \delta l_{w'}$, and hence G does not offer a payment to choose any policy other than $\tilde{x}(w'', g')$. By stationarity, M can therefore achieve policy $\tilde{x}(w'', g')$ with a type-w' incumbent for $g' \in \Gamma^2$ in any period, and thus $v_c(w')_{|g'\in\Gamma^2} \geq v_c(w'')_{|g'\in\Gamma^2}$.

Finally, note that for any w, $v_c(w) = \gamma(\Gamma^1)v_c(w)_{|g'\in\Gamma^1} + \gamma(\Gamma^2)v_c(w)_{|g'\in\Gamma^2}$. Combining results, we have $v_c(w') > v_c(w'')$, contradicting Lemma 1.

Now by the definition of \mathcal{D}_w , realized re-election choices are given by: $\eta(w,g) = 1(g \notin \mathcal{D}_w)$. Two comparative statics follow. First, Lemmas 1 and 2 implies that the left-hand side of (11) is non-decreasing in w. Thus, given g and w'' > w', if (11) holds for a type-w' incumbent, then it also holds for a type-w'' incumbent. Therefore, $\eta(w,g)$ is non-decreasing in w.

Second, by (4), $|\tilde{x}(w,g) - g|$ is non-decreasing in |m - g|. Concavity and symmetry of u^M then implies that $u^M(\tilde{x}(w,g)) - u^M(g)$ is increasing in |m - g|. It follows that for any g', g'' such that $|m - g''| > |m - g'|, g' \notin \mathcal{D}_w$ implies $g'' \notin \mathcal{D}_w$. Therefore, $\eta(w,g)$ is non-decreasing in g if u^M is concave and symmetric.

Proof of Comment 1. Suppose otherwise. Let $\overline{v}_s \equiv \int_{\Omega} v_s(\tilde{w}) f_w(\tilde{w}) d\tilde{w}$ represent the discounted expected payoff from electing a challenger. Then for some $w' \in \Omega$, either (i) $v_s(w') > \overline{v}_s$, or (ii) $v_s(w') < \overline{v}_s$. Suppose that (i) holds. By (6), $\rho_t^*(w', g_t, b_t, x_t) = 1$ for all g_t , b_t , and x_t . Given this voting strategy, G_t 's best response is: $\beta_t^*(w', g_t) = b_t(x_t)$, where $b_t(x_t) = \epsilon$ if $x_t = g_t$, and $b_t(x_t) = 0$ otherwise, for some $\epsilon > 0$ (*i.e.*, a contract of the form in (1)). Clearly, this induces policy choice $\chi_t^*(w', g_t, b_t) = g_t$. Note that retaining a type-w' politician in each period implies $v_s(w') = \frac{1}{1-\delta} \int_{\Gamma} u^M(\tilde{g}) f_g(\tilde{g}) d\tilde{g}$. But in any period t, M can achieve at least $u^M(g_t)$ by adopting strategy $\rho_t^*(\cdot) = 1$ for all g_t , b_t , and x_t ; thus, $\overline{v}_s \geq \frac{1}{1-\delta} \int_{\Gamma} u^M(\tilde{g}) f_g(\tilde{g}) d\tilde{g}$. Hence, $\overline{v}_s \geq v_s(w')$: contradiction.

Now suppose that (ii) holds. Let $\underline{\mathcal{W}} \equiv \{\tilde{w} \mid v_s(\tilde{w}) < \overline{v}_s\}$ denote the set of "below average" types, and $\omega(\underline{\mathcal{W}})$ the probability mass associated with it under f_w . If $\omega(\underline{\mathcal{W}}) > 0$, then there exists a non-empty set of types $\{\tilde{w}' \mid v_s(\tilde{w}') > \overline{v}_s\}$. By part (i), this set of types is empty: contradiction. We conclude that $\omega(\underline{\mathcal{W}}) = 0$.

Proof of Proposition 2. We first characterize a stationary equilibrium strategy profile, and then show that the policy outcomes cannot be improved upon. For M, since P's policy choice is constant with respect to the incumbent type w, she is indifferent amongst all candidates. Thus, any voting strategy ρ_t is optimal. We fix ρ_t^* as requiring re-election if and only if x_t is at least as good the optimal policy for M within $\mathcal{G}_t(-\frac{w}{1-\delta})$:

$$\rho_t^*(w, g_t, b_t, x_t) = \begin{cases} 1 & \text{if } m \notin \mathcal{G}_t(-\frac{w}{1-\delta}) \text{ and } u^M(x_t) \ge u^M(\hat{x}_t), \\ & \text{or } m \in \mathcal{G}_t(-\frac{w}{1-\delta}) \text{ and } x_t = m \\ 0 & \text{otherwise,} \end{cases}$$

where $\hat{x}_t = \arg \max_{x_t \in \mathcal{G}_t(-\frac{w}{1-\delta})} u^M(x_t)$. If $m \in \mathcal{G}_t(-\frac{w}{1-\delta})$, P must choose m; otherwise, she must choose \hat{x}_t , which by the single-peakedness of u^M is either $\underline{g}_t(-\frac{w}{1-\delta})$ or $\overline{g}_t(-\frac{w}{1-\delta})$.

Now consider G_t 's incentives. Given ρ_t^* and the specified policy choices, if $m \in \mathcal{G}_t(-\frac{w}{1-\delta})$, then G_t must offer at least $\frac{w}{1-\delta}$ to induce P to choose any policy other than m. But since $u^{G_t}(g_t) - u^{G_t}(m) < \frac{w}{1-\delta}$ by construction of $\mathcal{G}_t(\cdot)$, she is unwilling to do so and thus optimally offers $\beta^*(w, g_t) = b_t(x_t) = 0$. If $m \notin \mathcal{G}_t(-\frac{w}{1-\delta})$, then for all $x_t \in \mathcal{G}_t(-\frac{w}{1-\delta}) \setminus \{\hat{x}_t\}$, P is not re-elected and $u^{G_t}(g_t) - u^{G_t}(m) \leq \frac{w}{1-\delta}$, so G_t does not offer $\frac{w}{1-\delta}$ for such x_t . Clearly, $\hat{x}_t = \arg \max_{x_t \in \hat{x}_t \cup X \setminus \mathcal{G}_t(\cdot)} u^{G_t}(x_t)$, and P chooses \hat{x}_t if $b_t(\hat{x}_t) > b_t(x_t)$ for all $x_t \neq \hat{x}_t$. Thus G_t optimally offers:

$$\beta^*(w, g_t) = b_t(x_t) = \begin{cases} \epsilon & \text{if } x_t = \hat{x}_t \\ 0 & \text{otherwise} \end{cases}$$

for some $\epsilon > 0$. Letting $\epsilon \to 0$, we obtain the optimal contract.

Next, given ρ_t^* and β^* , if $m \in \mathcal{G}_t(-\frac{w}{1-\delta})$ P receives $\frac{w}{1-\delta} + \epsilon$ for choosing m and zero otherwise. If $m \notin \mathcal{G}_t(-\frac{w}{1-\delta})$, then P receives $\frac{w}{1-\delta} + \epsilon$ for choosing \hat{x}_t and either zero or $\frac{w}{1-\delta}$ otherwise. Thus her strategy is χ_t^* as specified. We claim that this strategy profile belongs in \mathcal{E}_s .

To show that these policy choices must hold in any optimal equilibrium, suppose that there exists another stationary equilibrium \mathcal{E}'_s with different policy choices and weakly higher payoffs for M. Recall that by Comment 1, $v_s(w)$ is constant with respect to w. In the preceding equilibrium, $x_t^* = m$ whenever $m \in \mathcal{G}_t(-\frac{w}{1-\delta})$. Therefore in \mathcal{E}'_s for a type-w incumbent there exists some g_t such that $m \notin \mathcal{G}_t(-\frac{w}{1-\delta})$, $\chi_t^*(w, g_t, b_t) > \overline{g}_t(-\frac{w}{1-\delta})$ ($\leq \underline{g}_t(-\frac{w}{1-\delta})$) for $m > (<) g_t$. Assume without loss of generality that $g_t > m$. Since $\chi_t^*(w, g_t, b_t) \notin \mathcal{G}_t(-\frac{w}{1-\delta})$, $u^{G_t}(\chi_t^*(w, g_t, b_t)) < -\frac{w}{1-\delta}$. Let G_t replace its equilibrium contract with: $b_t(x_t) = \frac{w}{1-\delta} + \epsilon$ if $x_t = g_t$, and zero otherwise. Then, letting $\epsilon \to 0$, P's optimal policy choice is g_t , and G_t receives $-\frac{w}{1-\delta}$. Thus, $\chi_t^*(w, g_t, b_t)$ cannot be an equilibrium policy: contradiction.

Proof of Proposition 3. We proceed in two steps. First, we establish the optimality of \mathcal{E}_c as a cooperative phase. Suppose otherwise; *i.e.*, that M attains higher discounted expected utility under another cooperative phase strategy profile. Denote this profile $\hat{\mathcal{E}}$, and let $\hat{\rho}_t(\cdot)$, $\hat{\beta}_t(\cdot)$, and $\hat{\chi}_t(\cdot)$ denote the associated cooperative phase strategies for M, G_t , and P, respectively. Suppose that in each period t of the contracting game M offers a contract identical to the cooperative phase voting strategy; $\rho_c(\cdot) = \hat{\rho}_1(h_1, \cdot)$. This contract strategy is clearly stationary. It is straightforward to verify that since $\hat{\beta}_t(\cdot)$ and $\hat{\chi}_t(\cdot)$ are best responses to each other and to $\hat{\rho}_t(\cdot)$ in a cooperative phase, they are also best responses to $\rho_c(\cdot)$ if the continuation payoffs for all players are identical in both games. By definition, $\hat{\beta}_t(\cdot)$ and $\hat{\chi}_t(\cdot)$ cannot induce a punishment phase, and thus they induce the same continuation games under $\rho_c(\cdot)$ and $\hat{\rho}_t(\cdot)$. Therefore, the policies realized under the stationary contract $\rho_c(\cdot)$ are identical to those under $\hat{\mathcal{E}}$ in all periods. This contradicts the optimality of \mathcal{E}_c in the contracting game.

Second, we show that trigger strategies of the form $(\mathcal{E}_c, \mathcal{E}_0, n)$ constitute a subgame perfect equilibrium for sufficiently large n. In the punishment phase, no action can affect the duration of the phase. For any period t in a punishment phase, given $\chi_t^*(\cdot) = g_t$, M is indifferent between electing and re-electing, and so chooses $\rho_t^*(\cdot) = 0$. Given $\rho_t^*(\cdot) = 0$, a bribing strategy $\beta_t^*(\cdot)$ as in (1), and $\chi_t^*(\cdot) = g_t$ are clearly optimal. Now consider the cooperative phase. Because $\beta_t^*(\cdot)$ and $\chi_t^*(\cdot)$ are best responses to $\rho_t^*(\cdot)$ in \mathcal{E}_c , we need check only that M casts re-election votes in accordance with \mathcal{E}_c . M will not re-elect an incumbent in equilibrium if:

$$\int_{\Omega} v_c(\tilde{w}) f_w(\tilde{w}) d\tilde{w} > \sum_{j=0}^{n-1} \delta^j \mathrm{E} u^M(g_t) + \delta^n \int_{\Omega} v_c(\tilde{w}) f_w(\tilde{w}) d\tilde{w}.$$
(15)

It is sufficient to show that $\int_{\Omega} v_c(\tilde{w}) f_w(\tilde{w}) d\tilde{w} > E u^m(g_t)/(1-\delta)$. To show this, note that for any w > 0, $u^M(\tilde{x}_t(w, g_t)) > u^M(g_t)$. M can therefore guarantee an expected payoff greater than $E u^m(g_t)/(1-\delta)$ with probability one under \mathcal{E}_c . Thus, there exists some n_1 (possibly infinite) such that (15) holds for all $n \ge n_1$.

Next, M re-elects an incumbent in equilibrium if:

$$v_c(w) \ge \sum_{j=0}^{n-1} \delta^j \mathbf{E} u^M(g_t) + \delta^n \int_{\Omega} v_c(\tilde{w}) f_w(\tilde{w}) d\tilde{w}.$$
 (16)

Clearly, under $\mathcal{E}_c, v_c(w) \geq \sum_{j=0}^{\infty} \delta^j \mathrm{E} u^M(g_t)$ for all $w \in \Omega$. Thus, there exists some n_2 (possibly infinite) such that (16) holds for all $n \geq n_2$. For all $n \geq \max\{n_1, n_2\}, \mathcal{E}_c$ is sustainable as the cooperative phase of a trigger equilibrium.

Proof of Proposition 4. Throughout the proof, we let $\omega(\{\cdot\})$ and $\gamma(\{\cdot\})$ denote the probability mass of sets of incumbent types and group ideal points associated with densities f_w and f_g , respectively. Let $\Gamma_{w,\theta} \equiv \{g \mid \rho^*(w,\theta,g,x^*_{w,\theta,g}) = 1\}$ denote the set of group ideal points such that a type- (w,θ) incumbent is re-elected. The *ex ante* re-election probability for a type- (w,θ) incumbent is then $\gamma(\Gamma_{w,\theta})$. We show that in an optimal equilibrium for M, $\Gamma_{w,\theta} \subseteq \Gamma_{w,\theta-1}$ for all θ $(1 < \theta \leq T)$.

We proceed via induction on $\{\Gamma_{w,T-\tau},\ldots,\Gamma_{w,T}\}$ $(\tau = 1,\ldots,T-1)$. Clearly, $\Gamma_{w,T} \subseteq \Gamma_{w,T-1}$, and so the desired relation holds for $\tau = 1$. Now suppose that $\Gamma_{w,\theta} \subseteq \Gamma_{w,\theta-1}$ for all θ between $T - \tau + 1$ and T, for some τ . We argue that this implies $\Gamma_{w,T-\tau} \subseteq \Gamma_{w,T-\tau-1}$, and thus $\Gamma_{w,\theta} \subseteq \Gamma_{w,\theta-1}$ for all θ between $T - \tau$ and T (*i.e.*, at $\tau + 1$). Observe first that if $\Gamma_{w,T-\tau} = \emptyset$, then $\Gamma_{w,T-\tau} \subseteq \Gamma_{w,T-\tau-1}$, thus establishing the result. Otherwise, by (8), for any $g \in \Gamma_{w,T-\tau}$, the induction hypothesis implies:

$$u^{M}(\tilde{x}(w,T-\tau,g)) + \delta v_{T}(w,T-\tau+1) \ge u^{M}(g) + \delta \int_{\Omega} v_{T}(\tilde{w},1) f_{w}(\tilde{w}) d\tilde{w}.$$
 (17)

To show that a type- $(w, T-\tau-1)$ incumbent is re-elected when G_t has ideal point g, it is sufficient to establish that $u^M(\tilde{x}(w, T-\tau-1, g)) + \delta v_T(w, T-\tau) \ge u^M(\tilde{x}(w, T-\tau, g)) + \delta v_T(w, T-\tau+1)$. To show that $v_T(w, T-\tau) \ge v_T(w, T-\tau+1)$, we use the following lemma.

Lemma 3 $v_T(w, \theta)$ is non-increasing in θ .

Proof. Suppose otherwise. Then there exist $\theta', \theta'' \in \{1, \ldots, T\}$ $(\theta' < \theta'')$ such that $v_T(w, \theta') < v_T(w, \theta'')$. Let $\rho^*(w, \theta, g_t, x)$ represent an optimal voting contract offered to a type- (w, θ) incumbent for choosing policy x when G_t has ideal point g_t . Let x^*_{w,θ,g_t} denote the equilibrium policy given w, θ , and g_t . Suppose that for all w, g_t, x , and $\tau = 0, \ldots, T - \theta''$, M offers the type- θ' incumbent the following contract at each period $t + \tau$:

$$\hat{\rho}(w,\theta'+\tau,g_t,x) = \begin{cases} 1 & \text{if } x = x_{w,\theta''+\tau,g_t}^* \text{ and } \rho^*(w,\theta''+\tau,g_t,x_{w,\theta''+\tau,g_t}^*) = 1\\ 0 & \text{otherwise.} \end{cases}$$
(18)

That is, M re-elects the type- $(w, \theta' + \tau)$ incumbent if and only if she chooses the equilibrium policy of a type- $(w, \theta'' + \tau)$ incumbent, when the type- $(w, \theta'' + \tau)$ incumbent is offered re-election according to her optimal contract.

It is clear that under $\hat{\rho}(\cdot)$, whenever a type- $(w, \theta'' + \tau)$ incumbent is not re-elected, the type- $(w, \theta' + \tau)$ incumbent is also not re-elected. In these cases G_t can write a contract of the form in (1), thus obtaining $x_t = g_t$.

Now consider cases in which a type- $(w, \theta''+\tau)$ incumbent is re-elected. By always choosing policy $x_{w,\theta''+\tau,g_t}^*$, the type- (w, θ') incumbent assures herself of a positive expected payoff. By not choosing x_{w,θ'',g_t}^* , P receives zero. G_t may therefore induce the type- (w, θ') incumbent to deviate to some policy $x^{\circ} \neq x_{w,\theta'',g_t}^*$ only by offering at least $w + \delta \hat{l}_{w,\theta'+1}$, where $\hat{l}_{w,\theta'+1} =$ $\sum_{i=\theta''+1}^{T} \delta^{i-\theta''-1} \gamma(\Gamma_{w,i}) w$ is the type- $(w, \theta'+1)$ incumbent's expected payoff under $\hat{\rho}(\cdot)$. This implies $u^{G_t}(x^{\circ}) - w - \delta \hat{l}_{w,\theta'+1} > u^{G_t}(x_{w,\theta'',g_t}^*)$. Now observe that the type- $(w, \theta''+1)$ incumbent's equilibrium expected lifetime payoff under $\rho^*(\cdot)$ is $l_{w,\theta''+1} = \sum_{i=\theta''+1}^{T} \delta^{i-\theta''-1} \gamma(\Gamma_{w,i}) w$. Since $\hat{l}_{w,\theta'+1} = l_{w,\theta''+1}$, G_t could have offered $w + \delta l_{w,\theta''+1}$ to the type- (w, θ'') incumbent to achieve x° . Thus x_{w,θ'',g_t}^* could not have been the type- (w, θ'') incumbent's equilibrium policy choice, a contradiction. Thus the contract $\hat{\rho}(\cdot)$ feasibly implements the policies $x_{w,\theta''+\tau,g_t}^*$ with a type- $(w, \theta'+\tau)$ incumbent for all $\tau = 0, \ldots, T - \theta''$. We conclude that in an optimal contracting equilibrium, a type- $(w, \theta' + \tau)$ incumbent can be induced to choose policies at least as good for M as those chosen by a type- $(w, \theta'' + \tau)$ incumbent. Thus, $v_T(w, \theta'') \leq v_T(w, \theta')$: contradiction.

Now it is sufficient to show that $u^M(\tilde{x}(w, T-\tau-1, g)) \ge u^M(\tilde{x}(w, T-\tau, g))$. By (7), this is true if $w + \delta l_{w,T-\tau} \ge w + \delta l_{w,T-\tau+1}$, or equivalently: $l_{w,T-\tau} \ge l_{w,T-\tau+1}$. Expanding terms, this is:

$$\sum_{k=0}^{\tau} \delta^k w \gamma(\Gamma_{w,k+T-\tau}) \ge \sum_{k=0}^{\tau-1} \delta^k w \gamma(\Gamma_{w,k+T-\tau+1}),$$
(19)

which holds if $\gamma(\Gamma_{w,k+T-\tau}) \geq \gamma(\Gamma_{w,k+T-\tau+1})$ over $0 \leq k \leq \tau-1$. This follows from the induction hypothesis. Therefore, $\Gamma_{w,T-\tau} \subseteq \Gamma_{w,T-\tau-1}$ and the induction hypothesis also holds at $\tau + 1$.

Proof of Proposition 5. Optimality follows from an analogous argument to that in the proof of Proposition 3.

We show that trigger strategies of the form $(\mathcal{E}_T, \mathcal{E}_0, n)$ constitute a subgame perfect equilibrium for sufficiently large n. In the punishment phase, no action can affect the duration of the phase. For any period t in a punishment phase, given $\chi_t^*(\cdot) = g_t$, M is indifferent between electing and re-electing, so M chooses $\rho_t^*(\cdot) = 0$. Given $\rho_t^*(\cdot) = 0$, a bribing strategy $\beta_t^*(\cdot)$ as in (1), and $\chi_t^*(\cdot) = g_t$ are clearly best responses.

Now consider the cooperative phase. Because $\beta_t(\cdot)$ and $\chi_t(\cdot)$ are best responses to $\{\rho_t(\cdot)\}$ in \mathcal{E}_T , we need check only that M casts re-election votes in accordance with \mathcal{E}_T . M will not re-elect an incumbent in equilibrium if:

$$\int_{\Omega} v_T(\tilde{w}, 1) f_w(\tilde{w}) d\tilde{w} > \sum_{j=0}^{n-1} \delta^j \mathrm{E} u^M(g_t) + \delta^n \int_{\Omega} v_T(\tilde{w}, 1) f_w(\tilde{w}) d\tilde{w}.$$
 (20)

It is sufficient to show that $\int_{\Omega} v_T(\tilde{w}, 1) f_w(\tilde{w}) d\tilde{w} > Eu^m(g_t)/(1-\delta)$. To show this, note that for any w > 0, $u^M(\tilde{x}_t(w, 1, g_t)) > u^M(g_t)$. Thus M can guarantee herself a payoff strictly higher than $Eu^M(g_t)$ in at least one period with probability one under \mathcal{E}_T . Thus, there exists some n_1 (possibly infinite) such that (20) holds for all $n \ge n_1$.

Next, M re-elects a type- (w, θ) incumbent in equilibrium if:

$$v_T(w,\theta+1) \ge \sum_{j=0}^{n-1} \delta^j \mathbf{E} u^M(g_t) + \delta^n \int_{\Omega} v_T(\tilde{w},1) f_w(\tilde{w}) d\tilde{w}.$$
 (21)

Clearly, under \mathcal{E}_T , $v_T(w, \theta + 1) \geq \sum_{j=0}^{\infty} \delta^j \mathrm{E} u^M(g_t)$ for all $w \in \Omega$ and θ . Thus, there exists some n_2 (possibly infinite) such that (16) holds for all $n \geq n_2$. For all $n \geq \max\{n_1, n_2\}$, \mathcal{E}_T is sustainable as the cooperative phase of a trigger equilibrium.

Proof of Proposition 6. Suppose first that P is of type \underline{w} . Since M removes this type in both games, G or G_t can costlessly induce a policy at g in both games.

Now suppose that P is of type \overline{w} . In the game with short-lived groups, M optimally induces P to choose a policy at $\tilde{x}(\overline{w},g) = g - \sqrt{\frac{\overline{w}}{1-\delta}}$. It is then easily calculated that M's expected utility is:

$$v_c(\overline{w}) = -\frac{\left(g - \sqrt{\frac{\overline{w}}{1-\delta}}\right)^2}{1-\delta}$$
(22)

$$v_c(\underline{w}) = -\frac{(1-\delta)g^2 + \delta(1-p)\left(g - \sqrt{\frac{\overline{w}}{1-\delta}}\right)^2}{(1-\delta)(1-\delta p)}$$
(23)

For an infinitely-lived group G, these strategies would imply expected payoffs of $v_c^G(\overline{w}) = -\frac{\overline{w}}{(1-\delta)^2}$ and $v_c^G(\underline{w}) = -\frac{\delta(1-p)\overline{w}}{(1-\delta)^2(1-\delta p)}$. Under these strategies, G can do better by paying $\overline{w}/(1-\delta) + \epsilon$ to a type- \overline{w} incumbent to choose a policy at g and leave office. This results in a new incumbent, and an expected payoff of:

$$\begin{aligned} -\frac{\overline{w}}{1-\delta} - \epsilon + \delta \left(-p \frac{\delta(1-p)\overline{w}}{(1-\delta)^2(1-\delta p)} - (1-p) \frac{\overline{w}}{(1-\delta)^2} \right) &\approx -\frac{\overline{w}}{1-\delta} \left(1 + \frac{\delta(1-p)}{(1-\delta)(1-\delta p)} \right) \\ &> -\frac{\overline{w}}{(1-\delta)^2}. \end{aligned}$$

Thus, to induce G not to buy out a type- \overline{w} incumbent P, G must be indifferent between buying out P and leaving her in office. Thus G's equilibrium value functions must solve the following system:

$$\begin{aligned} v_c^{G\infty}(\underline{w}) &= \delta \left(p v^{G\infty}(\underline{w}) + (1-p) v^{G\infty}(\overline{w}) \right) \\ v_c^{G\infty}(\overline{w}) &= -\frac{\overline{w}}{1-\delta} + \delta \left(p v^{G\infty}(\underline{w}) + (1-p) v^{G\infty}(\overline{w}) \right) \end{aligned}$$

Solving yields $v^{G\infty}(\underline{w}) = -\frac{\delta(1-p)\overline{w}}{(1-\delta)^2}$ and $v^{G\infty}(\overline{w}) = -\frac{(1-\delta p)\overline{w}}{(1-\delta)^2}$. Since a type- \overline{w} incumbent is always re-elected, this implies that its policy choice x is given by $-\frac{(g-x)^2}{1-\delta} = v^{G\infty}(\overline{w})$, or

 $x = g - \sqrt{\frac{(1-\delta p)\overline{w}}{1-\delta}}$, which is clearly closer to g and more distant from M's ideal than $\tilde{x}(\overline{w}, g)$, the policy chosen by a short-lived group. M's optimal contract therefore re-elects a type- \overline{w} incumbent if she chooses any policy in $[-g + \sqrt{\frac{(1-\delta p)\overline{w}}{1-\delta}}, g - \sqrt{\frac{(1-\delta p)\overline{w}}{1-\delta}}]$. G's best response contract is then to offer some $\epsilon > 0$ for choosing policy at $g - \sqrt{\frac{(1-\delta p)\overline{w}}{1-\delta}}$. Now M's expected utility from each type may be written:

$$v_c^{\infty}(\overline{w}) = -\frac{\left(g - \sqrt{\frac{(1-\delta p)\overline{w}}{1-\delta}}\right)^2}{1-\delta}$$
$$v_c^{\infty}(\underline{w}) = -\frac{(1-\delta)g^2 + \delta(1-p)\left(g - \sqrt{\frac{(1-\delta p)\overline{w}}{1-\delta}}\right)^2}{(1-\delta)(1-\delta p)}$$

It is easily verified that these values are strictly less than those of (22) and (23).

Proof of Proposition 7. We prove the result by construction. Suppose that P has type w. For convenience we drop time subscripts throughout.

(i) Suppose that P anticipates re-election in all future periods, and that M adopts the following contract: re-elect P if and only if she chooses x = m. Then if G^1 offers b(x) = 0 for all x, G^2 's best response is identical to the one-group case; *i.e.*, offer b(x) = 0 for all x. By symmetry, G^1 's best response is to offer b(x) = 0 for all x. P therefore chooses policy m. Since this outcome is clearly optimal for M for any draw of (g^1, g^2) , M always re-elects P; hence P's expected payoff at the beginning of each period is $w/(1-\delta)$. Thus, the contract to re-elect any incumbent if and only if she chooses m is part of a subgame perfect equilibrium.

(ii) Without loss of generality, let $m - g^1 > g^2 - m$. This implies that $m \notin \mathcal{G}^1(-\frac{w}{1-\delta})$. Additionally, let $u' \equiv u^{G^2}(g^1)$, and note that symmetry implies $u^{G^2}(g^1) = u^{G^1}(g^2)$.

Suppose that P anticipates re-election in all future periods, and that M adopts the contract: re-elect P if and only if she chooses x = m. Now let G^1 offer the contract:

$$b^{1}(x) = \begin{cases} -u'/(1-\delta) & \text{if } x = g^{1} \\ u^{G^{1}}(m) - u' & \text{if } x = m \\ 0 & \text{otherwise.} \end{cases}$$

Likewise, let G^2 offer the contract:

$$b^{2}(x) = \begin{cases} -u'/(1-\delta) & \text{if } x = g^{2} \\ -w - u^{G^{1}}(m) + \epsilon & \text{if } x = m \\ 0 & \text{otherwise.} \end{cases}$$

Under these strategies, P receives $-u'/(1-\delta)$ for choosing g^1 or g^2 , $(-u'+\epsilon)/(1-\delta)$ for choosing m, and 0 otherwise. P then chooses m.

Now consider whether better contracts exist for each G^i . G^i must offer at least $-u'/(1-\delta)$ to P to choose any $x \neq m$ (otherwise, P will strictly prefer choosing $x = g^{-i}$), so G^i receives strictly less than $u'/(1-\delta)$ from any $b^i(\cdot)$ inducing any $x \neq m, g^i$. G^i clearly cannot change P's choice of m by reducing $b^i(g^i)$. Finally, under $b^1(x)$ and $b^2(x)$, G^1 receives u' and G^2 receives $u^{G^2}(m) + u^{G^1}(m) + w - \epsilon$, which (by concavity) is strictly higher than u'. Letting $\epsilon \to 0$, reducing $b^i(m)$ results in a payoff of at most u' from a policy choice of either g^i or g^{-i} . Thus, $b^1(x)$ and $b^2(x)$ are best responses to M's voting contract.

Since this outcome is clearly optimal for M for any draw of (g^1, g^2) and w, M's re-election contract is optimal. These bribing and voting contracts are therefore part of a subgame perfect equilibrium.

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Figure 1: Policy choice. M can induce P to choose the closest policy in $\mathcal{G}(-\frac{w}{1-\delta})$. In this case, $m \neq \mathcal{G}(-\frac{w}{1-\delta})$, and so G acquiesces to policy $\underline{g}(-\frac{w}{1-\delta})$ and P is re-elected.