International Journal of Applied Electromagnetics and Mechanics 33 (2010) 1025–1032
DOI 10.3233/JAE-2010-1216
IOS Press

A new method of load identification in time domain

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Abstract. The main idea of the paper is to identify the load in time domain through inversely analyzing the Duhamel integral process, a time domain algorithm is developed without tedious theoretical derivation, the simulated results show that the proposed method is effective and of high accuracy.

1. Introduction

Some engineering structures inevitably suffered from dangerous dynamic loads, usually these dynamic loads are difficult to be measured. In contrast, the structure response generated by these load can easily be measured. Load identification is a task of identifying these dynamic loads according to the measured response signal data. Generally, load identification contains frequency domain method and time domain method [1,2]. Some time domain methods [3–5] need tedious theoretical derivation. This paper proposed a new load identification method in time domain. Its main idea originates from the analysis of Duhamel integral to calculate structure response, and the load could be inversely deduced from this discrete process with a compact form. Simulation results show that this new method is smart and effective.

2. Identification theory

2.1. Identification theory of single-degree-of-freedom system

As is known for all, the kinetic equation of an one-degree-of-freedom system suffering load $f(t)$ could be described as below

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \tag{1}$$

Its displacement response $x(t)$ in time domain could be calculated with Duhamel integral

$$x(t) = \int_0^t p(\tau)h_d(t-\tau)d\tau \tag{2}$$

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In which

$$h_d(t) = \frac{1}{\omega_d} e^{-\xi\omega_n t} \sin(\omega_d t) \tag{3}$$

$$\omega_n = \sqrt{\tfrac{k}{m}}\, \xi = \tfrac{c}{2\sqrt{km}}\, \omega_d = \omega_n \sqrt{1-\xi^2} \tag{4}$$

In discretization of time, Eq. (2) could be rewritten, for each time step, and because of

$$h_d(1) = 0 \tag{5}$$

$x(t)$ can be expressed as

$$\begin{cases} x(2) = p(1)h_d(2) \\ x(3) = p(1)h_d(3) + p(2)h_d(2) \\ x(4) = p(1)h_d(4) + p(2)h_d(3) + p(3)h_d(2) \\ \quad \vdots \\ x(i) = p(1)h_d(i) + p(2)h_d(i-1) + \ldots + p(i-1)h_d(2) \\ \quad \vdots \\ x(l-1) = p(1)h_d(l-1) + p(2)h_d(l-2) + \ldots + p(l-1)h_d(1) + p(l-2)h_d(2) \end{cases} \tag{6}$$

In Eq. (5), l refers to the length of time, i corresponding to time step $(i-1)dt$, dt refers to time interval, and

$$p(i) = f(i)dt \tag{7}$$

The matrix form of Eq. (6) is

$$x(t) = H(t)p(t) \tag{8}$$

In which

$$H(t) = \begin{bmatrix} h_d(2) & & \\ h_d(3) & h_d(2) & \\ \vdots & & \ddots \\ h_d(l-1) & h_d(l-2) \cdots & h_d(2) \end{bmatrix} \; p(t) = \begin{bmatrix} p(1) \\ p(2) \\ \vdots \\ p(l-1) \end{bmatrix} \; x(t) = \begin{bmatrix} x(2) \\ x(3) \\ \vdots \\ x(l-1) \end{bmatrix} \tag{9}$$

For load identification problem, the load could be calculated from the inversion of Eq. (8)

$$p(t) = H^{-1}(t)x(t) \tag{10}$$

$H(t)$is a lower triangular matrix and all its diagonal elements are the same, it has a good behaviour for its inversion calculation.

If the length of time data is not long, the size of $H(t)$ is not large, Eq. (10) is fast and effective, if the length of time data is rather long, the size of $H(t)$ is large, it may cause the inversion calculation lots of time, because of the characteristic of lower triangular matrix of $H(t)$, each time step of $p(t)$ could be obtained as below:

$$\begin{aligned} x(i) &= p(1)h_d(i) + p(2)h_d(i-1) + ... + p(i-1)h_d(2) \\ &= \sum_{j=0}^{i-1} p(j)h_d(i+1-j)(i=2,\dots,l-1) \end{aligned} \tag{11}$$

In fact, the essence of Eq. (11) is gauss elimination method. From Eq. (11), the load of each time step could be got as

$$p(i-1) = \frac{x(i) - \sum\limits_{j=1}^{i-2} p(j)h_d(i+1-j)}{h_d(2)}(i=2,\dots,t-1) \tag{12}$$

In most condition, the measured data is acceleration signal, in order to identificate the load from acceleration signal, an undamped single-degree-of-freedom system is assumed as

$$\ddot{x}(t) + x(t) = f_1(t) \tag{13}$$

In Eq. (13)

$$p_1(i) = f_1(i)dt \tag{14}$$

With the same treatment of Eq. (10)

$$p_1(t) = H_1^{-1}(t)x(t) \tag{15}$$

$$H_1(t) = \begin{bmatrix} h_1(2) & & & \\ h_1(3) & h_1(2) & & \\ \vdots & & \ddots & \\ h_1(l-1) & h_1(l-2) & \cdots & h_1(2) \end{bmatrix} \tag{16}$$

With the same analysis of Eq. (3), $h_1(t)$ could be simplified as

$$h_1(t) = \sin(t) \tag{17}$$

From Eq. (13), $\ddot{x}(t)$ could be expressed as

$$\ddot{x}(t) = f_1(t) - x(t) \tag{18}$$

Substitute Eqs (8) and (14) into Eq. (18)

$$\ddot{x}\left(t) = (\frac{H_1^{-1}(t)}{dt} - I\right)H(t)p(t) \tag{19}$$

Denoted T as

$$T = \left(\frac{H_1^{-1}(t)}{dt} - I\right)H(t) \tag{20}$$

The load identification equation from acceleration data could be written as

$$f(t) = \frac{T^{-1}\ddot{x}(t)}{dt} \tag{21}$$

2.2. Identification theory of multi-degree-of-freedom system

The kinetic equation of an multi-degree-of-freedom system suffering load P(t) could be described as below:

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = P(t) \tag{22}$$

M, C, K corresponding to the mass matrix, dumping matrix and stiffness matrix, assumption of the size of the system is n, and the acceleration response of v degree-of-freedom are measured.

Transferring Eq. (22) into canonical coordinate as below:

$$\Psi^T M\Psi\ddot{\eta}(t) + \Psi^T C\Psi\dot{\eta}(t) + \Psi^T K\Psi\eta(t) = \Psi^T P(t) \tag{23}$$

The size of Ψ is $v \times r$, and $r \leqslant v$ [6] its column elements are chosen from the first r order truncated canonical vibration modes, its row elements are chosen from these r modes corresponding to the degree-of-freedoms at which the acceleration response measured. In most condition, only obtaining first few order canonical mode is possible, in fact, the first few order canonical modes have the most significant effect of the response η is the canonical coordinate, after decoupling, Eq. (23) become

$$\ddot{\eta}(t) + C_z\dot{\eta}(t) + K_z\eta(t) = P_z(t) \tag{24}$$

And

$$\begin{aligned} C_z &= \Psi^T C\Psi = \begin{bmatrix} 2\xi_(1)\omega_(1) & & \\ & \ddots & \\ & & 2\xi_(r)\omega_(r) \end{bmatrix} \\ K_z &= \Psi^T K\Psi = \begin{bmatrix} \omega^2(1) & & \\ & \ddots & \\ & & \omega^2(r) \end{bmatrix} \\ P_z(t) &= \Psi^T P(t) \end{aligned} \tag{25}$$

So Eq. (22) decoupled into r one-degree-of-freedom systems, ξ is the first r order modal damping ratios, ω is the first r order nature frequencies, for each system, its canonical load P_z(t) could be calculated by Eqs (10) or (11). Because Ψ is not square matrix, with the method of left multiplication, the practical load P(t) could be calculated as below:

$$P(t) = (\Psi\Psi^T)^{-1}\Psi P_z(t) \tag{26}$$

Because of it is the acceleration signals are measured in most case, so this paper only studied the case of acceleration response.

3. Numerical simulation

3.1. Finite element model

Figure 1 shows the cantilever beam finite element model, it has 5 nodes and 4 beam elements, each element is 2 m long, and its section size is 100 mm $\times$ 100 mm, its density is 7800 kg/m^3 and the elastic modulus is 210e9 and poisson ratio is 0.3. The location of actual load P_1(t) and P_2(t) are shown in Fig. 1.

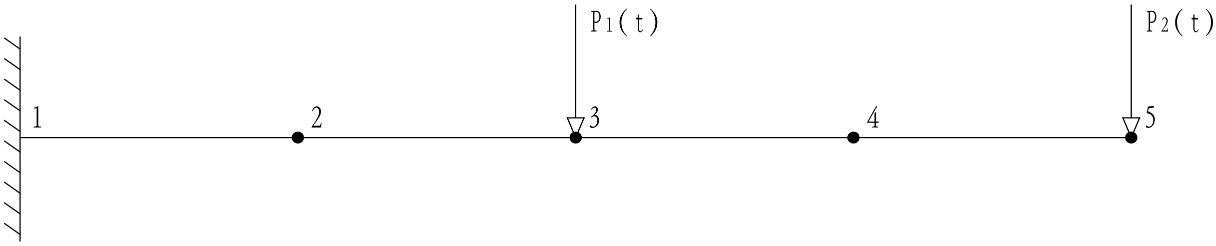


Fig. 1. The finite elements model of beam.

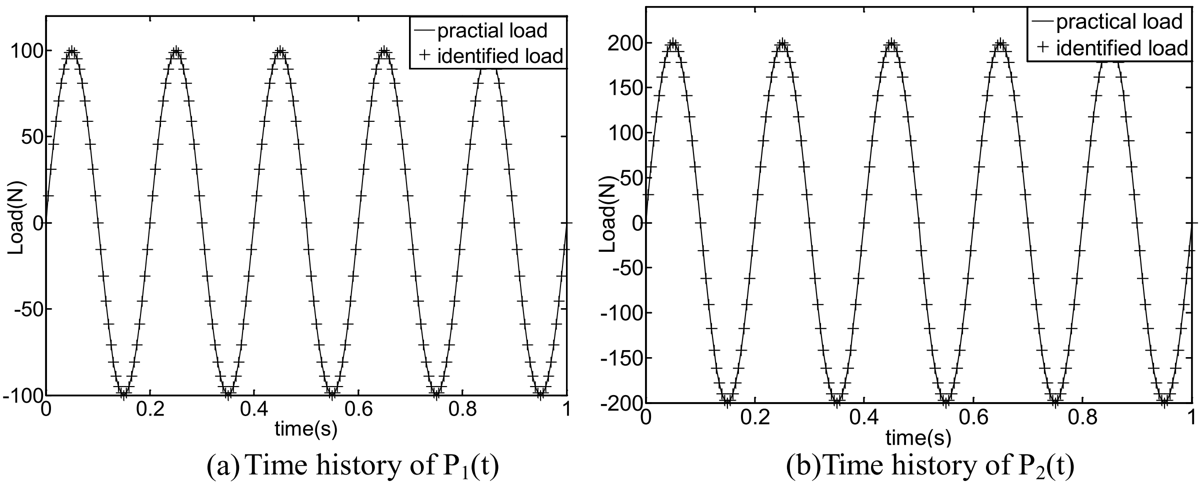


Fig. 2. Identification of sinusoidal load.

3.2. Identification of sinusoidal load

The assumption of actual sinusoidal load is expressed as:

$$P_1(t) = 100\sin(10\pi t)\ P_2(t) = 200\sin(10\pi t)\ (t = [0, 0.005, ..., 1]s)$$

The theoretical calculation vertical acceleration response results of node 2, 3, 4, 5 are used as measured signal data. The comparison of identified load and practical load are shown in Fig. 2, the solid line refers to the actual load, the '$+$' marked line refers to the identified result.

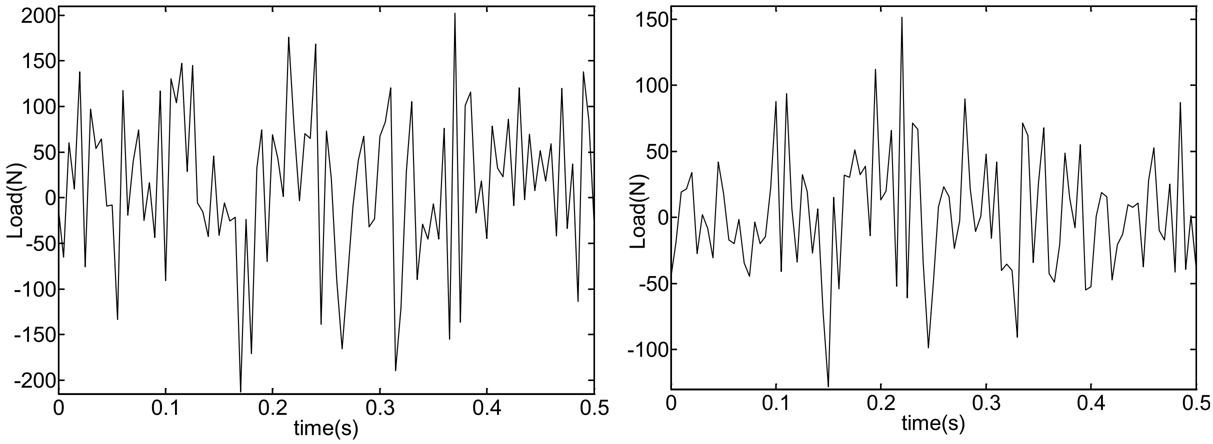
In Fig. 2, the recognized result matched very well with practical load, it confirmed the effectiveness of the proposed method.

3.3. Identification of random load without noise

The time history of actual load is shown in Fig. 3a and Fig. 3b, the time interval is 0.005s, the node number of measured signal are the same as Section 3.2.

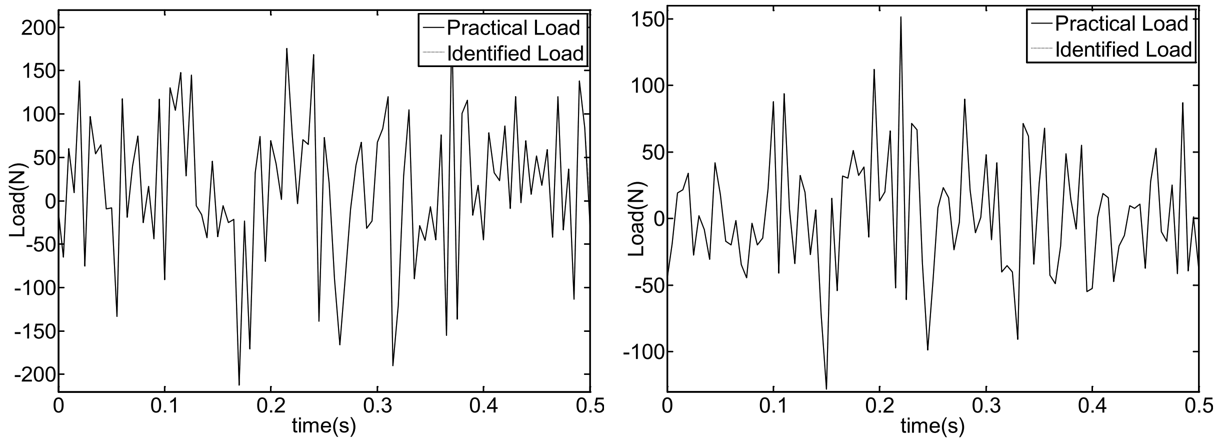
The identification result are shown in Fig. 4a and Fig. 4b, the solid line refers to the actual load, the '$+$' marked line refers to the identified result.

In Fig. 4, the recognized result matched perfect with the practical load.



(a) Time history of $P_1(t)$ (b) Time history of $P_2(t)$

Fig. 3. Actual random load time history.



(a) Time history of $P_1(t)$ (b) Time history of $P_2(t)$

Fig. 4. Comparison of random load identification.

3.4. *Identification of random excitation with white noise*

In fact, the measured signal is inevitably contaminated with noise, Fig. 5 and Fig. 5 are the identification result based on the assumption of that the measured signal contains white noise of 1% level and 2% level.

Defined identification error as

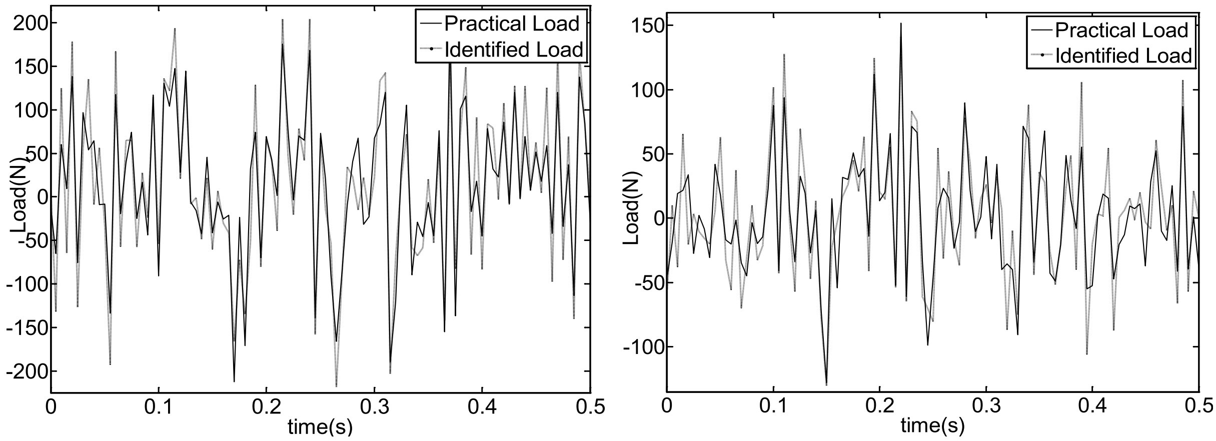
$$mse = \sum_{i=1}^{n} std(P_{ri} - P_{si})$$

P_r refers to practical load and P_s refers to identified load, ‘std’ is the standard deviation function, and n is the number of identified load.

The error is listed in Table 1.

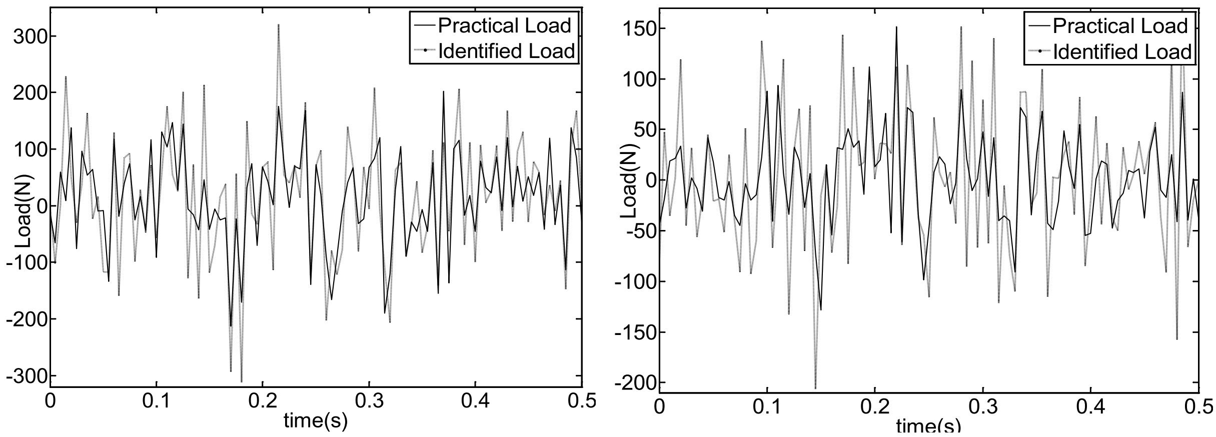
Table 1
Error comparison

Noise level	1%	2%
mse	65.83	130.97



(a) Time history of $P_1(t)$ (b) Time history of $P_2(t)$

Fig. 5. Comparison of random load identification with 1% noise level.



(a) Time history of $P_1(t)$ (b) Time history of $P_2(t)$

Fig. 6. Comparison of random load identification with 2% noise level.

From Table 1, it shows that with the contamination of noise, there is error generated between the practical load and identified load, with the increase of noise level, the error also become larger, some further study to improve the anti-noise ability of this algorithm are needed.

4. Conclusion

The process of time domain load identification method proposed in this paper is brief, and easy to be programmed. It avoids the problem caused by matrix inversion calculation. The simulation result of beam confirmed the effectiveness of this new method.

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