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Folding of Hyperbolic Manifolds

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Abstract

In this paper, we introduce the definition of hyperbolic manifold. The folding of hyperbolic manifold into itself is defined and discussed. Types of these foldings are deduced. Theorems governing these types are achieved.

Mathematics Subject Classification, 51H10, 57N20

Keywords: Folding, hyperbolic manifolds

Introduction

The word topology is derived from the Greek words tops and logos and means " the science of place ". Adopted from the French topologic , the word came into use in English in the 1600s.

Its original meaning … The branch of botany that deals with the localities of particular plants… has fallen into discuss, along with a host of later ones . (In the 1860s, the word primarily referred to the art of assisting the memory by associating the thing to be remembered with some well – known place). Topology was introduced as a mathematical term by the German mathematician Johann Benedict listing in 1847.

The first one who introduced the folding of Riemannian manifolds is S.A-Robertson 1977[9].More studies on the folding of real manifolds are studied by

E.El- kholy[8].and M.EL-Ghoul[1,2,3,4,5,6,7].In this article we will introduce the folding of hyperbolic manifold.

Definitions and background

We will give some definitions which we will need them in this paper:

(1) Map $f : M \to N$,where M, N are C^{∞} - Riemannian manifolds of dimensions m, n ,respectively is said to be an isometric folding of M into N ,if and only if for any piecewise geodesic path $\gamma : J \to N$,the induced path $f \circ \gamma : J \to M$ is piecewise geodesic and of the same length as $\gamma, \gamma = [0,1]$ [9] .

If f not preserves lengths then f is a topological folding.

(2) There exist two models of hyperbolic plane the first model is the geometry in the upper plane see Fig. (1).and the second model is the geometry interior the circle see Fig.(2).

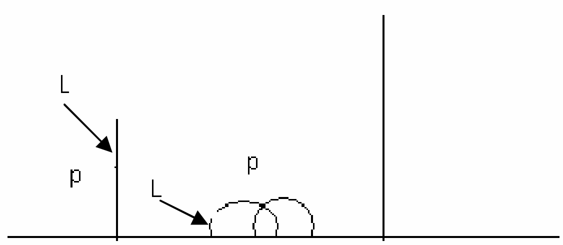


Fig. (1)

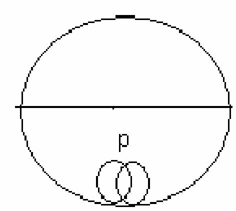


Fig. (2)

The main results

Aiming to our study we will introduce some definition:

[1]

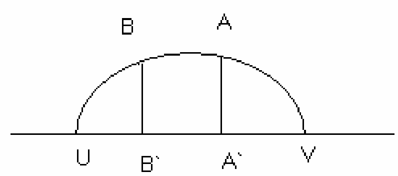


Fig.(4)

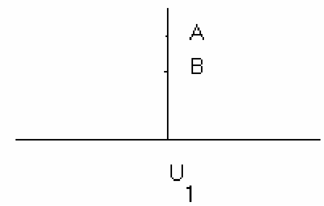


Fig.(3)

In Fig.(3) :

$$|AB|_n = Log\frac{|Au_1|_e}{|Bu_1|_e}$$

In Fig.(4) :

$$|AB|_n = \frac{1}{2} Log\frac{|A'u|_e \bullet |B'v|_e}{|B'u|_e \bullet |A'v|_e}$$

[2] Folding of hyperbolic plane is a map $f : H_1 \to H_2$ such that $H_2 \subset H_1$.

There exist three types of foldings of hyperbolic plane we will discuss these types:

(1) Folding which fold the hyperbolic plane into anther hyperbolic plane and preserving the basic properties of hyperbolic plane see Fig.(5)and Fig.(6) .

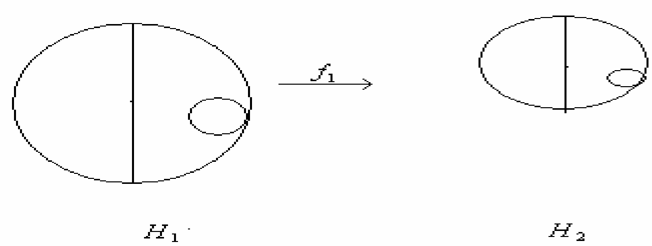


Fig . (5)

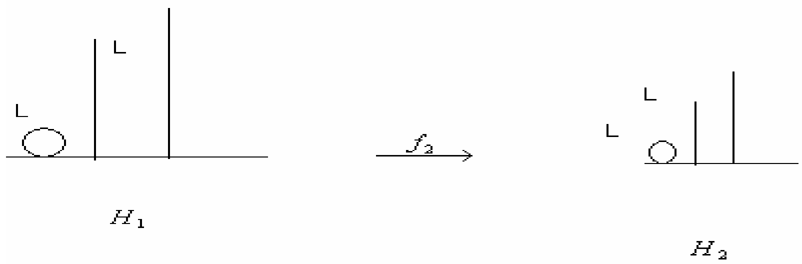


Fig . (6)

(2) folding which fold the hyperbolic plane to a subset of hyperbolic plane see Fig.(7)and Fig.(8) .

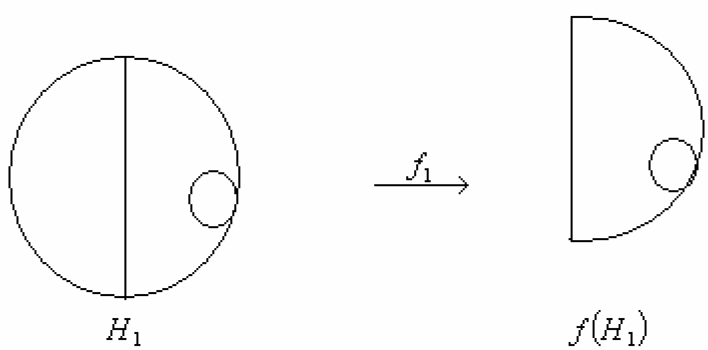


Fig.(7)

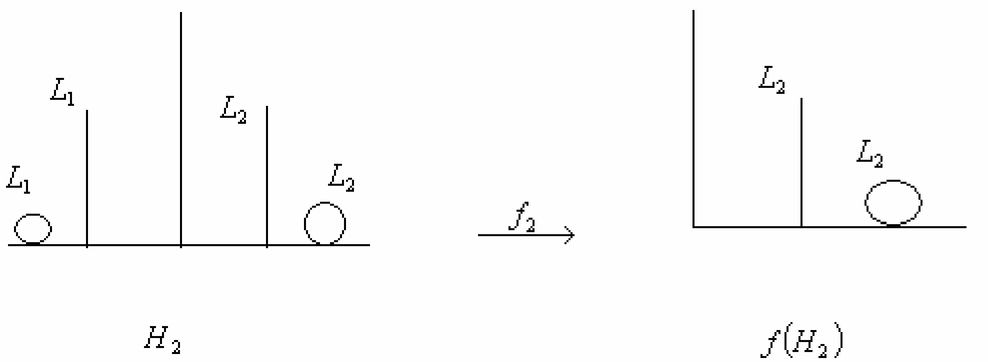


Fig.(8)

(3) Let $f_3 : H \to A$ which A is a subset of H with boundary see Fig.(9)and Fig.(10).

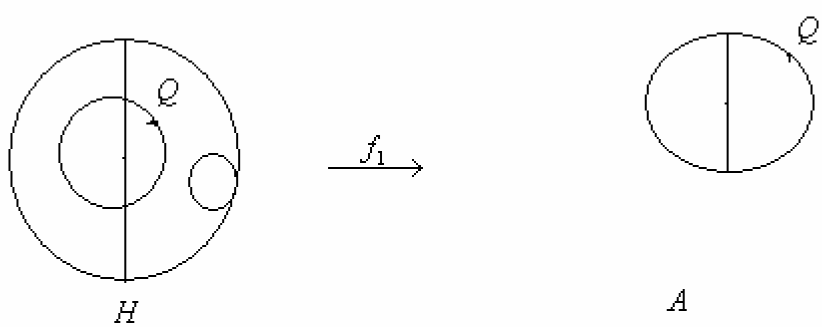


Fig.(9)

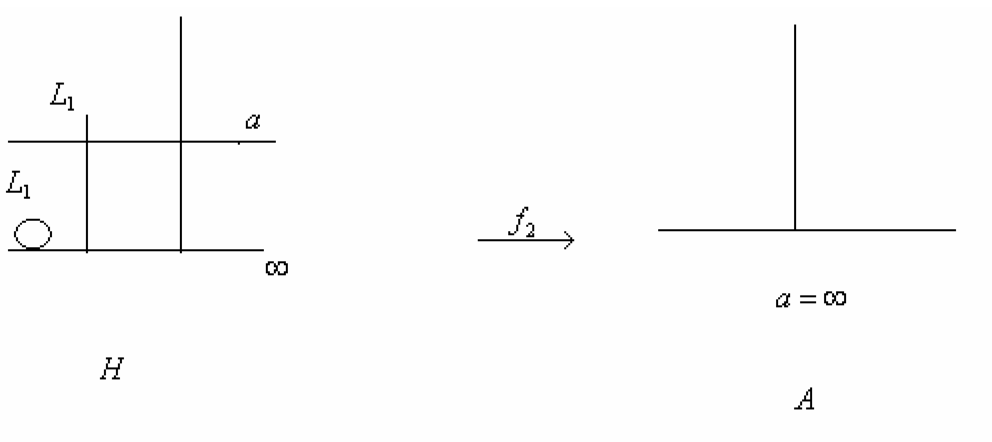


Fig.(10)

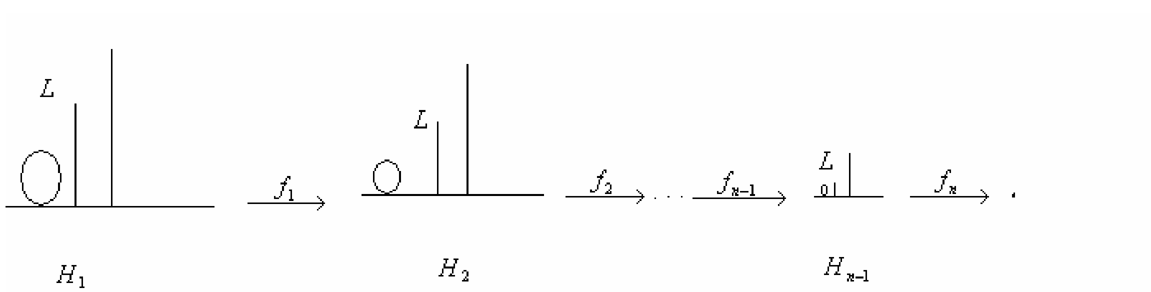
Theorem (1):

The limit of foldings of the hyperbolic plane into itself of the first type is a point.

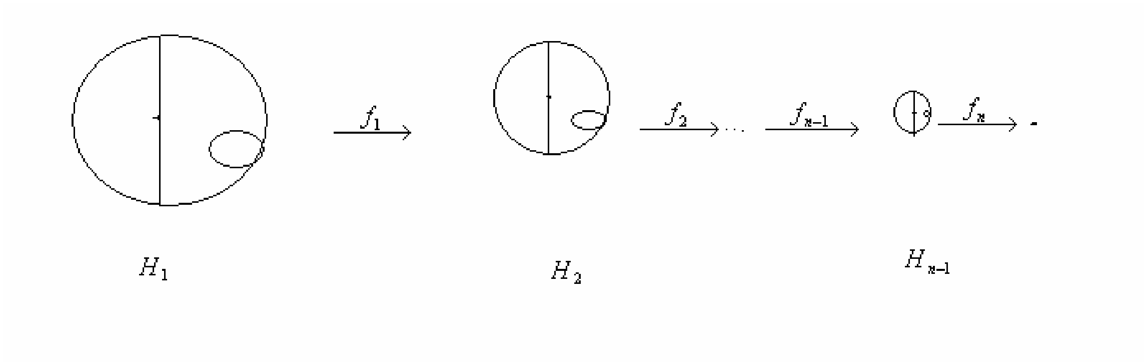
Proof:

This folding which fold the hyperbolic plane into itself is declination of the hyperbolic plane and declination of lines and can fold the hyperbolic plane any more as follows,

$$H_1 \xrightarrow{f_1} H_2 \xrightarrow{f_2} H_3 \xrightarrow{f_3} \cdots \xrightarrow{f_{n-1}} H_{n-1} \xrightarrow{f_n} point \text{ see Fig. (11)}$$



model (1)



model (2)

Fig.(11)

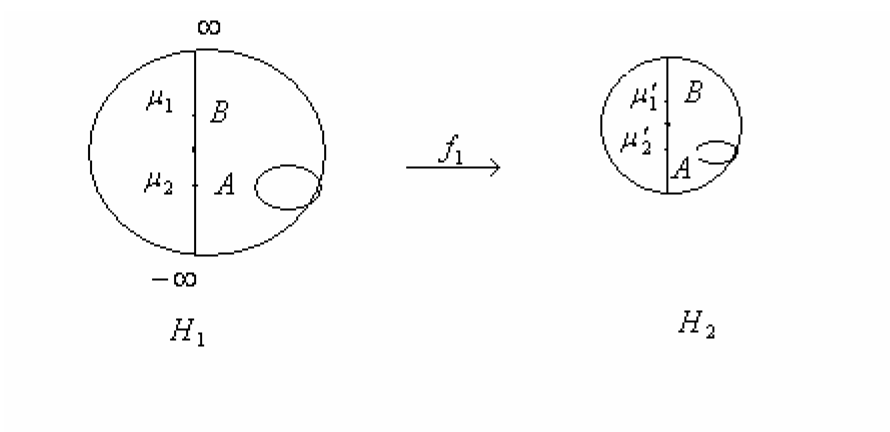
Theorem(2):

The image of the second and the third type of foldings of the hyperbolic plane is not hyperbolic plane.

Proof:

Let $f_1 : H_1 \to N$ is a folding of the hyperbolic plane H_1 to subset of itself N of the second type the image of this folding not satisfies the properties of the hyperbolic plane. And let $f_2 : H_2 \to A$ which is a folding of the third type which fold the hyperbolic plane H_2 to a subset A with boundary then the image of this folding A is a disk not hyperbolic plane.

Folding of fuzzy hyperbolic plane: is A map $\widetilde{f} : \widetilde{H}_1 \to \widetilde{H}_2$ such that $\widetilde{H}_2 \subset \widetilde{H}_1$ and $f(\mu_i) = \mu_j$ as fallows:



Theorem (3):

The isometric folding of Riemannian manifolds increasing the dimensions but the folding of hyperbolic plane preserve the dimensions.

Proof:

Let $f\ :M \to N$ be an isometric folding, M, N are two Riemannian manifolds, then dimensions M $\leq$ dimensions N .

If $g: H_1 \to H_2$ is isometric folding from hyperbolic plane H_1 to another one H_2, then dimensions H_1 = dimensions H_2, from the definition of length in hyperbolic plane.

$$\left|AB\right|_n = Log\frac{\left|Au_1\right|_e}{\left|Bu_1\right|_e} \qquad \text{if} \quad A = B \Rightarrow$$

$\left|AB\right|_n = Log1 = 0,$ And the another the definition of length in hyperbolic plane

$$\left|AB\right|_n = \frac{1}{2}Log\frac{\left|A'u\right|_e \bullet \left|B'v\right|_e}{\left|B'u\right|_e \bullet \left|A'v\right|_e} \Rightarrow \left|AB\right|_n = \frac{1}{2}Log1 = 0$$

No representation of a line to be a hyperbolic space.

Theorem(4):

The folding of the hyperbolic plane consider as the retraction of it.

Proof:

Let $r_1: H_1 \to H_2 \ , H_1 \subset H_2$

$r_2: H_2 \to H_3$

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$r_n: H_n \to H_{n+1}$

Then $r_i \equiv f_i$, there are homeomorphisms h_i such that:

$$
\begin{array}{ccccccccccc}
H_1 & \xrightarrow{r_1} & H_2 & \xrightarrow{r_2} & H_3 & \xrightarrow{r_3} & H_4 & \xrightarrow{r_4} & \cdots & \xrightarrow{r_n} & H_{n+1} \\
\downarrow h_1 & & \downarrow h_2 & & \downarrow h_3 & & \downarrow h_4 & & & & \downarrow h_{n+1} \\
\overline{H}_1 & \xrightarrow{f_1} & \overline{H}_2 & \xrightarrow{f_2} & \overline{H}_3 & \xrightarrow{f_3} & \overline{H}_4 & \xrightarrow{f_4} & \cdots & \xrightarrow{f_{n+1}} & \overline{H}_{n+1}
\end{array}
$$

such that $h_{n+1} \circ r_n = f_n \circ h_n$

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