DECOUPLING PROBLEM OF WEAKLY LINEAR COUPLED DOUBLE MINI-BETA-Y LATTICE OF TPS STORAGE RING

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Abstract

Three double mini-beta-y (3-DMBy) lattice design of the TPS storage ring is in progress to enhance the photon sources at three of the six long straight sections. For the estimation of Touschek beam lifetime, the TRACY code is used to calculate the momentum acceptance of the linear coupled TPS 3-DMBy lattice. The weak linear coupling was generated by adding some random skew quadrupoles at all quadrupole locations in order to create 1% emittance coupling. Using the Teng's symplectic rotation form in program may cause trouble in decoupling the one-turn coupled matrix. This report describes how we solve this decoupling problem and some useful references and comments are also presented.

INTRODUCTION

Taiwan Photon Source (TPS) under construction is a low emittance X-ray light source machine at NSRRC. The bare lattice of TPS storage ring is a 6-fold structure with 24 double bend cells [1]. The betatron tunes of the bare lattice designed for TPS storage ring are 26.18 in horizontal plane and 13.28 in vertical plane. Except one of the six 12m long straight sections is for beam injection, the 3-DMBy lattice design of the TPS storage ring is in progress to improve light source benefit at three of the rest long straight sections (see figure 1). Due to the difficulty of nonlinear sextupole scheme in low emittance lattice design of TPS, it is tried to preserve the achieved non-linear optimization result by applying the π -trick [2] algorithm.

In most cases of the lattice design, the betatron tunes are chosen to avoid the sum resonances than the difference resonances. Once the horizontal and vertical betatron motions are unexpectedly perturbed by some coupling components, the betatron amplitudes may growth infinity for sum resonances but they are still limited for difference resonances [3]. However this criterion does not limit the existence of stable solution for lattice design with one of the selected betatron tunes below half integer and the other one above half integer.

The first DMBy lattice design for TPS [4] includes two different options, the symmetry DMBy with betatron tunes (26.21, 13.32) and the anti-symmetry DMBy with betatron tunes (26.18, 13.71). During the double waist lattice design of vertical betatron function, some cases have the vertical tune above half integer indeed. For 3-DMBy lattice design with applying the π -trick for each DMBy, the symmetry type of DMBy is adopted. We still have two options for study: the first option has the vertical tune changed from near 13.28 directly to about 14.78 above half integer; the other one is using a re-matched bare lattice with vertical tune above half integer such that it keeps the vertical tune below half integer after adding the three DMBy sets.

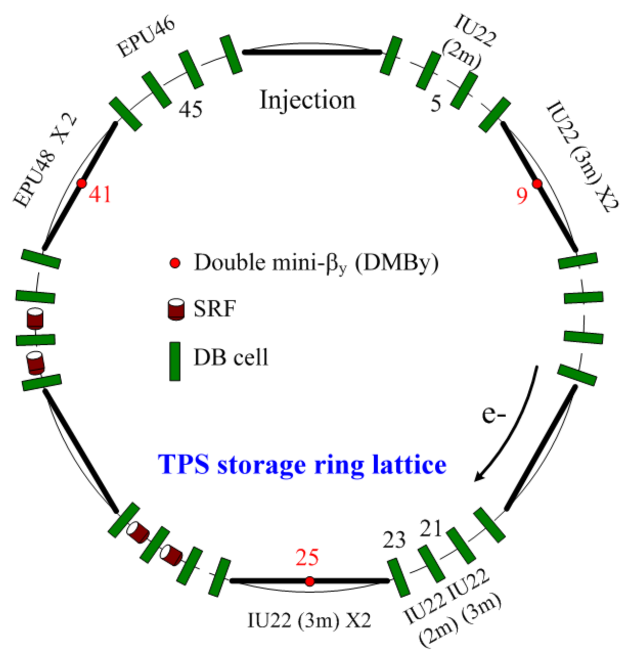

Figure 1: The distribution of 3-DMBy in TPS storage ring.

We tried to estimate the momentum acceptance with the controlled 1% emittance coupling for the Touschek beam lifetime calculation. For those studied cases with vertical betatron tune above half integer, there always have no decoupling solution of the linear coupled one-turn matrix using the TRACY code with the built-in formula of the Teng's symplectic rotation form [5]. In this report, we will describe and prove why the Teng's symplectic rotation form adopted in the TRACY code didn't work. And we will advertise and give the solution we found from some useful references. Finally we solve this decoupling problem by adding a modified decoupling algorithm in TRACY program.

LINEAR COUPLING CONTROL

Linear optics matching and the dynamic aperture study with tuning the tune-shift with amplitude and tune-shift with momentum deviation are the basic process of the 3-DMBy lattice design. Once the results are fine, the nonlinear optimization may be tried to improve the beam

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dynamics properties with the frequency map analysis (FMA). Finally the Touschek beam lifetime should be estimated by putting some specified emittance coupling.

For all the linear coupled TPS lattice studies, we generate the linear coupling by tilting the pure quadrupole components around the storage ring. The emittance coupling is under control to be 1% by the decoupling analysis using the "Coupling_Edward_Teng" function in TRACY code.

If the decoupling analysis does not work, we do not know the realistic coupling rate has been achieved. Although it may not influence the calculation process of momentum acceptance in TRACY, but one does not know how much emittance coupling is corresponding the obtained momentum acceptance.

Unfortunately, when we executed the TRACY code, we got the problem that there is no solution of Teng's symplectic rotation form in decoupling the single-turn transfer matrix. We tried to trace this problem in program and doubted the completeness of the Teng's form. In the following section, we can prove it and show you the problem by a simplified linear coupled one-turn matrix with one of the betatron tunes above half integer.

DECOUPLING PROBLEM

Teng's symplectic rotation form [5] of decoupling the single-turn transfer matrix presented with its original notations

$$T = \begin{pmatrix} M & n \\ m & N \end{pmatrix} = RUR^{-1}, \tag{1}$$

where

$$R = \begin{pmatrix} I\cos\phi & D^{-1}\sin\phi \\ -D\sin\phi & I\cos\phi \end{pmatrix}, \quad U = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix},$$
$$R^{-1} = \begin{pmatrix} I\cos\phi & -D^{-1}\sin\phi \\ D\sin\phi & I\cos\phi \end{pmatrix}; \tag{2}$$

and

$$D = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, ad - bc = 1;$$

$$A = \begin{pmatrix} \cos\mu_1 + \alpha_1 \sin\mu_1 & \beta_1 \sin\mu_1 \\ -\gamma_1 \sin\mu_1 & \cos\mu_1 - \alpha_1 \sin\mu_1 \end{pmatrix}$$
$$= M - D^{-1}\cdot m \cdot \tan\phi, \tag{3}$$
$$B = \begin{pmatrix} \cos\mu_2 + \alpha_2 \sin\mu_2 & \beta_2 \sin\mu_2 \\ -\gamma_2 \sin\mu_2 & \cos\mu_2 - \alpha_2 \sin\mu_2 \end{pmatrix}$$
$$= N + D\cdot n \cdot \tan\phi.$$

Here we can prove that there is no real solution ϕ as following: consider a decoupled linear one-turn matrix with one of the betatron tunes above half integer perturbed by a thin-lens skew quadrupole as below (Eq. 4)

$$T = \begin{pmatrix} A_0 & 0 \\ 0 & B_0 \end{pmatrix} \cdot Q_s. \tag{4}$$

Where the transfer matrix of thin-lens skew quadrupole can be generated by rotating the thin-lens quadrupole Q_k with 45 degree

$$Q_s = \Theta^{-1} Q_k \Theta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{f} & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{1}{f} & 0 & 0 & 1 \end{pmatrix}. \tag{5}$$

Assume $\mu_1 < \pi$ and $\mu_2 > \pi$ and solve the decoupling parameters, we have

$$|\cos 2\phi| = \left| \frac{Tr(M-N)}{2(\cos\mu_1 - \cos\mu_2)} \right| > 1 \tag{6}$$

using the notations in reference [5]. Or, by the definition of fundamental coupling matrix $h \equiv m^+ + n$ in reference [10] (note that, in references [5] and [10], the symbols, *n* and *m*, used in the one-turn matrices are exchanged; and the definitions of *D*'s are different), we obtain

$$\det(h) = \frac{\beta_1 \sin\mu_1 \beta_2 \sin\mu_2}{f^2} < 0$$
$$\tan 2\phi = \frac{2\sqrt{\det(h)}}{Tr(N-M)}. \tag{7}$$

Such that there is no real solution ϕ of Teng's symplectic rotation form. It causes the problem when one analysis the emittance coupling by running programs built-in with Teng's formula. It seems to us that Teng's formula should be extended to include the other possible solution.

COMPLETE DECOUPLING SOLUTION

After searching the related materials on the coupling and decoupling topics, some noticeable references should be mentioned. Since 1987, D. Rubin, et al, studied the transverse coupling problem at Wilson Laboratory in Cornell University. They have given an extended decoupling form in their papers [8, 9, 11, 12]

$$T = \begin{pmatrix} M & \overline{m} \\ \overline{n} & N \end{pmatrix} = VUV^{-1}. \tag{8}$$

Where *V* is symplectic such that

$$V = \begin{pmatrix} rI & C \\ -C^+ & rI \end{pmatrix},$$
$$V^{-1} = V^+. \tag{9}$$

The possible solutions of decoupling parameter *r* are

$$(2r^2 - 1)^2 = \frac{(Tr[M-N])^2}{(Tr[M-N])^2 + 4\det(H)}, \tag{10}$$

i.e.

$$r = \sqrt{\frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{(Tr[M-N])^2}{(Tr[M-N])^2 + 4\det(H)}}}, \qquad (11)$$

where $H \equiv \overline{m} + \overline{n}^+$. And the decoupling solutions are

$$C = \frac{\mp H \cdot \operatorname{sgn}(Tr[M-N])}{r\sqrt{(Tr[M-N])^2 + 4\det(H)}}.$$

$$A = r^2 M - r(Cn + mC^+) + CNC^+, \qquad (12)$$

$$B = r^2 N - r(nC + C^+ m) + C^+ MC.$$

It shows there still is possible stable solution for our study cases of 3-DMBy with the first option $\det(H) < 0$. We use the extended decoupling formula to modify the "Coupling_Edward_Teng" function and make it as an additional function in TRACY code such that we have solved this problem successfully.

SUMMARY AND DISCUSSIONS

As we mentioned in the introduction section, the comments said it is better to choose the betatron tunes to avoid the sum resonances. With re-matching the bare lattice to have the vertical tune above half integer, similar to the single DMBy design in reference [4], the final betatron tunes keep below half integer in both planes after applying the π-trick for the 3-DMBY. We have obtained a better 3-DMBy lattice for TPS storage ring [17] with the chosen of betatron tunes (26.18, 14.28) to avoid the sum resonances than the difference resonances.

During the study of 3-DMBy lattice design, we found the Teng's symplectic rotation form of decoupling the one-turn matrix is an incomplete solution for all possible coupling cases and it has been redeemed by the extended decoupling form studied by D. Rubin and his co-workers. Some useful references up to 2009 we found to trace the coupling and decoupling problem are listed in references [5-16] to complete the progress of this topic.

Finally, after solving this decoupling problem in the TRACY code, we can control the coupling one-turn matrix to calculate the momentum acceptance around the ring for the estimation of Touschek beam lifetime. Hopefully this revised function of decoupling one-turn matrix can be considered in the future updated TRACY code.

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