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Free Convective Unsteady MHD Flow of Newtonian Fluid in a Channel with Adiabatic

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ABSTRACT
In this paper, we investigated an unsteady free convection MHD flow of an incompressible viscous electrically conducting fluid under the action of transverse uniform magnetic field between two heated vertical plates by keeping one plate is adiabatic. The governing equations of velocity and temperature fields with appropriate boundary conditions are solved by using perturbation technique. The effects of various physical parameters on the velocity and temperature fields are discussed in detail with the help of graphs.
Keywords: MHD flow, Unsteady Flow, Adiabatic Plate, Heat Transfer

I. Introduction

The influence of magnetic field on viscous incompressible flow of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of rayon and nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. In many process industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled electively to achieve final products of desired characteristics by drawing threads, etc. in the presence of an electrically conducting fluid subject to a magnetic field. The unsteady flow and heat transfer through a viscous incompressible fluid in the presence of transverse magnetic field between two horizontal plates, lower plate being a stretching sheet and upper being porous was studied by Sharma and Kumar (1998). Borkakati and Chakrabarty (2000) have investigated unsteady free convection MHD flow between two heated vertical plates. The unsteady transient free convection flow of an incompressible dissipative viscous fluid past an infinite vertical plate on taking into account the viscous dissipative heat under the influence of a uniform transverse magnetic field was analyzed by Sreekant et al. (2001). Gourla and Katoch (1991) have studied the unsteady free convection MHD flow between two heated vertical plates. But, they did not discuss about the thermodynamic case on the boundary condition on which the plate is adiabatic.

In view of these, we studied the unsteady free convection MHD flow of an incompressible viscous electrically conducting fluid under the action of transverse uniform magnetic field between two heated vertical plates by keeping one plate is adiabatic. The governing equations of velocity and temperature fields with appropriate boundary conditions are solved by using perturbation technique. The effects of various physical parameters on the velocity and temperature fields are discussed in detail with the help of graphs.

II. Mathematical formulation

Let us consider free convective unsteady MHD flow of a viscous incompressible electrically conducting fluid between two heated vertical parallel plates. Let x-axis be taken along the vertically upward direction through the central line of the channel and the y-axis is perpendicular to the x-axis. The plates of the channel are kept at $y = \pm h$ distance apart. A uniform magnetic field B_0 is applied in the plane of y-axis and perpendicular to the both x axis and y-axis. u' is the velocity in the direction of flow of fluid, along the x-axis and v' is the velocity along the y-axis. Consequently u' is a function of y' and t', but v' is independent of y'. The fluid is assumed to be of low conductivity, such that the induced magnetic field is negligible.

In order to derive the equations of the problem, we assume that the fluid is finitely conducting and the viscous dissipation the Joule heats are neglected. The polarization effect is also neglected.

At time $t > 0$, the temperature of the plate at $y = h$ changes according to the temperature function: $T = T_0 + (T_w - T_0)(1 - e^{-n't'})$, where T_w and T_0 are the temperature at the plates $y = h$ and at $y = -h$ respectively, and $n^{'}(\geq 0)$ is a real number, denoting the decay factor.
Hence the flow field is seen to be governed by the following equations:

$$\frac{\partial v^{'}}{\partial y^{'}} = 0 \tag{2.1}$$

$$\frac{\partial u'}{\partial t^{'}} = \nu \frac{\partial^2 u^{'}}{\partial y^{'2}} + g\beta(T^{'} - T_0) - \frac{\sigma B_0^2}{\rho} u^{'} \tag{2.2}$$

$$\frac{\partial T'}{\partial t^{'}} = \frac{K}{\rho C_p} \frac{\partial^2 T^{'}}{\partial y^{'2}} \tag{2.4}$$

here ρ is the density of the fluid, B_0 is the magnetic field strength, σ is the electrical conductivity of the fluid, ν is the co-efficient of kinematic viscosity, K is the thermal conductivity of the fluid, C_p is the specific heat at constant pressure, β is the co-efficient of thermal expansion, g is the acceleration due to gravity and T' is the temperature of the fluid.
The initial and boundary conditions for the problem are:

$$t^{'} = 0 : u' = 0, T^{'} = T_0 \quad \text{for all } y^{'} \in [-h, h]$$

$$t^{'} > 0 : u' = 0, T^{'} = T_0 + (T_w - T_0)(1 - e^{-n^{'}t^{'}}) \quad \text{for} \quad y' = h$$

$$u' = 0, \frac{\partial T'}{\partial y^{'}} = 0 \quad \text{for} \quad y' = -h \tag{2.4}$$

We now introduce the following non-dimensional quantities:

$$u = \frac{\nu u^{'}}{\beta g h^2 (T_w - T_0)}, y = \frac{y^{'}}{h}, T = \frac{T^{'} - T_0}{T_w - T_0},\ t = \frac{\nu t^{'}}{h^2}, P_r = \frac{\mu C_p}{K}, n = \frac{h^2 n^{'}}{\nu}, M = B_0 h\sqrt{\frac{\sigma}{\mu}} \tag{2.5}$$

in which $\Pr$ is the Prandtl number and M is the Hartmann number.
Using the quantities (2.5) in the equations (2.2) and (2.3),we obtain

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - M^2 u + T \tag{2.6}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 T}{\partial y^2} \tag{2.7}$$

Under the above non-dimensional quantities, the corresponding boundary conditions redues to

$$t = 0 : \ u = 0, T = 0 \quad \text{for all } y \in [-1, 1]$$

$$t > 0 : u = 0, T = (1 - e^{-nt}) \quad \text{for} \quad y = 1$$

$$u = 0, \frac{\partial T}{\partial y} = 0 \quad \text{for} \quad y = -1 \tag{2.8}$$

III. Solution of the problem

We seek a regular perturbation series solution to solve the Equations (2.6) and (2.7) of the form

$$u = u_o(y) + e^{-nt} u_1(y) \tag{3.1}$$

$$T = T_0(y) + e^{-nt} T_1(y) \tag{3.2}$$

Substituting Equations (3.1) and (3.2) into the Equations (2.6) – (2.8) and solving the resultant Equations, we get

$$u=\frac{1}{M^2}\left(1-\frac{\cosh My}{\cosh M}\right)+\left(\begin{array}{l}\left(\dfrac{-1-\cosh 2m_1}{2\left(m_1^2-m_2^2\right)\cos m_2\cosh 2m_1}\right)\cosh m_2 y\\ +\left(\dfrac{-\cosh 2m_1}{2\left(m_1^2-m_2^2\right)\sin m_2\cosh 2m_1}\right)\sin m_2 y\\ +\left(\dfrac{1}{m_1^2-m_2^2}\right)\dfrac{\cosh m_1(1+y)}{\cosh 2m_1}\end{array}\right)e^{-nt} \tag{3.3}$$

and

$$T=1-\frac{\cosh m_2(1+y)}{\cosh 2m_1}e^{-nt} \tag{3.4}$$

IV. Results and Discussions

Fig. 1 shows the variation of velocity u with Hartmann number M for $\Pr=0.71$, $n=1$ and $t=1$. It is found that the velocity u decreases with increasing M.

The variation of velocity u with decay parameter n for $\Pr=0.71$, $M=2$ and $t=1$ is shown in Fig. 2. It is observed that the velocity u decreases with an increase in n.

Fig. 3 depicts the variation of velocity u with Prandtl number $\Pr$ for $n=1$, $M=2$ and $t=1$. It is noted that the velocity u decreases with increasing $\Pr$.

The variation of temperature T with decay parameter n for $\Pr=0.71$ and $t=1$ is depicted in Fig. 4. It is found that the temperature T decreases with increasing n.

Fig. 5 shows the variation of temperature T with Prandtl number $\Pr$ for $n=1$ and $t=1$. It is observed that the temperature T decreases with increasing Prandtl number $\Pr$.

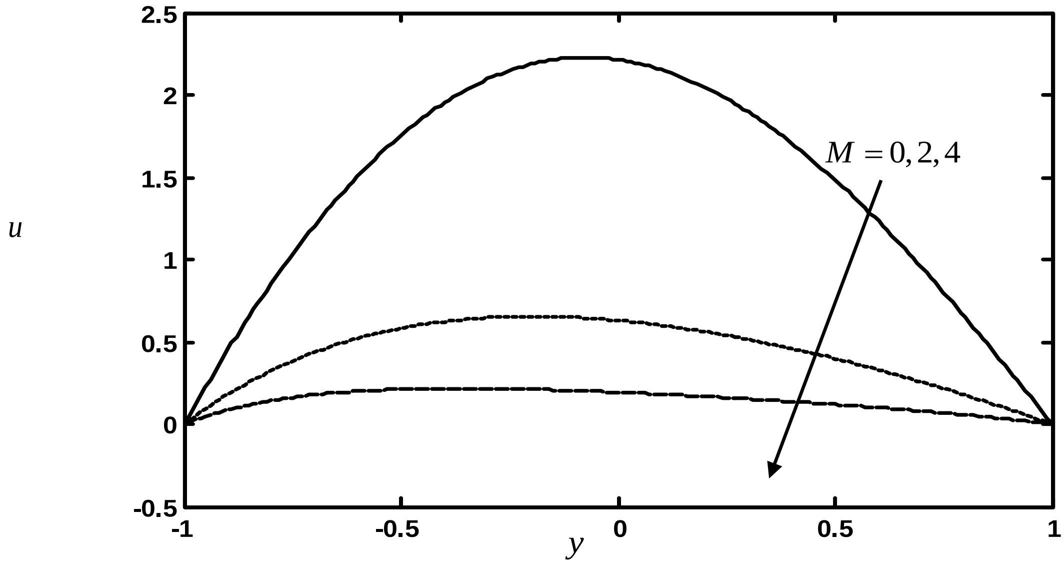

Fig. 1 The variation of velocity u with Hartmann number M for $\Pr=0.71$, $n=1$ and $t=1$.

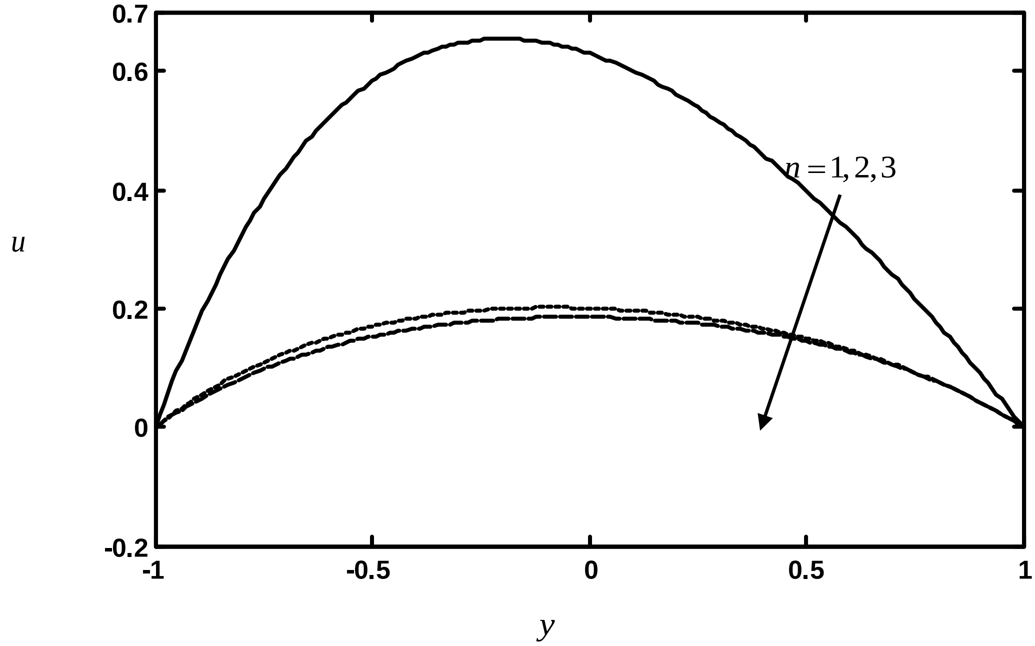

Fig. 2 The variation of velocity u with decay parameter n for $\Pr = 0.71$, $M = 2$ and $t = 1$.

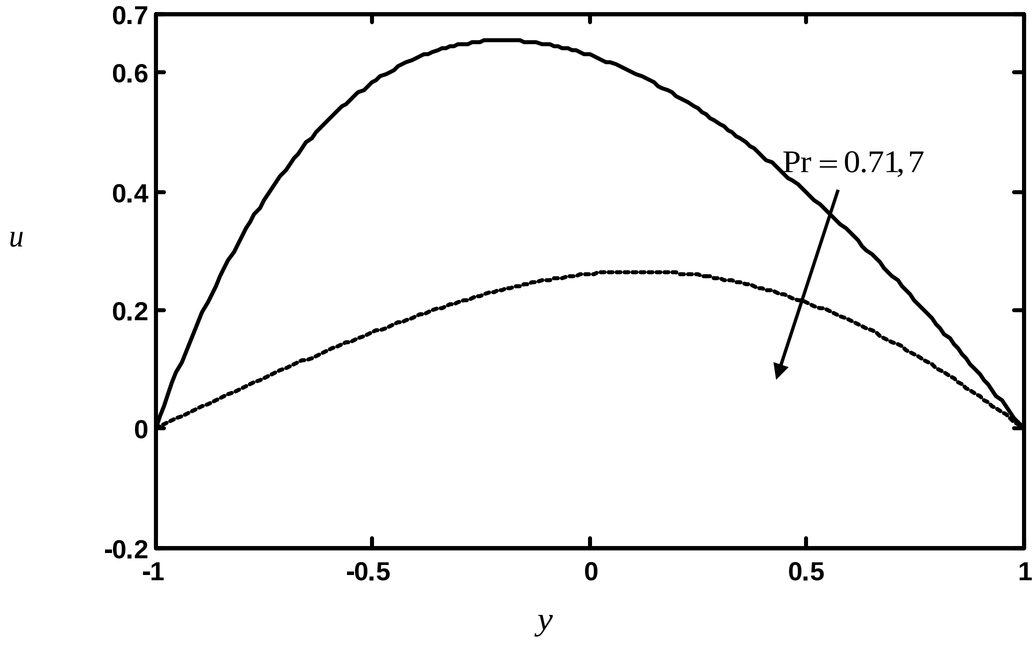

Fig. 3 The variation of velocity u with Prandtl number $\Pr$ for $n = 1$, $M = 2$ and $t = 1$.

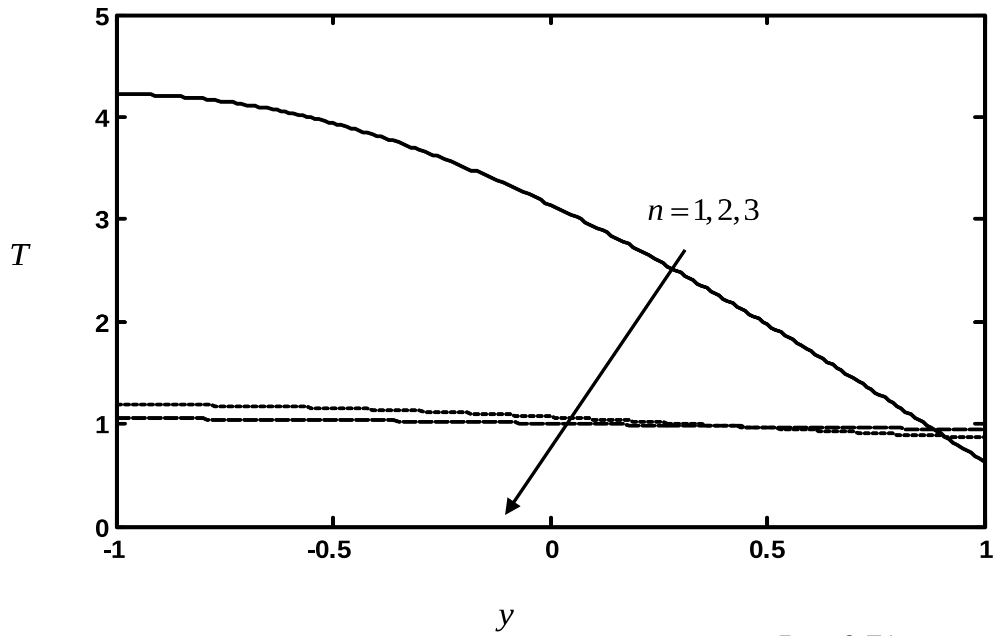

Fig. 4 The variation of temperature T with decay parameter n for $\mathrm{Pr}=0.71$ and $t=1$.

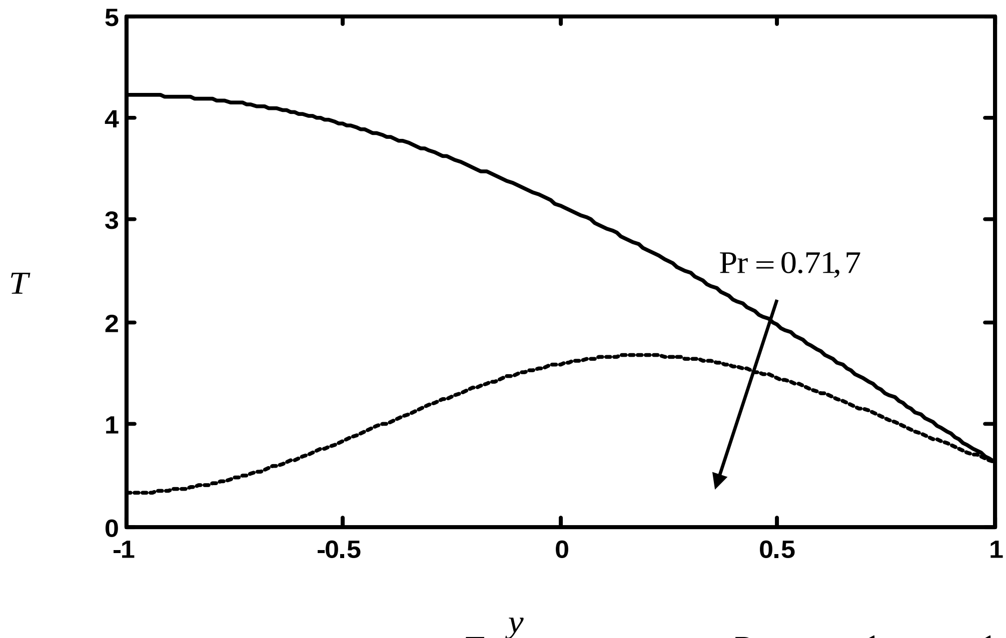

Fig. 5 The variation of temperature T with Prandtl number Pr for $n=1$ and $t=1$.

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