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A new generalization of fuzzy ideals in LA-semigroups

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Abstract. In this article, the concept of $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroups, $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left(right) ideals, $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideals and $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy bi-ideals of an LA-semigroup are introduced. The given concept is a generalization of $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy LA-subsemigroups, $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy left(right) ideals, $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy generalized bi-ideals and $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy bi-ideals of an LA-semigroup. We also give some examples of $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroups (left, right, generalized bi- and bi) ideals of an LA-semigroup. We prove some fundamental results of these ideals. We characterize $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left(right) ideals, $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideals and $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy bi-ideals of an LA-semigroup by the properties of level sets.

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1. Introduction

The concept of LA-semigroup was first introduced by Kazim and Naseerudin [19, 26]. A non-empty set S with binary operation $*$ is said to be an LA-semigroup if identity $(x * y) * z = (z * y) * x$ for all $x, y, z \in S$ holds. Later, Mushtaq and others have been investigated the structure further and added many useful results to theory of LA-semigroups see [20, 21, 22]. Ideals of LA-semigroups were defined by Mushtaq and Khan in his paper [23]. In [17], Khan and Ahmad characterized LA-semigroup by their ideals. Zadeh introduced the fundamental concept of a fuzzy set [35] in 1965. On the basis of this concept, mathematicians initiated a natural framework for generalizing some basic notions of algebra, e.g group theory, set theory, ring theory, topology, measure theory and semigroup theory etc. The importance of fuzzy technology in information processing is increasing day by day. In granular computing, the information is represented in the form of aggregates, called granules. Fuzzy logic is very useful in modeling the granules as fuzzy sets. Bargeila and Pedrycz considered this new computing methodology in [7]. Pedrycz and Gomide in [27] considered the presentation of update trends in fuzzy set theory and it's applications. Foundations of fuzzy groups are laid by Rosenfeld in [29]. Abdullah et al, used the concept of fuzzy set to different algebraic structures [2, 3, 4, 5]. In [24] Murali gave the concept of belongingness of a fuzzy point to a fuzzy subset under a natural equivalence on a fuzzy subset. In [28] the idea of quasi-coincidence of a fuzzy point with a fuzzy set is defined. These two ideas played a vital role in generating some different types of fuzzy subgroups. Using these ideas Bhakat and Das [8, 12] gave the concept of (α, β)-fuzzy subgroups, where $\alpha, \beta \in \{\in, q, \in \vee q, \in \wedge q\}$ and $\alpha \neq \in \wedge q$. These fuzzy subgroups are further studied in [10, 9]. The concept of $(\in, \in \vee q)$-fuzzy subgroups is a viable generalization of Rosenfeld's fuzzy subgroups, $(\in, \in \vee q)$-fuzz subrings and ideals are defined In [11], Bhakat and Das introduced the $(\in, \in \vee q)$-fuzzy subrings and ideals. Davvaz gave the concept of $(\in, \in \vee q)$-fuzzy subnearrings and ideals of a near ring in [13]. Jun and Song initiated the study of (α, β)-fuzzy interior ideals of a semigroup in [15]. In [16] Kazanci and Yamak studied $(\in, \in \vee q)$-fuzzy bi-ideals of a semigroup. In [30] regular semigroups are characterized by the properties of $(\in, \in \vee q)$-fuzzy ideals. Aslam et al defined generalized fuzzy Γ-ideals in Γ-LA-semigroups [6]. In [1], Abdullah et al give new generalization of fuzzy normal subgroup and fuzzy coset of groups. Generalizing the idea of the quasi-coincident of a fuzzy point with a fuzzy subset Jun [14] defined $(\in, \in \vee q_k)$-fuzzy subalgebras in BCK/BCI-algebras. In [31], $(\in, \in \vee q_k)$-fuzzy ideals of semigroups are introduced.

The concepts of "γ-belongingness $\in_\gamma$"and "δ-quasi-coincidence (q_δ)" of a fuzzy point with a fuzzy set were introduced in [32] by Yin and Zhan and and then studied extensively different characterizations of hemirings in terms of fuzzy soft h-ideals in [33]. In [34], Yin and Zhan characterized order semigroup by fuzzy soft ideals by using γ-belongingness $\in_\gamma$"and "δ-quasi-coincidence (q_δ)" of a fuzzy point with a fuzzy set. Recently, Yin and Zhan defined $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy ideals of near rings [36]. In [25], $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy ideals of BCI-algebras were introduced.

In this article, the concept of $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroups, $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left(right) ideals, $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideals and $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy bi-ideals of an LA-semigroup are introduced. The given concept is a generalization of $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy LA-subsemigroups, $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy left(right) ideals, $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy generalized bi-ideals and $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy bi-ideals of an LA-semigroup. We also give some examples of $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroups (left, right, generalized bi- and bi) ideals of an LA-semigroup. We prove some fundamental results of these ideals. We characterize $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left(right) ideals, $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideals and $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy bi-ideals of an LA-semigroup by the properties of level sets.

2. Preliminaries

An LA-subsemigroup of S means a non-empty subset A of S such that $A^2 \subseteq A$. By a left (right) ideals of S we mean a non-empty subset I of S such that $SI \subseteq I (IS \subseteq I)$. An ideal I is said to be two sided or simply ideal if it is both left and right ideal. An LA-subsemigroup A is called bi-ideal if $(BS)\,B \subseteq A$. A non-empty subset B is called generalized bi-ideal if $(BS)\,B \subseteq A$. A non-empty subset Q is called a quasi-ideal if $QS \cap SQ \subseteq Q$. A non-empty subset A is called interior ideal if it is LA-subsemigroup of S and $(SA)\,S \subseteq A$. An LA-semigroup S is called regular if for each $a \in S$ there exists $x \in S$ such that $a = (ax)a$. An LA-semigroup S is called intra-regular if for each $a \in S$ there exist $x, y \in S$ such that $a = (xa^2)y$. In an LA-semigroup S, the following law hold, (1) $(ab)\,c = (ab)\,c$, for all $a, b, c \in S$. (2) $(ab)\,(cd) = (ac)\,(bd)$, for all $a, b, c, d \in S$. If an LA-semigroup S has a left identity e, then the following law holds, (3) $(ab)\,(cd) = (db)\,(ca)$, for all $a, b, c, d \in S$. (4) $a(bc) = b(ac)$, for all $a, b, c \in S$.

A fuzzy subset μ of the form

$$\mu(y) = t(\neq 0) \text{ if } y = x$$
$$\mu(y) = 0 \qquad \text{if } y \neq x$$

is said to be a fuzzy point with support x and value t and is denoted by x_t.A fuzzy point x_t is said to be "belong to"(res.,"quasicoincident with") a fuzzy set μ, written as $x_t \in \mu$(repectively, $x_t q \mu$) if $\mu(x) \geq t$ (repectively, $\mu(x) + t > 1$). We write $x_t \in \vee q\mu$ if $x_t \in \mu$ or $x_t q\mu$. If $\mu(x) < t$(respectively, $\mu(x) + t \leq 1$), then we write $x_t \overline{\in} \mu$(repectively, $x_t \overline{q} \mu$). We note that $\overline{\in \vee q}$ means that $\in \vee q$ does not hold. Generalizing the concept of $x_t q \mu$, Y. B. Jun [14] defined $x_t q_k \mu$, where $k \in [0, 1]$ as $x_t q_k \mu$ if $\mu(x) + t + \dot{k} > 1$ and $x_t \in \vee q_k \mu$ if $x_t \in \mu$ or $x_t q_k \mu$.

A fuzzy subset μ of S is a function $\mu : S \to [0, 1]$. For any two fuzzy subsets μ and ν of S, $\mu \subseteq \nu$ means $\mu(x) \leq \nu(x)$ for all x in S. The fuzzy subsets $\mu \cap \nu$ and $\mu \cup \nu$ of S are defined as

$$(\mu \cap \nu)\,(x) = \min\{\mu(x), \nu(x)\} = \mu(x) \wedge \nu(x)$$
$$(\mu \cup \nu)\,(x) = \max\{\mu(x), \nu(x)\} = \mu(x) \vee \nu(x)$$

for all x in S. If $\{\mu_i\}_{i \in I}$ is a faimly of fuzzy subsets of S, then $\bigwedge_{i \in I} \mu_i$ and $\bigvee_{i \in I} \mu_i$ are fuzzy subsets of S defined by

$$\left(\bigwedge_{i \in I} \mu_i\right)(x) = \min\{\mu_i\}_{i \in I}$$
$$\left(\bigvee_{i \in I} \mu_i\right)(x) = \max\{\mu_i\}_{i \in I}$$

For any two subsets μ and ν of S, the product $\mu \circ \nu$ is defined as

$$(\mu \circ \nu)\,(x) = \bigvee_{x = yz} \{\mu(y) \wedge \nu(z), \text{ if there exist } y, z \in S, \text{ such that } x = yz$$
$$(\mu \circ \nu)\,(x) = 0 \qquad \text{otherwise}$$

Definition 2.1 ([18]). A fuzzy subset μ of an LA-semigroup S is called fuzzy LA-subsemigroup S if $\mu(xy) \geq \mu(x) \wedge \mu(y)$ for all $x, y \in S$.

Definition 2.2 ([18]). A fuzzy subset μ of an LA-semigroup S is called fuzzy left(right) ideal of S if $\mu(xy) \geq \mu(y)(\mu(xy) \geq \mu(x))$ for all $x, y \in S$.

Definition 2.3 ([18]). An LA-subsemigroup μ of an LA-semigroup S is called fuzzy bi-ideal of S if $\mu((xy)\,z) \geq \mu(x) \wedge \mu(z)$ for all $x, y \in S$.

Definition 2.4 ([18]). A fuzzy subset μ of an LA-semigroup S is called fuzzy generalized bi-ideal of S if $\mu((xy)\,z) \geq \mu(x) \wedge \mu(z)$ for all $x, y \in S$.

Definition 2.5 ([18]). Let μ be a fuzzy subset of an LA-semigroup S, then for all $t \in (0, 1]$, the set $\mu_t = \{x \in S \,|\mu(x) \geq t\}$ is called a level subset of S.

In this paper we defined to the study of $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy ideals,$(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideals, $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy bi-ideals, $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy interior ideals and $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy quasi-ideals. We have also characterized different classes of LA-semigroups. J. Zhan and Y. Yin [36] gave meaning to the symbols $x_r \overline{\in}_\gamma \mu$ and $x_r \overline{q}_\delta \mu$. Let $\gamma, \delta \in [0, 1]$ be such that $\gamma < \delta$. For a fuzzy point x_r and fuzzy subset μ of S, we say $x_r \overline{\in}_\gamma \mu$, if $\mu(x) < r$ and $x_r \overline{q}_\delta \mu$, if $\mu(x) + r \leq 2\delta$. We say $x_r \overline{\in}_\gamma \vee \overline{q}_\delta$, if $x_r \overline{\in}_\gamma \mu$ or $x_r \overline{q}_\delta \mu$ and $x_r \overline{\in}_\gamma \wedge \overline{q}_\delta$ if $x_r \overline{\in}_\gamma \mu$, and $x_r \overline{q}_\delta \mu$.

3. Major section

Definition 3.1. A fuzzy subset μ of an LA-semigroup S is called an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroup of S if for all $x, y \in S$ and $t, r \in (\gamma, 1]$, $(xy)_{t \wedge r} \,\overline{\in}_\gamma \mu$ implies that $x_t \overline{\in}_\gamma \vee \overline{q}_\delta \mu$ or $y_r \overline{\in}_\gamma \vee \overline{q}_\delta \mu$.

Remark 3.2. Every $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy LA-subsemigroup is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroup of S but the converse is not true.

Example 3.3. Let $S = \{a, b, c, d\}$ with the following Clayey table;

$*$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	c	a	a	a

Define a fuzzy set μ by

$\mu(a) = 0.6$, $\mu(b) = 0.8$, $\mu(c) = 0.9$, $\mu(d) = 0.5$.

Then,

(i) μ is an $(\overline{\in}_{0.5}, \overline{\in}_{0.5} \vee \overline{q}_{0.8})$-fuzzy LA-subsemigroup of S.

(ii) μ is not an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy LA-subsemigroup of S. Because $(bc)_{0.7 \wedge 0.8} \,\overline{\in} \mu$ but $(b)_{0.7} \in \vee q \mu$ and $(c)_{0.8} \in \vee q \mu$.

Theorem 3.4. *Let A be an LA-subsemigroup of S. Then, the fuzzy subset μ of S defined by*

$$\mu(a) = 1 \text{ if } a \in A$$
$$\mu(a) \leq \delta \text{ if } a \notin A$$

is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroup of S.

Proof. Let A be an LA-subsemigroup of S and $a, b \in S$ and $t, r \in (\gamma, 1]$, such that $(ab)_{t\wedge r}\overline{\in}_\gamma\mu$. Then, $\mu(ab) < t \wedge r$ and $t \wedge r \leq 1$, which implies that $\mu(ab) < 1$. Thus, $\mu(ab) \leq \delta$. Hence, $ab \notin A$, which implies that either $a \notin A$ or $b \notin A$. If $t, r > \delta$ and $a \notin A$. Then, $\mu(a) \leq \delta$. This implies that $\mu(a) \leq \delta < t$ or $\mu(a) < t$, so $a_t\overline{\in}_\gamma\mu$. Similarly, if $b \notin A$, then $b_r\overline{\in}_\gamma\mu$. Now, if $t, r \leq \delta$ and $a \notin A$, then $\mu(a) \leq \delta$, which implies that $\mu(a) + t \leq 2\delta$. Thus, $a_t\overline{q}_\delta\mu$. Similarly, if $b \notin A$, then $b_r\overline{q}_\delta\mu$. Thus, $a_t, b_r\overline{\in}_\gamma \vee \overline{q}_\delta\mu$. Hence, μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroup of S. □

Corollary 3.5. *Let A be an LA-subsemigroup of S. Then, χ_A the characteristic function of A is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroup of S.*

Theorem 3.6. *A fuzzy subset μ of an LA-semigroup S is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroup of S if and only if for all $a, b \in S$, $\mu(ab) \vee \delta \geq \mu(a) \wedge \mu(b)$.*

Proof. Let μ be an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroup of S. We are going to show that $\mu(ab) \vee \delta \geq \mu(a) \wedge \mu(b)$ for all $a, b \in S$. On contrary, assume that there exist some $a, b \in S$, such that, $\mu(ab) \vee \delta < \mu(a) \wedge \mu(b)$. Choose $r \in (\gamma, 1]$, such that $\mu(ab) \vee \delta < r \leq \mu(a) \wedge \mu(b)$. Which implies that $\mu(ab) < r$ but $\mu(a) \geq r$ and $\mu(b) \geq r$. This implies that $(ab)_r\,\overline{\in}_\gamma\mu$ but $a_r \in_\gamma \vee q_\delta\mu$ and $b_r \in_\gamma \vee q_\delta\mu$. Which is a contradiction. Hence, for all $a, b \in S$, we have $\mu(ab) \vee \delta \geq \mu(a) \wedge \mu(b)$. Thus, μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroup of S.

Conversely, assume that $\mu(ab)\vee\delta \geq \mu(a)\wedge\mu(b)$ for all $a, b \in S$. To show that μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroup of S. Case(a): If $\mu(ab) \geq \delta$, then let $(ab)_r\,\overline{\in}_\gamma\mu$, which implies that $\mu(ab) < r$. Since $\mu(ab) \vee \delta \geq \mu(a) \wedge \mu(b)$, then $\mu(ab) \geq \mu(a) \wedge \mu(b)$, also $r > \mu(ab) \geq \mu(a) \wedge \mu(b)$, implies that $\mu(a) \wedge \mu(b) < r$. This implies that $\mu(a) < r$ or $\mu(b) < r$. Thus, $a_t\overline{\in}_\gamma \vee \overline{q}_\delta\mu$ or $b_r\overline{\in}_\gamma \vee \overline{q}_\delta\mu$. Hence, μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroup of S. Case(b): If $\mu(ab) < \delta$, then $(ab)_\delta\,\overline{\in}_\gamma\mu$. Since, $\mu(ab) \vee \delta \geq \mu(a) \wedge \mu(b)$. So, $\mu(a) \wedge \mu(b) \leq \delta$, which implies that $\mu(a) \leq \delta$ or $\mu(b) \leq \delta$. Thus, $\mu(a) + \delta \leq 2\delta$ or $\mu(b) + \delta \leq 2\delta$. Thus, $a_\delta\overline{\in}_\gamma \vee \overline{q}_\delta\mu$ or $b_\delta\overline{\in}_\gamma \vee \overline{q}_\delta\mu$. Hence, μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroup of S. □

Remark 3.7. For any $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroup μ of S, we can conclude that if $\delta = 0.5$, then μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy LA-subsemigroup of S.

Theorem 3.8. *Let μ be a fuzzy subset of S. Then,*

(i) μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroup of S if and only if $\mu_r^\gamma(\neq \phi)$ is an LA-subsemigroup of S for all $r \in (\delta, 1]$.

(ii) μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroup of S if and only if $\mu_r^\delta(\neq \phi)$ is an LA-subsemigroup of S for all $r \in (\gamma, \delta]$.

Proof. (i) Let μ be an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroup of S and $a, b \in \mu_r^\gamma$, for all $r \in (\delta, 1]$. Then, $a_r, b_r \in_\gamma \mu$, that is $\mu(a) \geq r > \gamma$, $\mu(b) \geq r > \gamma$. By hypothesis, $\mu(ab) \vee \delta \geq \mu(a) \wedge \mu(b)$, which implies that $\mu(ab) \vee \delta \geq r \wedge r = r$. Since $r \in (\delta, 1]$, so $r > \delta$. Thus, $\mu(ab) \geq r > \gamma$, implies that $ab \in \mu_r^\gamma$. Hence, μ_r^γ is an LA-subsemigroup of S.

Conversely, assume that $\mu_r^\gamma(\neq \Phi)$ is an LA-subsemigroup of S for all $r \in (\gamma, \delta]$. Let $a, b \in S$, such that $\mu(ab) \vee \delta < \mu(a) \wedge \mu(b)$. Select $r \in (\gamma, \delta]$, such that $\mu(ab) \vee \delta < r \leq \mu(a) \wedge \mu(b)$, which implies that $\mu(a) \wedge \mu(b) \geq r$ and $\mu(ab) < r$. This implies that $\mu(a) \geq r$ or $\mu(b) \geq r$ but $\mu(ab) < r$. Hence, $(ab)_r\,\overline{\in}_\gamma\mu$ but $a_r, b_r \in_\gamma \vee q_\delta\mu$. Thus,

$a, b \in \mu_r^{\gamma}$ but $ab \notin \mu_r^{\gamma}$. Which is a contradiction. Hence, $\mu(ab) \vee \delta \geq \mu(a) \wedge \mu(b)$. That is μ is an $(\overline{\in}_{\gamma}, \overline{\in}_{\gamma} \vee \overline{q}_{\delta})$-fuzzy LA-subsemigroup of S.

(ii) Let μ be an $(\overline{\in}_{\gamma}, \overline{\in}_{\gamma} \vee \overline{q}_{\delta})$-fuzzy LA-subsemigroup of S and let $a, b \in \mu_r^{\delta}$ for all $r \in (\gamma, \delta]$. Then, $\mu(a)+r > 2\delta$ and $\mu(b)+r > 2\delta$ or $\mu(a) > 2\delta - r$ and $\mu(b) > 2\delta - r$. By hypothesis, $\mu(ab) \vee \delta \geq \mu(a) \wedge \mu(b)$, which implies that $\mu(ab) \vee \delta > (2\delta - r) \wedge (2\delta - r)$. Now, $\mu(ab) \vee \delta > (2\delta - r) > \delta$, implies that $\mu(ab) > 2\delta - r$ or $\mu(ab) + r > 2\delta$. That is $(ab)_r\, q_{\delta} \mu$. Thus, $ab \in \mu_r^{\delta}$. Hence, μ_r^{δ} is an LA-subsemigroup of S.

Conversely, assume that $\mu_r^{\delta} (\neq \Phi)$ is an LA-subsemigroup of S. We are going to show that μ is an $(\overline{\in}_{\gamma}, \overline{\in}_{\gamma} \vee \overline{q}_{\delta})$-fuzzy LA-subsemigroup of S. On contrary, assume that $\mu(ab) \vee \delta < \mu(a) \wedge \mu(b)$. Select $r \in (\gamma, \delta]$, such that $\mu(ab) \vee \delta < r \leq \mu(a) \wedge \mu(b)$. Then, $\mu(a) \wedge \mu(b) \geq r$ and $\mu(ab) < r$. Let $r = \delta \in (\gamma, \delta]$, then $\mu(a) \wedge \mu(b) \geq \delta$ and $\mu(ab) < \delta$ implies that $\mu(a) \geq \delta$ or $\mu(b) \geq \delta$. Thus, $\mu(a) + \delta \geq 2\delta$ or $\mu(b) + \delta \geq 2\delta$. Similarly, $\mu(ab) + \delta < 2\delta$. Thus, $a_{\delta}, b_{\delta} q_{\delta} \mu$, but $(ab)_{\delta}\, \overline{q}_{\delta} \mu$, implies that $a, b \in \mu_r^{\delta}$ but $ab \notin \mu_r^{\delta}$. Which is a contradiction. Hence, $\mu(ab) \vee \delta \geq \mu(a) \wedge \mu(b)$. Hence, μ is an $(\overline{\in}_{\gamma}, \overline{\in}_{\gamma} \vee \overline{q}_{\delta})$-fuzzy LA-subsemigroup of S. □

If we take $\gamma = 0$ and $\delta = 0.5$ in above theorem we can conclude the following results:

Corollary 3.9. *Let μ be a fuzzy set of S.Then,*

(i) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy LA-subsemigroup of S if and only if $\mu_r (\neq \phi)$ is an LA-subsemigroup of S for all $r \in (0.5, 1]$.

(ii) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy LA-subsemigroup of S if and only if $\mu_r^{\delta} (\neq \phi)$ is an LA-subsemigroup of S for all $r \in (0, 0.5]$.

Lemma 3.10. *(i) The intersection of any family of $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$-fuzzy LA-subsemigroups is an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$-fuzzy LA-subsemigroup .*

Proof. (i) Let $\{\mu_i\}_{i \in I}$ be a family of $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$-fuzzy LA-subsemigroups of S and $x, y \in S$. Then,

$$
\begin{aligned}
((\bigwedge_{i \in I} \mu_i)(xy)) \vee \delta &= (\bigwedge_{i \in I} \mu_i(xy)) \vee \delta \\
&= \bigwedge_{i \in I} \{(\mu_i(xy)) \vee \delta\} \\
&\geq \bigwedge_{i \in I} (\mu_i(x) \wedge \mu_i(y)) \\
&= (\bigwedge_{i \in I} \mu_i(x)) \wedge (\bigwedge_{i \in I} \mu_i(y)) \\
&= (\bigwedge_{i \in I} \mu_i)(x)) \wedge (\bigwedge_{i \in I} \mu_i)(y)).
\end{aligned}
$$

Hence, $\bigwedge_{i \in I} \mu_i$ is an $(\overline{\in}_{\gamma}, \overline{\in}_{\gamma} \vee \overline{q}_{\delta})$-fuzzy LA-subsemigroups of S. □

4. $(\overline{\in}_{\gamma}, \overline{\in}_{\gamma} \vee \overline{q}_{\delta})$-FUZZY LEFT(RIGHT) IDEALS

Definition 4.1. A fuzzy subset μ of an LA-semigroup S is called an $(\overline{\in}_{\gamma}, \overline{\in}_{\gamma} \vee \overline{q}_{\delta})$-fuzzy left (right) ideal of S if for all $a, s \in S$ and $r \in (\gamma, 1]$, $(sa)_r\, \overline{\in}_{\gamma} \mu$ implies that $a_r \overline{\in}_{\gamma} \vee \overline{q}_{\delta} \mu$. $((as)_r\, \overline{\in}_{\gamma} \mu$ implies that $a_r \overline{\in}_{\gamma} \vee \overline{q}_{\delta} \mu)$.

Remark 4.2. Every $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy left (right) ideal is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left(right) ideal of S but the converse is not true.

Example 4.3. Let $S = \{e, f, g, h\}$ with the following multiplication table:

$*$	e	f	g	h
e	h	g	h	h
f	e	h	h	e
g	h	h	h	h
h	h	h	h	h

Define a fuzzy subset μ by

$\mu(1) = 0.4$, $\mu(2) = 0$, $\mu(3) = 0$, $\mu(4) = 0.6$.

Then,

(i) μ is an $(\overline{\in}_{0.4}, \overline{\in}_{0.4} \vee \overline{q}_{0.7})$-fuzzy left ideal of S.

(ii) μ is not $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy left ideal of S. Because $(f * h)_{0.5}\, \overline{\in} \mu$ but $h_{0.5} \in \vee q\mu$.

Theorem 4.4. *Let A be a left ideal of S. Then, the fuzzy subset μ of S defined by*

$$\mu(a) = 1 \text{ if } a \in A$$
$$\mu(a) \leq \delta \text{ if } a \notin A$$

is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left ideal of S.

Proof. Let A be a left ideal of S and $a, s \in S$ and $t \in (\gamma, 1]$, such that $(sa)_t \overline{\in}_\gamma \mu$. Then, $\mu(sa) < t$ and $t \leq 1$, which implies that $\mu(sa) < 1$. Thus, $\mu(sa) \leq \delta$. Hence, $sa \notin A$, which implies that $a \notin A$. If $t, r > \delta$ and $a \notin A$. Then, $\mu(a) \leq \delta$. This implies that $\mu(a) \leq \delta < t$ or $\mu(a) < t$, so $a_t \overline{\in}_\gamma \mu$. Now, if $t \leq \delta$ and $a \notin A$, then $\mu(a) \leq \delta$, which implies that $\mu(a) + t \leq 2\delta$. Thus, $a_t \overline{q}_\delta \mu$. Thus, $a_t \overline{\in}_\gamma \vee \overline{q}_\delta \mu$. Hence, μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left ideal of S. □

Corollary 4.5. *Let A be a left(right) of S. Then, χ_A the characteristic function of A is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left(right) of S.*

Theorem 4.6. *Let μ be a fuzzy subset of an LA-semigroup S. Then, μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left(right) ideal of an LA-semigroup S if and only if $\mu(as) \vee \delta \geq \mu(a)$ for all $a, s \in S$.*

Proof. Let μ be an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left ideal of an LA-semigroup S. We are going to show that $\mu(sa) \vee \delta \geq \mu(a)$ for all $a, s \in S$. On contrary, assume that there exist $a, s \in S$, such that, $\mu(sa) \vee \delta < \mu(a)$. Select $t \in (\gamma, 1]$, such that, $\mu(sa) \vee \delta < t \leq \mu(a)$. Then, $(sa)_t \overline{\in}_\gamma \mu$ but $a_t \in_\gamma \vee q_\delta \mu$. Which is a contradiction. Hence, $\mu(sa) \vee \delta \geq \mu(a)$ for all $a, s \in S$.

Conversely, assume that $\mu(sa) \vee \delta \geq \mu(a)$ for all $a, s \in S$. To show that μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left ideal of an LA-semigroup S. Let $(sa)_t \overline{\in}_\gamma \mu$, then $\mu(sa) < t$. By hypothesis, $\mu(sa) \vee \delta \geq \mu(a)$. If $t \in (\gamma, \delta]$, then $t \leq \delta$ and so $\mu(sa) \leq \delta$, implies that $\mu(sa) \vee \delta = \delta$. Thus, $\mu(sa) \vee \delta \geq \mu(a)$, implies that $\mu(a) \leq \delta$ or $\mu(a) + \delta \leq 2\delta$. Since, $t \leq \delta$, so $\mu(a) + t \leq \mu(a) + \delta \leq 2\delta$ or $\mu(a) + t \leq 2\delta$. Hence, $a_t \overline{\in}_\gamma \vee \overline{q}_\delta \mu$. If $t \in (\delta, 1]$, then $t > \delta$. Now, if $\mu(sa) > \delta$, then $\mu(sa) \vee \delta \geq \mu(a)$, implies that $\mu(sa) \geq \mu(a)$. As $t > \mu(sa)$, so $\mu(sa) \geq \mu(a) \implies t > \mu(a)$. Hence, $a_t \overline{\in}_\gamma \vee \overline{q}_\delta \mu$. If $\mu(sa) \leq \delta$, then $\mu(sa) \vee \delta \geq \mu(a)$, implies that $\delta \geq \mu(a)$, that is $t > \delta \geq \mu(a)$

or $t > \mu(a)$. Hence, $a_t \overline{\in}_\gamma \vee \overline{q}_\delta \mu$. Thus, μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left ideal of an LA-semigroup S. (Similarly, we can prove for right ideal). □

Remark 4.7. For any $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left ideal, we can conclude that if $\delta = 0.5$, then μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy left ideal of an LA-semigroup of S.

Theorem 4.8. *Let μ be a fuzzy set of S. Then,*

(i) μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left(right) ideal of S if and only if $\mu_r^\gamma (\neq \phi)$ is left(right) ideal of S for all $r \in (\delta, 1]$.

(ii) μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left(right) ideal of S if and only if $\mu_r^\delta (\neq \phi)$ is left(right) ideal of S for all $r \in (\gamma, \delta]$.

Proof. (i) Let μ be an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left ideal of S and $a \in \mu_r^\gamma$ for all $r \in (\delta, 1]$ and $s \in S$. Then, $a_r \in_\gamma \mu$, that is $\mu(a) \geq r > \gamma$. By hypothesis, $\mu(sa) \vee \delta \geq \mu(a)$. Which implies that $\mu(sa) \vee \delta \geq r$. Since $r \in (\delta, 1]$, so $r > \delta$. Thus, $\mu(sa) \geq r > \gamma$, implies that $sa \in \mu_r^\gamma$. Hence, μ_r^γ is left ideal of S.

Conversely, assume that $\mu_r^\gamma (\neq \Phi)$ is a left ideal of S for all $r \in (\delta, 1]$. Let $a, s \in S$, such that, $\mu(sa) \vee \delta < \mu(a)$. Select $r \in (\delta, 1]$, such that, $\mu(sa) \vee \delta < r \leq \mu(a)$. Then, $\mu(a) \geq r$ and $\mu(sa) < r$. This implies that $\mu(a) \geq r$ and $\mu(sa) < r$. Thus, $a_r \in_\gamma \vee q_\delta \mu$ but $(sa)_r \overline{\in}_\gamma \mu$. Hence, $a \in \mu_r^\gamma$ but $sa \notin \mu_r^\gamma$. Which is a contradiction. Hence, $\mu(sa) \vee \delta \geq \mu(a)$. Thus, μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left ideal of S.

(ii) Let μ be an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left ideal of S and $a \in \mu_r^\delta$ for all $r \in (\gamma, \delta]$ and $s \in S$. Then, $\mu(a) + r > 2\delta$ or $\mu(a) > 2\delta - r$. By hypothesis, $\mu(sa) \vee \delta \geq \mu(a)$, which implies that $\mu(sa) \vee \delta > (2\delta - r)$. Also, $(2\delta - r) > \delta$. Therefore, $\mu(sa) > 2\delta - r$ or $\mu(sa) + r > 2\delta$. That is $(sa)_r \, q_\delta \mu$. Thus, $sa \in \mu_r^\delta$. Hence, μ_r^δ is a left ideal of S.

Conversely, assume that $\mu_r^\delta (\neq \Phi)$ is a left ideal of S. We are going to show that μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left ideal of S. On contrary, assume that $\mu(sa) \vee \delta < \mu(a)$. Select $r \in (\gamma, \delta]$, such that $r = \delta$, then $2\delta - r \in (\gamma, \delta]$. So, if we select $2\delta - r \in (\gamma, \delta]$, such that $\mu(sa) \vee \delta < 2\delta - r \leq \mu(a)$. Then, $\mu(sa) < 2\delta - r$ and $2\delta - r \leq \mu(a)$ or $\mu(sa) + r < 2\delta$ and $\mu(a) + r \geq 2\delta$. This implies that $a \in \mu_r^\delta$, but $as \notin \mu_r^\delta$. Which is a contradiction. Hence, $\mu(sa) \vee \delta \geq \mu(a)$. That is μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left ideal of S. (Similarly, we can prove for right ideal). □

If we take $\gamma = 0$ and $\delta = 0.5$ in above theorem we can conclude the following results:

Corollary 4.9. *Let μ be a fuzzy set of S. Then,*

(i) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy left(right) ideal of S if and only if $\mu_r (\neq \phi)$ is a left(right) ideal of S for all $r \in (0.5, 1]$.

(ii) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy left(right) ideal of S if and only if $\mu_r^\delta (\neq \phi)$ is a left(right) ideal of S for all $r \in (0, 0.5]$.

5. $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideals

Definition 5.1. A fuzzy subset μ of an LA-semigroup S is called a $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal if for all $a, b, s \in S$ and $t, r \in (\gamma, 1]$, $((as)\, b)_{t \wedge r} \overline{\in}_\gamma \mu$ implies that $a_t \overline{\in}_\gamma \vee \overline{q}_\delta \mu$ or $b_r \overline{\in}_\gamma \vee \overline{q}_\delta \mu$.

Remark 5.2. Every $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy generalized bi-ideal is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of S but the converse is not true.

Example 5.3. Let $S = \{a, b, c, d\}$ with the following multiplication table

$*$	a	b	c	d
a	c	b	c	b
b	b	b	b	b
c	b	b	b	b
d	b	b	b	b

Define a fuzzy set μ by $\mu(a) = 0$, $\mu(b) = 0.5$, $\mu(c) = 0.7$, $\mu(d) = 0.6$. Then,

(i) μ is an $(\overline{\in}_{0.5}, \overline{\in}_{0.5} \vee \overline{q}_{0.7})$-fuzzy generalized bi-ideal of S.

(ii) μ is not an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy generalized bi-ideal of S because

$$((c * b) * d)_{0.6\wedge 0.6}\, \overline{\in}\mu \text{ but } c_{0.6} \in \vee q\mu \text{ and } d_{0.6} \in \vee q\mu.$$

Theorem 5.4. *Let A be a generalized bi-ideal of S. Then, the fuzzy subset μ of S defined by*

$$\mu(a) = 1 \text{ if } a \in A$$
$$\mu(a) \leq \delta \text{ if } a \notin A$$

is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of S.

Proof. Let A be a generalized bi-ideal of S and $a, b \in S$ and $t, r \in (\gamma, 1]$, such that $((as)\, b)_{t\wedge r}\overline{\in}_\gamma\mu$. Then, $\mu((as)\, b) < t\wedge r$ and $t\wedge r \leq 1$, which implies that $\mu((as)\, b) < 1$. Thus, $\mu((as)\, b) \leq \delta$. Hence, $(as)\, b \notin A$, which implies that either $a \notin A$ or $b \notin A$. If $t, r > \delta$ and $a \notin A$. Then, $\mu(a) < \delta$. This implies that $\mu(a) < \delta < t$ or $\mu(a) < t$, so $a_t\overline{\in}_\gamma\mu$. Similarly, if $b \notin A$, then $b_r\overline{\in}_\gamma\mu$. Now, if $t, r \leq \delta$ and $a \notin A$, then $\mu(a) < \delta$, which implies that $\mu(a) + t < 2\delta$. Thus, $a_t\overline{q}_\delta\mu$. Similarly, if $b \notin A$, then $b_r\overline{q}_\delta\mu$. Thus, $a_t, b_r\overline{\in}_\gamma \vee \overline{q}_\delta\mu$. Hence, μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of S. □

Corollary 5.5. *Let A be a generalized bi-ideal of S. Then, χ_A the characteristic function of A is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of S.*

Theorem 5.6. *A fuzzy set μ of S is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of S if and only if for all $a, b \in S$, $\mu((as)\, b) \vee \delta \geq \mu(a) \wedge \mu(b)$.*

Proof. Let μ be an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of S . We are going to show that for all $a, b, s \in S$, $\mu((as)\, b) \vee \delta \geq \mu(a) \wedge \mu(b)$. On contrary, assume that there exist some $a, b \in S$, such that $\mu((as)\, b)\vee\delta < \mu(a)\wedge\mu(b)$. Choose $r \in (\gamma, 1]$, such that $\mu((as)\, b) \vee \delta < r \leq \mu(a) \wedge \mu(b)$. Which implies that $\mu((as)\, b) < r$ but $\mu(a) \geq r$ and $\mu(b) \geq r$. This implies that $((as)\, b)_r\, \overline{\in}_\gamma\mu$ but $a_r \in_\gamma \vee q_\delta\mu$ and $b_r \in_\gamma \vee q_\delta\mu$. Which is a contradiction. Hence, for all $a, b, s \in S$, $\mu((as)\, b) \vee \delta \geq \mu(a) \wedge \mu(b)$.

Conversely, assume that for all $a, b, s \in S$, $\mu((as)\, b) \vee \delta \geq \mu(a) \wedge \mu(b)$. To show that μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of S. Let $((as)\, b)_t\, \overline{\in}_\gamma\mu$. Then, $\mu\left((as)\, b\right) < t$. By hypothesis, $\mu\left((as)\, b\right)\vee\delta \geq \mu\left(a\right)\wedge\mu\left(b\right)$. If $t \in (\gamma, \delta]$, then $t \leq \delta$ and so $\mu\left((as)\, b\right) \leq \delta$, implies that $\mu\left((as)\, b\right) \vee \delta = \delta$. Thus, $\mu\left((as)\, b\right) \vee \delta \geq \mu\left(a\right) \wedge \mu\left(b\right)$, implies that $\mu\left(a\right) \wedge \mu\left(b\right) \leq \delta$. Thus, $\mu\left(a\right) \leq \delta$ or $\mu\left(b\right) \leq \delta$, implies that $\mu\left(a\right) + \delta \leq 2\delta$ or $\mu\left(b\right) + \delta \leq 2\delta$. Since, $t \leq \delta$ so $\mu\left(a\right) + t \leq \mu\left(a\right) + \delta \leq 2\delta$, implies that $\mu\left(a\right) + t \leq 2\delta$. Similarly, $\mu\left(b\right) + t \leq 2\delta$. Hence, $a_t\overline{\in}_\gamma \vee \overline{q}_\delta\mu$ or $b_t\overline{\in}_\gamma \vee \overline{q}_\delta\mu$. If $t \in (\delta, 1]$, then $t > \delta$. Now, if $\mu\left((as)\, b\right) > \delta$, then $\mu\left((as)\, b\right) \vee \delta \geq \mu\left(a\right) \wedge \mu\left(b\right)$, implies that $\mu\left((as)\, b\right) \geq \mu\left(a\right) \wedge \mu\left(b\right)$. As $t > \mu\left((as)\, b\right)$, so $\mu\left((as)\, b\right) \geq \mu\left(a\right) \wedge \mu\left(b\right)$, implies that $t > \mu\left(a\right) \wedge \mu\left(b\right)$. Thus, $\mu\left(a\right) < t$ or $\mu\left(b\right) < t$. Hence, $a_t\overline{\in}_\gamma \vee \overline{q}_\delta\mu$ or $b_t\overline{\in}_\gamma \vee \overline{q}_\delta\mu$. Now, if $\mu\left((as)\, b\right) \leq \delta$, then $\mu\left((as)\, b\right) \vee \delta \geq \mu\left(a\right) \wedge \mu\left(b\right)$, implies that $\delta \geq \mu\left(a\right) \wedge \mu\left(b\right)$.

Thus, $t > \delta \geq \mu(a) \wedge \mu(b)$, implies that $t > \mu(a)$ or $t > \mu(b)$. Hence, $a_t \overline{\in}_\gamma \vee \overline{q}_\delta \mu$ or $b_t \overline{\in}_\gamma \vee \overline{q}_\delta \mu$. Thus, μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of an LA-semigroup S. □

Remark 5.7. For any $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of S, we can conclude that if $\delta = 0.5$, then μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy generalized bi-ideal of S.

Theorem 5.8. *Let μ be a fuzzy set of S. Then,*

(i) μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of S if and only if $\mu_r^\gamma (\neq \phi)$ is a generalized bi-ideal of S for all $r \in (\delta, 1]$.

(ii) μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of S if and only if $\mu_r^\delta (\neq \phi)$ is a generalized bi-ideal of S for all $r \in (\gamma, \delta]$.

Proof. (i) Let μ be an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of S and $a, b \in \mu_r^\gamma$, for all $r \in (\delta, 1]$ and $s \in S$. Then, $a_r, b_r \in_\gamma \mu$, that is $\mu(a) \geq r > \gamma$ and $\mu(b) \geq r > \gamma$. By hypothesis, $\mu((as)b) \vee \delta \geq \mu(a) \wedge \mu(b)$. Which implies that$(\gamma, \delta]$. $\mu((as)b) \vee \delta \geq r \wedge r = r$. Since $r \in (\delta, 1]$, so $r > \delta$. Thus, $\mu((as)b) \geq r > \gamma$, implies that $(as)b \in \mu_r^\gamma$. Hence, μ_r^γ is a generalized bi-ideal of S.

Conversely, assume that $\mu_r^\gamma (\neq \Phi)$ is a generalized bi-ideal of S for all $r \in (\gamma, \delta]$. Let $a, b, s \in S$, such that, $\mu((as)b) \vee \delta < \mu(a) \wedge \mu(b)$. Select $r \in (\gamma, \delta]$, such that, $\mu((as)b) \vee \delta < r \leq \mu(a) \wedge \mu(b)$. Then, $\mu(a) \wedge \mu(b) \geq r$ and $\mu((as)b) < r$. This implies that $\mu(a) \geq r$ or $\mu(b) \geq r$ and $\mu((as)b) < r$. Thus, $a_r, b_r \in_\gamma \vee q_\delta \mu$ but $((as)b)_r \overline{\in}_\gamma \mu$. This implies, $a, b \in \mu_r^\gamma$ but $((as)b) \notin \mu_r^\gamma$. Which is a contradiction. Hence, $\mu((as)b) \vee \delta \geq \mu(a) \wedge \mu(b)$. Thus, μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of S.

(ii) Let μ be an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of S and let $a, b \in \mu_r^\delta$ for all $r \in (\gamma, \delta]$ and $s \in S$. Then, $\mu(a) + r > 2\delta$ and $\mu(b) + r > 2\delta$ or $\mu(a) > 2\delta - r$ and $\mu(b) > 2\delta - r$. By hypothesis, $\mu((as)b) \vee \delta \geq \mu(a) \wedge \mu(b)$, which implies that $\mu((as)b) \vee \delta > (2\delta - r) \wedge (2\delta - r)$. Since $2\delta - r > \delta$, therefore $\mu((as)b) > 2\delta - r$ or $\mu((as)b) + r > 2\delta$, that is $((as)b)_r q_\delta \mu$. Thus, $(as)b \in \mu_r^\delta$.Hence, μ_r^δ is a generalized bi-ideal of S.

Conversely, assume that $\mu_r^\delta (\neq \Phi)$ is a generalized bi-ideal of S. We are going to show that μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of S. On contrary, assume that $\mu((as)b) \vee \delta < \mu(a) \wedge \mu(b)$. Select $r \in (\gamma, \delta]$, such that, $r = \delta$, then $2\delta - r \in (\gamma, \delta]$. So, if we select $2\delta - r \in (\gamma, \delta]$, such that, $\mu((as)b) \vee \delta < 2\delta - r \leq \mu(a) \wedge \mu(b)$. This implies, $\mu((as)b) < 2\delta - r$ and $2\delta - r \leq \mu(a)$ or $2\delta - r \leq \mu(b)$, which implies that $\mu((as)b) + r < 2\delta$ but $\mu(a) + r \geq 2\delta$ or $\mu(b) + r \geq 2\delta$. This implies, $a_r, b_r \in \mu_r^\delta$, but $((as)b)_r \notin \mu_r^\delta$. Which is a contradiction. Hence, $\mu((as)b) \vee \delta \geq \mu(a) \wedge \mu(b)$. Thus, μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideal of S. □

If we take $\gamma = 0$ and $\delta = 0.5$ in above theorem we can conclude the following results:

Corollary 5.9. *Let μ be a fuzzy set of S.Then,*

(i) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy generalized bi-ideal of S if and only if $\mu_r (\neq \phi)$ is generalized bi-ideal of S for all $r \in (0.5, 1]$.

(ii) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy generalized bi-ideal of S of S if and only if $\mu_r^\delta (\neq \phi)$ is a generalized bi-ideal of S for all $r \in (0, 0.5]$

6. $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-FUZZY BI-IDEALS

Definition 6.1. A fuzzy subset μ of an LA-semigroup S is called an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy bi-ideal of S if for all $a, b, s \in S$ and $t, r \in (\gamma, 1]$,
(i) $(ab)_{t\wedge r}\,\overline{\in}_\gamma \mu$ implies that $a_t \overline{\in}_\gamma \vee \overline{q}_\delta \mu$ or $b_r \overline{\in}_\gamma \vee \overline{q}_\delta \mu$.
(ii) $((as)\,b)_{t\wedge r}\,\overline{\in}_\gamma \mu$ implies that $a_t \overline{\in}_\gamma \vee \overline{q}_\delta \mu$ or $b_r \overline{\in}_\gamma \vee \overline{q}_\delta \mu$.

Remark 6.2. Every $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy bi-ideal is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy bi-ideal of S but the converse is not true.

Example 6.3. Let $S = \{l, m, p, q\}$ with the following multiplication table:

$*$	l	m	p	q
l	m	q	q	q
m	m	m	m	m
p	q	m	q	q
q	m	m	m	m

Define a fuzzy subset μ by
$\mu(l) = 0$, $\mu(m) = 0.3$, $\mu(p) = 0.5$, $\mu(q) = 0.6$.
Then,
(i) μ is an $(\overline{\in}_{0.5}, \overline{\in}_{0.5} \vee \overline{q}_{0.6})$-fuzzy bi-ideal of S.
(ii) μ is not an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy bi-ideal of S.Because $((q * q) * q)_{0.6\wedge 0.6}\,\overline{\in}\mu$ but $q_{0.6} \in \vee q\mu$.

Theorem 6.4. *Let A be a bi-ideal of S. Then, the fuzzy subset μ of S defined by*

$$\mu(a) = 1 \text{ if } a \in A$$
$$\mu(a) \leq \delta \text{ if } a \notin A$$

is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy bi-ideal of S.

Proof. Proof follows from Theorem 5.4. □

Corollary 6.5. *Let A be a bi-ideal of an LA-semigroup S. Then, χ_A the characteristic function of A is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy bi-ideal of S.*

Theorem 6.6. *A fuzzy subset μ of S is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy bi-ideal of S if and only if for all $a, b, s \in S$ and $t, r \in (\gamma, 1]$,*
(i) $\mu(ab) \vee \delta \geq \mu(a) \wedge \mu(b)$.
(ii) $\mu((as)\,b) \vee \delta \geq \mu(a) \wedge \mu(b)$.

Proof. Proof follows from Theorem 5.6. □

Remark 6.7. For any $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy bi-ideal μ of S, we can conclude that if $\delta = 0.5$, then μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy bi-ideal of S.

Theorem 6.8. *Let μ be a fuzzy subset of S. Then,*
(i) μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy bi-ideal of S if and only if $\mu_r^\gamma (\neq \Phi)$ is a bi-ideal of S for all $r \in (\delta, 1]$.
(ii) μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy bi-ideal of S if and only if$\mu_r^\gamma (\neq \Phi)$ is a bi-ideal of S for all $r \in (\gamma, \delta]$.

Proof. Proof follows from Theorem 5.8. □

If we take $\gamma = 0$ and $\delta = 0.5$ in above theorem we can conclude the following results:

Corollary 6.9. *Let μ be a fuzzy set of S. Then,*

(i) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy bi-ideal of S of S if and only if $\mu_r(\neq \phi)$ is a bi-ideal of S of S for all $r \in (0.5, 1]$.

(ii) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy bi-ideal of S if and only if $\mu_r^\delta(\neq \phi)$ is a bi-ideal of S for all $r \in (0, 0.5]$

7. $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy quasi-ideals

Definition 7.1. A fuzzy subset μ of an LA-semigroup S is called an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy quasi-ideal of S, if it satisfies,

$\mu(x) \vee \delta \geq (\mu \circ 1)(x) \wedge (1 \circ \mu)(x)$.

Theorem 7.2. *Let μ be an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy quasi-ideal of S, then the set $\mu_\delta = \{a \in S \mid \mu(a) > \delta\}$ is a quasi-ideal of S.*

Proof. In order to prove that μ_δ is a quasi-ideal of S, we have to show that $S\mu_\delta \cap \mu_\delta S \subseteq \mu_\delta$. Let $x \in S\mu_\delta \cap \mu_\delta S$. This implies that $x \in S\mu_\delta$ and $x \in \mu_\delta S$. So $x = sa$ and $x = bt$ for some $a, b \in \mu_\delta$ and $s, t \in S$. Thus, $\mu(a) > \delta$ and $\mu(b) > \delta$. Since

$$\begin{aligned} (1 \circ \mu)(x) &= \bigvee_{x=yz} \{1(y) \wedge \mu(z)\} \\ &\geq \{1(s) \wedge \mu(a)\} \text{ because } x = sa \\ &= \mu(a). \end{aligned}$$

Similarly, $(\mu \circ 1)(x) \geq \mu(b)$. Thus,

$$\begin{aligned} \mu(x) \vee \delta &\geq (\mu \circ 1)(x) \wedge (1 \circ \mu)(x) \\ &\geq \mu(a) \wedge \mu(b) \\ &> \delta \text{ because } \mu(a) > \delta,\ \mu(b) > \delta. \end{aligned}$$

Which implies that $\mu(x) > \delta$. Thus, $x \in \mu_\delta$. Hence, μ_γ is a quasi-ideal of S. □

Remark 7.3. Every $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy quasi-ideal of S is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy quasi-ideal of S but the converse is not true.

Lemma 7.4. *Let Q be a quasi-ideal of an LA-semigroup S then, the characteristic function χ_Q of Q is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy quasi-ideal of S.*

Proof. Let Q be a quasi-ideal of S and χ_Q is the characteristic function of Q. If $x \notin Q$, then $x \notin SQ$ or $x \notin QS$. Thus, $(1 \circ \chi_Q)(x) = 0$ or $(\chi_Q \circ 1)(x) = 0$ and so, $(1 \circ \chi_Q)(x) \wedge (\chi_Q \circ 1)(x) = 0 \leq \chi_Q(x) \vee \delta$. If $x \in Q$, then $\chi_Q(x) \vee \delta = 1 \vee \delta = 1 \geq (1 \circ \chi_Q)(x) \wedge (\chi_Q \circ 1)(x)$. Hence, χ_Q is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy quasi-ideal of S. □

Theorem 7.5. *Every $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left(right) ideal μ of S is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy quasi-ideal of S.*

Proof. Let $a \in S$, then

$$(1 \circ \mu)(a) = \bigvee_{a=xy} \{1(x) \wedge \mu(y)\} = \bigvee_{a=xy} \mu(y).$$

This implies that

$$\begin{aligned}(1 \circ \mu)(a) &= \bigvee_{a=xy} \mu(y) \\ &\leq \bigvee_{a=xy} \{\mu(xy) \vee \delta\}, \ (\because \mu \text{ is an } (\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)\text{-fuzzy left ideal of } S.) \\ &= \mu(a) \vee \delta.\end{aligned}$$

Thus, $(1 \circ \mu)(a) \leq \mu(a) \vee \delta$. Hence, $\mu(a) \vee \delta \geq (1 \circ \mu)(a) \geq (\mu \circ 1)(a) \wedge (1 \circ \mu)(a)$. Thus, μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy quasi-ideal of S □

8. Conclusions

In algebraic structures, the LA-semigroup has a vital role in non-associative structures. We notice that the notion of fuzzy point has prominent role in theory of fuzzy LA-semigroups. We generalized the notion of fuzzy ideals of LA-semigroups by using the notions of fuzzy point to fuzzy ideals in LA-semigroups. We introduced the concept of $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroups, $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left(right) ideals, $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideals and $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy bi-ideals of an LA-semigroup are introduced. The given concept is a generalization of $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy LA-subsemigroups, $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy left(right) ideals, $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy generalized bi-ideals and $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy bi-ideals of an LA-semigroup. We also give some examples of $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy LA-subsemigroups (left, right, generalized bi- and bi) ideals of an LA-semigroup. we prove some fundamental results of these ideals. We characterize $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy left(right) ideals, $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy generalized bi-ideals and $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$-fuzzy bi-ideals of an LA-semigroup by the properties of level sets. In future we will apply this concept to other algebraic structures.

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