International Journal of Thermal Sciences 50 (2011) 2078–2083

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Thermophysical continuous profiles and their discretization

Ph. Grossel*, F. Depasse

Groupe de Recherche en Sciences Pour l'Ingénieur, EA 4301, URCA/GRESPI/CATHERM, Université de Reims, BP 1039, 51687 REIMS Cedex 2, France

A R T I C L E I N F O

Article history:
Received 28 September 2010
Received in revised form
3 March 2011
Accepted 20 March 2011
Available online 13 July 2011

Keywords:
Thermal waves
Thermal quadrupoles
Continuous profiles
Discretization
Riccati equation

A B S T R A C T

Most of the time the surface of materials presents profiles of physical properties that are variable along the depth. The heat diffusion equation of current thermophysical profiles rarely accepts analytical solutions that can be introduced in a rapid minimization scheme. A thermal Riccati equation is defined in case of any continuous profiles, the solutions of which are shown to be the limit of the exact solutions for simple staircase multilayer modellings when the layers are taken thinner and thinner. Thanks to the use of iterative methods based on the thermal wave or the quadrupole descriptions, it is shown that the choice of the multilayer approach to describe continuous depth profiles of materials leads to exact solutions of the problem.

1. Introduction

On a general ground a studied sample has an unknown depth structure and its analysis can begin with the choice of some trial profile accepting exact analytical solutions for the heat diffusion equation (HDE) [1,2]; a fitting is made for some parameters, along a minimization scheme. More generally, a way to do that is to use a modelling consisting in a numerous multilayer, which gives exact numerical or analytical solutions, so that the discretization finely approaches the unknown continuous profile. Among all the possibilities, one is very interesting due to its clear and easy interpretation: it is obtained by using a staircase discretization of the model as a numerous multilayer constituted with homogeneous thermophysical layers, with continuity of temperature and heat flux at the interfaces.

Sometimes a question is rising about the reliability of the multilayer modelling with a continuous profile: in what extent are we sure that the results obtained by increasing the number of layers, and decreasing their individual width correspond to the limit of those of the current profile ? Some numerical indications have been obtained with the observation of the convergence in case of profiles permitting analytical solutions. This is the case for profiles having a thermal conductivity or heat capacity varying linearly or exponentially with depth [3–5]. A clear link between a very thin multilayer and the WKB thermal wave approximation has also been obtained [4].

The present work deals with the general study of the convergence between any continuous profile and the staircase modelling, in the case of a one-dimensional profile (along the depth of the sample). It consists in considering the multilayer that can be tangent to the continuous profile. Among the possibilities for solving the parabolic heat equation for the multilayer different methods can be used as the powerfull thermal quadrupole analysis using a vector the components of which are the temperature and the heat flux at each point [6–8]. Another method is the analytical iterative one based on transfer matrix using *forward* and *backward* thermal waves, connected between them by the *generalized reflection coefficient* [5,9–12] that verifies a very attractive iterative law along the multilayer. This leads to the introduction of the Riccati equation that manages the reflection coefficient along the continuous profile [4]. In this study the convergence will be proved in showing that this (continuous) generalized reflection coefficient leads to the same iterative expression than its equivalent in the discretized approach.

The iterative method can be used in the Fourier domain, which is chosen in case of an alternative heat excitation for non-destructive control at very different scales [1,13–22]. It must be emphasized that the present results can be rewritten in terms of the Laplace variables which can be used to describe experiments working with flash photothermal excitations among many other experiments ([23] and references therein). In this paper we restricted the study to the parabolic heat equation, although it

* Corresponding author. Tel.: +33 3 26 91 31 08.
E-mail address: philippe.grossel@univ-reims.fr (Ph. Grossel).

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doi:10.1016/j.ijthermalsci.2011.03.031