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Research of Cubic Bezier Curve NC Interpolation Signal Generator

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Abstract: Interpolation technology is the core of the computer numerical control (CNC) system, and the precision and stability of the interpolation algorithm directly affect the machining precision and speed of CNC system. Most of the existing numerical control interpolation technology can only achieve circular arc interpolation, linear interpolation or parabola interpolation, but for the numerical control (NC) machining of parts with complicated surface, it needs to establish the mathematical model and generate the curved line and curved surface outline of parts and then discrete the generated parts outline into a large amount of straight line or arc to carry on the processing, which creates the complex program and a large amount of code, so it inevitably introduce into the approximation error. All these factors affect the machining accuracy, surface roughness and machining efficiency. The stepless interpolation of cubic Bezier curve controlled by analog signal is studied in this paper, the tool motion trajectory of Bezier curve can be directly planned out in CNC system by adjusting control points, and then these data were put into the control motor which can complete the precise feeding of Bezier curve. This method realized the improvement of CNC trajectory controlled ability from the simple linear and circular arc to the complex project curve, and it provides a new way for economy realizing the curve surface parts with high quality and high efficiency machining. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Bezier curve interpolation, Bezier curve, Signal generator, Stepless interpolation, CNC.

1. Introduction

Numerical control technology is a kind of automatic control technology developed in modern times. It is a key link in modern manufacturing technology. The numerical control interpolation technology is the core technology of the CNC system. The advantages of interpolation algorithm will directly affect the machining precision and speed of numerical control system. Free-form curve interpolation can not only make the curve processing speed uniform, improve the quality of surface processing, but also can improve the processing speed, so the free-form curve interpolation algorithm for the CNC system is of great significance. The interpolation method of the NC machining system is divided into two main categories: the pulse increment interpolation and data sampling interpolation. The pulse increment interpolation by assigned to each movement axis drive pulse to control the machine tool axes coordinated motion, so as to work out a certain contouring shape. The pulse equivalent determines the machining precision of the numerical control system. This will inevitably produce the interpolation error that less than one pulse equivalent. Data sampling interpolation divides the contouring

curve of the parts into a series of head-tail tiny straight segment, and then outputs the position incremental data corresponds to the tiny straight segment. Controlling the servo system will implement the feed of the coordinate axis. Now all existing interpolations are the approximations of broken line approximation interpolation. In the free-form curve interpolation, it shows that the shortcoming of complex procedure, large amount of calculation, low processing precision and low efficiency. While machining a freeform surface, the motion trails is not a smooth curve surface. All over the world has already begun on the research of the direct interpolation curve and obtained some achievements in recent years. For example, University of Zurich, Switzerland, with the company Fides has developed Ozelot system [1] which can be directly machine on the Bezier surface; Chou and Yang [2] apply the parametric curve interpolation method to the five-axis CNC machine tools and the CMM (Coordinate Measuring those) system; Shpitalni [3] designed the first and second order NURBS curve interpolation based on the Taylor expansion, studies have shown that it has a better performance than the constant parameter interpolation, smaller amount of calculation, but bigger error; Under the condition of three axis processing, Bedi [4] developed a system using a six Tran sputter processor network parallel computing. It implements real-time intervention treatment, but only solved the processing problem of the second order continuous single surface of the ball end cutter for cutting plane method, has yet to consider the reasonable path interval effective feed speed control and the error compensation of the technology system; In the terms of five axis machining, Koren Yoram [5] developed a five-axis curved surface interpolation, but which only can realize the end milling of the single without interference surface and the side milling of straight grain surface; Han [6] proposed a high-speed processing algorithms with a prospective configurator which was used to reduce the error of NC process and increase processing speed; Behnam [7] came up with a three order forward difference algorithm which can achieve the fast forecast for the three times parametric curve interpolation points. This algorithm can avoid the cumulative error of interpolation through proper approximate treatment, analyze the relationship between the step pitch of parameter and the interpolation error; Tsai [8] proposed an interpolation method of planning the track of the cutting contact point on the surface directly. Compared with the traditional method that planning the tool path on the offset surface, this new method can guarantee the stability of the cutting feed speed, but if it works at a constant feed speed way, the sharp corner of the curve will have a big chord error; Yau [9] put forward a real-time predictive algorithms, it adopted the Bezier curve which based on some kind of error tolerance to replace the continuous short line segment for the interpolation arithmetic. Wang [10] carried

out velocity planning through Taylor expansion method to calculate the next interpolation points, but it has the truncation error, which can produce significant speed fluctuations. Huazhong University of Science and Technology, Ye Bosheng [11] put forward three b-spline curve high speed interpolation algorithms. Du Juan and Tian Xitian [12] put forward the equal arc length interpolation algorithm that can achieve the uniform interpolation. Xu Hong put forward spline curve interpolation algorithm based on the redundancy error control. But the algorithm presented in this paper is the stepless interpolation controlled by the analog signal, which is the opposite of the above-mentioned algorithm. And it will greatly improve the smoothness of the curve.

2. The Mathematical Model of the Cubic Bezier Curve

The Bezier curve is first proposed in 1962 by a French engineer Pierre Bézier, it mainly for the automobile body design, and it is important parameter curve in computer graphics. The cubic Bezier curve [14] is defined as: four points in space p_0, p_1, p_2, p_3 the direction is $p_0 \rightarrow p_1$, $p_1 \rightarrow p_2$, $p_2 \rightarrow p_3$, the parameter expression of the curve is $B(t) = p_0(1-t)^3 + 3p_1t(1-t)^2 + 3p_2t^2(1-t) + p_3t^3$, $t \in [0,1]$. The definition of basic functions-Bernstein polynomial is $BEZ_{i,n}(t) = C_n^i t^i (1-t)^{n-i}$, $t \in [0,1]$. The basis functions of the cubic Bezier curve are $(1-t)^3$, $3t(1-t)^2$, $3t^2(1-t)$, t^3, the corresponding theory graphics as shown in Fig. 1.
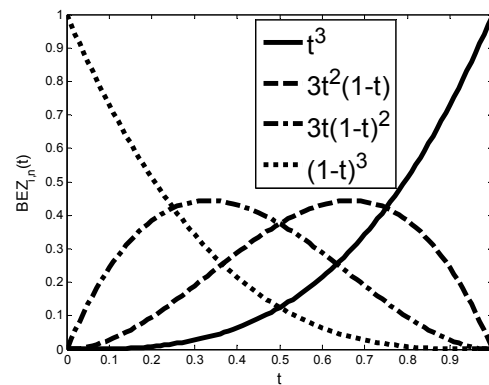

Fig. 1. The Cubic Bezierbasis function image.

3. The Research of the Composition Modules of the Signal Generator

3.1. Sawtooth Wave Signal Generator

In order to intuitive display the curve changes of the analog signal, parameter transformation strategy are used which converts parameter t into a voltage

change over time, that is to say, to use the change of the voltage indicates the change of the parameters, the range is $u_t \in [0,1]$ V, the $1-t$ can be expressed by the voltage $u_{\max} - u_t$, therefore the linear sawtooth wave was studied to generate the circuit of the 0-1 V voltage.

3.1.1. Bootstrap Voltage Type Sawtooth Wave Generator

The sawtooth generator has a variety of circuits. The higher requires the linearity, the more complex circuit will be. In this paper, the sawtooth wave generator adopts the 555 chip for the base circuit, the cost is low, and it adopts the method provided by the literature [15] to produce sawtooth wave with a high linearity. Using the 555 chip as the base circuit to constitute a sawtooth wave generator, the key condition to build a sawtooth wave generator with a property of good linearity is: charge the timing capacitor with a constant current and discharge at tremendous speed. In this paper, the Self-boosting voltage type sawtooth wave generator was used and the schematic diagram is shown in Fig. 2. The principle is in the process of capacitor C_1 charge and discharge, at the same time, the emitter follower output sawtooth wave, through the bootstrap capacitance C_3 positive feedback to the top of R_2, so during charging of the capacitance C_1, the pressure drop on the resistance R_2 remains unchanged, and the charging current remains the same.

So the voltage on the capacitor C_1 rises with a linear trend that makes the output of the sawtooth wave has good linearity. But the voltage variations of

the sawtooth range of $\frac{1}{3}v_{cc} - \frac{2}{3}v_{cc}$, after produce the sawtooth wave with good linearity, connect an operation circuit to the follow-up circuit, set the voltage between 0 and 1 V. If it needs to achieve this only need to set V_{cc} to 3 V, and then make a subtraction circuit with a constant voltage of $\frac{1}{3}v_{cc}$. And the operation will not affect the linearity of the sawtooth.
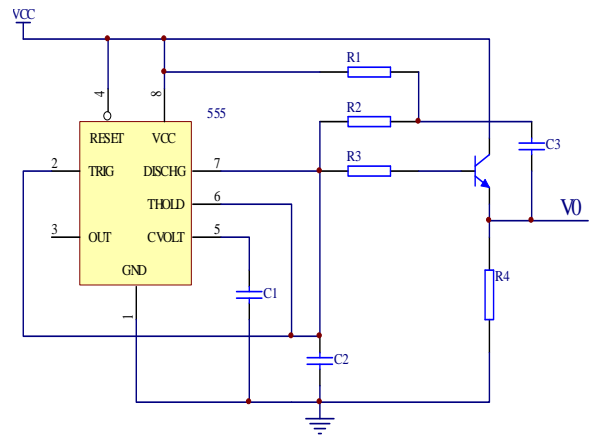

Fig. 2. The principle diagram of the generating circuit for the Bootstrap linear sawtooth.

3.1.2. Circuit Diagram and Simulation Effect

Based on the schematic above to build a circuit diagram and conduct a simulation. The simulation circuit diagram of the sawtooth wave generator is shown in Fig. 3, and the simulation effect is shown in Fig. 4.
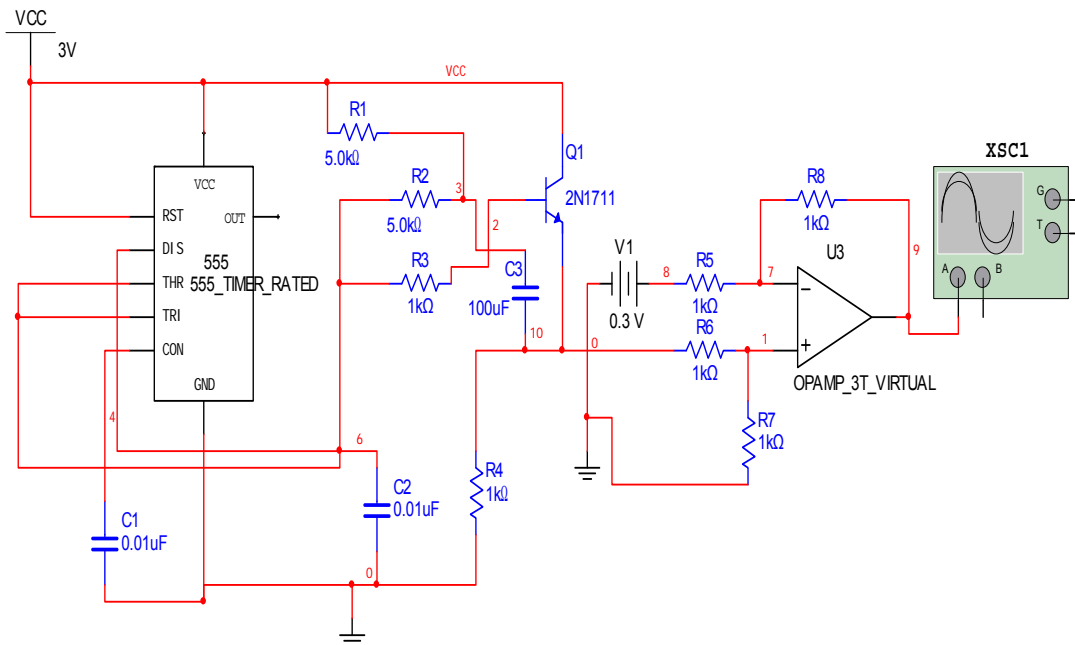

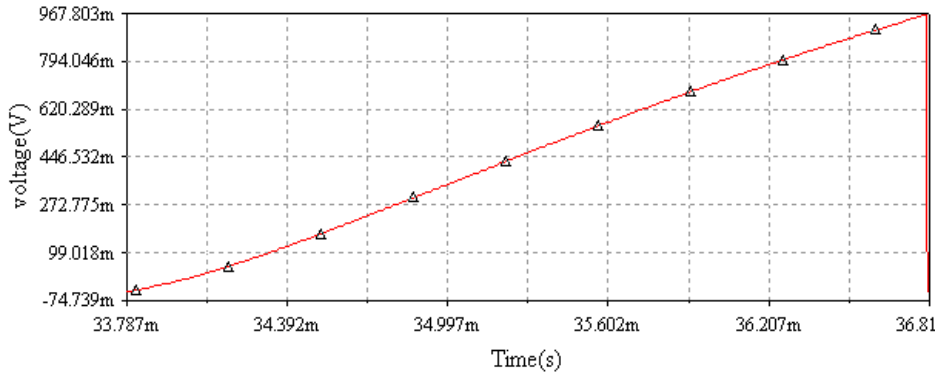**Fig. 3.** The simulation circuit diagram of the sawtooth wave generator.

Fig. 4. A cycle simulation waveform of the sawtooth wave generator.

3.1.3. The Simulation Results and Analysis

A 0-1 V cycle sawtooth wave with good linearity can be obtained through the above circuit, and it provides the good wave source for the subsequent simulation operation circuit.

3.2. The Signal Generator of the Bezier Basis Function

With the good linear wave source, it can build the signal generator of the Bezier basis function, then it can analyze the signal waveform of the basis function and compared with the theoretical curve.

3.2.1. The Basic Mathematical Model

The definition of the Bezier basis function – Bernstein polynomial is: $BEZ_{i,n}(t) = C_n^i t^i (1-t)^{n-i}$, $t \in [0,1]$. The image of the basis function is shown in Fig. 1.

3.2.2. Implement the Circuit Module

First, construct the representation of the circuit $1-t$, $u_{max} - u_t$, the schematic of the circuit is shown in Fig. 5. Fig. 5 illustrates that:

$$\frac{u_t - u_N}{R_1} = \frac{u_N - u_{out}}{R_4} \qquad (1)$$

$$\frac{u_{max} - u_p}{R_2} = \frac{u_p}{R_3} \qquad (2)$$

So

$$u_N = u_p \qquad (3)$$

By the formula (1), (2), (3), we can deduce the formula (4)

$$u_{out} = (1 + \frac{R_4}{R_1})(\frac{R_3}{R_2 + R_3})u_{max} - \frac{R_4}{R_1}u_t \qquad (4)$$

For the formula (4), when $R_1 = R_2 = R_3 = R_4$,

$$u_{out} = u_{max} - u_t \qquad (5)$$

After here got the output voltage value $u_{max} - u_t$ by formula (5), then construct the representation of the voltage $(1-t)^3$ with the multiplier, that is $u_{max} - u_t$. For the output voltage $u_{out} = u_{max} - u_t$ through the Fig. 5, we use the multiplier operation twice, and then we can get the output voltage $(u_{max} - u_t)^3$. The schematic is shown in Fig. 6. Likewise it can get the Bezier basis function circuit which representations of the voltage are $3t(1-t)^2$, $3t^2(1-t)$, t^3.
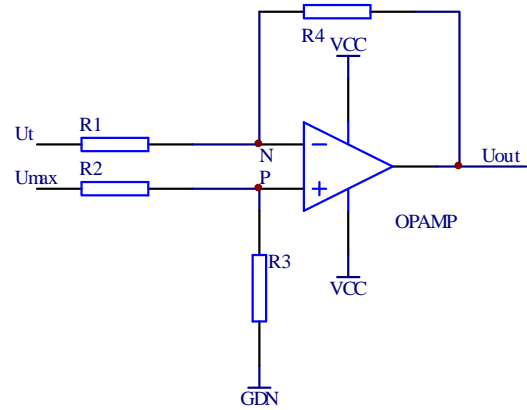

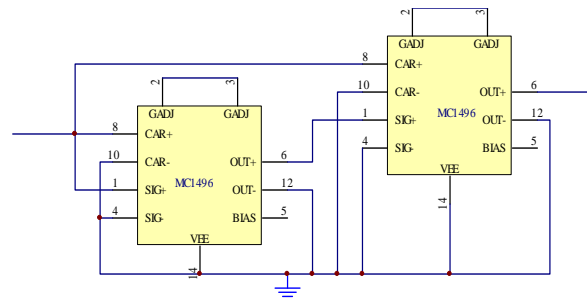**Fig. 5.** $u_{max} - u_t$ the schematic of the circuit.

Fig. 6. $(u_{max} - u_t)^3$ the schematic of the circuit.

3.2.3. The Simulation Results and Analysis

A 0-1V cycle sawtooth wave with good linearity can be obtained through the circuit, and it provides the good wave source for the subsequent simulation operation circuit.

The simulation curve of the Bezier basis function is shown in Fig. 7. From the Fig. 7, it can be found that the waveform simulation diagram of the Bezier basis function simulated by the circuit is basically identical to the diagram of the Bezier basis function in theory.
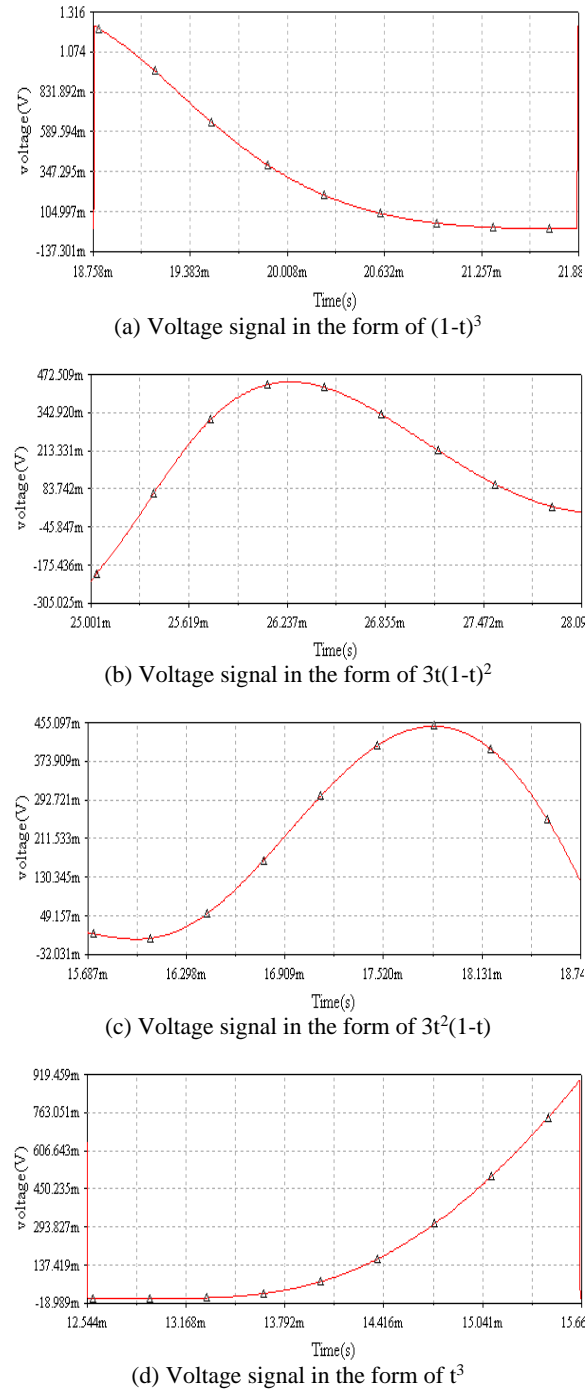

(a) Voltage signal in the form of $(1-t)^3$

(b) Voltage signal in the form of $3t(1-t)^2$

(c) Voltage signal in the form of $3t^2(1-t)$

(d) Voltage signal in the form of t^3

Fig. 7. The simulation diagram of the Bezier basis function.

But there is some gap between the actual waveform peak of the $(1-t)^3$, t^3 with the theoretical value. The theoretical peak of the $(1-t)^3$, t^3 shall be 1 V, while the actual waveform is a slightly greater than 1 V, a slightly smaller than 1 V (Fig. 7 (a) and Fig. 7 (b)). The reason is that the output sawtooth wave is not a precise 0-1 V, the error within 0.04 V, and the error will be gradually exaggerated after the multiplier operation circuit.

4. Cubic Bezier Curve Signal Generator

4.1. The Basic Building Ideas

Using the above design circuit, here can develop the basis function circuit of the Bezier curve, and only need to do the multiplication calculation between the four voltages of the control vertex and the waveform of the basis function corresponding. Then it can show the cubic Bezier curve signal. It can be easily realized by using the analog multiplier. We assume that the product of four voltages of the control vertex and the waveform of the basis function corresponding are u_0, u_1, u_2, u_3. Add these four values, and then we can obtain the cubic Bezier curve generator.

4.2. The Composition of the Circuit's Realization

The schematic of the circuit's realization is shown in Fig. 8. From the circuit diagram we know that:

$$i = i_0 + i_1 + i_2 + i_3 \qquad (6)$$

In the formula, $i_0 = \dfrac{u_0}{R_5}$, $i_1 = \dfrac{u_1}{R_6}$, $i_2 = \dfrac{u_2}{R_7}$, $i_3 = \dfrac{u_3}{R_8}$.

$$u = -iR_{10} \qquad (7)$$

Take the (6) into (7):

$$u = -(\frac{u_0}{R_5} + \frac{u_1}{R_6} + \frac{u_2}{R_7} + \frac{u_3}{R_8})R_{10} \qquad (8)$$

For the formula (8), when the $R_5 = R_6 = R_7 = R_8 = R_{10}$, after a reverse operation on $u = -(u_1 + u_2 + u_3 + u_4)$ here can obtain the voltage waveform of the $u = u_0 + u_1 + u_2 + u_3$. The whole simulation circuit diagram is shown in Fig. 9. In the Fig. 9, the left-most rectangle is the subcircuit of the sawtooth wave generator.
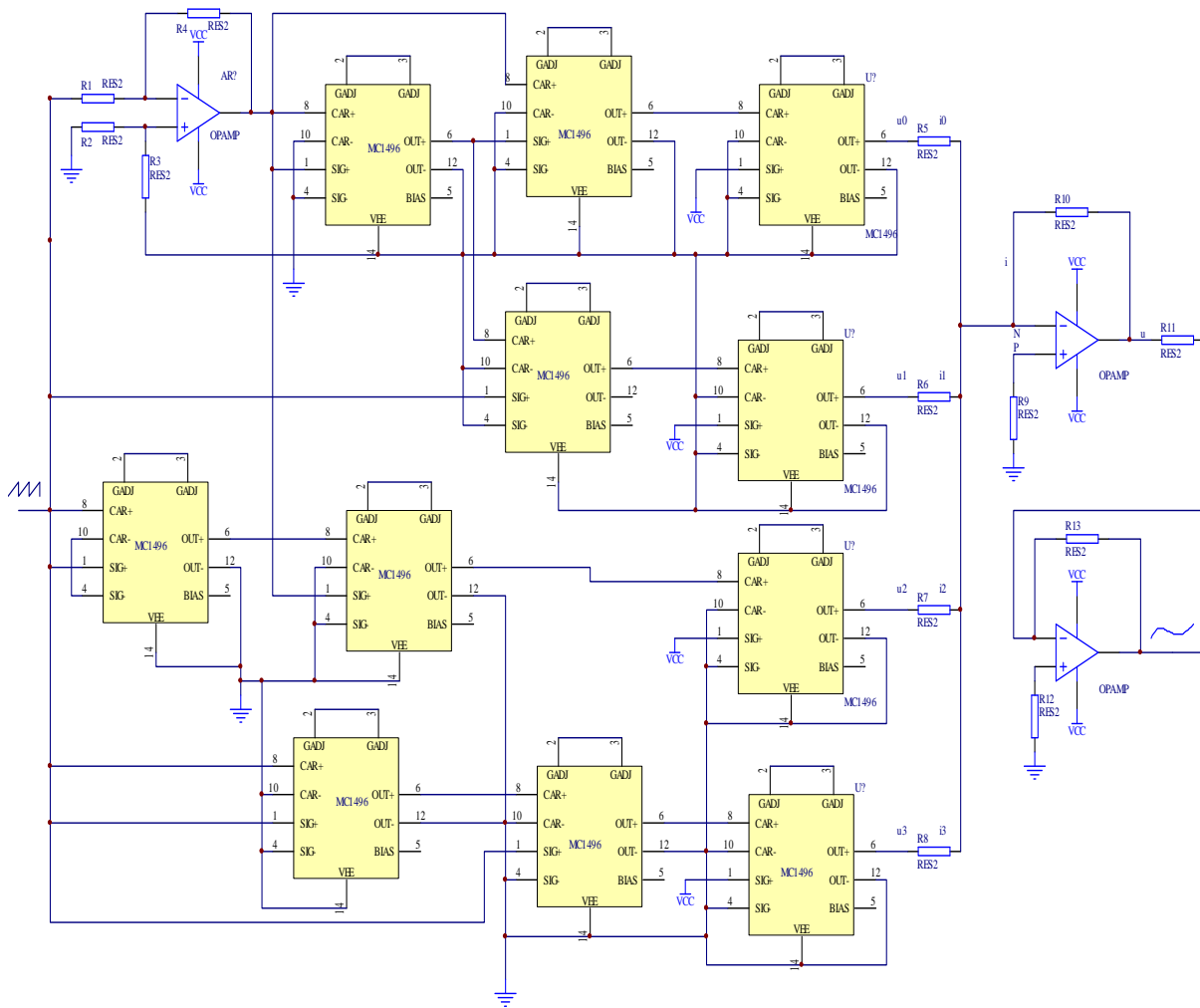
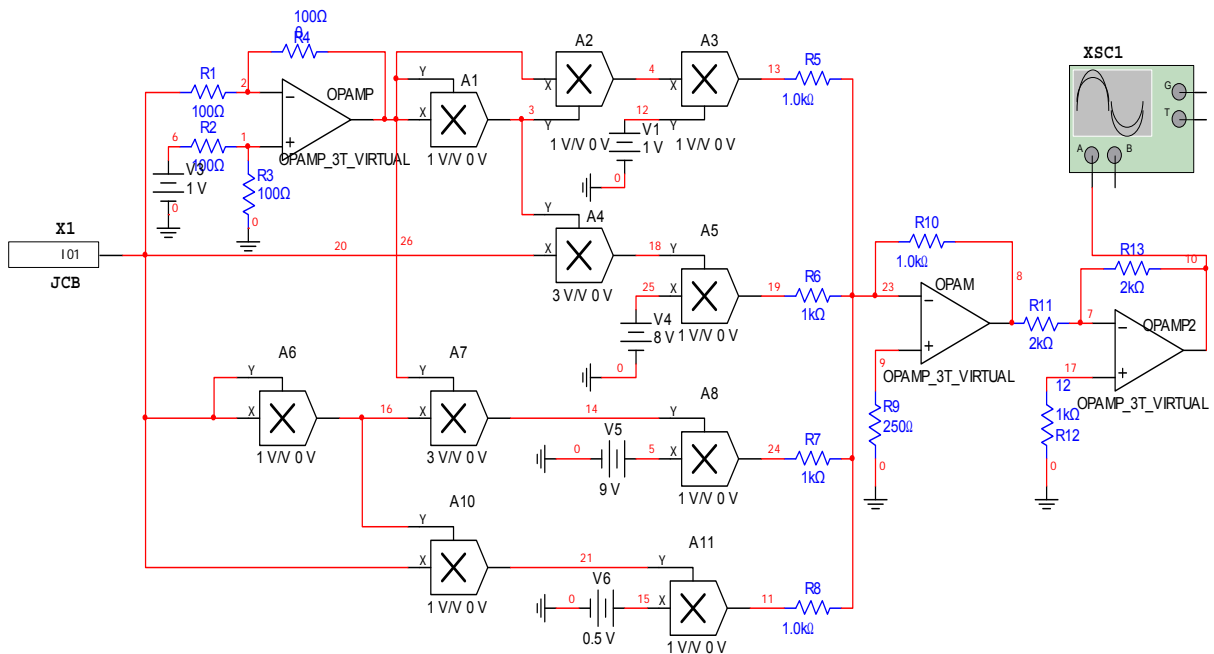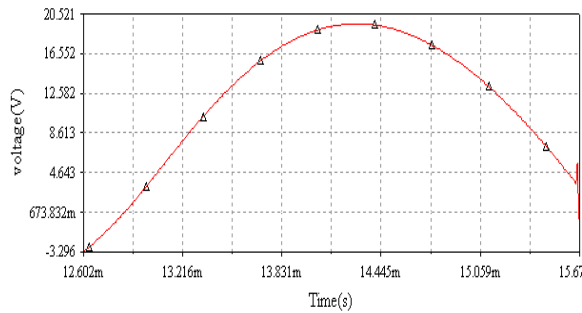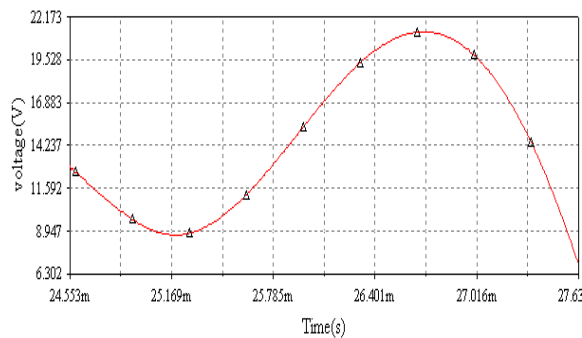**Fig. 8.** The schematic of the circuit's realization of the cubic Bezier curve.

Fig. 9. The simulation circuit diagram of the cubic Bezier curve generator.

4.3. The Results Analysis of the Simulation and Experiment

Here only needs to change the voltage of the four control points and can obtain the cubic Bezier curve waveform. When the voltage of the control vertices p_0, p_1, p_2, p_3 are 0.5 V, 9 V, 8 V, 1 V and 2 V, 15 V, 0.1 V, 10 V, the simulation waveform of the cubic Bezier curve are Waveform1 and Waveform 2, which are shown in Fig. 10.

(a) Waveform 1

(b) Waveform 2

Fig. 10. The simulation circuit diagram of the cubic Bezier curve generator.

5. The Basic Principle of the Realization of the Interpolation

Put the analog signal we developed into the control of DC motor in the form of current, it can know that:

$$T = K_t \Phi I_a \qquad (9)$$

$$n = \frac{U}{K_e \Phi} - \frac{R_a}{K_e K_t \Phi^2} T \qquad (10)$$

Here can calculate the electromagnetic torque by formula (9), and further calculate the speed of the motor through the relationship between the torque and the speed of motor by formula (10). Put the analog signal developed above into the stator coil of the three-phase asynchronous motor in the form of current, it can be obtained that:

$$T = K_T \Phi_m I \cos\phi \qquad (11)$$

$$n = \frac{60f}{p}\{1 - s_m[\frac{T_{\max}}{T} - \sqrt{(\frac{T_{\max}}{T})^2 - 1}]\} \qquad (12)$$

In the formula, $s_m = \dfrac{R_2}{X_{20}}$, $T_{\max} = K\dfrac{U^2}{2X_{20}}$

It can calculate the electromagnetic torque by formula (11), and further calculate the speed of the motor through the relationship between the torque and the speed of motor by formula (12).

6. Conclusions

1) Put forward the idea of using the analog signal to realize stepless interpolation. Build the simulation circuit of the cubic Bezier curve stepless interpolation, which plays a complementary and perfection role in complex curved surface parts processing controlled by existing circular and linear interpolation.

2) Through the analysis for the construction of the basic circuit and the simulation of the integrated circuit, it can be found that the stepless interpolation technology has a good adaptability. It is only need to change the voltage values of the four control points and can change the shape of the interpolation.

3) The analog signals produced by signal generator can be directly input to the drive equipment such as DC motor and three-phase asynchronous motors to control it achieve the complex contour machining.

Acknowledgements

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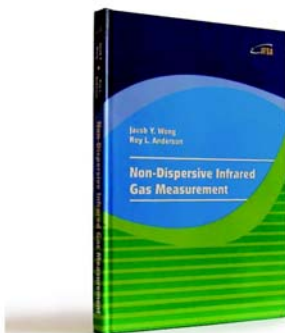
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