2013 IEEE/RSJ International Conference on
Intelligent Robots and Systems (IROS)
November 3-7, 2013. Tokyo, Japan

Iterative Learning Identification and Computed Torque Control of Robots

M. Gautier, A. Jubien and A. Janot

***Abstract*— This paper deals with a new iterative learning dynamic identification and control method of robot. The robot is closed-loop controlled with a Computed Torque Control (*CTC*). The parameters of the Inverse Dynamic Model (*IDM*), which calculates the *CTC* are calculated to minimize the quadratic error between the actual joint force/torque and a joint force/torque calculated with the Inverse Dynamic Identification Model (*IDIM*), linear in relation to the parameters. Usually the parameters are off-line linear least squares estimated ((*IDIM-LS*) where the *IDIM* is calculated with the joint position and its noisy derivatives, which cannot take into account variations of the parameters. The new method called *IDIM-ILIC* (*IDIM* with Iterative Learning Identification and Control) overcomes these 2 drawbacks. The parameters are periodically calculated over a moving time window to update the *IDM* of the *CTC*, and the *IDIM* is calculated with the noise-free data of the trajectory generator, which avoids using the noisy derivatives of the actual joint position. A study of convergence of the method is performed in simulation and an experimental setup with stationary parameters and with a variation of the payload on a prismatic joint validates the procedure.**

I. Introduction

The best tracking performances for robots use feed-forward inverse dynamic model-based controller. It requires accurate knowledge of the robot parameters. In the case of stationary parameters, the usual off-line method based on the Inverse Dynamic Identification Model and linear Least Squares estimation (*IDIM-LS*) have given good results [1].

In the case of non-stationary parameters (friction) or variation of the payload, an online procedure updating the parameters, is required. Most of online estimation methods minimize the trajectory tracking error to estimate the parameters and are related to adaptive and reconfigurable control [2-8]. The optimal values of the parameters are calculated using non-linear programming algorithms to solve a non-linear least-squares problem. Difficulties arise from the choice of initial conditions and loss of persistent excitation, resulting in loss of stability. In [9] the authors propose a combination of model-based and iterative learning but their method requires the error on velocity to estimate the parameters. Online identification methods proposed in [10] and [11] are interesting because they avoid the use of joint accelerations and velocities however the measurement of joint position is still necessary.

This paper presents a new iterative learning dynamic identification method, which avoids the drawbacks of the usual online estimation methods. The robot is closed-loop controlled with a *CTC* law [12][13] that linearizes and decouples the non-linear and coupled dynamics of the robot. The actual optimal parameters of the Inverse Dynamic Model (*IDM*) of the controller are periodically calculated to minimize the quadratic error between the actual joint force/torque and a joint force/torque calculated with the Inverse Dynamic Identification Model (*IDIM*), over a moving time window. The regressor matrix of *IDIM* is calculated with noise-free data of the trajectory generator, which avoids using the noisy derivatives of the actual joint position. The updating of the dynamic parameters of *IDM* in the *CTC* law allows taking into account the variation of non-stationary parameters as friction parameters or payload parameters. The estimated parameters are optimal with respect both to the Least Squares (*LS*) identification (minimize the quadratic error on joint force/torque) and to the *CTC* law (decrease the tracking error).

This paper is divided into 7 sections. Section II describes the dynamic modeling. Section III presents the usual offline *IDIM-LS* method for dynamic identification of robots. Section IV presents the *CTC* law. Section V presents the proposed Iterative Learning Identification and Control method called *IDIM-ILIC*. Section VI is devoted to the study of convergence of the method in simulation and on experimental identification of one prismatic joint manipulator without and with stationary parameters and with a variation of payload. The values identified with *IDIM-ILIC* are compared with those identified with the usual *IDIM-LS* method. Finally, section VII gives the conclusion.

II. MODELING

The *IDM* of a rigid robot calculates the motor force/torque τ_{idm} as a function the joint positions, velocities and accelerations of the n moving links. It can be obtained from the Newton-Euler equations [14] as following:

$$\tau_{idm} = M(q)\ddot{q} + N(q,\dot{q}) \quad (1)$$

Where q, $\dot{q}$ and $\ddot{q}$ are respectively the $(n \text{ x } 1)$ vectors of joint positions, velocities and accelerations; $M(q)$ is the

Manuscript received July 31, 2013.

M. Gautier are with the University of Nantes and the IRCCyN (Institut de Recherche en Communications et Cybernétique de Nantes), Nantes, France (e-mail: maxime.gautier@irccyn.ec-nantes.fr).

A. Jubien is with IRCCyN, Nantes, France and ONERA (the French Aerospace Lab), Toulouse, France (e-mail: anthony.jubien@irccyn.ec-nantes.fr).

A. Janot is with ONERA (the French Aerospace Lab), Toulouse, France (e-mail: alexandre.janot@onera.fr).

$(n \text{ x n})$ robot inertia matrix; $N(q,\dot{q})$ is the $(n \text{ x } 1)$ vector of centrifugal and frictions force/torque.

The choice of the Modified Denavit and Hartenberg (*MDH*) frames attached to each link allows to calculate a dynamic model that is linear in relation to a set of standard dynamic parameters χ_{st} [15]:

$$\tau_{idm} = IDM_{st}(q,\dot{q},\ddot{q})\chi_{st} \text{ with } \chi_{st} = \begin{bmatrix} \chi_{st1}^T & \chi_{st2}^T & \dots & \chi_{stn}^T \end{bmatrix}^T \quad (2)$$

Where $IDM_{st}(q,\dot{q},\ddot{q})$ is the $(n \text{ x } Ns)$ Jacobian matrix of τ_{idm}, with respect to the $(Ns \text{ x } 1)$ vector χ_{st} of the standard parameters. χ_{stj} is composed of standard dynamic parameters of axis j:

$$\begin{aligned} \chi_{stj} = [& XX_j \quad XY_j \quad XZ_j \quad YY_j \quad YZ_j \quad ZZ_j \\ & MX_j \quad MY_j \quad MZ_j \quad M_j \quad Ia_j \quad Fv_j \quad Fc_j \quad \tau_{off_j}]^T \end{aligned} \quad (3)$$

Where: $XX_j, XY_j, XZ_j, YY_j, YZ_j, ZZ_j$ are the six components of the inertia matrix of link j; MX_j, MY_j, MZ_j are the components of the first moments of link j; M_j is the mass of link j; Ia_j is a total inertia moment for rotor of actuator and gear of joint j; Fv_j and Fc_j are the viscous and Coulomb friction parameters of joint j; τ_{off_j} is an offset parameter which takes into account the dissymmetry of Coulomb friction of joint j and the motor current amplifier offset of joint j; $Ns = 14 \text{ x } n$ is the number of standard parameters.

III. *IDIM-LS*: INVERSE DYNAMIC IDENTIFICATION MODEL WITH LEAST SQUARES METHOD

Because of perturbations due to noise measurement and modeling errors, the actual force/torque τ differs from τ_{idm} by an error e, such that:

$$\tau = \tau_{idm} + e = IDM_{st}(q,\dot{q},\ddot{q})\chi_{st} + e \quad (4)$$

The vector $\hat{\chi}_{st}$ is the least squares (*LS*) solution of an over determined system built from the sampling of (4), while the robot is tracking exciting trajectories [16]:

$$\mathrm{Y} = \mathrm{W}_{st}(q,\dot{q},\ddot{q})\chi_{st} + \rho \quad (5)$$

Where: Y is the $(r\text{x}1)$ measurement vector, W_{st} the $(r\text{x}n_{st})$ observation matrix, and ρ is the $(r\text{x}1)$ vector of errors. The number of rows is $r = n * n_e$, where the number of recorded samples is n_e. When W_{st} is not a full rank matrix, the *LS* solution is not unique. The system (5) is rewritten:

$$\mathrm{Y} = \mathrm{W}(q,\dot{q},\ddot{q})\chi + \rho \quad (6)$$

Where a subset W of b independent columns of W_{st} is calculated, which defines the vector χ of b base parameters [15][17]. Standard deviations $\sigma_{\hat{\chi}_i}$, are estimated assuming that W is a deterministic matrix and ρ, is a zero-mean additive independent Gaussian noise, with a covariance matrix [18]:

$$C_{\rho\rho} = E(\rho\rho^{\mathrm{T}}) = \sigma_\rho^2 I_r \quad (7)$$

Where E is the expectation operator and I_r, the $(r \times r)$ identity matrix. An unbiased estimation of the standard deviation σ_ρ is the following:

$$\hat{\sigma}_\rho^2 = \|Y - W\hat{\chi}\|^2 / (r - b) \quad (8)$$

The covariance matrix of the estimation error is given by:

$$C_{\hat{\chi}\hat{\chi}} = E[(\chi - \hat{\chi})(\chi - \hat{\chi})^{\mathrm{T}}] = \hat{\sigma}_\rho^2 (\mathrm{W}^{\mathrm{T}}\mathrm{W})^{-1} \quad (9)$$

The relative standard deviation $\%\sigma_{\hat{\chi}_{ri}}$ is given by:

$$\%\sigma_{\hat{\chi}_{ri}} = 100\,\sigma_{\hat{\chi}_i} / |\hat{\chi}_i|, \text{ for } |\hat{\chi}_i| \neq 0 \quad (10)$$

Where $\sigma_{\hat{\chi}_i}^2 = C_{\hat{\chi}\hat{\chi}}(i,i)$ is the i$^{\text{th}}$ diagonal coefficient of $C_{\hat{\chi}\hat{\chi}}$. Calculating the *LS* solution of (6) from perturbated data in W and Y may lead to bias if W is correlated to ρ. Then, it is essential to filter data in Y and W before computing the *LS* solution. Velocities and accelerations are estimated by means of a band-pass filtering of the positions. To eliminate high frequency noises and force/torque ripples, a parallel decimation is performed on Y and on each column of W. More details about data filtering can be found in [18] and [19].

IV. COMPUTED TORQUE CONTROL – TRACKING CONTROL

To improve performance of the control, it is necessary to take into account the dynamic interaction force/torque. Linearizing and decoupling control is based on feed-forward compensation of the nonlinearities in the robot dynamics [12-14]. It is called *CTC* because it uses the *IDM*, which computes the force/torque control input. The input control τ is defined such that:

$$\begin{aligned} \tau = \hat{M}(q_r)w + \hat{N}(q_r,\dot{q}_r) \text{ with } q_r &= \begin{bmatrix} q_{r1} & \cdots & q_{rn} \end{bmatrix}^T \\ \text{and } \dot{q}_r &= \begin{bmatrix} \dot{q}_{r1} & \cdots & \dot{q}_{rn} \end{bmatrix}^T \end{aligned} \quad (11)$$

Where $\hat{M}(q_r)$ and $\hat{N}(q_r,\dot{q}_r)$ are the estimations of the actual $M(q_r)$ and $N(q_r,\dot{q}_r)$ matrices of the robot respectively, q_r and $\dot{q}_r$ are the reference position and velocity. Ideally without error modeling and no error in actual joint data, after substituting (11) in (1), the problem reduces to the linear closed-loop control of n decoupled double integrators:

$$\ddot{q} = w \text{ with } \ddot{q} = \begin{bmatrix} \ddot{q}_1 & \cdots & \ddot{q}_n \end{bmatrix}^T \quad (12)$$

Where w is a new input control, like Proportionnal-Derivative control:

$$w = K_p e_q + K_v \dot{e}_q + \ddot{q}_r \text{ with } e_q = q_r - q \text{ and } \dot{e}_q = \dot{q}_r - \dot{q} \quad (13)$$

Where K_p et K_v are $(n\text{x}n)$ positive diagonal matrices of proportional and derivative gains.

From (11), the closed-loop system response is determined by the following decoupled linear error equation :

$$K_p e_q + K_v \dot{e}_q + \ddot{e}_q = 0 \text{ with } \ddot{e}_q = \ddot{q}_r - \ddot{q} \quad (14)$$

The solution $e_q(t)$ is the free response of a second order system. The j^{th} gains of K_p and K_v are tuned to give 2 stable and fast poles for joint j given by the damping coefficient ξ_j and a natural frequency ω_j :

$$K_{pj} = \omega_j^2,\ K_{vj} = 2\xi_j \omega_j \quad (15)$$

The control input force/torque includes three components; the first compensates the Coriolis, centrifugal, gravity and friction effects; the second is a proportional and derivative control with variables gains $\hat{M}(q_r)K_p$ and $\hat{M}(q_r)K_v$ respectively and the third provides a predictive action of the desired acceleration forces/torques $\hat{M}(q_r)\ddot{q}_r$.

The force/torque τ can be rewritten in linear relation the estimation of χ at iteration k, $\hat{\chi}^k$:

$$\tau(\hat{\chi}^k) = IDM(q_r, \dot{q}_r, w)\hat{\chi}^k \quad (16)$$

Equation (16) is the *IDM* (2) where $\ddot{q}$ is replaced by the input control noted w. q and $\dot{q}$ are replaced by q_r and $\dot{q}_r$.

The columns of $IDM(q, \dot{q}, w)$ corresponding to the coulomb frictions contain the non linear $sign(\dot{q}_j)$ function. To avoid discontinuities on the force/torque $\tau(\hat{\chi}^k)$ during the crossing 0 velocity $\dot{q}_j$, the non linear $sign(\dot{q}_j)$ function is replaced by the continuous $(2/\pi)\arctan(\alpha_j\ \dot{q}_j)$ function in (4). α_j is a ratio of the maximum joint velocity $\dot{q}_{j\max}$.

V. *IDIM-ILIC* METHOD

A. Theoretical approach

The proposed method is a new iterative learning dynamic identification method with control procedure. The identification process does not need the joint position measurement or velocity and acceleration estimation. At each iteration, the parameters of the *CTC* are updated with the last identified parameters. In the presence of modeling errors, the error equation (14) becomes from (11) and (1):

$$\begin{aligned} & K_p e_q + K_v \dot{e}_q + \ddot{e}_q \\ & = \hat{M}^{-1}(q_r)\left(M(q)\ddot{q} + N(q,\dot{q}) - \hat{M}(q_r)\ddot{q} - \hat{N}(q_r,\dot{q}_r)\right) \end{aligned} \quad (17)$$

Without modeling error nor error on the parameter values, the error equation (17) becomes the free second order differential equation (14), where $e_q(t) \to 0$ with dynamics depending on the gains K_p and K_v. Usually, in robotics, these gains are high enough to get fast dynamics and good robustness to error modeling. Unfortunately, the perfect model hypothesis is implausible in practice. Indeed, the values of parameters are not perfectly known and there are always small errors in the model. So, in this case, $e_q(t) \to 0$ may not hold because of the right member of (17).

For strong nonlinear systems such as robots, it is impossible to analyze the effects of such errors. However, with practical considerations, we can make some well founded approximations. First, in recent papers, [20], we have shown that the crucial component is $\Delta M\ddot{q} = (M - \hat{M})\ddot{q}$ because the vectors of centrifugal and frictions force/torque N and $\hat{N}$ can be considered as a perturbation. Second, in order to show that $e_q(t)$ is bounded, we consider one degree of freedom (*dof*) robot with only one parameter M in (17), which reduces to:

$$K_p e_q + K_v \dot{e}_q + \ddot{e}_q = (\alpha - 1)\ddot{q} \text{ with } \alpha = M/\hat{M} \quad (18)$$

Finally, with $\ddot{q} = \ddot{q}_r - \ddot{e}_q$, we have:

$$K_p e_q + K_v \dot{e}_q + \alpha \ddot{e}_q = (\alpha - 1)\ddot{q}_r \quad (19)$$

Thus, with α close to 1, with appropriate gains and because $\ddot{q}_r$ is a bounded trajectory, $e_q(t)$ is also bounded.

Then if $\ddot{q}$ remains close to $\ddot{q}_r$, (17) becomes:

$$\begin{aligned} & \left(K_p e_q + K_v \dot{e}_q + \ddot{e}_q\right) \\ & = \hat{M}^{-1}(q_r)\left(IDM(q,\dot{q},\ddot{q})\chi^{robot} - IDM_r(q_r,\dot{q}_r,\ddot{q}_r)\chi\right) \end{aligned} \quad (20)$$

Where χ^{robot} contains the "real" parameters of the robot. In accordance with a learning process where $\chi = \hat{\chi}^k$ in (16) at iteration k, (20) becomes:

$$\begin{aligned} & \left(K_p e_q(\hat{\chi}^k) + K_v \dot{e}_q(\hat{\chi}^k) + \ddot{e}_q(\hat{\chi}^k)\right) \\ & = \left(\hat{M}^k(q_r)\right)^{-1}(IDM\left(q(\hat{\chi}^k),\dot{q}(\hat{\chi}^k),\ddot{q}(\hat{\chi}^k)\right)\chi^{robot} \\ & \qquad - IDM_r(q_r,\dot{q}_r,\ddot{q}_r)\chi) \end{aligned} \quad (21)$$

To satisfy (14), it is necessary to minimize the following error between the actual joint torque of the robot and its *IDIM* model:

$$\left[IDM\left(q(\hat{\chi}^k),\dot{q}(\hat{\chi}^k),\ddot{q}(\hat{\chi}^k)\right)\chi^{robot} - IDM_r(q_r,\dot{q}_r,\ddot{q}_r)\chi\right] \quad (22)$$

It can be rewritten introducing an error e:

$$\tau(\hat{\chi}^k) = IDM_r(q_r,\dot{q}_r,\ddot{q}_r)\chi + e \quad (23)$$

Thus the vector $\hat{\chi}^{k+1}$ at iteration $k+1$ is the least squares (*LS*) solution of an over determined system built from the sampling and filtering of (23):

$$\begin{aligned} & Y(\hat{\chi}^k) = W_r(q_r,\dot{q}_r,\ddot{q}_r)\chi + \rho \\ & \text{with } \hat{\chi}^{k+1} = \arg\min_{\chi}\left\|Y(\hat{\chi}^k) - W_r(q_r,\dot{q}_r,\ddot{q}_r)\chi\right\|^2 \end{aligned} \quad (24)$$

Where: $Y(\hat{\chi}^k)$ is a vector obtained by filtering and down-sampling the actual sampled force/torque $y = \tau(\hat{\chi}^k)$ at iteration k, $W_r(q_r,\dot{q}_r,\ddot{q}_r)$ is the observation matrix, and ρ is the vector of errors. It should be noted that $W_r(q_r,\dot{q}_r,\ddot{q}_r)$

is calculated with the *IDM* (2) where the actual noisy position, velocity and acceleration are replaced by the noise free data of the reference trajectory.

B. Bias in the estimation of parameters

In practice, the velocity is estimated by backward difference of the measured position. In (19) $\dot{e}_q$ must be replaced by $\hat{\dot{e}}_q = \dot{q}_r - \hat{\dot{q}}$, with:

$$\hat{\dot{q}}(z) = q(z)\left(1 - z^{-1}\right) f_e \text{ with } z^{-1} = e^{-s/f_e} \quad (25)$$

Where f_e is the sampling frequency of the controller, $q(z)$ is the z transform of q, z^{-1} is the delay operator. So the z transform of (19) becomes:

$$K_p e_q + K_v\left(\dot{q}_r - \hat{\dot{q}}\right) + \alpha \ddot{e}_q = (\alpha - 1)\ddot{q}_r \quad (26)$$

In order to compare (19) and (26) with the same left side expression, (26) is rewritten as:

$$K_p e_q + K_v \dot{e}_q + \alpha \ddot{e}_q = (\alpha - 1)\ddot{q}_r - K_d\left(\dot{q} - \hat{\dot{q}}\right) \quad (27)$$

The error $K_d\left(\dot{q} - \hat{\dot{q}}\right)$ on velocity estimation introduces a small bias in the estimation of parameters which decreases with increasing the frequency f_e. In practice, a simulation study shows that the bias is less than 10^{-4} with $f_e > 500$(Hz) and that it does not increase the tracking error.

C. Scheme of the IDIM-ILIC procedure

The structure of the online identification scheme is shown in 1. The identification is performed with a moving time window of size T_{obs} (s) and the parameters are updated with frequency f_i (Hz). If $f_i = 1/T_{obs} = 1/T_{cycle}$, the parameters are updated at each cycle time T_{cycle} of the trajectory [9].

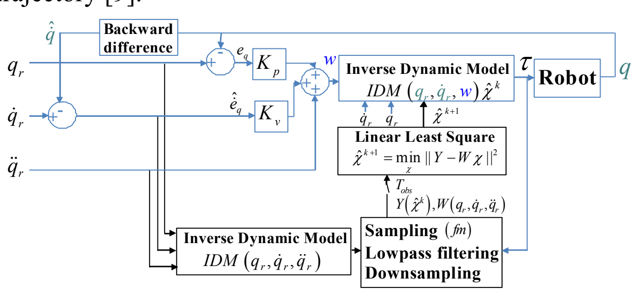

Figure 1. *IDIM-ILIC* scheme

D. Initialization

The algorithm is sensitive to the initial conditions because the bandwidth of the closed-loop should be large enough to get fast dynamics and good robustness to error modeling. Thus the initial values of inertia parameters need to be close to the actual ones for $\hat{M}$ to be close enough to the good inertia matrix. Consequently the initial parameters are chosen as:

$$\hat{\chi}^0 = 0 \text{ except } Ia_j = Ia_j^{ap} \text{ for } j = 1, n. \quad (28)$$

Ia_j^{ap} can be well estimated with the rotor inertia of the motor given by the manufacturer of the robot.

E. Update of identified parameters in the IDIM of controller

In case of loss of persistent exciting trajectories, all parameters may not be excited during tracking trajectory at iteration k. It is necessary to avoid loss of performances of the controller between the iterations k and $k+1$ by updating bad identified parameters. Only the essential parameters are updated. They are calculated with respect to their relative standard deviation. If the relative standard deviation $\%\sigma_{\hat{\chi}_{ri}}$ of parameter $\hat{\chi}_i^{k+1}$ is greater than a value $\%\sigma_{MAX}$, the parameters $\hat{\chi}_i^{k+1}$ is not essential and not updated in the controller (16). $\%\sigma_{MAX}$ can be chosen between 10 and 30.

If the reference trajectory does not excite all parameters, this allows keeping the best possible control law without increasing the computing time significantly at each iteration.

F. Discussion

For usual *IDIM-LS* method, the observation matrix depends on the estimation of actual positions, velocities and accelerations $\left(q, \hat{\dot{q}}, \hat{\ddot{q}}\right)$. But the computation of velocities and accelerations needs a well-tuned filter. If the filter is not well-tuned, the *LS* estimation is biased.

In the proposed iterative learning identification method with control procedure the observation matrix depends on the noise-free reference positions, velocities and accelerations $\left(q_r, \dot{q}_r, \ddot{q}_r\right)$. The measurement of position and the estimation of velocity are only necessary to compute the new input control w, the bias in identified parameters is very small after convergence and does not increase the tracking error.

Other interests of the method are the following:

-the actual force/torque at iteration k is used at each iteration of the algorithm. It allows identifying the non-stationary parameters of robot, especially for friction coefficients and for the actual inertia/mass value if the payload changes during the task cycle of the robot.

-only the *IDM* is needed for the *CTC* (16) law and for the identification to compute the matrix W_r,

-only the well identified parameters (essential parameters) are updated in the controller (see section V-E) compared to usual online estimation methods. It avoids the loss of stability in the case of loss of excitation,

-the estimated parameters are optimal both with respect to the *LS* identification and to the *CTC* law, by decreasing the tracking error and minimizing the quadratic error on joint force/torque.

VI. Experimental Validation

A. Presentation of the prismatic joint robot

The *EMPS* is a high-precision linear Electro-Mechanical Positioning System (see figure 2). Its main components are a Maxon DC motor equipped with an incremental encoder, a

Star high-precision low-friction ball screw drive positioning unit (with negligible backlash) and a carriage which moves in translation. A payload (10(Kg)) can be added on the carriage.

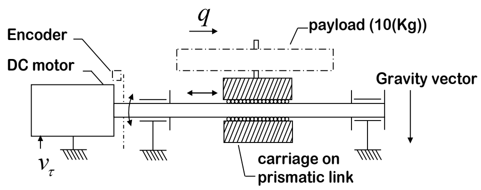

Figure 2. EMPS components

All variables and parameters are given in *SI* units on the load side. The motor force (unit on load side) is proportional to the motor torque (unit on motor side) with factor equal to the gear ratio of the ball screw.

B. Inverse Dynamic Model of the prismatic joint robot

The robot is a rigid structure:

$$\tau_{idm} = M\ddot{q} + F_v\dot{q} + F_c sign(\dot{q}) \tag{29}$$

Where: M (kg) is the total mass (rotor of motor, screw, nut and carriage), F_v (N/(m/s)) and F_c (N) are the viscous and Coulomb friction parameters respectively.

The motor force of the rigid robot τ_{idm} can be written in a linear relation to the dynamic parameters as follows:

$$\begin{aligned} &\tau_{idm} = IDM_{st}\ \chi_{st} \\ &\text{with: } IDM_{st} = \left(\ddot{q} \quad \dot{q} \quad sign(\dot{q})\right), \chi_{st} = \left(M \quad F_v \quad F_c\right)^T \end{aligned} \tag{30}$$

There are $n_{st} = 3$ parameters to be identified.

C. Control of the prismatic joint robot

From equations (16) and (30), the control law $\tau\left(\hat{\chi}^k\right)$ is noted in relation to estimation of *EMPS* parameters at iteration *k*:

$$\begin{aligned} &\tau\left(\hat{\chi}^k\right) = \left(w \quad \dot{q}_1 \quad 2\arctan\left(\alpha\dot{q}_1\right)/\pi\right)\chi^k \\ &\text{with: } \hat{\chi}^k = \left(\hat{M}_1^k \quad \hat{F}_{v1}^k \quad \hat{F}_{c1}^k\right)^T \end{aligned} \tag{31}$$

D. Data acquisition

Motor position is measured by means of high precision encoder working in quadrature count mode (accuracy of 12500 counts per revolution). The sample acquisition frequency for joint position and current reference (drive force) is 1 (KHz). We calculate the motor force using the relation:

$$\tau = {}^{ap}g_\tau v_\tau \tag{32}$$

where v_τ is the current reference of the amplifier current loop, and ${}^{ap}g_\tau$ is *a priori* value of the gain of the joint drive chain, which is taken as a constant value in the frequency range of the robot (less than 30Hz) because of the large bandwidth of the current loop (700 Hz). In practice, (31) is divided by ${}^{ap}g_\tau$ to compute the input current reference $v_\tau\left(\hat{\chi}^k\right)$. Exciting tracking trajectory consists to a concatenation of trapezoidal velocity signals with different amplitudes and times T_{obs} = T_{cycle} =6.2(s). The α value is fixed at 1000. The damping coefficient ξ is $\sqrt{2}/2$ and the control bandwidth ω is 20Hz. The cut-off frequency low pass filter and the frequency of down sampling are fixed at 5Hz (*FIR* decimate filter block function of Matlab).

E. Study of convergence in simulation with stationary parameters

The proposed identification algorithm is carried out in simulation to study the convergence and to study the bias described in section V-B.

The parameters of the simulated robot are given by the following values:

$$M_1 = 95 \text{ (kg)}; \quad Fv_1 = 200 \text{ (Ns/m)}; \quad Fc_1 = 20 \text{ (Ns/m)} \tag{33}$$

$M^0 = f(Ia^{ap})$ is equal to 100(kg) and the other parameters are initialized with 0. At each new identification with $f_i = 1/T_{obs}$, the previous identified parameters are updated with the new ones in the computed torque controller. Results are given in table I at iterations *k* =1 and *k* =2.

TABLE I. IDENTIFIED PARAMETERS WITH THE LEARNING IDENTIFICATION METHOD AND WITH *IDIM-LS* METHOD

Parameter	$\hat{\chi}^1$	$\sigma_{\hat{\chi}_r^1}$ (%)	$\hat{\chi}^2$	$\sigma_{\hat{\chi}_r^2}$ (%)
M_1 (kg)	**94.4**	0.07	**95**	0.01
Fv_1 (Ns/m)	**199**	0.26	**200**	$1\ 10^{-3}$
Fc_1 (N)	**20.1**	0.21	**20**	$1\ 10^{-3}$

As a reference, offline identification with *IDIM-LS* method is performed using the simulated position, velocity and acceleration. For the three iterations the parameters with *IDIM-LS* method are identified with an accuracy of 10^{-16}. While the identified parameters with the learning identification method are identified with an accuracy of 10^{-4} for *k*=3. The bias due to the estimation of velocity (25) the controller is very small in simulation.

F. IDIM-ILIC method with stationary parameters

The proposed iterative learning identification algorithm is carried out on the real robot. In this case, identification is performed at each cycle time T_{cycle} of trajectory with $f_i = 1/T_{obs}$. The parameters are initialized to the same values of section VI-E. At each new identification, the previous identified parameters are updated with the new ones in the computed torque controller if $\%\sigma_{\hat{\chi}_{ri}}$ is smaller than a value $\%\sigma_{MAX}$ equals to 15.

Moreover, offline identification with *IDIM-LS* method is performed. Because *IDIM-LS* method uses the actual position, velocity and acceleration, the *IDIM-LS* identified parameters are considered as references for the study of the proposed *IDIM-ILIC* method. The numerical values of the identified parameters are given in Table I.

The proposed algorithm converges after only 2 iterations on the real robot, all parameters are well identified. The identified values with *IDIM-ILIC* method are close to *IDIM-LS* identified values for $k=2$. However a bias is present, the relative error between both method is less 1.5%.

TABLE II. TABLE I. IDENTIFIED PARAMETERS WITH *IDIM-ILIC* METHOD AND WITH *IDIM-LS* METHOD

	IDIM-ILIC method					*IDIM-LS method*			
Parameter	$\hat{\chi}^0$	$\hat{\chi}^1$	$\sigma_{\hat{\chi}^1_r}$ (%)	$\hat{\chi}^2$	$\sigma_{\hat{\chi}^2_r}$ (%)	$\hat{\chi}^1$	$\sigma_{\hat{\chi}^1_r}$ (%)	$\hat{\chi}^2$	$\sigma_{\hat{\chi}^2_r}$ (%)
M (Kg)	**100**	**92.2**	2.9	**94.2**	0.4	**93.8**	0.4	**93.1**	0.4
Fv (Ns/m)	**0**	**298**	3.4	**263**	1.5	**262**	1.4	**265**	1.4
Fc (N)	**0**	**22.3**	3.2	**18.9**	1.6	**18.6**	1.5	**18.7**	1.5
$\Vert Y-WX\Vert / \Vert Y\Vert$	-	10.2%		4.6%		4.2%		4.3%	
$\Vert e_q\Vert / \Vert q-\bar{q}\Vert$	9.0 10^{-4}	1.1 10^{-4}		7.0 10^{-5}		7.2 10^{-5}		7.2 10^{-5}	

The decrease of the tracking error with *IDIM-ILIC* method is important and validates the proposed theoretical approach (see figure 3 and last line of Table I).

The *IDIM-LS* identified values are well identified from the first iteration but in our case the filter is well tuned. If it is not the case, the parameters are biased and the *CTC* law is not good and the tracking error increases.

G. *IDIM-ILIC method with variation of the payload*

After this experiment, the mass is changing with a payload of 10(kg) attached to the carriage at time 60(s) (see figure 4). After convergence, $\hat{M}$ is 104(kg) with *IDIM-ILIC* method and 103Kg with *IDIM-LS* method. The payload $\Delta\hat{M}$ is well identified (9.7(kg))). Thus the controller is adapted and optimized online according to the variation of the payload.

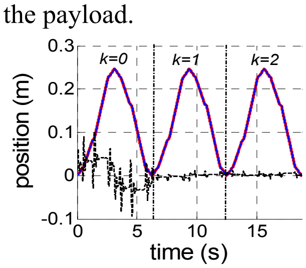

Figure 3. Position (Red: reference, Blue: measurement, Black :error –for position error is multiply by 100-)

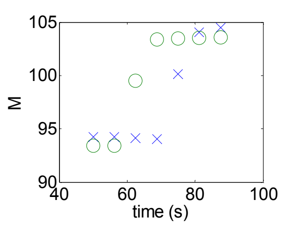

Figure 4. Identified mass (cross: *IDIM-ILIC* method, circle: *IDIM-LS* method)

VII. CONCLUSION

This paper has presented a new iterative learning identification method with control procedure using the force/torque data. It avoids using the noisy derivatives of the actual joint position by using the noise-free data of the trajectory generator to calculate the inverse dynamic model feedforward componenet in the *CTC* control law, expect for the computation of the feedback component (new control input w). The proposed method converges faster, is robust to initial conditions and is very efficient for non-stationary parameters (specially the load and friction parameters) during the execution of the trajectory. It allows keeping good *CTC* if the tracking trajectory does not excite all parameters. The application of this method on a simulation and on a real prismatic joint robot shows the effectiveness of our approach. Future works concern the use of the proposed method on a multi-*dof* rigid robot and the study of the moving time window size.

REFERENCES

[1] J. Hollerbach, W. Khalil, and M. Gautier, "Model Identification," in *Springer Handbook of Robotics*, Springer, 2008.

[2] J. Craig, *Adaptive Control of Mechanical Manipulators*. Addison-Wesley Publishing Company, 1988.

[3] N. Sadegh and R. Horowitz, "Stability and Robustness Analysis of a Class of Adaptive Controllers for Robotic Manipulators," *International Journal of Robotics Research*, vol. 9, no. 3, pp. 74–92, 1990.

[4] M. Stoline, "Adaptive manipulator control: A case study," *IEEE Transactions on Automatic Control*, vol. 33, pp. 995–1003, 1988.

[5] R. Middleton and A. Goodwin, "Adaptive computed torque control for rigid link manipulators," *Systems and Control Letters*, vol. 10, pp. 9–16, 1988.

[6] W. Li and J.-J. Stoline, "An indirect adaptive robot controller," *Systems and Control Letters*, vol. 12, pp. 259–266, 1989.

[7] K. El Serafi and W. Khalil, "Energy based indirect adaptive control of robots," IEEE International Conference on Robotics and Automation, 1991, vol. 3, pp. 2142–2147.

[8] I. D. Landau, R. Horowitz, and F. De Bruyne, "Synthesis of adaptive controllers for robot manipulators using a passive feedback systems approach," IEEE Conference on Robotics and Automation, 1988, vol. 2, pp. 1028–1033.

[9] B. Bukkems, D. Kostic, B. De Jager, and M. Steinbuch, "Learning-based identification and iterative learning control of direct-drive robots," *IEEE Transactions on Control Systems Technology*, vol. 13, no. 4, pp. 537–549, 2005.

[10] P. Bao, P. Jiang, and H.-T. . Chen, "A learning scheme for the parameter identification of robot dynamics," IEEE International Conference on Industrial Technology, 1996, pp. 651–655.

[11] F. Zhang, D. M. Dawson, M. S. De Queiroz, and W. E. Dixon, "Global adaptive output feedback tracking control of robot manipulators," *IEEE Transactions on Automatic Control*, vol. 45, no. 6, pp. 1203–1208, 2000.

[12] C. Canudas de Wit, B. Siciliano, and G. Bastin, *Theory of Robot Control*, 1st edition. London: Springer, 1996.

[13] J. Luh, M. Walker, and R. Paul, "Resolved-acceleration control of mechanical manipulators," *IEEE Transactions on Automatic Control*, vol. 25, no. 3, pp. 468–474, 1980.

[14] W. Khalil and E. Dombre, *Modeling, Identification and Control of Robots*, 3rd edition. New York: Taylor and Francis Group, 2002.

[15] M. Gautier and W. Khalil, "Direct calculation of minimum set of inertial parameters of serial robots," *IEEE Transactions on Robotics and Automation*, pp. 368–373, 1990.

[16] M. Gautier and W. Khalil, "Exciting trajectories for the identification of base inertial parameters of robots," in *Proc. 30th IEEE Conf. Decision and Control*, 1991, pp. 494–499.

[17] H. Mayeda, K. Yoshida, and K. Osuka, "Base parameters of manipulator dynamic models," *Ieee Trans. Robot. Autom.*, vol. 6, no. 3, pp. 312–321, 1990.

[18] M. Gautier, "Dynamic identification of robots with power model," IEEE International Conference on Robotics and Automation, 1997, vol. 3, pp. 1922–1927.

[19] M. T. Pham, M. Gautier, and P. Poignet, "Identification of joint stiffness with bandpass filtering," IEEE International Conference on Robotics and Automation, 2001, vol. 3, pp. 2867–2872.

[20] M. Gautier, A. Janot, and P.-O. Vandanjon, "A New Closed-Loop Output Error Method for Parameter Identification of Robot Dynamics," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 2, pp. 428–444, 2012.