The role of counterparty risk and collateral in longevity swaps*

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PRELIMINARY AND INCOMPLETE

Abstract

Longevity swaps allow pension schemes and annuity providers to swap out longevity risk, but introduce counterparty credit risk, which can be mitigated or eliminated by collateralization. In this study, we examine the impact of bilateral default risk and collateralization rules on the marking to market of longevity swaps. In particular, we show how different rules for posting collateral during the life of the swap may affect longevity swap rates.

Keywords: longevity swap, default risk, longevity risk, collateral, marking-to-market.

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1 Introduction

The market for longevity-linked securities and derivatives has recently experienced a surge in transactions in longevity swaps. These are agreements between two parties to exchange fixed payments against variable payments linked to the number of survivors in a reference population (see Dowd *et al.*, 2006). Table 1 presents a list of recent deals that have been publicly disclosed. So far, deals have mainly involved pension funds and annuity providers wanting to hedge their exposure to longevity risk but without having to bear any basis risk; this means that the variable payments in longevity swaps are driven by the mortality experience of each individual hedger (hence the name indemnity-based, or bespoke, longevity swaps). This type of transaction is essentially a form of longevity risk insurance, similar to annuity reinsurance in reinsurance markets.

A fundamental difference, however, is that longevity swaps are typically collateralized, whereas insurance/reinsurance transactions are not.[1] The reason is that hedgers have been placing increasing emphasis on the issue of counterparty risk[2] and look to the fixed-income markets to provide a reference model. In swap markets, for example, the most common form of credit enhancement is the posting of collateral. According to ISDA (2010b), almost every swap at major financial institutions is 'bilaterally' collateralized, meaning that either party is required to post collateral depending on whether the market value of the swap is positive or negative.[3] The vast majority of transactions is collateralized according to the Credit Support Annex to the Master Swap Agreement introduced by the International Swap and Derivatives Association (ISDA) (see ISDA,

[1]One rationale for this is that reinsurers pool several uncorrelated risks and diversification benefits compensate for the absence of collateral.

[2]Basel II (2006, Annex 4) defines counterpary default risk as "the risk that the counterparty to a transaction could default before the final settlement of the transaction's cash flows". The recent Solvency II proposal makes explicit allowance for a counterparty risk module in its 'standard formula' approach; see CEIOPS (2009).

[3]"Unlike a firm's exposure to credit risk through a loan, where the exposure to credit risk is unilateral and only the lending bank faces the risk of loss, counterparty credit risk creates a bilateral risk of loss: the market value of the transaction can be positive or negative to either counterparty to the transaction. The market value is uncertain and can vary over time with the movement of underlying market factors." (Basel II, 2006, Annex 4).

1994). The Global Banking Crisis of 2008-09 highlighted the importance of bilateral counterparty risk and collateralization for over-the-counter markets, spurring a number of responses (e.g, ISDA, 2009; Brigo and Capponi, 2009; Brigo *et al.*, 2010). The Dodd-Frank Wall Street Reform and Consumer Protection Act (signed into law by President Barack Obama on July 21, 2010) is likely to have a major impact on the way financial institutions will manage counterparty risk in the coming years.[4] The recently founded Life and Longevity Markets Association[5] has collateralization rules at the center of its agenda, and will certainly draw extensively from the experience garnered in fixed-income and credit markets.

The design of collateralization strategies addresses the concerns aired by pension trustees regarding the efficacy of longevity swaps. At the same time, it introduces another dimension in the traditional pricing framework used for insurance transactions. The 'insurance premium' embedded in a longevity swap rate reflects not only the aversion (if any) of the hedge supplier to the risk taken on or the regulatory capital needed to support the transaction, but also the expected costs (gains) to be incurred (made) from posting (holding) collateral during the life time of the swap. To understand the role of collateral, let us first take the perspective of a hedger (pension fund or annuity provider) acquiring protection through a collateralized longevity swap: whenever the swap is (sufficiently) in the money, the hedge supplier (reinsurer or investment bank) is required to post collateral, which can be used by the hedger to mitigate losses in the event of default. Although interest on collateral is (partially) rebated, there is a gain from holding collateral since the hedger benefits from capital relief in regulatory valuations and may re-pledge collateral for other purposes.[6] On the other hand, whenever the swap is sufficiently out of the money, the hedger will have to deposit collateral with

[4]See, for example, "Berkshire may scale back derivative sales after Dodd-Frank", *Bloomberg*, August 10, 2010.

[5]See `http://www.llma.org`.

[6]In interest-rate swap markets, the vast majority of collateral is indeed rehypothecated for other purposes (e.g., ISDA, 2010b). Currently, collateral can be re-pledged under the New York Credit Support Annex, but not under the English Credit Support Deed (see ISDA, 2010a).

the counterparty, thus incurring an opportunity cost. This opportunity cost is particularly relevant if we take the symmetric perspective of the hedge supplier, as longevity protection is very capital intensive.

The objective of our study is to provide a valuation framework for longevity swaps in the presence of (bilateral) counterparty default risk, and to show how collateralization rules affect longevity swap rates. In particular, we discuss tools to quantify the trade-off between the cost of a longevity swap as measured by the swap rate and the credit enhancement offered by tighter collateralization rules. Our results are consistent with the empirical observation that hedge suppliers are able to outbid competitors on longevity swap rates, and still secure a deal, by committing to tighter collateralization rules.

We show that, in the presence of longevity risk neutrality and absence of collateral, longevity swap rates depend on best estimate survival probabilities and on the degree of covariation between the floating leg and the defaultable term structure of interest rates facing both the hedger and the hedge supplier. This means that when the hedger is a pension plan, a proper analysis of the longevity swap cannot disregard the sponsor's covenant (see The Pensions Regulator, 2009, and Section 4 below). When collateral is introduced, longevity swap rates are also shaped by the expected gains/costs from holding/posting collateral. We show that collateralization means that the valuation of longevity swaps needs to allow for a discount rate that reflects the opportunity cost of collateral. This means, in particular, that default-free valuation formulae are not appropriate even in the presence of full collateralization and the corresponding absence of default losses.

We devote part of the article to examining relevant special cases, in order to understand how different collateral rules might affect longevity swap rates. A number of studies have recently addressed the issue of how to quantify longevity swap rates by calibrating to primary/secondary market prices or using approximate hedging methods (e.g., Dowd *et al.*, 2006; Bauer *et al.*, 2010; Biffis *et al.*, 2010; Chen and Cummins, 2010;

Cox *et al.*, 2010). As it is by no means clear how risk aversion plays a role in these transactions, we abstract from longevity risk aversion and focus on how counterparty default risk and collateral requirements might shape longevity swap rates. Our analysis shows that in stylized but realistic situations, longevity swap rates embed a margin for the cost of collateral.

The article is organized as follows. In the next section, we formalize the payoffs on longevity swaps, providing expressions for swap rates in the case of both indemnity-based and index-based swaps. In section 3, we examine the marking to market of a longevity swap during its life time to show the impact of default risk on its hedge effectiveness. Section 4 introduces bilateral counterparty risk into the longevity swap valuation formulae, identifying the main channels through which default risk may affect swap rates. Section 5 introduces credit enhancement in the form of collateral and shows how longevity swap rates are affected even in the presence of full collateralization. Several stylized examples are provided to understand how different collateralization rules may affect swap rates. Concluding remarks are offered in section 6. Further details and technical remarks are collected in an appendix.

2 Longevity swaps

We consider a hedger (pension fund, insurer), referred to as party A, and a hedge supplier (reinsurer, investment bank), referred to as counterparty B. Agent A has the obligation to pay a unitary amount to the survivors at some future time $T > 0$ of an initial population of n individuals (annuitants or pensioners) alive at time zero. Party A's liability is therefore given by $n - N_T$, where the random variable N_T counts the number of deaths experienced by the population during the period $[0, T]$. Assuming that the individuals have death times with common intensity $(\mu_t)_{t \geq 0}$,[7] the expected numbers of survivors at

[7] Intuitively, μ_t represents the instantaneous conditional death probability for an individual alive at time t.

T can be written as $E^{\mathbb{P}}[n - N_T] = np_T^{\mathbb{P}}$, with $p_T^{\mathbb{P}}$ given by (see appendix A for details)

$$p_T^{\mathbb{P}} := E^{\mathbb{P}}\left[\exp\left(-\int_0^T \mu_t \mathrm{d}t\right)\right]. \tag{2.1}$$

Here and in the following, $\mathbb{P}$ denotes the real-world probability measure. The intensity could be modeled explicitly, for instance by using any of the stochastic mortality models in Cairns *et al.* (2009). For our examples, we will rely on the simple Lee-Carter model.

Let us now consider a financial market and introduce the risk-free rate process $(r_t)_{t\geq 0}$. We assume that a market-consistent price of the liability can be computed by using a risk-neutral measure $\mathbb{Q}$, equivalent to $\mathbb{P}$, such that the death times still admit the intensity $(\mu_t)_{t\geq 0}$; see Biffis *et al.* (2010) for details. The liability therefore has time-zero price

$$E^{\mathbb{Q}}\left[\exp\left(-\int_0^T r_t \mathrm{d}t\right)(n - N_T)\right] = nE^{\mathbb{Q}}\left[\exp\left(-\int_0^T (r_t + \mu_t)\mathrm{d}t\right)\right]. \tag{2.2}$$

We consider two instruments which A can enter into with B to hedge its exposure: an indemnity-based longevity swap and an index-based longevity swap. In this section, we ignore default risk and for simplicity we consider single payment instruments (i.e., forward contracts); the extension to multiple payments is immediate and covered in some of our numerical examples. In what follows, we always take the perspective of the hedger.

2.1 Indemnity-based longevity swap

This instrument allows A to pay a fixed rate $\overline{p}^N \in (0, 1)$ against the realized survival rate experienced by the population between time zero and time T. Assuming a notional amount equal to the initial population size, n, the net payout to the hedger at time T is

$$n\left(\frac{n - N_T}{n} - \overline{p}^N\right),$$

i.e., the difference between the realized number of survivors and the fixed rate $n\overline{p}^N$ locked in at inception. Letting S_0 denote the market value of the swap at inception, we have

$$\begin{aligned} S_0 &= nE^{\mathbb{Q}}\left[\exp\left(-\int_0^T r_t \mathrm{d}t\right)\left(\frac{n-N_T}{n} - \overline{p}^N\right)\right] \\ &= nE^{\mathbb{Q}}\left[\exp\left(-\int_0^T (r_t+\mu_t)\mathrm{d}t\right)\right] - nB(0,T)\overline{p}^N, \end{aligned} \tag{2.3}$$

with $B(0,T)$ denoting the time-zero price of a zero-coupon bond maturing at T. Setting $S_0 = 0$, we obtain the following expression for the swap rate:

$$\overline{p}^N = p_T^{\mathbb{Q}} + B(0,T)^{-1}\mathrm{Cov}^{\mathbb{Q}}\left(\exp\left(-\int_0^T r_t \mathrm{d}t\right), \exp\left(-\int_0^T \mu_t \mathrm{d}t\right)\right), \tag{2.4}$$

where the risk-adjusted survival probability $p_T^{\mathbb{Q}}$ is defined analogously to (2.1).

2.2 Index-based longevity swap

This standardized instrument allows A to pay a fixed rate $\overline{p} \in (0,1)$ against the realized value of a survival index at time T. The latter might reflect the mortality experience of a reference population closely matching that of the liability portfolio. Examples are represented by the LifeMetrics index developed by J.P. Morgan.[8] We assume that the index admits the representation $\exp(-\int_0^t \mu_s^I \mathrm{d}s)$, where $(\mu_t^I)_{t\geq 0}$ is the intensity of mortality of the reference population. Expression (2.4), for example, is then replaced by

$$\overline{p}^I = p_T^{I,\mathbb{Q}} + B(0,T)^{-1}\mathrm{Cov}^{\mathbb{Q}}\left(\exp\left(-\int_0^T r_t \mathrm{d}t\right), \exp\left(-\int_0^T \mu_t^I \mathrm{d}t\right)\right). \tag{2.5}$$

The relative advantages and disadvantages of this instrument with respect to the indemnity-based swap are discussed in Biffis and Blake (2010b).

[8] See `www.lifemetrics.com`.

2.3 Swap rates

Expressions (2.4)-(2.5) show that if the intensity of mortality is uncorrelated with the bond/swap market (a reasonable first-order approximation), swap rates are just the risk-adjusted survival probabilities $p_T^{\mathbb{Q}}$ and $p_T^{I,\mathbb{Q}}$. If longevity risk is not priced under $\mathbb{Q}$, we simply set $\overline{p} = p_T^{\mathbb{P}}$ and $\overline{p}^I = p_T^{I,\mathbb{P}}$. A number of studies have recently addressed the issue of how to quantify $p_T^{I,\mathbb{Q}}$, for example, by calibration to annuity market prices, books of policies traded in secondary markets, or by use of approximate hedging methods (see references in Section 1). As it is not clear how longevity risk is priced in a longevity swap transaction (there is essentially no publicly available information on swap rates), we will suppose a baseline case in which $p_T^{\mathbb{P}} = p_T^{\mathbb{Q}}$ or $p_T^{I,\mathbb{Q}} = p_T^{I,\mathbb{P}}$ and focus on how counterparty default risk and collateral requirements may shape longevity swap rates. Similarly, Biffis and Blake (2010a, 2009) endogenize longevity risk premia by introducing asymmetric information and capital requirements in a risk-neutral setting.

2.4 More general structures

In practice, the floating payment of a longevity swap has a LIBOR component which typically makes the covariance term appearing in (2.4)-(2.5) non null. In what follows, we mainly concentrate on longevity risk and will typically ignore the interest-rate component of the variable payment. To keep the setup general, however, we will consider instruments with a generic variable payment, P, which may include a LIBOR component, as well as survival indexation rules different from the ones considered above. In this case, we will write the market value of the swap and the swap rates as

$$S_0 = nE^{\mathbb{Q}}\left[\exp\left(-\int_0^T r_t \mathrm{d}t\right)(P - \overline{p})\right], \tag{2.6}$$

$$\overline{p} = E^{\mathbb{Q}}[P] + B(0,T)^{-1}\mathrm{Cov}^{\mathbb{Q}}\left(\exp\left(-\int_0^T r_t \mathrm{d}t\right), P\right). \tag{2.7}$$

3 Marking to market

Longevity swaps are not currently exchange traded and so there is no commonly accepted framework for counterparties to mark to market their positions. The presence of counterparty default risk and collateralization rules, however, makes the marking-to-market procedure an important feature of these transactions. The role of collateral is examined later on; here, we show how the hedging instrument operates from the point of view of the hedger. In the case of an indemnity-based solution, at each time t in $[0, T]$, the value of the swap can be computed by using the valuation formula

$$\begin{aligned} S_t &= nE_t^{\mathbb{Q}}\left[\exp\left(-\int_t^T r_s \mathrm{d}s\right)\left(\frac{n-N_T}{n} - \overline{p}^N\right)\right] \\ &= nE_t^{\mathbb{Q}}\left[\exp\left(-\int_t^T r_s \mathrm{d}s\right)\left(\frac{n-N_t}{n}\exp\left(-\int_t^T \mu_s \mathrm{d}s\right)\right)\right] - nB(t,T)\overline{p}^N, \end{aligned} \tag{3.1}$$

where $B(t, T)$ denotes the market value of a ZCB with time to maturity $T - t$. The extension to multiple payments or to index-based swaps is immediate.

The analysis of the market value of a longevity swap over its life time is important for at least three reasons. First, at each payment date, the difference between the variable and fixed payment generates a cash inflow or outflow to the hedger, depending on the evolution of mortality. In the absence of basis risk (which is the case for indemnity-based solutions), these differences show a pure 'cashflow hedge' of the longevity exposure in operation. Second, as market conditions change (e.g., mortality patterns, counterparty default risk), the marking-to-market procedure will result in the swap qualifying as an asset or a liability in the hedger's balance sheet. This may have the implication that, even if the swap payments are expected to provide a good hedge against longevity risk, the hedger's position may still turn into a liability if, for example, deterioration in the hedge supplier's credit quality shrinks the expected present value of the variable payments. Third, for solvency requirements, it is important to value a longevity swap under extreme market/mortality scenarios ('stress testing'). This means, for example, that even if a longevity swap qualifies as a liability on a market-consistent basis, it might still provide considerable capital relief when valued on a regulatory basis.

To illustrate these points, let us consider the hypothetical situation of an insurer A with a liability represented by a group of ten thousand 65-year-old annuitants drawn from the population of England & Wales in 1980. We assume that A enters a 15-year pure longevity swap in 1980 and we follow the evolution of the contract until maturity. The population is assumed to evolve according to the death rates reported in the Human Mortality Database (HMD) for England & Wales. We assume that interest-rate risk is hedged away through interest rate swaps, locking in a rate of 5% throughout the life of the swap. As a simple benchmark case, we assume that longevity swap rates at each marking-to-market date (including inception) are based on Lee-Carter forecasts using the latest HMD information available. Figure 1 illustrates the evolution of swap survival rates for an England & Wales cohort tracked from age 65 in 1980 to age 80 in 1995 (see Dowd *et al.*, 2010a,b, for a comprehensive analysis of different mortality models).

< Figure 1 about here >

It is clear that the systematic underestimation of mortality improvements by the Lee-Carter model in this particular example will mean that the hedger's position will become increasingly in the money as the swap matures. This is shown in Figure 2. In practice the contract may allow the counterparty to cancel the swap / reset the fixed leg for a nonnegative fee; we ignore these features in this example. Figure 2 also reports the sequence of cash inflows and outflows generated by the swap, which are lower ex-post than what was anticipated from an examination of the marking-to-market basis. As interest rate risk is hedged - and again ignoring default risk for the moment - cash inflows/outflows arising in the backtesting exercise only reflect the difference between realized survival rates and swap rates. On the other hand, the swap's market value reflects changes in survival swap rates, which follow the updated Lee-Carter forecasts depicted in Figure 1 and differ from realized survival rates. Marking-to-market profits and losses can jeopardize a well structured hedging position. As a simple example which predicts the next section, let us introduce credit risk and assume that in 1988 the credit spread of the hedge supplier widens across all maturities by 50 and 100 basis points. The impact of such a scenario on the hedger's balance sheet is dramatic, as shown in Figure 3.

< Figure 2 about here >

< Figure 3 about here >

4 Counterparty default risk

The backtesting exercise of the previous section has emphasized the importance of marking to market and default risk in assessing the value of a longevity swap to the hedger. As was emphasized in the introduction, however, a proper valuation should allow for bilateral counterparty default risk. This is the case even when the hedger is a pension plan. Private sector defined benefit pension plans in countries such as the UK rely on a promise by the sponsoring employer to pay the benefits to plan members. This promise is known as the 'sponsor covenant'. The Actuarial Profession (2005, par. 3.2) defined the sponsor covenant as: "the combination of (a) the ability and (b) the willingness of the sponsor to pay (or the ability of the trustees to require the sponsor to pay) sufficient advance contributions to ensure that the scheme's benefits can be paid as they fall due." The strength of the sponsor covenant therefore depends on both the financial strength of the employer and the employer's commitment to the scheme. The sponsor covenant plays the same role in defined benefit pension plans as, say, capital in life insurance company annuity provision or collateral and margin payments in derivatives contracts (such as swaps and options). This is why pension funds in the UK do not have formal capital requirements or collateralization agreements in place. It is also why they have a different regulator - The Pensions Regulator - from the rest of the financial services industry - which is regulated by the Financial Services Authority. However, the regulatory capital requirements for insurers are precisely laid down. Life insurance companies need to have sufficient capital in place to remain solvent over a 12-month period with 99.5% probability (i.e., they need to have sufficient capital to survive a 1-in-200-year event). By contrast, the sponsor's obligations to make contributions into a pension plan is typically not well defined in the trust deeds. This is why The Actuarial Profession (2005, p. 4) admits that its definition refers to vague and difficult-to-measure concepts such as 'willingness' or 'ability'. In June 2009, The Pensions Regulator (2009) issued a statement inviting trustees to consider the sponsor covenant when setting prudent funding targets and suitable recovery plans in response to lower asset values and higher deficits following the Global Banking Crisis. In June 2010, The Pensions Regulator (2010) launched a campaign to improve the monitoring of the sponsor covenant by scheme trustees on an ongoing basis.

The following analysis shows that a proper valuation of default risk in longevity swaps must take into account the value of the sponsor covenant. For the latter, we use the sponsor's default intensity, and refer to it as party A's intensity of default. For large corporate pension plans, the intensity can be derived/extrapolated from spreads observed in corporate bond and CDS markets. For smaller plans, an analysis of the funding level and strategy of the scheme is required.

Assume that both A (the hedger) and B (the hedge supplier) may default at random times τ^A, τ^B admitting default intensities $(\lambda_t^A)_{t\geq 0}$, $(\lambda_t^B)_{t\geq 0}$. Assume further that on the default event $t = \min(\tau^A, \tau^B) \leq T$, the nondefaulting counterparty receives a fraction $\psi^i \in [0, 1]$ $(i \in \{A, B\})$ of the market value of the swap before default, S_{t-}, if she is in the money, otherwise she has to pay the full pre-default market value S_{t-} to the defaulting counterparty. We can then write the market value of the swap as (e.g., Duffie and Huang, 1996):

$$S_0 = nE_0^{\mathbb{Q}}\left[\exp\left(-\int_0^T (r_t + (1-\psi^A)\lambda_t^A + (1-\psi^B)\lambda_t^B)\mathrm{d}t\right)(P - \overline{p})\right], \qquad (4.1)$$

where P denotes the variable payment (see section 2.4). The swap rate then admits the representation

$$\overline{p} = E^{\mathbb{Q}}[P] + \frac{\mathrm{Cov}^{\mathbb{Q}}\left(\exp\left(-\int_0^T (r_t + (1-\psi^A)\lambda_t^A + (1-\psi^B)\lambda_t^B)\mathrm{d}t\right), P\right)}{E^{\mathbb{Q}}\left[\exp\left(-\int_0^T (r_t + (1-\psi^A)\lambda_t^A + (1-\psi^B)\lambda_t^B)\mathrm{d}t\right)\right]}, \qquad (4.2)$$

showing that swap rates depend in a complex way on the interaction between the variable payments and economic variables such as interest rates, default intensities and recovery rates. When P does not include a demographic component, as in interest rate swaps, the covariance term is typically negative (e.g., Johannes and Sundaresan, 2007). When P only includes a demographic component, as in Sections 2.1-2.2, we may expect the covariance term also to be negative, as longevity-linked payments are likely to be positively correlated with the yields on the bonds issued by companies with significant pension liabilities.[9] The joint case of floating payments linked to both mortality and interest rates would appear to suggest a swap rate $\overline{p} < E^{\mathbb{Q}}[P]$. In the next section, we will show that, consistent with what is observed in the longevity swap market, this is not necessarily the case. To understand why, observe that in the case of full recovery ($\psi^A = \psi^B = 1$), expression (4.1) reduces to a default-free risk-neutral valuation formula, irrespective of the default intensities of the counterparties. This is misleading, as different credit enhancement strategies/tools carry a cost that is not explicitly captured by (4.1)-(4.2). The next section addresses this issue.

[9] On the hedge supplier's side, this is a reasonable assumption for monoline insurers such as pension buyout firms. The assumption is clearly questionable for well diversified reinsurers. In the latter case, however, the covariance is still likely to be negative due to the positive dependence on the hedger's side.

5 Collateralization

Counterparty risk can be mitigated through a number of credit enhancement techniques, such as termination rights (e.g., credit puts and break clauses) or credit derivatives (e.g., CDSs and credit spread options). Here we focus on collateralization, a form of direct credit support requiring each party to post cash or securities when either party is out of the money. For simplicity, we consider the case of cash, which is by far the most common type of collateral (e.g., ISDA, 2010a) and allows us to disregard close-out risk (i.e., the risk that the value of collateral may change at default).

Collateral agreements reflect the amount of acceptable credit exposure each party agrees to take on. We will consider simple collateral rules capturing the main features of the problem. Formally, let us introduce a collateral process $(C_t)_{t\in[0,T]}$ indicating how much cash C_t to post at each time t in response to changes in market conditions and, in particular, the market value of the swap (we provide explicit examples below). Again, we develop our analysis from the point of view of the hedger, so that $C_t > 0$ ($C_t < 0$) means that agent A is holding (posting) collateral. For simplicity, we assume that each party recovers/loses nothing more than the collateral held/posted upon default of the counterparty:

- On $\{\tau^A \leq \min(\tau^B, T)\}$ (hedger's default), party B seizes any collateral received by the hedger an instant prior to default, $\max(-C_{\tau^A-}, 0)$, and looses any collateral posted with A; hence A recovers $C_{\tau^A-} = \max(C_{\tau^A-}, 0) + \min(C_{\tau^A-}, 0)$.

- On $\{\tau^B \leq \min(\tau^A, T)\}$ (hedge supplier's default), party A seizes any collateral received by B an instant prior to default, $\max(C_{\tau^B-}, 0)$, and looses any collateral posted with B; hence A recovers $C_{\tau^B-} = \max(C_{\tau^B-}, 0) + \min(C_{\tau^B-}, 0)$.

To obtain neater results, it is convenient to express the collateral before default of either party as[10]

$$C_t = \left(c_t^1 1_{\{S_t \geq 0\}} + c_t^2 1_{\{S_t < 0\}}\right) S_t, \tag{5.1}$$

where c^1, c^2 are two nonnegative processes expressing collateral as a fraction of the market value of the swap, when marking to market an asset or a liability for A. Finally, we introduce two nonnegative processes $(\delta_t^1)_{t \geq 0}$, $(\delta_t^2)_{t \geq 0}$ representing the yield on and opportunity cost of collateral (they are assumed to be the same for both parties), in the sense that holding/posting collateral of amount C_t yields/costs

$$\left(\delta_t^1 c_t^1 1_{\{S_t \geq 0\}} + \delta_t^2 c_t^2 1_{\{S_t < 0\}}\right) S_t.$$

As shown in the appendix, under our assumptions the market value of the swap can be written as

$$S_0 = n E^{\mathbb{Q}} \left[\exp\left(- \int_0^T (r_t + \Gamma_t) \mathrm{d}t \right) (P - \overline{p}) \right], \tag{5.2}$$

where the spread $(\Gamma_t)_{t \in [0,T]}$ admits the explicit expression

$$\begin{aligned} \Gamma_t = & \lambda_t^A (1 - c_t^1 1_{\{S_t \geq 0\}} - c_t^2 1_{\{S_t < 0\}}) + \lambda_t^B (1 - c_t^1 1_{\{S_t \geq 0\}} - c_t^2 1_{\{S_t < 0\}}) \\ & - \left(\delta_t^1 c_t^1 1_{\{S_t \geq 0\}} + \delta_t^2 c_t^2 1_{\{S_t < 0\}}\right), \end{aligned} \tag{5.3}$$

and the swap rate can be written as

$$\overline{p} = E^{\mathbb{Q}}[P] + \frac{\mathrm{Cov}^{\mathbb{Q}}\left(\exp\left(- \int_0^T (r_t + \Gamma_t) \mathrm{d}t \right), P \right)}{E^{\mathbb{Q}}\left[\exp\left(- \int_0^T (r_t + \Gamma_t) \mathrm{d}t \right) \right]}. \tag{5.4}$$

We elaborate on formula (5.4) by examining simple special cases.

[10]The indicator function 1_H takes the value of unity if the event H is true, zero otherwise.

5.1 Full collateralization

Consider the collateral rule $c_t^1 = 1, c_t^2 = 1$, implying that the full market value of the swap is received/posted as collateral depending on whether S_t is positive/negative. As there is full recovery of collateral, default is immaterial. In contrast to expression (4.1), however, expression (5.2) does not reduce to the usual default-free, risk-neutral valuation formula, unless collateral is costless. For example, assuming that collateral yields/costs are symmetric and equal to δ_t, we obtain

$$S_0 = nE^{\mathbb{Q}}\left[\exp\left(-\int_0^T (r_t - \delta_t)\mathrm{d}t\right)(P - \overline{p})\right], \tag{5.5}$$

which reduces to the usual default-free valuation formula only if collateral costs are zero. If the net cost of collateral is positively related to interest rates and default intensities, we expect the swap rate to be higher than the one given by expression (4.2), reflecting the fact that (costly) collateralized protection commands a premium (see Johannes and Sundaresan, 2007, for the case of interest rate swaps). As in the longevity space the cost of collateral is positively dependent on mortality improvements, and typically much higher than the short rate, we expect the covariance term in (5.4) to be positive, giving $\overline{p} > E^{\mathbb{Q}}[P]$.

5.2 Collateral rules

According to ISDA (2010a), it is very common for collateral agreements to specify collateral triggers based on the market value of the swap crossing pre-specified threshold levels. The following are relevant (although stylized) examples for our discussion:

a) $c_t^1 = 1_{\{S_t > \underline{s}\}}$ and $c_t^2 = 1_{\{S_t < \overline{s}\}}$ (with $\underline{s} < \overline{s}$), meaning that the hedge supplier (hedger) is required to post full collateral if the swap's market value is above (below) a given threshold $\overline{s}$ ($\underline{s}$).

b) $c_t^1 = 1_{\{N_t<\alpha\}}$ and $c_t^2 = 1_{\{N_t>\beta\}}$ (with $0 \leq \alpha < \beta \leq n$), meaning that the hedge supplier (hedger) is required to post full collateral if realized deaths are below (above) a given threshold α (β). The strategy can be used for an index-based swap by setting $c_t^1 = 1_{\{\int_0^t \mu_s^I \mathrm{d}s<a\}}$ and $c_t^2 = 1_{\{\int_0^t \mu_s^I \mathrm{d}s>b\}}$ (with $0 \leq a < b$), meaning that collateral posting is triggered when the path of the longevity index exits a pre-specified range $[\exp(-b), \exp(-a)]$.

c) As the severity of counterparty risk depends on the credit quality of the counterparties, collateralization agreements typically set collateral thresholds that explicitly depend on credit ratings or CDS spreads.[11] A simple example of this practice is the collateralization rule $c_t^1 = 1_{\{N_t<\alpha\}\cup\{\lambda_t^B>\gamma\}}$, $c_t^2 = 1_{\{N_t>\beta\}}$ (with $\gamma \geq 0$), meaning that the hedger receives collateral when either realized deaths fall below a given level $\alpha < \beta$ or the hedge supplier's default intensity overshoots a threshold γ.

As was evident from the examples in section 3, the credit exposure of a longevity swap is close to zero at inception and at maturity, but may be sizable during the life of the swap, depending on the trade-off between changes in market/mortality conditions and the residual swap payments (amortization effect). In practice, the threshold levels $\alpha, \beta, a, b, \gamma$ will be set so as to ensure that the size and dynamics of the credit risk exposure are acceptable for both parties.

5.3 Some numerical examples

As a simple example, consider the case in which the short rate is a constant $r > 0$, both parties have the same default intensity $\lambda > 0$, and collateral yields/costs are symmetric and equal to $\delta > 0$. In this setting the market value of the swap can only change in response to the evolution of mortality. The collateralization rule described in example (b)

[11] According to responses collected by ISDA in the 2010 Margin Survey, 86% of firms use credit ratings to set collateral thresholds, 12% use CDS spreads. The percentages increase to 100% and 27% for the 14 largest dealer banks. See ISDA (2010b).

above is therefore appropriate to proxy changes in the swap's market value. Assuming that longevity risk is not priced under $\mathbb{Q}$ (i.e., $p_T^{\mathbb{Q}} = p_T^{\mathbb{P}}$) and that collateral is posted if $N_t < \alpha$ or $N_t > \beta$ (with $0 \leq \alpha < \beta \leq n$), we can write expression (5.2) as

$$\begin{aligned} S_0 =& n \exp\left(-T(r+2\lambda)\right) \\ & E^{\mathbb{P}}\left[\exp\left((2\lambda+\delta)\int_0^T \left(1_{\{N_t<\alpha\}}1_{\{S_t\geq 0\}} + 1_{\{N_t>\beta\}}1_{\{S_t<0\}}\right)\mathrm{d}t\right)\left(\frac{n-N_T}{n} - \overline{p}^N\right)\right], \end{aligned} \tag{5.6}$$

obtaining the following expression for the longevity swap rate:

$$\overline{p}^N = p_T^{\mathbb{P}} + \frac{\mathrm{Cov}^{\mathbb{P}}\left(\exp\left((2\lambda+\delta)\int_0^T \left(1_{\{N_t<\alpha\}}1_{\{S_t\geq 0\}} + 1_{\{N_t>\beta\}}1_{\{S_t<0\}}\right)\mathrm{d}t\right), \frac{n-N_T}{n}\right)}{E^{\mathbb{P}}\left[\exp\left((2\lambda+\delta)\int_0^T \left(1_{\{N_t<\alpha\}}1_{\{S_t\geq 0\}} + 1_{\{N_t>\beta\}}1_{\{S_t<0\}}\right)\mathrm{d}t\right)\right]}. \tag{5.7}$$

Depending on how the thresholds α and β are set, the covariance term can have different sign. Table 2 reports some examples for different values of α, β, λ and δ.

In practice, it is not uncommon for hedge suppliers to agree to one-way collateralization to secure a deal, meaning that they will bear the burden of posting collateral if the swap's market value moves against them. Setting $\beta = n$ (so that $1_{N_t>\beta} = 0$ almost surely) in the above formula yields:

$$\overline{p}^N = p_T^{\mathbb{P}} + \frac{\mathrm{Cov}^{\mathbb{P}}\left(\exp\left((2\lambda+\delta)\int_0^T \left(1_{\{N_t<\alpha\}}1_{\{S_t\geq 0\}}\right)\mathrm{d}t\right), \frac{n-N_T}{n}\right)}{E^{\mathbb{P}}\left[\exp\left((2\lambda+\delta)\int_0^T \left(1_{\{N_t<\alpha\}}1_{\{S_t\geq 0\}}\right)\mathrm{d}t\right)\right]}. \tag{5.8}$$

The covariance term is clearly positive for $\alpha > 0$, and hence the longevity swap rate embeds a positive margin reflecting the hedge supplier's opportunity cost of collateral.

In general, there may be asymmetry in collateral costs for hedgers and hedge suppliers, because pension plans need to satisfy solvency requirements far less stringent than insurers (see Biffis and Blake, 2009, for a discussion). Without formalizing this situation, it is clear that it would have the effect of making the covariance term in (5.7) more likely to be positive, and hence lead to a swap rate $\overline{p}^N > p_T^{\mathbb{P}}$.

...To be completed ...

6 Conclusion

...To be completed ...

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A Details on the setup

We take as given a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, \mathbb{P})$, and model the death times in a population of n individuals (annuitants or pensioners) as stopping times $\tau^1, \dots, \tau^n$. This means that at each time t the information carried by $\mathcal{F}_t$ allows us to state whether each individual has died or not. We assume that death times cannot occur simultaneously. The hedger's liability is given by the random variable $\sum_{i=1}^n 1_{\tau^i > T}$, which can be equivalently written as $n - \sum_{i=1}^n 1_{\tau^i \le T} = n - N_T$ (recall that the indicator function 1_H takes the value of unity if the event H is true, zero otherwise). We assume that death times coincide with the first jumps of n conditionally Poisson processes with common random intensity of mortality $(\mu_t)_{t\ge 0}$ under both $\mathbb{P}$ and an equivalent martingale measure $\mathbb{Q}$ (see Biffis *et al.*, 2010, for details). The expected number of survivors over $[0, T]$ under the two measures can then be expressed as $E^{\mathbb{P}}\left[\sum_{i=1}^{n} 1_{\tau^i > T}\right] = np_T^{\mathbb{P}}$ and $E^{\mathbb{Q}}\left[\sum_{i=1}^{n} 1_{\tau^i > T}\right] = np_T^{\mathbb{Q}}$, with (say) $p_T^{\mathbb{Q}}$ a risk-adjusted survival probability given by

$$p_T^{\mathbb{Q}} = E^{\mathbb{Q}}\left[\exp\left(-\int_0^T \mu_t \mathrm{d}t\right)\right].$$

Consider any stopping time τ^i satisfying the above assumptions, an integrable random variable $Y \in \mathcal{F}_T$ and a bounded process $(X_t)_{t\in[0,T]}$ such that each X_t is measurable with respect to $\mathcal{F}_{t-}$. Then a security paying Y at time T in case $\tau^i > T$ and X_{τ^i} at time τ^i in case $\tau^i \leq T$ has time-zero price (e.g., Bielecki and Rutkowski, 2002)

$$E^{\mathbb{Q}}\left[\int_0^T \exp\left(-\int_0^s (r_t + \mu_t)\mathrm{d}t\right) X_s \mu_s \mathrm{d}s + \exp\left(-\int_0^T (r_t + \mu_t)\mathrm{d}t\right) Y\right].$$

Consider now two stopping times τ^i, τ^j, with intensities μ^i, μ^j, jointly satisfying the above assumptions (i.e., they are the first jump times of the components of a bivariate conditionally Poisson process). A security paying Y at time T in case neither stopping time has occurred (i.e., $\min(\tau^i, \tau^j) > T$) and X_t in case the first occurrence is at time $t \in [0, T]$ (i.e., $t = \min(\tau^i, \tau^j)$) has time-zero price given by the same formula, with μ_t replaced by $\mu_t^i + \mu_t^j$. This follows from the fact that the stopping time $\min(\tau^i, \tau^j)$ is the first jump time of a conditionally Poisson process with intensity $(\mu_t^i + \mu_t^j)_{t\geq 0}$ (e.g., Bielecki and Rutkowski, 2002). The expressions presented in sections 2-4 all follow from these simple results.

Proof of expression (5.2). Let $(\delta_t^1)_{t\geq 0}$ denote the yield on holding collateral and $(\delta_t^2)_{t\geq 0}$ the opportunity cost of posting collateral for both parties, meaning that holding collateral of amount C_t provides an instantaneous yield $\delta_t^1 C_t^+ - \delta_t^2 C_t^-$ (we use the notation $a^+ := \max(a, 0)$, $a^- := -\min(a, 0)$). We assume that collateral is bounded and for all $t \in [0, T]$ C_t is $\mathcal{F}_{t-}$ measurable. Using the properties of τ^A, τ^B reviewed above, we can then write:

$$
\begin{aligned}
S_0 =& E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^T (r_t+\lambda_t^A+\lambda_t^B)\mathrm{d}t\right)(P-\overline{p})\right]\\
&+E_0^{\mathbb{Q}}\left[\int_0^T \exp\left(-\int_0^s (r_t+\lambda_t^A+\lambda_t^B)\mathrm{d}t\right)\left(\lambda_s^A C_s^+ - \lambda_s^B C_s^-\right)\mathrm{d}s\right]\\
&+E_0^{\mathbb{Q}}\left[\int_0^T \exp\left(-\int_0^s (r_t+\lambda_t^A+\lambda_t^B)\mathrm{d}t\right)(\delta_s^1 C_s^+ - \delta_s^2 C_s^-)\mathrm{d}s\right].
\end{aligned}
\tag{A.1}
$$

Using representation (5.1) we can finally write the above as

$$
\begin{aligned}
S_0 =& E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^T (r_t+\lambda_t^A+\lambda_t^B)\mathrm{d}t\right)(P-\overline{p})\right]\\
&+E_0^{\mathbb{Q}}\left[\int_0^T \exp\left(-\int_0^s (r_t+\lambda_t^A+\lambda_t^B)\mathrm{d}t\right)\left((\lambda_s^A+\delta_s^1)c_s^1 S_s^+ - (\lambda_s^B+\delta_s^2)c_s^2 S_s^-\right)\mathrm{d}s\right]\\
=& E_0^{\mathbb{Q}}\left[\exp\left(-\int_0^T (r_t+\Gamma_t)\mathrm{d}t\right)(P-\overline{p})\right],
\end{aligned}
\tag{A.2}
$$

which is nothing other than the usual risk-neutral valuation formula for a security with terminal payoff $S_T = P - \overline{p}$ paying continuously a dividend equal to a fraction

$$(\lambda_s^A+\delta_s^1)c_s^1 1_{S_t\geq 0} + (\lambda_s^B+\delta_s^2)c_s^2 1_{S_t<0}$$

of the security's market value, S_t, for each $t\in[0,T]$. The result then follows.

B Tables and figures

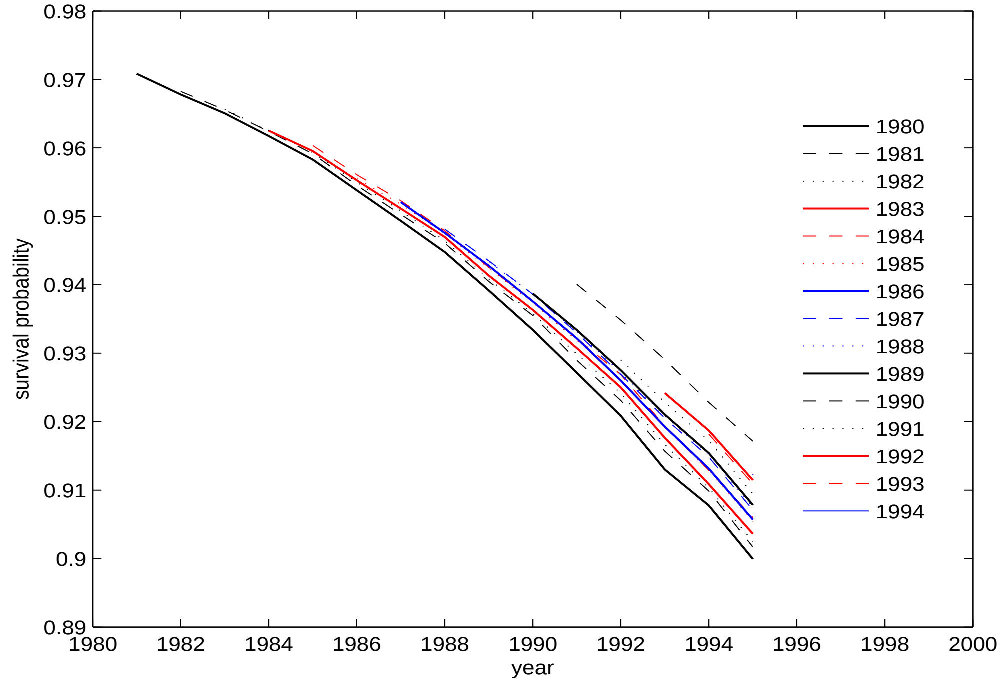

Figure 1: Survival probabilities $65 + t$-year old males from England & Wales in year $1980 + t$, based on Lee-Carter forecasts using the latest HMD data avaiable.

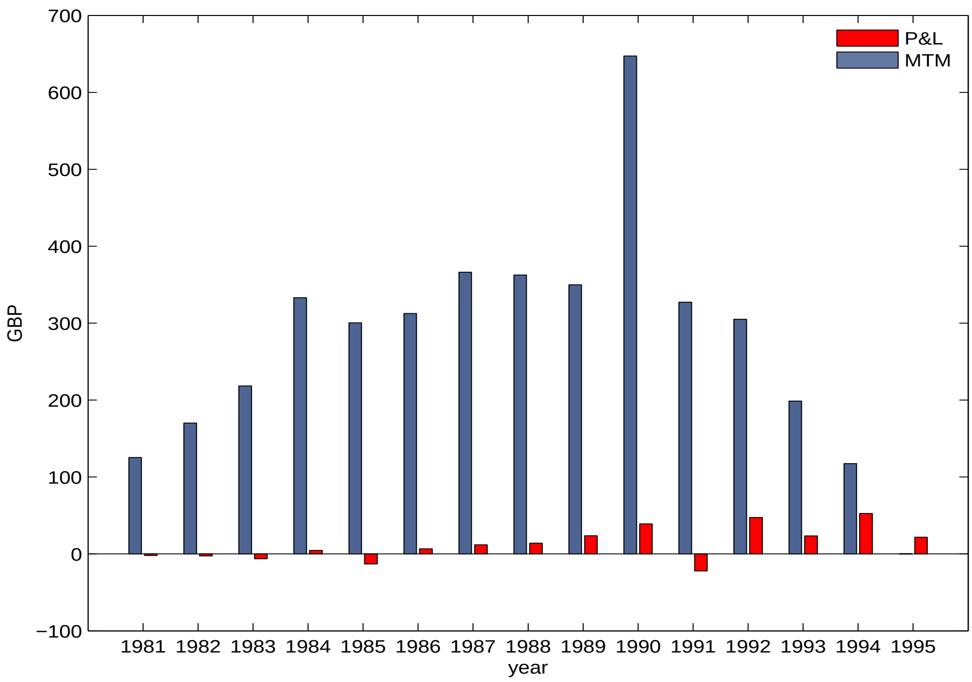

Figure 2: Market value of the longevity swap and stream of cashflows with no credit risk.

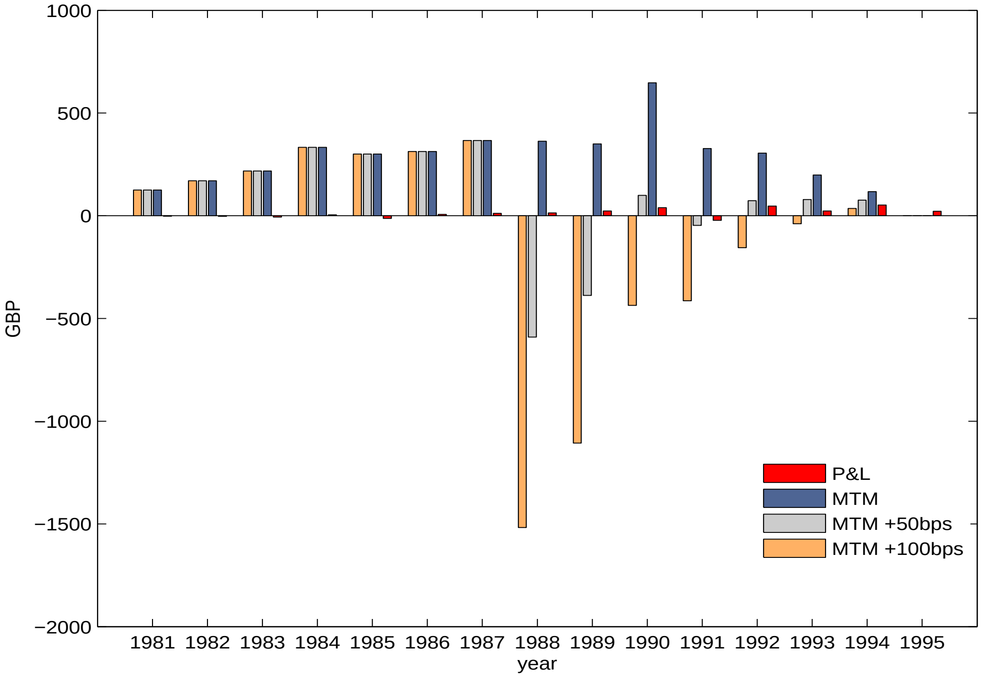

Figure 3: Market value of the longevity swap and stream of cashflows with no credit risk (MTM), and with counterparty B's credit spreads widening by 50 and 100 basis points over 1988-1995.

Date	**Hedger**	**Size**	**Term (yrs)**	**Type**
Jan 08	Lucida	Not disclosed	10	indexed
Jul 2008	Canada Life	GBP 500m	40	indemnity
Feb 2009	Abbey Life	GBP 1.5bn	run off	indemnity
Mar 2009	Aviva	GBP 475m	10	indemnity
Jun 2009	Babcock International	GBP 750m	50	indemnity
Jul 2009	RSA	GBP 1.9bn	run off	indemnity
Dec 2009	Royal County of Berkshire	GBP 750m	run off	indemnity
Feb 2010	BMW UK	GBP 3bn	run off	indemnity

Table 1: Publicly announced longevity swap transactions. Source: Coughlan (2010).