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Research Article

Model Predictive Control for Continuous-Time Singular Jump Systems with Incomplete Transition Rates

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This paper is concerned with model predictive control (MPC) problem for continuous-time Markov Jump Systems (MJSs) with incomplete transition rates and singular character. Sufficient conditions for the existence of a model predictive controller, which could optimize a quadratic cost function and guarantee that the system is piecewise regular, impulse-free, and mean square stable, are given in two cases at each sampling time. Since the MPC strategy is aggregated into continuous-time singular MJSs, a discrete-time controller is employed to deal with a continuous-time plant and the cost function not only refers to the singularity but also considers the sampling period. Moreover, the feasibility of the MPC scheme and the mean square admissibility of the closed-loop system are deeply discussed by using the invariant ellipsoid. Finally, a numerical example is given to illustrate the main results.

1. Introduction

As a class of stochastic hybrid systems, Markov Jump Systems (MJSs) are suitable to describe many practical systems whose structures and parameters suffer from random abrupt changes [1–5]. Furthermore, MJSs, which contain a non-full rank system matrix, are usually referred to as singular MJSs, and they have obtained much attention from mathematical and control community. It should be mentioned that a singular MJS can better describe some physical systems than standard MJS model and has extensive applications, such as microelectronic circuits and economics [6–9].

In recent years, an ever increasing number of scholars have paid much attention in the study of singular MJSs [6, 10–12]. In [13], Boukas investigated the state feedback control for singular MJSs with norm-bounded uncertainties. By considering the full and partial knowledge of transition rates, sufficient conditions which guarantee that the closed-loop system is piecewise regular, impulse-free, and mean square stable are given in [14]. Due to the regularity and pulse phenomenon, the stability and stabilizability of singular MJSs are intensively studied [10]. In [11], the authors aimed to study the mean square stability of singular MJSs and extended the main criterion to frequency domain. The robust mean square stability problem for uncertain singular MJSs with actuator saturation is further considered [15], in which sufficient conditions for such systems to be regular, causal, and mean square stable are derived in LMI form.

On another research frontline, model predictive control (MPC), which is also named as receding horizon control, has received tremendous attention in practical applications [16–19]. At each time instant, MPC requires the online solution of an optimization problem and then gets the optimized control inputs. We calculate a control sequence, but only the first element is implemented. At the next sampling time, we resolve the optimization problem again. Therefore, the control gain is updated at each sampling time and can naturally compensate the model uncertainties and exogenous disturbance to some extent. MPC has been successfully employed in industry since it can cope with hard constraints, control moves, system states, and outputs. As MJS is considered, MPC strategy has also received considerable research efforts on its analysis and synthesis from a theoretical point of view [20–23].

Almost all of the above-mentioned studies are based on the assumption that the transition probabilities are completely known. However, in practice, obtaining the complete knowledge of transition probabilities is difficult and costly. Until very recently, some efforts start to focus on MJSs with incomplete transition probabilities. Zhang and Boukas firstly proposed an analysis and design method for MJSs with partly unknown transition probability, which does not require the information of unknown elements [24]. Subsequently, necessary and sufficient conditions for the stability analysis of MJSs with incomplete transition descriptions are further given in [25]. Based on the results of [24, 25], Zheng and Yang [26] investigated the robust stabilization problem with a sliding mode control approach. Shen et al. proposed a new method for the H_∞ state feedback control [27] and established sufficient conditions for the existence of the H_∞ static output feedback controller [28].

It is worth noting that MPC for MJSs are usually based on discrete-time version. However, in the real world, we take the controlled objects as continuous-time models. To the best of our knowledge, few results about continuous-time singular MJSs are addressed due to the difficulty in stability analysis and controller design, especially further considering incomplete transition rates. In this paper, a new MPC method is developed for continuous-time singular MJSs with incomplete transitions rates. Our main contributions include the following three aspects. (1) A sampled-period MPC law is introduced to deal with the continuous-time plant. (2) A modified Lyapunov function approach is employed to deal with analysis and design problems for the underlying system concerning incomplete information. The approach not only refers to the knowledge of system state but also considers the sampling period and the singularity. (3) The feasibility of the MPC scheme for the continuous-time singular MJS and the mean square admissibility of the closed-loop system are discussed by using invariant ellipsoid.

2. Problem Statement

Consider a continuous-time singular MJS with uncertainties described as follows:

$$\begin{aligned} E\dot{x}(t) &= (A(r(t)) + \Delta A(r(t)))\,x(t) \\ &\quad + (B(r(t)) + \Delta B(r(t)))\,u(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the control input, E is a real constant matrix with $\operatorname{rank}(E) = r(r < n)$, $A(r(t))$ and $B(r(t))$ are known mode-dependent constant matrices with appropriate dimensions, and $\Delta A(r(t))$ and $\Delta B(r(t))$ are matrices with unknown uncertainties satisfying

$$\begin{aligned} \Delta A(r(t)) &= DF(r(t),t)\,H_1, \\ \Delta B(r(t)) &= DF(r(t),t)\,H_2, \end{aligned} \tag{2}$$

where $D \in \mathbb{R}^{n\times p}$, $H_1 \in \mathbb{R}^{q\times n}$, and $H_2 \in \mathbb{R}^{q\times m}$ are known constant matrices and $F(r(t),t) \in \mathbb{R}^{p\times q}$ is a time-varying matrix function satisfying $F^T(r(t),t)F(r(t),t) \le I$. $\{r(t), t \ge 0\}$ is a continuous-time Markov process which takes values in a finite set $\mathbb{M} = \{1,2,\ldots,N\}$ with the following transition rate:

$$\begin{aligned} &P_r\{r(t+h) = j \mid r(t) = i\} \\ &\quad = \begin{cases} \lambda_{ij}h + o(h), & i \ne j \\ 1 + \lambda_{ij}h + o(h), & i = j, \end{cases} \\ &\lambda_{ii} = -\sum_{j\in\mathbb{M}, i\ne j} \lambda_{ij}, \quad \lambda_{ij} \ge 0, \end{aligned} \tag{3}$$

where $h > 0$ and $\lim_{h\to 0} o(h)/h \to 0$ and λ_{ij} is the switching rate from mode i at time t to mode j at time $t + h$.

For the sake of convenience, $A(r(t))$, $B(r(t))$, $\Delta A(r(t))$, and $\Delta B(r(t))$ could be denoted by A_i, B_i, ΔA_i, and ΔB_i, respectively, where $r(t) = i \in \mathbb{M}$.

Moreover, in this paper, the transition rates are considered to be partly available; namely, some elements in matrix Λ are unknown. Taking singular MJS (1) with four operation modes as an example, the transition rate matrix could be expressed as

$$\widehat{\Lambda} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \widehat{\lambda}_{13} & \widehat{\lambda}_{14} \\ \widehat{\lambda}_{21} & \widehat{\lambda}_{22} & \lambda_{23} & \lambda_{24} \\ \lambda_{31} & \widehat{\lambda}_{32} & \lambda_{33} & \widehat{\lambda}_{34} \\ \widehat{\lambda}_{41} & \lambda_{42} & \widehat{\lambda}_{43} & \lambda_{44} \end{bmatrix}, \tag{4}$$

where the elements with "^" represent the unknown transition rate.

In this paper, we denote $\mathbb{M} = \mathbb{M}_k^i + \mathbb{M}_{uk}^i$, $\forall i \in \mathbb{M}$, with

$$\begin{aligned} \mathbb{M}_k^i &\triangleq \{j : \lambda_{ij} \text{ is known}\}, \\ \mathbb{M}_{uk}^i &\triangleq \{j : \lambda_{ij} \text{ is unknown}\}. \end{aligned} \tag{5}$$

In addition, if $\mathbb{M}_k^i \ne \phi$, it can be written as

$$\mathbb{M}_k^i = \{K_1^i, \ldots, K_m^i\}, \quad \forall 1 \le m \le N, \tag{6}$$

where $K_m^i \in \mathbb{N}^+$ represent the mth known element in the ith row of matrix Λ. Also, we denote $\lambda_k^i \triangleq \sum_{j\in\mathbb{M}_k^i} \lambda_{ij}$.

In the case that $\widehat{\lambda}_{ii}$ is unknown, providing a lower bound λ_d^i with $\lambda_d^i \le -\lambda_k^i$ is necessary.

MPC for (1) is studied over an infinite horizon. At each sampling time kT for $k = 0, 1, \ldots$, we could obtain control gains by making use of the LMI techniques and we will only implement the first calculated control input $u(kT) = F_i x(kT)$, where T represents the fixed sampling interval. At the next sampling time, we will calculate the state $x((k+1)T)$ and then compute the feedback control gain F_i once again. Let $x(kT, kT)$ denote the state measured at sampling time kT; the predicted state at time $kT + \tau$ is denoted by $x(kT + \tau, kT)$; $u(kT + \tau, kT)$ is the control action at time $kT + \tau$. $x(kT, kT)$ can be noted as $x(kT)$ for simplicity.

About the singular MJS (1), we give the following definitions.

Definition 1 (see [10]). The continuous-time singular MJSs (1) are said to be

(1) regular if $\det(sE - A_i)$ is not identically zero for each $i \in \mathbb{M}$;
(2) impulse-free if $\deg(\det(sE - A_i)) = \text{rank}(E)$ for each $i \in \mathbb{M}$;
(3) mean square stable if there exists a scalar $M(x_0, r_0) > 0$, for any $x_0 \in \mathbb{R}^n$, $r_0 \in \mathbb{M}$, such that

$$\mathbf{E}\left\{\int_0^\infty \|x(t)\|^2 \, dt \mid x_0, r_0\right\} \leq M(x_0, r_0); \qquad (7)$$

(4) mean square admissible if it is regular, impulse-free, and mean square stable.

Definition 2 (see [29] (invariant ellipsoid)). Given a continuous-time dynamical system $\dot{x}(t) = f(x(t))$, a subset $\mathbb{E} = \{x \in R^n \mid x^T Q^{-1} x \leq 1\}$ of the state space $\mathbb{R}^n$ is said to be an invariant ellipsoid, if it has the property that whenever $x(kT) \in \mathbb{E}$, then $x(NT) \in \mathbb{E}$ for all times $NT > kT$.

Before giving the main results of the singular MJS (1), some useful lemmas are presented below.

Lemma 3 (see [30]). *Given matrices G_1 and G_2 with appropriate dimensions and a symmetric matrix Ω, $\Omega + G_1 F(t) G_2 + G_2{}^T F(t)^T G_1{}^T < 0$, for all $F(t)^T F(t) \leq I$, if and only if there exists a scalar $\delta > 0$ such that*

$$\Omega + \delta G_1 G_1{}^T + \frac{1}{\delta} G_2{}^T G_2 < 0. \qquad (8)$$

Lemma 4 (see [31]). *Given symmetric and positive definite matrices M and N, if $M \geq N > 0$, then $N^{-1} \geq M^{-1} > 0$.*

Lemma 5 (see [14]). *If a mode-dependent symmetric matrix P_i and a real constant matrix E with $\text{rank}(E) = r (r < n)$ are given a priori, then we will have $E^T P_i \leq (1/4)\widetilde{\varepsilon}_i^{-1} I + \widetilde{\varepsilon}_i E^T P_i P_i^T E$ for any $\varepsilon_i > 0$, where I presents the identity matrices with compatible dimensions.*

In this paper, at each sampling time kT, we aim to design a state feedback controller

$$u(kT + \tau, kT) = F_i x(kT + \tau, kT) \quad \tau \geq 0, \qquad (9)$$

by solving the following optimization problem:

$$\begin{aligned} &\min_{F_i} J_k, \\ &J_k := \int_0^\infty \left(x^T (kT + \tau, kT) Q_c x(kT + \tau, kT)\right. \\ &\qquad \left. + u^T (kT + \tau, kT) R u(kT + \tau, kT)\right) d\tau, \end{aligned} \qquad (10)$$

where $Q_c > 0$ and $R > 0$ are symmetric weighting matrices, J_k is the performance index of system (1) at time kT, and F_i is the state feedback control gain.

3. Model Predictive Controller Design

In this section, we will design MPC controller for the singular MJSs with incomplete transition rates by presenting sufficient conditions, which could be efficiently solved by LMI toolbox.

Theorem 6. *Consider the singular MJS (1) with incomplete transition rates and let $x(kT)$ be the state of uncertain singular system (1) at sampling time kT. One will obtain Q_i and Y_i by solving the following optimization problem:*

$$\min_{\gamma, Q_i, Y_i, \varepsilon_i, \delta_i} \gamma \qquad (11)$$

satisfying

$$\begin{bmatrix} 1 & x^T(kT) & x^T(kT) E^T \\ * & 4\varepsilon_i I & 0 \\ * & * & Q_i + Q_i^T - \varepsilon_i I \end{bmatrix} \geq 0 \quad i \in \mathbb{M}, \qquad (12)$$

$$\begin{bmatrix} \Omega_1 & Q_i^T & Y_i^T & (H_1 Q_i + H_2 Y_i)^T & \Omega_2 & \Omega_4 & \sqrt{-\lambda_k^i} Q_i^T & \sqrt{-\lambda_k^i} Q_i^T E^T \\ * & -\gamma Q_c^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma R^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\delta_i & 0 & 0 & 0 & 0 \\ * & * & * & * & -\Omega_3 & 0 & 0 & 0 \\ * & * & * & * & * & -\Omega_5 & 0 & 0 \\ * & * & * & * & * & * & -4\varepsilon_j I & 0 \\ * & * & * & * & * & * & * & -Q_j - Q_j^T + \varepsilon_j I \end{bmatrix} \leq 0 \quad \forall j \in \mathbb{M}_{uk}^i, \ \text{if } i \in \mathbb{M}_k^i, \qquad (13)$$

$$\begin{bmatrix} \Omega_1^* & Q_i^T & Y_i^T & (H_1Q_i + H_2Y_i)^T & \Omega_2 & \Omega_4 & \sqrt{-\lambda_d^i - \lambda_k^i}Q_i^T & \sqrt{-\lambda_d^i - \lambda_k^i}Q_i^T E^T \\ * & -\gamma Q_c^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma R^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\delta_i & 0 & 0 & 0 & 0 \\ * & * & * & * & -\Omega_3 & 0 & 0 & 0 \\ * & * & * & * & * & -\Omega_5 & 0 & 0 \\ * & * & * & * & * & * & -4\varepsilon_j I & 0 \\ * & * & * & * & * & * & * & -Q_j - Q_j^T + \varepsilon_j I \end{bmatrix} \leq 0 \quad \forall j \in \mathbb{M}_{uk}^i, \ \textit{if } i \in \mathbb{M}_{uk}^i, \tag{14}$$

where

$$\Omega_1 = A_iQ_i + B_iY_i + (A_iQ_i + B_iY_i)^T + \delta_i DD^T + \lambda_{ii}Q_i^T E^T,$$

$$\Omega_1^* = A_iQ_i + B_iY_i + (A_iQ_i + B_iY_i)^T + \delta_i DD^T - \lambda_k^i Q_i^T E^T,$$

$$\Omega_2 = \left[\sqrt{\lambda_{i\mathbb{M}_1^i}}Q_i^T \ \cdots \ \sqrt{\lambda_{i\mathbb{M}_m^i}}Q_i^T\right], \tag{15}$$

$$\Omega_3 = \operatorname{diag}\left\{4\varepsilon_{\mathbb{M}_1^i} I \ \cdots \ 4\varepsilon_{\mathbb{M}_m^i} I\right\},$$

$$\Omega_4 = \left[\sqrt{\lambda_{i\mathbb{M}_1^i}}Q_i^T E^T \ \cdots \ \sqrt{\lambda_{i\mathbb{M}_m^i}}Q_i^T E^T\right],$$

$$\Omega_5 = \operatorname{diag}\left\{Q_{\mathbb{M}_1^i} + Q_{\mathbb{M}_1^i}^T - \varepsilon_{\mathbb{M}_1^i} I \ \cdots \ Q_{\mathbb{M}_m^i} + Q_{\mathbb{M}_m^i}^T - \varepsilon_{\mathbb{M}_m^i} I\right\}.$$

The mode-dependent state feedback gain F_i is given by

$$F_i = Y_iQ_i^{-1}. \tag{16}$$

Proof. Define a quadratic function

$$V(x(t)) = x^T(t)E^T P_i x(t), \quad E^T P_i = P_i^T E \geq 0. \tag{17}$$

Here, we use Γ to denote the weak infinitesimal generator of random process $\{x(t), i, t \geq 0\}$ for each mode $i \in \mathbb{M}$; it is defined as

$$\Gamma(V(x(t), r(t) = i)) = \lim_{\delta \to 0^+} \frac{1}{\delta}\{\mathbf{E}[V(x(t+\delta), r(t+\delta)) \mid x(t), r(t) = i] - V(x(t), i)\}. \tag{18}$$

At each sampling time kT, suppose that V satisfy the following inequality:

$$\Gamma(V(x(kT+\tau, kT), i)) \leq -\left(x^T(kT+\tau, kT)Q_c x(kT+\tau, kT) + u^T(kT+\tau, kT)Ru(kT+\tau, kT)\right). \tag{19}$$

Because the robust cost function will eventually become $x(\infty, kT) = 0$, we will get

$$J_k \leq V(x(kT), i) = x^T(kT)E^T P_i x(kT), \tag{20}$$

by integrating both the left and the right sides of inequality (19) with $\tau \in [0 \quad \infty)$.

Considering Lemma 5, for any $\tilde{\varepsilon}_i \geq 0$, we will have

$$V(x(kT), i) = x^T(kT)E^T P_i x(kT) \leq x^T(kT)\left(\frac{1}{4}\tilde{\varepsilon}_i^{-1} I + \tilde{\varepsilon}_i E^T P_i P_i^T E\right)x(kT); \tag{21}$$

let

$$x^T(kT)\left(\frac{1}{4}\tilde{\varepsilon}_i^{-1} I + \tilde{\varepsilon}_i E^T P_i P_i^T E\right)x(kT) \leq \gamma, \quad \tilde{\varepsilon}_i = \gamma^{-1}\varepsilon_i, \quad P_i = \gamma Q_i^{-1}; \tag{22}$$

then, we have

$$1 - x^T(kT)\left(\frac{1}{4}\varepsilon_i^{-1} I\right)x(kT) - x^T(kT)\left(\varepsilon_i E^T Q_i^{-1} Q_i^{-T} E\right)x(kT) \geq 0. \tag{23}$$

Using the fact that $\varepsilon_i^{-1}Q_iQ_i^T \geq Q_i + Q_i^T - \varepsilon_i I$ for any $\varepsilon_i > 0$ and Lemma 4, we will obtain

$$\varepsilon_i Q_i^{-1} Q_i^{-T} \leq \left(Q_i + Q_i^T - \varepsilon_i I\right)^{-1}. \tag{24}$$

Obviously, via the well-known Schur complement, it is easy to get that conditions (12) and (24) imply (23).

Applying the control law (9) and the definition of weak infinitesimal generator, inequality (19) becomes

$$\left((A_i + \Delta A_i) + (B_i + \Delta B_i)F_i\right)^T P_i + P_i^T\left((A_i + \Delta A_i) + (B_i + \Delta B_i)F_i\right) + \sum_{j \in \mathbb{M}} \lambda_{ij} E^T P_j + Q_c + F_i^T R F_i \leq 0. \tag{25}$$

Pre- and postmultiplying (25) by P_i^{-T} and its transpose, replacing P_i and Y_i, by γQ_i^{-1} and $F_i Q_i$, respectively, one will get

$$\begin{aligned}&(A_iQ_i+B_iY_i)+(A_iQ_i+B_iY_i)^T+\Delta A_iQ_i+\Delta B_iY_i\\&\quad+Q_i^T\Delta A_i^T+Y_i^T\Delta B_i^T+\gamma^{-1}\sum_{j\in\mathbb{M}}Q_i^T\lambda_{ij}E^TP_jQ_i\\&\quad+\gamma^{-1}Q_i^TQ_cQ_i+\gamma^{-1}Y_i^TRY_i\le 0;\end{aligned}\tag{26}$$

considering the unknown uncertainty (2), (26) becomes

$$\begin{aligned}&(A_iQ_i+B_iY_i)+(A_iQ_i+B_iY_i)^T\\&\quad+DF_i(t)(H_1Q_i+H_2Y_i)\\&\quad+(H_1Q_i+H_2Y_i)^TF_i^T(t)D^T\\&\quad+\gamma^{-1}\sum_{j\in\mathbb{M}}Q_i^T\lambda_{ij}E^TP_jQ_i+\gamma^{-1}Q_i^TQ_cQ_i\\&\quad+\gamma^{-1}Y_i^TRY_i\le 0.\end{aligned}\tag{27}$$

According to Lemma 3, (27) holds if and only if there exists $\delta_i>0$ satisfying

$$\begin{aligned}&(A_iQ_i+B_iY_i)+(A_iQ_i+B_iY_i)^T+\delta_iDD^T\\&\quad+\frac{1}{\delta_i}(H_1Q_i+H_2Y_i)^T(H_1Q_i+H_2Y_i)\\&\quad+\gamma^{-1}\sum_{j\in\mathbb{M}}Q_i^T\lambda_{ij}E^TP_jQ_i+\gamma^{-1}Q_i^TQ_cQ_i\\&\quad+\gamma^{-1}Y_i^TRY_i\le 0.\end{aligned}\tag{28}$$

According to Lemma 5, (28) holds if (29) is satisfied for any $\tilde{\varepsilon}_j>0$ satisfying

$$\begin{aligned}&(A_iQ_i+B_iY_i)+(A_iQ_i+B_iY_i)^T+\delta_iDD^T\\&\quad+\frac{1}{\delta_i}(H_1Q_i+H_2Y_i)^T(H_1Q_i+H_2Y_i)\\&\quad+\gamma^{-1}\sum_{j\in\mathbb{M}}Q_i^T\lambda_{ij}\left(\frac{1}{4}\tilde{\varepsilon}_j^{-1}I+\tilde{\varepsilon}_jE^TP_jP_j^TE\right)Q_i\\&\quad+\gamma^{-1}Q_i^TQ_cQ_i+\gamma^{-1}Y_i^TRY_i\le 0.\end{aligned}\tag{29}$$

Replacing P_j and $\tilde{\varepsilon}_j^{-1}$ by γQ_j^{-1} and $\gamma\varepsilon_j^{-1}$, respectively, we will have

$$\begin{aligned}&(A_iQ_i+B_iY_i)+(A_iQ_i+B_iY_i)^T+\delta_iDD^T\\&\quad+\frac{1}{\delta_i}(H_1Q_i+H_2Y_i)^T(H_1Q_i+H_2Y_i)\\&\quad+\sum_{j\in\mathbb{M}}\frac{1}{4}\lambda_{ij}Q_i^T\varepsilon_j^{-1}IQ_i\\&\quad+\sum_{j\in\mathbb{M}}\lambda_{ij}Q_i^TE^T\left(Q_j^{-1}Q_j^{-T}\varepsilon_j\right)EQ_i+\gamma^{-1}Q_i^TQ_cQ_i\\&\quad+\gamma^{-1}Y_i^TRY_i\le 0.\end{aligned}\tag{30}$$

Subsequently, we take transition rate into account and separate it into two cases, $i\in\mathbb{M}_k^i$ and $i\in\mathbb{M}_{uk}^i$, respectively.

Case 1 ($i\in\mathbb{M}_k^i$). Considering that $\sum_{j\in\mathbb{M}}\lambda_{ij}=\sum_{j\in\mathbb{M}_k^i}\lambda_{ij}+\sum_{j\in\mathbb{M}_{uk}^i}\lambda_{ij}$, the left side of (30) is rewritten as

$$\begin{aligned}\theta_i\triangleq{}&(A_iQ_i+B_iY_i)+(A_iQ_i+B_iY_i)^T+\delta_iDD^T\\&+\frac{1}{\delta_i}(H_1Q_i+H_2Y_i)^T(H_1Q_i+H_2Y_i)\\&+\gamma^{-1}Q_i^TQ_cQ_i+\gamma^{-1}Y_i^TRY_i+\alpha_k^i\\&+\frac{1}{4}\sum_{j\in\mathbb{M}_{uk}^i}\hat{\lambda}_{ij}Q_i^T\varepsilon_j^{-1}IQ_i+\beta_k^i\\&+\sum_{j\in\mathbb{M}_{uk}^i}\hat{\lambda}_{ij}Q_i^TE^T\left(Q_j^{-1}Q_j^{-T}\varepsilon_j\right)EQ_i,\end{aligned}\tag{31}$$

where the elements $\hat{\lambda}_{ij}$ ($\forall j\in\mathbb{M}_{uk}^i$) are unknown, $\alpha_k^i\triangleq(1/4)\sum_{j\in\mathbb{M}_k^i}\lambda_{ij}Q_i^T\varepsilon_j^{-1}IQ_i$, and $\beta_k^i\triangleq\sum_{j\in\mathbb{M}_k^i}\lambda_{ij}Q_i^TE^T(Q_j^{-1}Q_j^{-T}\varepsilon_j)EQ_i$. Noting that, in this case, λ_{ii} is known, $\lambda_k^i=\lambda_{ii}+\sum_{j\in\mathbb{M}_k^i,i\neq j}<0$, and $\sum_{j\in\mathbb{M}}Q_i^T\lambda_{ij}E^T(Q_j^{-1})Q_i=\lambda_{ii}Q_i^TE^T$, $j=i$, then

$$\begin{aligned}\theta_i={}&(A_iQ_i+B_iY_i)+(A_iQ_i+B_iY_i)^T+\delta_iDD^T\\&+\frac{1}{\delta_i}(H_1Q_i+H_2Y_i)^T(H_1Q_i+H_2Y_i)\\&+\gamma^{-1}Q_i^TQ_cQ_i+\gamma^{-1}Y_i^TRY_i+\lambda_{ii}Q_i^TE^T+\alpha_k^{i*}\\&+\frac{1}{4}\sum_{j\in\mathbb{M}_{uk}^i}\frac{\hat{\lambda}_{ij}}{-\lambda_k^i}\left(-\lambda_k^i\right)Q_i^T\varepsilon_j^{-1}IQ_i+\beta_k^{i*}\\&+\sum_{j\in\mathbb{M}_{uk}^i}\frac{\hat{\lambda}_{ij}}{-\lambda_k^i}\left(-\lambda_k^i\right)Q_i^TE^T\left(Q_j^{-1}Q_j^{-T}\varepsilon_j\right)EQ_i,\end{aligned}\tag{32}$$

where $\alpha_k^{i*}=(1/4)\sum_{j\in\mathbb{M}_k^i,j\neq i}\lambda_{ij}Q_i^T\varepsilon_j^{-1}IQ_i$ and $\beta_k^{i*}=\sum_{j\in\mathbb{M}_k^i,j\neq i}\lambda_{ij}Q_i^TE^T(Q_j^{-1}Q_j^{-T}\varepsilon_j)EQ_i$, because of $0\le\hat{\lambda}_{ij}/-\lambda_k^i\le 1$ and $\sum_{j\in\mathbb{M}_{uk}^i}\hat{\lambda}_{ij}/-\lambda_k^i=1$; we will obtain that

$$\begin{aligned}\theta_i={}&\sum_{j\in\mathbb{M}_{uk}^i}\frac{\hat{\lambda}_{ij}}{-\lambda_k^i}\Bigg((A_iQ_i+B_iY_i)+(A_iQ_i+B_iY_i)^T\\&+\delta_iDD^T+\frac{1}{\delta_i}(H_1Q_i+H_2Y_i)^T(H_1Q_i+H_2Y_i)\\&+\gamma^{-1}Q_i^TQ_cQ_i+\gamma^{-1}Y_i^TRY_i+\lambda_{ii}Q_i^TE^T+\alpha_k^{i*}\\&+\left(-\lambda_k^i\right)Q_i^TE^T\left(Q_j^{-1}Q_j^{-T}\varepsilon_j\right)EQ_i\\&+\frac{1}{4}\left(-\lambda_k^i\right)Q_i^T\varepsilon_j^{-1}IQ_i+\beta_k^{i*}\Bigg).\end{aligned}\tag{33}$$

Since $\theta_i < 0$, we can have the following inequality:

$$\begin{aligned}&(A_iQ_i+B_iY_i)+(A_iQ_i+B_iY_i)^T+\delta_iDD^T\\&\quad+\frac{1}{\delta_i}(H_1Q_i+H_2Y_i)^T(H_1Q_i+H_2Y_i)\\&\quad+\gamma^{-1}Q_i^TQ_cQ_i+\gamma^{-1}Y_i^TRY_i+\alpha_k^{i*}\\&\quad+\frac{1}{4}(-\lambda_k^i)Q_i^T\varepsilon_j^{-1}IQ_i+\beta_k^{i*}\\&\quad+(-\lambda_k^i)Q_i^TE^T(Q_j^{-1}Q_j^{-T}\varepsilon_j)EQ_i+\lambda_{ii}Q_i^TE^T\\&<0.\end{aligned}\tag{34}$$

Using (24), we can easily get (34), if

$$\begin{aligned}&(A_iQ_i+B_iY_i)+(A_iQ_i+B_iY_i)^T+\delta_iDD^T\\&\quad+\frac{1}{\delta_i}(H_1Q_i+H_2Y_i)^T(H_1Q_i+H_2Y_i)\\&\quad+\lambda_{ii}Q_i^TE^T+\gamma^{-1}Q_i^TQ_cQ_i+\gamma^{-1}Y_i^TRY_i+\alpha_k^{i*}\\&\quad+\sum_{j\in\mathbb{M}_k^i,j\neq i}\lambda_{ij}Q_i^TE^T(Q_j+Q_j^T-\varepsilon_jI)^{-1}EQ_i\\&\quad+\frac{1}{4}(-\lambda_k^i)Q_i^T\varepsilon_j^{-1}IQ_i\\&\quad+(-\lambda_k^i)Q_i^TE^T(Q_j+Q_j^T-\varepsilon_jI)^{-1}EQ_i<0\end{aligned}\tag{35}$$

holds. Applying Schur complement to the above inequality, one can obtain (13).

Case 2 ($i\in\mathbb{M}_{uk}^i$). Likewise, considering $\sum_{j\in\mathbb{M}}\lambda_{ij}=\sum_{j\in\mathbb{M}_k^i}\lambda_{ij}+\sum_{j\in\mathbb{M}_{uk}^i}\lambda_{ij}$, we rewrite the left side of (30) as

$$\begin{aligned}\theta_i&\triangleq(A_iQ_i+B_iY_i)+(A_iQ_i+B_iY_i)^T+\delta_iDD^T\\&\quad+\frac{1}{\delta_i}(H_1Q_i+H_2Y_i)^T(H_1Q_i+H_2Y_i)\\&\quad+\gamma^{-1}Q_i^TQ_cQ_i+\gamma^{-1}Y_i^TRY_i+\widehat{\lambda}_{ii}Q_i^TE^T\\&\quad+\frac{1}{4}\sum_{j\in\mathbb{M}_k^i}\lambda_{ij}Q_i^T\varepsilon_j^{-1}IQ_i+\frac{1}{4}\sum_{j\in\mathbb{M}_{uk}^i,j\neq i}\widehat{\lambda}_{ij}Q_i^T\varepsilon_j^{-1}IQ_i\\&\quad+\sum_{j\in\mathbb{M}_k^i}\lambda_{ij}Q_i^TE^T(Q_j^{-1}Q_j^{-T}\varepsilon_j)EQ_i\\&\quad+\sum_{j\in\mathbb{M}_{uk}^i,j\neq i}\widehat{\lambda}_{ij}Q_i^TE^T(Q_j^{-1}Q_j^{-T}\varepsilon_j)EQ_i.\end{aligned}\tag{36}$$

Noting that, in this case, $\widehat{\lambda}_{ii}$ is unknown, $\lambda_k^i=-\widehat{\lambda}_{ii}-\sum_{j\in\mathbb{M}_{uk}^i,j\neq i}\lambda_{ij}>0$, then

$$\begin{aligned}\theta_i&=(A_iQ_i+B_iY_i)+(A_iQ_i+B_iY_i)^T+\delta_iDD^T\\&\quad+\frac{1}{\delta_i}(H_1Q_i+H_2Y_i)^T(H_1Q_i+H_2Y_i)\\&\quad+\gamma^{-1}Q_i^TQ_cQ_i+\gamma^{-1}Y_i^TRY_i+\alpha_k^i+\frac{1}{4}(-\widehat{\lambda}_{ii}-\lambda_k^i)\\&\quad\cdot\sum_{j\in\mathbb{M}_{uk}^i,j\neq i}\frac{\widehat{\lambda}_{ij}}{-\widehat{\lambda}_{ii}-\lambda_k^i}Q_i^T\varepsilon_j^{-1}IQ_i+(-\widehat{\lambda}_{ii}-\lambda_k^i)\\&\quad\cdot\sum_{j\in\mathbb{M}_{uk}^i,j\neq i}\frac{\widehat{\lambda}_{ij}}{-\widehat{\lambda}_{ii}-\lambda_k^i}Q_i^TE^T(Q_j^{-1}Q_j^{-T}\varepsilon_j)EQ_i\\&\quad+\widehat{\lambda}_{ii}Q_i^TE^T+\beta_k^i.\end{aligned}\tag{37}$$

Similarly, since we have $0\le\widehat{\lambda}_{ij}/(-\widehat{\lambda}_{ii}-\lambda_k^i)\le 1$ and $\sum_{j\in\mathbb{M}_{uk}^i,j\neq i}(\widehat{\lambda}_{ij}/(-\widehat{\lambda}_{ii}-\lambda_k^i))=1$, we can easily obtain

$$\begin{aligned}\theta_i&=\sum_{j\in\mathbb{M}_{uk}^i,j\neq i}\frac{\widehat{\lambda}_{ij}}{-\widehat{\lambda}_{ii}-\lambda_k^i}\Big((A_iQ_i+B_iY_i)\\&\quad+(A_iQ_i+B_iY_i)^T+\delta_iDD^T\\&\quad+\frac{1}{\delta_i}(H_1Q_i+H_2Y_i)^T(H_1Q_i+H_2Y_i)\\&\quad+\gamma^{-1}Q_i^TQ_cQ_i+\gamma^{-1}Y_i^TRY_i+\widehat{\lambda}_{ii}Q_i^TE^T+\alpha_k^i+\beta_k^i\\&\quad+(-\widehat{\lambda}_{ii}-\lambda_k^i)Q_i^TE^T(Q_j^{-1}Q_j^{-T}\varepsilon_j)EQ_i\\&\quad+\frac{1}{4}(-\widehat{\lambda}_{ii}-\lambda_k^i)Q_i^T\varepsilon_j^{-1}IQ_i\Big).\end{aligned}\tag{38}$$

Therefore, $\theta_i<0$ is equivalent to $\forall j\in\mathbb{M}_{uk}^i$, $j\neq i$,

$$\begin{aligned}&(A_iQ_i+B_iY_i)+(A_iQ_i+B_iY_i)^T+\delta_iDD^T\\&\quad+\frac{1}{\delta_i}(H_1Q_i+H_2Y_i)^T(H_1Q_i+H_2Y_i)\\&\quad+\gamma^{-1}Q_i^TQ_cQ_i+\gamma^{-1}Y_i^TRY_i+\widehat{\lambda}_{ii}Q_i^TE^T+\alpha_k^i\\&\quad+\frac{1}{4}(-\widehat{\lambda}_{ii}-\lambda_k^i)Q_i^T\varepsilon_j^{-1}IQ_i\\&\quad+(-\widehat{\lambda}_{ii}-\lambda_k^i)Q_i^TE^T(Q_j^{-1}Q_j^{-T}\varepsilon_j)EQ_i+\beta_k^i<0.\end{aligned}\tag{39}$$

As $\widehat{\lambda}_{ii}$ is lower bounded by λ_d^i, we know that $\lambda_d^i\le\widehat{\lambda}_{ii}<-\lambda_k^i$. Thus, (39) holds if the following inequality holds:

$$\begin{aligned}&(A_iQ_i+B_iY_i)+(A_iQ_i+B_iY_i)^T+\delta_iDD^T\\&\quad+\frac{1}{\delta_i}(H_1Q_i+H_2Y_i)^T(H_1Q_i+H_2Y_i)\\&\quad+\gamma^{-1}Q_i^TQ_cQ_i+\gamma^{-1}Y_i^TRY_i-\lambda_k^iQ_i^TE^T+\alpha_k^i\\&\quad+\frac{1}{4}(-\lambda_d^i-\lambda_k^i)Q_i^T\varepsilon_j^{-1}IQ_i\\&\quad+(-\lambda_d^i-\lambda_k^i)Q_i^TE^T(Q_j^{-1}Q_j^{-T}\varepsilon_j)EQ_i+\beta_k^i<0.\end{aligned}\tag{40}$$

Similarly, (40) is implied by (24) and

$$\begin{aligned}&(A_iQ_i+B_iY_i)+(A_iQ_i+B_iY_i)^T+\delta_iDD^T\\&\quad+\frac{1}{\delta_i}(H_1Q_i+H_2Y_i)^T(H_1Q_i+H_2Y_i)\\&\quad+\gamma^{-1}Q_i^TQ_cQ_i+\gamma^{-1}Y_i^TRY_i-\lambda_k^iQ_i^TE^T+\alpha_k^i\\&\quad+\frac{1}{4}(-\lambda_d^i-\lambda_k^i)Q_i^T\varepsilon_j^{-1}IQ_i\end{aligned}$$

$$+\left(-\lambda_d^i-\lambda_k^i\right)Q_i^TE^T\left(Q_j+Q_j^T-\varepsilon_jI\right)^{-1}EQ_i + \sum_{j\in\mathbb{M}_k^i}\lambda_{ij}Q_i^TE^T\left(Q_j+Q_j^T-\varepsilon_jI\right)^{-1}EQ_i<0. \tag{41}$$

Applying Schur complement to (41), we can obtain (14).

Certainly, MPC for singular MJSs without incomplete transition descriptions and the normal MJS with incomplete transition descriptions can be viewed as two special cases of singular MJS. Then, we have the following corollaries. □

Corollary 7. *Consider the singular MJS (1) without incomplete transition rate and let $x(kT)$ be the state of singular system (1) at sampling time kT. One will obtain Q_i and Y_i by solving the following optimization problem:*

$$\min_{\gamma,Q_i,Y_i,\varepsilon_i,\delta_i}\gamma \tag{42}$$

satisfying

$$\begin{bmatrix}1 & x^T(kT) & x^T(kT)E^T\\ * & 4\varepsilon_iI & 0\\ * & * & Q_i+Q_i^T-\varepsilon_iI\end{bmatrix}\geq 0\quad i\in\mathbb{M},$$

$$\begin{bmatrix}\Omega_1 & Q_i^T & Y_i & (H_1Q_i+H_2Y_i)^T & \Omega_2 & \Omega_4\\ * & -\gamma Q_c^{-1} & 0 & 0 & 0 & 0\\ * & * & -\gamma R^{-1} & 0 & 0 & 0\\ * & * & * & -\delta_i & 0 & 0\\ * & * & * & * & -\Omega_3 & 0\\ * & * & * & * & * & -\Omega_5\end{bmatrix}\leq 0\quad i\in\mathbb{M}, \tag{43}$$

where

$$\begin{aligned}
\Omega_1&=A_iQ_i+B_iY_i+\left(A_iQ_i+B_iY\right)^T+\delta_iDD^T+\lambda_{ii}Q_i^TE^T,\\
\Omega_2&=\left[\sqrt{\lambda_{i1}}Q_i^T\ \cdots\ \sqrt{\lambda_{i,i-1}}Q_i^T\ \sqrt{\lambda_{i,i+1}}Q_i^T\ \cdots\ \sqrt{\lambda_{i,N}}Q_i^T\right],\\
\Omega_3&=\operatorname{diag}\left\{4\varepsilon_1I\ \cdots\ 4\varepsilon_{i-1}I\ 4\varepsilon_{i+1}I\ \cdots\ 4\varepsilon_NI\right\},\\
\Omega_4&=\left[\sqrt{\lambda_{i1}}Q_i^TE^T\ \cdots\ \sqrt{\lambda_{i,i-1}}Q_i^TE^T\ \sqrt{\lambda_{i,i+1}}Q_i^TE^T\ \cdots\ \sqrt{\lambda_{i,N}}Q_i^TE^T\right],\\
\Omega_5&=\operatorname{diag}\left\{Q_1+Q_1^T-\varepsilon_1I\ \cdots\ Q_{i-1}+Q_{i-1}^T-\varepsilon_{i-1}I\ Q_{i+1}+Q_{i+1}^T-\varepsilon_{i+1}I\ \cdots\ Q_N+Q_N^T-\varepsilon_NI\right\}.
\end{aligned} \tag{44}$$

Then, the mode-dependent state feedback gain F_i can be obtained by (16).

Corollary 8. *When E is nonsingular and can be simplified as an identity matrix, system (1) becomes*

$$\dot{x}(t)=\left(A(r(t))+\Delta A(r(t))\right)x(t)+\left(B(r(t))+\Delta B(r(t))\right)u(t); \tag{45}$$

let $x(kT)$ be the state of uncertain MJS (45) at sampling time kT. One will obtain Q_i and Y_i by solving the following optimization problem:

$$\min_{\gamma,Q_i,Y_i,\delta_i}\gamma \tag{46}$$

satisfying

$$\begin{bmatrix}1 & x^T(kT)\\ * & Q_i\end{bmatrix}\geq 0,\quad i\in\mathbb{M},$$

$$\begin{bmatrix}\Omega_1 & Q_i^T & Y_i^T & (H_1Q_i+H_2Y_i)^T & \Omega_2 & \sqrt{-\lambda_k^i}Q_i^T\\ * & -\gamma Q_c^{-1} & 0 & 0 & 0 & 0\\ * & * & -\gamma R^{-1} & 0 & 0 & 0\\ * & * & * & -\delta_i & 0 & 0\\ * & * & * & * & -\Omega_3 & 0\\ * & * & * & * & * & -Q_j\end{bmatrix}\leq 0\quad \forall j\in\mathbb{M}_{uk}^i,\ \text{if } i\in\mathbb{M}_k^i,$$

$$\begin{bmatrix}\Omega_1^* & Q_i^T & Y_i^T & (H_1Q_i+H_2Y_i)^T & \Omega_2 & \sqrt{-\lambda_d^i-\lambda_k^i}Q_i^T\\ * & -\gamma Q_c^{-1} & 0 & 0 & 0 & 0\\ * & * & -\gamma R^{-1} & 0 & 0 & 0\\ * & * & * & -\delta_i & 0 & 0\\ * & * & * & * & -\Omega_3 & 0\\ * & * & * & * & * & -Q_j\end{bmatrix}\leq 0\quad \forall j\in\mathbb{M}_{uk}^i,\ \text{if } i\in\mathbb{M}_{uk}^i, \tag{47}$$

where

$$\begin{aligned}\Omega_1 &= A_iQ_i + B_iY_i + \left(A_iQ_i + B_iY\right)^T + \delta_i DD^T \\ &\quad + \lambda_{ii}Q_i^T, \\ \Omega_1^* &= A_iQ_i + B_iY_i + \left(A_iQ_i + B_iY\right)^T + \delta_i DD^T \\ &\quad + \lambda_k^i Q_i^T, \\ \Omega_2 &= \left[\sqrt{\lambda_{i\mathbb{M}_1^i}}Q_i^T \ \cdots \ \sqrt{\lambda_{i\mathbb{M}_m^i}}Q_i^T\right], \\ \Omega_3 &= \operatorname{diag}\left\{Q_{\mathbb{M}_1^i} \ \cdots \ Q_{\mathbb{M}_m^i}\right\}.\end{aligned} \tag{48}$$

The mode-dependent state feedback gain F_i is obtained by (16).

4. Mean Square Admissibility Analysis

Previous section gives sufficient conditions, which could guarantee the existence of model predictive controller. In this section, we mainly study the mean square stability of the closed-loop singular MJS (1).

Lemma 9 (feasibility). *The solution of the optimization problem in Theorem 6 is feasible over any time interval $[NT, (N+1)T)$ with $N > k$, if it has the property that the solution is feasible over the time interval $[kT, (k+1)T)$.*

Proof. At sampling time kT, from inequality (12), we limit the measured state $x(kT)$ in the ellipsoid region $x^T[(1/4)\varepsilon_i^{-1}I + E^T(Q_i + Q_i^T - \varepsilon_i I)^{-1}E]x \leq 1$. According to the proof of Theorem 6, we can get

$$\Gamma\left(V\left(x\left(t\right), r\left(t\right) = i\right)\right) < 0. \tag{49}$$

Then, for any sampling time NT with $N > k$, the future state $x(NT)$ remains in the ellipsoid set. Therefore, by referring to Definition 2, $x^T[(1/4)\varepsilon_i^{-1}I + E^T(Q_i + Q_i^T - \varepsilon_i I)^{-1}E]x \leq 1$ is an invariant ellipsoid.

The solution of the optimization problem in Theorem 6 is feasible over the time interval $[kT, (k+1)T)$. By observing (12), (13), and (14), it could be concluded that the feasibility of the MPC strategy only depends on (12). Thus, to prove Lemma 3, we only need to prove that the LMI (12) is feasible for all future states $x(NT)$, $N > k$. Considering the invariance of the ellipsoid, we obtain

$$\begin{aligned}&x^T\left((k+1)T\right)\left[\frac{1}{4}\varepsilon_i^{-1}I + E^T\left(Q_i + Q_i^T - \varepsilon_i I\right)^{-1}E\right] \\ &\quad \cdot x\left((k+1)T\right) \leq 1,\end{aligned} \tag{50}$$

which means LMI (12) is feasible with state $x((k+1)T)$ and then the feasibility of optimization problem (11) could be achieved over the time interval $[(k+1)T, (k+2)T)$. Similarly, we can complete the proof on the time intervals $[(k+2)T, (k+3)T)$, $[(k+3)T, (k+4)T)$,.... □

Theorem 10. *The feedback predictive control gain computed from Theorem 6 guarantees that the closed-loop singular MJS (1) is mean square admissible.*

Proof. Firstly, we show that the singular MJS (1) is regular and impulse-free. There exist two orthogonal matrices $U = [U_1 \ \ U_2]$ and $V = [V_1 \ \ V_2]$ such that

$$\overline{E} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} = U^T E V^{-T}. \tag{51}$$

Based on singular value decomposition, system (1) can be written as the following differential and algebraic form:

$$\begin{aligned}\dot{x}_1\left(t\right) &= A_{ic1}^* x_1\left(t\right) + A_{ic2}^* x_2\left(t\right), \\ 0 &= A_{ic3}^* x_1\left(t\right) + A_{ic4}^* x_2\left(t\right),\end{aligned} \tag{52}$$

where $A_{ic} = (A_i + \Delta A_i) + (B_i + \Delta B_i)F_i$, and it can be decomposed as $A_{ic} = \left[\begin{smallmatrix} A_{ic1} & A_{ic2} \\ A_{ic3} & A_{ic4} \end{smallmatrix}\right]$.

Assuming the optimal solution of optimization problem (11) at sampling time kT can be denoted as $P_i^*(kT)$ and $F_i^*(kT)$, from the proof of Theorem 6, we can obtain that there exists $P_i^*(kT)$ satisfying (25) and

$$E^T P_i^*\left(kT\right) = P_i^{T*}\left(kT\right)E \geq 0. \tag{53}$$

Let

$$\begin{aligned}\overline{A}_i &= \begin{bmatrix} A_{ic1}^* & A_{ic2}^* \\ A_{ic3}^* & A_{ic4}^* \end{bmatrix} = U^T A_{ic}^* V^{-T}, \\ \overline{P}_i^*\left(kT\right) &= \begin{bmatrix} P_{i1}^*\left(kT\right) & P_{i2}^*\left(kT\right) \\ P_{i3}^*\left(kT\right) & P_{i4}^*\left(kT\right) \end{bmatrix} = U^T P_i^*\left(kT\right)V^{-T}, \\ \overline{Q}_c &= \begin{bmatrix} Q_{c1} & Q_{c2} \\ Q_{c2}^T & Q_{c3} \end{bmatrix} = V^{-1}Q_c V^{-T}, \\ \overline{F}_i^*\left(kT\right) &= \left[F_{i1}^*\left(kT\right) \ \ F_{i2}^*\left(kT\right)\right] = F_i^*\left(kT\right)V^{-T}.\end{aligned} \tag{54}$$

By (53), we have $P_{i1}^*(kT) > 0$ (because $P_i^*(kT)$ is nonsingular) and $P_{i2}^*(kT) = 0$. Pre- and postmultiplying the left and right sides of (25) by V^{-1} and V^{-T}, we obtain

$$\begin{aligned}&A_{ic4}^{T*}P_{i4}^*\left(kT\right) + P_{i4}^{T*}\left(kT\right)A_{ic4}^* + Q_{c3} \\ &\quad + F_{i2}^{T*}\left(kT\right)RF_{i2}^*\left(kT\right) < 0.\end{aligned} \tag{55}$$

From (55), we can easily get that A_{ic4}^{T*} is invertible. Hence, for any $t \geq 0$,

$$\begin{aligned}\det\left(sE - A_{ic}^*\right) &= \det U^{-T}\det V^T \det\left(A_{ic4}^*\right) \\ &\quad \cdot \det\left(sI - A_{ic1}^* - A_{ic2}^* A_{ic4}^{(-1)*}A_{ic3}^*\right), \\ \deg\left(\det\left(sE - A_{ic}^*\right)\right) &= \operatorname{rank}\left(E\right),\end{aligned} \tag{56}$$

from which we can see that $\det(sE - A_{ic}^*)$ is not identically zero; by Definition 1, the closed-loop system is regular and impulse-free.

What is more, we consider the mean square stability of the singular MJS (1). Here, the piecewise Lyapunov function is described as

$$\begin{aligned}V^*\left(x\left(t\right), i\right) &= x^T\left(t\right)E^T P_i^*\left(kT\right)x\left(t\right), \\ &\qquad t \in \left[kT, \left(k+1\right)T\right).\end{aligned} \tag{57}$$

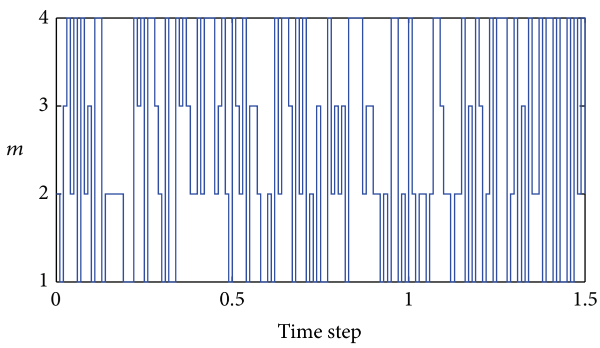

Figure 1: Jump mode.

From (25), we have

$$\begin{aligned}&\left(\left(A_i+\Delta A_i\right)+\left(B_i+\Delta B_i\right)F_i\right)^T P_i^* \\ &\quad + P_i^{T*}\left(\left(A_i+\Delta A_i\right)+\left(B_i+\Delta B_i\right)F_i\right) \\ &\quad + \sum_{j\in\mathbb{M}}\lambda_{ij}E^T P_j^* \le 0\end{aligned} \tag{58}$$

which yields

$$\Gamma\left(V^*\left(x\left(t\right),i\right)\right)<0, \quad t\in\left[kT,\left(k+1\right)T\right). \tag{59}$$

Therefore, $V(x(t),i)$ is strictly decreasing over each time interval $[kT,(k+1)T)$.

Note that $P_i^*(kT)$ represents a feasible solution at time $(k+1)T$ while $P_i^*((k+1)T)$ is an optimal solution at time $(k+1)T$. Since $V^*(x(t),i)$ is a piecewise function, we further consider

$$\begin{aligned}&\lim_{t\to(k+1)T^-}V^*\left(x\left(t\right),i\right) \\ &= x^T\left(\left(k+1\right)T\right)EP_i^*\left(kT\right)x\left(\left(k+1\right)T\right) \\ &\ge x^T\left(\left(k+1\right)T\right)EP_i^*\left(\left(k+1\right)T\right)x\left(\left(k+1\right)T\right) \\ &= V^*\left(x\left(\left(k+1\right)T\right)\right),\end{aligned} \tag{60}$$

for all $k \ge 0$. Inequalities (59) and (60) guarantee that $V(x(t),i)$ is strictly decreasing when $t\in[0,\infty)$; system (1) is mean square stable.

Therefore, according to the property of being regular and impulse-free, the closed-loop singular MJS is mean square admissible. □

5. Numerical Example

To illustrate the efficiency of the proposed MPC scheme for continuous-time MJSs, a numerical example is presented in the following.

Consider a system with the form of (1), where $r(t) = 1,2,3,4$. The system matrices are $A_1 = \left[\begin{smallmatrix}0.75 & -0.75\\ 0 & -1.5\end{smallmatrix}\right]$, $A_2 = \left[\begin{smallmatrix}0.15 & -0.49\\ 0 & -2.1\end{smallmatrix}\right]$, $A_3 = \left[\begin{smallmatrix}0.3 & -0.15\\ 0 & -1.8\end{smallmatrix}\right]$, $A_4 = \left[\begin{smallmatrix}0.9 & -0.34\\ 0 & -1.65\end{smallmatrix}\right]$, $B_1 = \left[\begin{smallmatrix}5\\ -1\end{smallmatrix}\right]$, $B_2 = \left[\begin{smallmatrix}2\\ -1\end{smallmatrix}\right]$, $B_3 = \left[\begin{smallmatrix}1\\ -1\end{smallmatrix}\right]$, $B_4 = \left[\begin{smallmatrix}3\\ -1\end{smallmatrix}\right]$, $D = \left[\begin{smallmatrix}0.1\\ 0\end{smallmatrix}\right]$, $E = \left[\begin{smallmatrix}1 & 0\\ 0 & 0\end{smallmatrix}\right]$, $H_1 = [0.1 \quad 0]$, and $H_2 = 0.1$. The weighting matrices are $Q_c = \left[\begin{smallmatrix}1 & 0\\ 0 & 1\end{smallmatrix}\right]$ and $R = 1$.

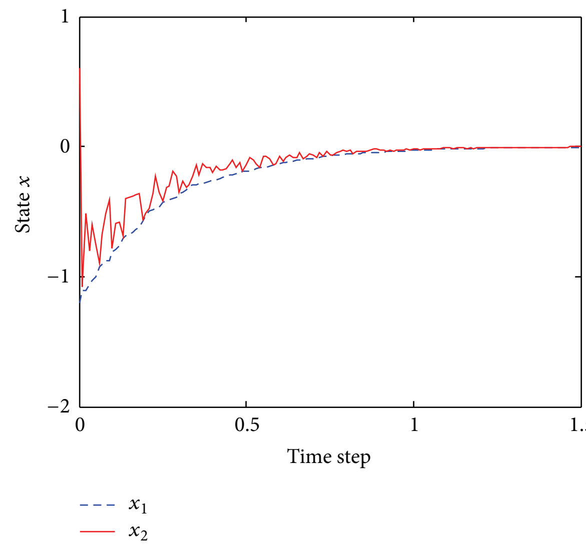

Figure 2: State responses under MPC.

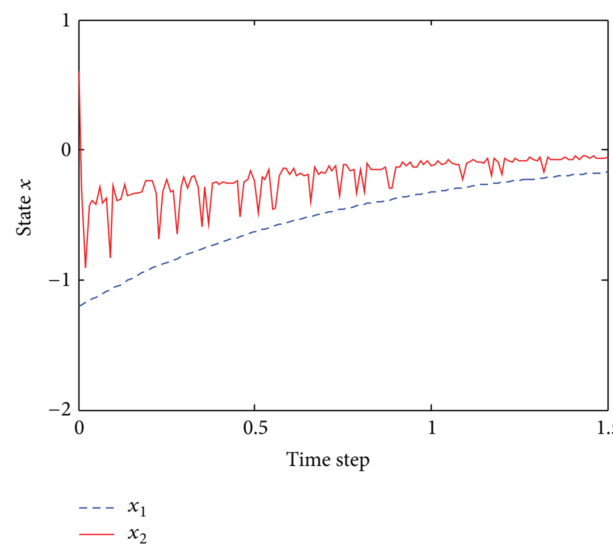

Figure 3: State responses under normal state feedback control.

The TRM is given as follows:

$$\Lambda = \begin{bmatrix}-1.3 & 0.2 & \hat{\lambda}_{13} & \hat{\lambda}_{14}\\ \hat{\lambda}_{21} & \hat{\lambda}_{22} & 0.3 & 0.3\\ 0.1 & \hat{\lambda}_{32} & -2.5 & \hat{\lambda}_{34}\\ \hat{\lambda}_{41} & 0.2 & \hat{\lambda}_{43} & -1.2\end{bmatrix}, \tag{61}$$

where the diagonal element $\hat{\lambda}_{22}$ is unknown and its lower bound $\lambda_d^2 = -5$ is a priori given value. The initial value of the state is $x_0 = [-1.2 \quad 0.6]^T$ and the initial mode is $r_0 = 1$.

The simulation step is taken as 150 time units and each unit length is taken as 0.01. We get the jump mode (**Figure 1**) and apply the designed MPC algorithm and normal state

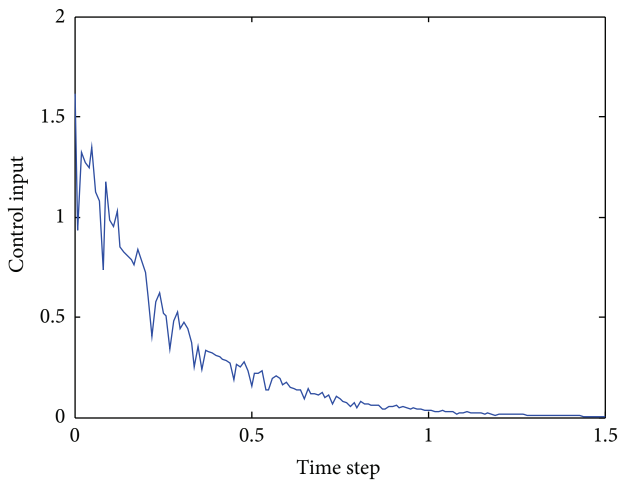

Figure 4: Control input under MPC.

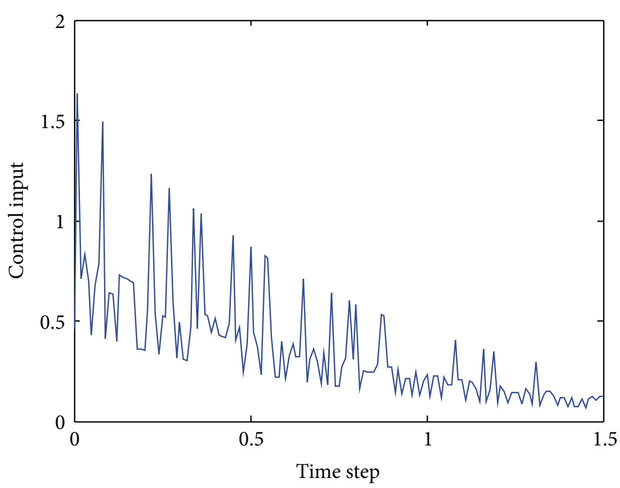

Figure 5: Control input under normal state feedback control.

feedback control to system (1), separately. Then, we get the simulation results in Figures 2–5.

Obviously, the unstable system (1) becomes more easily stable by using MPC strategy.

6. Conclusions

In this paper, MPC strategy is presented for continuous-time singular MJSs with incomplete transition rates. The controller design problem is formulated as LMI optimization algorithm. The predictive control strategy is proved to be feasible at every sampling time and it also can guarantee the mean square admissibility of the closed-loop system.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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