LRS Bianchi Type V Universe in Creation-Field Cosmology

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Abstract. The Hoyle-Narlikar C-field cosmology with LRS Bianchi type-V
space-time has been studied. Using methods of Narlikar and Padmanabham [1],
the solutions have been obtained when the creation field C is a function of time
t only. The geometrical and physical aspects of the model are also examined.

1 Introduction

The phenomenon of expanding universe, primordial nucleon-synthesis and the
observed isotropy of cosmic microwave background radiation (CMBR) are the
three very important observations in astronomy. These were successfully ex-
plained by big-bang cosmology based on Einstein's field equations. Smoot *et
al.* [2] revealed that the earlier predictions of the Friedman-Robertson-Walker
type of models do not always exactly match with our expectations. Some puz-
zling results regarding the red shifts from the extra galactic objects continue
to contradict the theoretical explanations given from the big-bang type of the
model. Also, CMBR discovery did not prove it to be a outcome of big-bang
theory. In fact, Narlikar *et al.* [3] have proved the possibility of non-relic in-
terpretation of CMBR. To explain such phenomenon, many alternative theories
have been proposed from time to time. Hoyle [4], Bondi and Gold [5] pro-
posed steady state theory in which the universe does not have singular begin-
ning nor an end on the cosmic time scale. To overcome this difficulty Hoyle and
Narlikar [6] adopted a field theoretic approach by introducing a massless and
chargeless scalar field C in the Einstein-Hilbert action to account for the mat-
ter creation. In the C-field theory introduced by Hoyle and Narlikar there is no
big-bag type of singularity as in the steady state theory of Bondi and Gold [5].
A solution of Einstein's field equations admitting radiation with negative energy
massless scalar creation field C was obtained by Narlikar and Padmanabhan [1].

The study of Hoyle and Narlikar theory [6–8] to the space-times with higher dimensions was carried out by Chatterjee and Banerjee [9]. RajBali and Tikekar [10] studied C-field cosmology with variable G in the flat Friedmann-Robertson-Walker model and with non-flat FRW space-time by RajBali and Kumawat [11]. The solutions of Einstein's field equations in the presence of creation field have been obtained for different Bianchi type universes by Singh and Chaubey [12].

We have examined a LRS Bianchi type-V cosmological model in Hoyle and Narlikar C-field cosmology. We have assumed that the creation field C is a function of time t only, *i.e.* $C(x,t) = C(t)$.

2   Hoyle and Narlikar *C*-field Cosmology

Introducing a massless scalar field called as creation field viz. C-field, Einstein's field equations were modified by Hoyle and Narlikar (1963, 1964 and 1966). The modified field equations are

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi \left({}^{m}T_{ij} + {}^{c}T_{ij} \right),\qquad(1)$$

where ${}^{m}T_{ij}$ is a matter tensor of Einstein's theory and ${}^{c}T_{ij}$ is a matter tensor due to the C-field which is given by

$$ {}^{c}T_{ij} = -f \left(C_i C_j - \frac{1}{2}g_{ij}C^k C_k \right),\qquad(2)$$

where $f > 0$ and $C_i = \dfrac{\partial C}{\partial x^i}$.

Because of the negative value of T^{00} $\left(T^{00} < 0 \right)$, the C-field has negative energy density producing repulsive gravitational field which causes the expansion of the universe. Hence, the energy conservation equation reduces to

$$ {}^{m}T^{ij}_{;j} = -{}^{c}T^{ij}_{;j} = fC^i C^j_{;j},\qquad(3)$$

i.e. the matter creation through non-zero left hand side is possible while conserving the over all energy and momentum.

The Above equation is similar to

$$mg_{ij}\frac{dx^i}{ds} - C_j = 0,\qquad(4)$$

which implies that the 4-momentum of the created particle is compensated by the 4-momentum of the C-field. In order to maintain the balance, the C-field must have negative energy.

3 Metric and Field Equations

The LRS Bianchi Type-V line element can be written as

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 e^{-2mx} \left(dy^2 + dz^2 \right) , \tag{5}$$

where the scale factors a_1 and a_2 are functions of t only and m is a constant.

We have assumed that creation field C is a function of time t only, *i.e.*

$$C(x,t) = C(t) \quad \text{and} \quad {}^m T^i_j = \text{diag}(\rho, -p, -p, -p). \tag{6}$$

We have assumed that the velocity of light and the gravitational constant are equal to one unit.

Now, the Hoyle-Narlikar field equations (1) for metric (5) with the help of equations (2), (3), and (6) will reduce to the following set of equations:

$$2\frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2^2}{a_2^2} - \frac{3m^2}{a_1^2} = 8\pi \left(\rho - \frac{1}{2} f \dot{C}^2 \right) , \tag{7}$$

$$2\frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_2^2}{a_2^2} - \frac{m^2}{a_1^2} = 8\pi \left(-p + \frac{1}{2} f \dot{C}^2 \right) , \tag{8}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} - \frac{m^2}{a_1^2} = 8\pi \left(-p + \frac{1}{2} f \dot{C}^2 \right) , \tag{9}$$

$$\frac{\dot{a}_1}{a_1} = \frac{\dot{a}_2}{a_2} , \tag{10}$$

$$\dot{\rho} + \left(\frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2} \right)(\rho + p) = f\dot{C}\left[\ddot{C} + \left(\frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2} \right)\dot{C} \right] , \tag{11}$$

where dot $(\cdot)$ indicates the derivative with respect to t.

From equation (10), we get

$$a_1 = c a_2 , \tag{12}$$

where c is the constant of integration.

The spatial volume is given as

$$V = a_1 a_2^2 . \tag{13}$$

From equation (12) and equation (13), we get

$$a_1 = c^{\frac{2}{3}} V^{\frac{1}{3}} . \tag{14}$$

The above equation (11) can be written in the form

$$\frac{d}{dV}(V\rho + p = f\dot{C}(V)\frac{d}{dV}[V\dot{C}(V)]. \tag{15}$$

In order to obtain a unique solution, one has to specify the rate of creation of matter-energy (at the expense of the negative energy of the C-field). Without loss of generality, we assume that the rate of creation of matter energy density is proportional to the strength of the existing C-field energy-density, *i.e.* the rate of creation of matter energy density per unit proper-volume is given by

$$\frac{d}{dV}(V\rho) + p = \alpha^2 \dot{C}^2 \equiv \alpha^2 g^2(V), \tag{16}$$

where α is a proportionality constant and we have defined $\dot{C}(V) \equiv g(V)$.

Substituting it in equation (15), we get

$$\frac{d}{dV}(V\rho) + p = fg(V)\frac{d}{dV}(Vg). \tag{17}$$

Comparing right hand sides of equations (16) and (17), we get

$$g(V)\frac{d}{dV}(gV) = \frac{\alpha^2}{f}g^2(V). \tag{18}$$

Integrating, which gives

$$g(V) = A_1 V^{(\alpha^2/f - 1)}, \tag{19}$$

where A_1 is arbitrary constant of integration.

We consider the equation of state of matter as

$$p = \gamma\rho. \tag{20}$$

Substituting equations (19) and (20) in the equation (16), we get

$$\frac{d}{dV}(V\rho) + \gamma\rho = \alpha^2 A_1^2 V^{2(\alpha^2/f - 1)}, \tag{21}$$

which further yields

$$\rho = \frac{\alpha^2 A_1^2}{\left(2\dfrac{\alpha^2}{f} - 1 + \gamma\right)} V^{2(\alpha^2/f - 1)}. \tag{22}$$

From equation (20), we get

$$p = \frac{\alpha^2 A_1^2 \gamma}{\left(2\dfrac{\alpha^2}{f} - 1 + \gamma\right)} V^{2(\alpha^2/f - 1)}. \tag{23}$$

Adding equations (8), 2 times (9) and 3 times equation (7), we get

$$\left(\frac{\ddot{a}_1}{a_1} + 2\frac{\ddot{a}_2}{a_2}\right) + 2\left(2\frac{\dot{a}_1\dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2^2}{a_2^2}\right) - \frac{6m^2}{a_1^2} = 12\pi(\rho - p). \tag{24}$$

From equation (13) we have

$$\frac{\ddot{V}}{V} = \left(\frac{\ddot{a}_1}{a_1} + 2\frac{\ddot{a}_2}{a_2}\right) + 2\left(2\frac{\dot{a}_1\dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2^2}{a_2^2}\right). \tag{25}$$

From equations (24), (25) and (20), we get

$$\frac{\ddot{V}}{V} - \frac{6m^2}{a_1^2} = 12\pi(1-\gamma)\rho. \tag{26}$$

Substituting equation (14) in equation (26), we get

$$\frac{\ddot{V}}{V} - \frac{6m^2}{V^{2/3}} = 12\pi(1-\gamma)\rho. \tag{27}$$

Substituting equation (22) in equation (27), we get

$$\frac{\ddot{V}}{V} - \frac{6m^2}{V^{2/3}} = 12\pi(1-\gamma)\frac{\alpha^2 A_1^2}{\left(2\frac{\alpha^2}{f} - 1 + \gamma\right)}V^{2(\alpha^2/f-1)}. \tag{28}$$

This further gives

$$\int \frac{dV}{\sqrt{\frac{12\pi(1-\gamma)A_1^2 f}{\left(2\frac{\alpha^2}{f} - 1 + \gamma\right)}V^{2\alpha^2/f} + 9m^2 V^{4/3} + k_1}} = t, \tag{29}$$

where k_1 is integration constant.

For $\gamma = 1$ (Zeldovich fluid or stiff fluid) and $k_1 = 0$, the above equation gives

$$V = m^3 t^3, \tag{30}$$

Substituting equation (30) in equation (19), we get

$$g = A_1 m^{3(\alpha^2/f-1)} t^{3(\alpha^2/f-1)}. \tag{31}$$

Also, from equation $\dot{C}(V) = g(V)$, we get the expression for creation field as

$$C = \frac{A_1 m^{3(\alpha^2/f-1)} t^{(3\alpha^2/f-2)}}{\left(3\frac{\alpha^2}{f} - 2\right)}. \tag{32}$$

Substituting equation (30) in equation (22), the homogeneous mass density becomes

$$\rho = \frac{1}{2}A_1^2 f m^{6(\alpha^2/f-1)} t^{6(\alpha^2/f-1)}. \tag{33}$$

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Using equation (23) and $\gamma = 1$, pressure becomes

$$p = \frac{1}{2}A_1^2 fm^{6(\alpha^2/f-1)}t^{6(\alpha^2/f-1)}. \tag{34}$$

From equations (33) and (34), it is observed that for $f = \alpha^2$, there is no singularity in density and pressure.

Using equation (12), (14) and (30), we get

$$a_1(t) = c^{2/3}mt, \tag{35}$$

$$a_2(t) = c^{-1/3}mt. \tag{36}$$

Substituting the above values, the line element (5) reduces to

$$ds^2 = dt^2 - c^{4/3}m^2t^2dx^2 - c^{-2/3}m^2t^2e^{-2mx}(dy^2 + dz^2). \tag{37}$$

4 Physical Properties

We define $a = (a_1a_2^2)^{1/3}$ as the average scale factor so that the Hubble's parameter in our anisotropic models may be defined as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\sum_{i=1}^{3} H_i\,,$$

where $H_i = \dfrac{\dot{a}_i}{a_i}$ are directional Hubble's factors in the direction of x^i, respectively.

The expansion scalar, θ, is given by

$$\theta = 3H,$$
$$\theta = \frac{3}{t}. \tag{38}$$

The mean anisotropy parameter, $\bar{h}$, is given by

$$\bar{h} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right), \tag{39}$$
$$\bar{h} = 0.$$

The shear scalar, σ^2, is given by

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3}H_i^2 - 3H^2\right) = \frac{3}{2}AH^2, \tag{40}$$
$$\sigma^2 = 0.$$

The deceleration parameter q is given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1,$$
$$q = 0. \tag{41}$$

5 Conclusion

In this paper we have studied the space-time geometry corresponding to LRS Bianchi type-V in Hoyle-Narlikar creation field theory of gravitation [6–8]. The scale factors a_1 and a_2 are proportional to time t. The deceleration parameter $(q = 0)$ indicates that the universe is expanding with a constant velocity.

The expansion scalar θ starts with the infinite value at $t = 0$ and then gradually decreases. The expansion halts at $t = \infty$. The mean anisotropy parameter is found to be zero. It means that the universe becomes isotropic throughout its evolution. Here we get the ratio $\sigma/\theta = 0$. Therefore, the model is isotropic. In general, we have observed that the creation field C is proportional to the time t. That is, the creation of matter increases as time increases.

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