Acoustic Response of a Thin Poroelastic Plate

Kirill V. Horoshenkov* and Kimihiro Sakagami**

* *Department of Civil and Environmental Engineering, University of Bradford, Bradford, BD7 1DP, UK*

** *Environmental Acoustics Laboratory, Faculty of Engineering, Kobe University, Rokko, Nada, Kobe 657 Japan*

Abstract: The Helmholtz integral equation formulation is used to produce the solution for the sound field reflected from an infinite, thin, porous, elastic plate. The effect of an air cavity behind the plate is considered. A parametric study is performed to predict the effect of variations in microscopic and structural parameters of the poroelastic plate.

INTRODUCTION

Thin plate and membrane structures find the increasing applications in building large public halls and leisure centres in Japan as light and earthquake robust materials. It has been observed [1] that the acoustic properties of multi-leaf structures can be largely improved by a combination of permeable membranes and porous absorbing materials. In a similar way, the desirable combination of structural motion and "viscothermal" acoustic boundary layer absorption in the pores of the material may be exploited, in a single structure, represented in a form of a thin, porous, elastic plate. The microstructure of consolidated granular materials used to manufacture the plate can readily be designed since it is a function of the granules and the consolidation process.

THEORETICAL ASSUMPTIONS

As the first step toward the design of a porous, elastic plate absorber it should be possible to assume that the plate is large enough to neglect the diffraction on its edges. Let us assume that the problem is two-dimensional and that the plate's face is in the plane $z = 0$ at a distance $d \to \infty$ from a rigid wall, and that it is exposed to an incident sound wave $p_i = e^{i[k_o(x\sin\theta + z\cos\theta) - \omega t]}$. Using the Helmholtz integral formulation and including the velocity of the plate υ_p in the boundary conditions it can be shown that the acoustic pressure on the front of the plate is given by

$$p_s(x) = 2p_i(x) + \tfrac{i}{2}\int_{-\infty}^{\infty} \{ik_o\beta_{ins}p_s(x_o) + i\omega\rho_o\upsilon_p(x_o)\}\, H_o^{(1)}(k_o|x - x_o|)\, dx_o. \qquad (1)$$

and the pressure on the back by

$$p_t(x) = -\tfrac{i}{2}\int_{-\infty}^{\infty} \{ik_o\beta_{in}\,\eta p_s(x_o) + i\omega\rho_o\upsilon_p(x_o)\}\}\, H_o^{(1)}(k_o\,|x - x_o|))\, dx_o\;, \qquad (2)$$

where β_{in} is the acoustic surface normalised admittance of the porous plate which can be easily predicted, for example from Pade approximations given in [2]. The Hankel function representing the Green's function for 2-D sound propagation in the first term in equation (1) is replaced by p_i. It is assumed here that the source is in the far field and that the incident wave is plane. This assumption allow us to obtain an analytical solution of (1) and provides a simple way of estimating the absorption coefficient $\alpha = 1 - |p_s - p_i|^2$ and the transmission loss $TL = 10\log_{10}|p_t|^{-2}$. Since the plate is porous, i.e. its acoustic properties are described by a finite value of the acoustic admittance, then the pressure on the back of the plate can be related to the pressure on the front via a complex coefficient $p_t = \eta p_s$. The coefficient, η, can be determined from the formulation of the boundary conditions on the front and on the back of the plate in the case when $\upsilon_p = 0$.

If oscillations of the plate result from the differential pressure applied, then the displacement of the plate can be found from the convolution integral

$$w_p(x) = \int_{-\infty}^{\infty} \{p_s(x_o) - p_t(x_o)\}\, u(x - x_o)\, dx_o\,. \qquad (3)$$

where the function $u(x-x_o)$ is the response of an elastic plate to a unit force $\delta(x-x_o)$. The solutions for p_s, p_t and w_p, can be found analytically by applying the Fourier transform in wavenumber space, by re-arranging the resulting terms and then taking the inverse Fourier transform of the obtained functions of k, noting that $\tilde{u}(k) = [2\pi D(k_o^4 - k_f^4)]^{-1}$, $\tilde{p}_i(k) = \delta(k - k_o\sin\theta)$ and $K = \sqrt{k_o^2 - k^2}$. This yields

$$w_p(x) = \varphi(k_o\sin\theta)e^{ikx\sin\theta}, \qquad (4)$$

$$p_s(x) = (2k_o\cos\theta + i\omega^2\rho_o\ \psi(k_o\sin\theta))[k_o(\cos\theta + \beta_{in})]^{-1}e^{ik_o\sin\theta x}, \qquad (5)$$

$$p_t(x) = -\left(\frac{i\omega^2\rho_o\psi(k_o\sin\theta)}{k_o\cos\theta} - \beta_{in}\eta\frac{2k_o\cos\theta + i\omega^2\rho_o\psi(k_o\sin\theta)}{k_o\cos\theta(\cos\theta + \beta_{in})}\right)e^{ik_o\sin\theta x} \qquad (6)$$

where

$$\psi(k) = \frac{4\pi\tilde{u}(k)(K - k_o\beta_{in}\eta)K}{K(K + k_o\beta_{in}) - 2\pi i\omega^2\rho_o\tilde{u}(k)(2K + k_o\beta_{in}(1-\eta))}. \qquad (7)$$

Similar expression, but with different values of β_{in} and η can be obtained for the case when the plate is separated from an impermeable wall by an air gap of finite width d.

PARAMETERIC STUDY

The derived expressions (4)-(7) have been used to perform a parameteric study of the behaviour of a thin elastic, porous plate made from melamine-based elastic foam exposed to an acoustic loading. The results are presented in Figures 1 and 2.

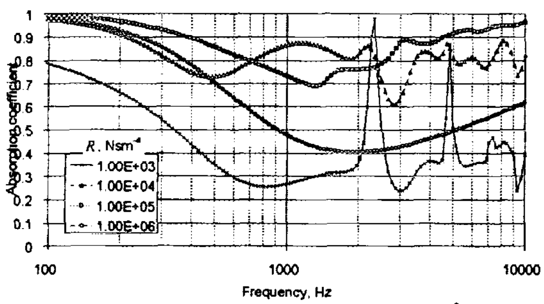

Figure 1. Effect of flow resistivity (ρ_{pl} = 10 kg/m^3).

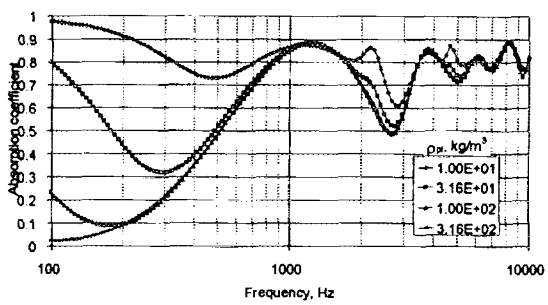

Figure 2. Effect of material's density (R = 10^4 Nsm^{-4}).

The selected parameters of the plate: porosity, $\Omega = 0.98$, tortuosity, $q = 1.06$, Young's modulus, $E = 10^5$ N/m^2, Poisson ratio, $\nu = 0.2$, thickness, $h = 0.02$ m, the air gap, $d = 0.05$ m and the angle of incidence $\theta = 0$ deg.

CONCLUSIONS

Although the theoretical model is for an infinite plate, it can also be extended to the case of a plate with finite dimensions. The boundary conditions on the edges of the plate can be accoutered for by the modification of the plate's response function $\tilde{u}(k)$ and by changing the integration limits in the original expressions.

The obtained results indicate that a carefully chosen combination of microscopic and structural parameters can improve the acoustic absorption in a thin, porous, elastic plate. In particular, the reduced density of the material results in the greater absorption at the lower frequencies, whereas the flow resistivity affects the absorption coefficient in a much broader frequency range.

REFERENCES

1.Takahashi, D., Sakagami, K., Morimoto, M., Acoustic properties of permeable membranes, J. Acoust. Soc. Am., 99, No.5, 3003-3009 (1996).

2. Horoshenkov, K. V., *Control of Road Traffic Noise in City Streets*, PhD thesis, University of Bradford, 1996, ch. 3.