Metastable supersymmetry breaking in N=2 non-linear sigma-models

Jean-Claude Jacot and **Claudio A. Scrucca**

Institut de Théorie des Phénomènes Physiques
Ecole Polytechnique Fédérale de Lausanne
CH-1015 Lausanne, Switzerland

Abstract

We perform a general study of the issue of metastability for supersymmetry-breaking vacua in theories with $N = 1$ and $N = 2$ global supersymmetry. This problem turns out to capture all the important qualitative features of the corresponding question in theories with local supersymmetry, where gravitational effects induce only quantitative modifications. Moreover, it allows to directly compare the conditions arising in the $N = 1$ and $N = 2$ cases, since the latter becomes particular case of the former in the rigid limit. Our strategy consists in a systematic investigation of the danger of instability coming from the sGoldstini scalars, whose masses are entirely due to supersymmetry breaking mass-splitting effects. We start by reviewing the metastability conditions arising in general $N = 1$ non-linear sigma-models with chiral and vector multiplets. We then turn to the case of general $N = 2$ non-linear sigma-models with hyper and vector multiplets. We first reproduce and clarify the known no-go theorems applying to theories with only Abelian vector multiplets and only hyper multiplets, and then derive new results applying to more general cases. To make the comparison with $N = 1$ models as clear as possible, we rely on a formulation of $N = 2$ models where one of the supersymmetries is manifestly realized in terms of ordinary superfields, whereas the other is realized through non-trivial transformations. We give a self-contained account of such a construction of $N = 2$ theories in $N = 1$ superspace, generalizing previous work on various aspects to reach a general and coordinate-covariant construction. We also present a direct computation of the supertrace of the mass matrix.

1   Introduction

One of the main issues in supersymmetric theories aiming at describing real fundamental interactions is how supersymmetry is spontaneously broken. Indeed, this breaking induces mass splittings between ordinary particles and their superpartners, and the details of this process are thus of crucial importance. It turns out that the structure of theses splittings is strongly constrained, and this causes some difficulties in phenomenological applications. The perhaps most spectacular incarnation of this phenomenon is provided by the supertrace sum rule, which concerns the average of all the mass splittings. This implies for instance that renormalizable and anomaly-free supersymmetric extensions of the standard model cannot directly accommodate a viable way of spontaneously breaking supersymmetry. The standard way out to this problem is to assume that supersymmetry is broken in a hidden sector, which communicates with the visible sector only in a way that is suppressed by some mass scale. Spontaneous supersymmetry breaking can then be designed in a much more flexible way within the hidden sector, while supersymmetry breaking effects communicated to the visible sector are encoded in soft supersymmetry breaking terms. The only strong constraints on supersymmetry breaking that one is left with are then the metastability of the vacuum and the value of the cosmological constant. There are then more phenomenological constraints concerning the mediation of supersymmetry breaking to the visible sector and the structure of the soft terms.

The problem of understanding under which conditions vacua that break spontaneously supersymmetry may by at least metastable clearly emerges as one of the most relevant possible discrimination tools on the structure of the hidden sector. While the stability of supersymmetry-preserving vacua is guaranteed, that of supersymmetry-breaking ones is not, and whether they can be metastable or even absolutely stable depends on certain particular aspects of the theory. By now it has been well appreciated that requiring only metastability, rather than absolute stability, is perfectly satisfactory, as long as the life-time of the vacuum is sufficiently large, say larger than the age of the universe. Moreover, a supersymmetric theory generically admits both stable supersymmetry-preserving vacua and metastable supersymmetry-breaking vacua, but generically no absolutely stable supersymmetry-breaking vacua, unless some extra features are imposed, like for instance the existence of a global R-symmetry [1]. This clearly means that metastability of supersymmetry-breaking vacua is the relevant minimal requirement to impose, rather than absolute stability. More specifically, the requirement of metastability translates into the requirement that the mass matrix of the scalar field fluctuations, given by the Hessian matrix of the scalar potential at the stationary point defining the vacuum, should be positive definite. This obviously constrains the theory, but at first sight in a rather indirect and mild way. It turns however out that one can deduce from this requirement a quite simple and sharp necessary condition.

The main observation that allows to translate the condition of metastability into an interesting information is the following. To a large extent, one can adjust the overall masses of the particles belonging to each multiplet independently of the splittings induced by the process of spontaneous supersymmetry breaking, by tuning those parameters of the theory that are unrelated to the latter process. This allows to make the square mass of most of the scalar fields arbitrarily large and positive. There is however one exception to

this fact, represented by the Goldstino would-be multiplet. Indeed, for that multiplet there is an obstruction against changing the overall mass, due to Goldstone's theorem applied to the spontaneous breaking of supersymmetry. In rigid supersymmetry, this implies that the Goldstino is strictly massless, and the masses of its scalar partners, the sGoldstini, are thus entirely controlled by the mass-splitting effects due to supersymmetry breaking. In local supersymmetry, the Goldstino is absorbed by the gravitino through a super-Higgs mechanism, but it remains true that the masses of the sGoldstini are determined by the process of supersymmetry breaking. This means that the only scalar fields for which there may be a potential obstruction against achieving a positive square mass are the sGoldstini. If there are several supersymmetries, there are just several Goldstini and thus also a larger number of sGoldstini to look at.

The above strategy was first developed and applied to $N = 1$ supergravity theories with only chiral multiplets in [2, 3].[1] The main outcome is that the average mass of the two real sGoldstini is controlled by the holomorphic sectional curvature of the Kähler manifold spanned by the scalar fields along the complex Goldstino direction. To achieve metastability, one then needs first of all that the scalar geometry admits directions along which the curvature is sufficiently small, and then that the Goldstino direction be sufficiently aligned towards those preferred directions. On the other hand, the adjustment of the value of the cosmological constant constrains the length of the Goldstino direction. Subsequently, this analysis was extended in [5] to more general $N = 1$ theories involving both chiral and vector multiplets. The main conclusion is that gaugings by vector multiplets improve the situation occurring for just chiral multiplets, and make the bounds on the curvature milder. These metastability conditions have then been further elaborated and applied in [6, 7] for particular classes of $N = 1$ supergravity theories emerging as low-energy effective theories of string models, like for instance no-scale models. It has however become clear that in the context of string models, an analysis based on minimal $N = 1$ supersymmetry may fail to capture all of the potentially relevant information, due to the fact that the structure of the low-energy effective supergravity theories underlying these models is strongly constrained by its higher-dimensional origin. More specifically, although for interesting models one gets a theory with minimal supersymmetry in four dimensions, the moduli sector emerging through the compactification of the extra space-time dimensions, which is the most natural candidate to represent the hidden sector, actually displays many of the features of theories with extended supersymmetry in four dimensions. As a first step towards gaining an understanding of the impact on the metastability condition of such additional peculiarities in $N = 1$ theories, one may then try to study the question of metastability in $N = 2$ theories. The case of $N = 2$ supergravity theories with only hyper multiplets was studied in [8]. The result of this analysis is that out of the four sGoldstini arising in this case, one is absorbed by the graviphoton and is thus not dangerous, whereas the other three have an average square mass which is negative when the cosmological constant is positive, meaning that it is impossible to achieve metastability. A similar no-go theorem has been known for a long time to arise also in $N = 2$ theories involving only Abelian vector multiplets [9]. On the other hand, it has been shown through the construction of particular examples that more general $N = 2$ theories involving non-

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[1]See also [4] for an analysis of similar spirit applied to the ideas of distribution and landscape of vacua.

Abelian vector multiplets and/or both hyper and vector multiplets can admit metastable supersymmetry-breaking vacua with positive cosmological constant [10, 11]. A natural step to take is then to try to understand the metastability condition applying for general $N = 2$ theories, with the aim of figuring out which are the truly necessary ingredients to go in business. Such an analysis is however quite challenging from a technical point of view [12].

The aim of this work is to study the question of metastability in theories with $N = 1$ and $N = 2$ global supersymmetry. This rigid limit of the problem turns out to capture all the qualitatively important aspects of the corresponding problem in local supersymmetry, gravitational effects being responsible only for a quantitative deformation of the results. Moreover, besides yielding a much simpler and more transparent setting, the rigid limit also offers the very interesting possibility of directly comparing the results for $N = 2$ theories to those of $N = 1$ theories. This is due to the fact that in global supersymmetry $N = 2$ theories with hyper and vector multiplets are just particular cases of $N = 1$ theories with chiral and vector multiplets, whereas on the contrary in local supersymmetry this is not the case, due to the effects of the spin-3/2 multiplet describing the degrees of freedom needed to complete the $N = 1$ gravitational multiplet to the $N = 2$ one. To perform this study and make the comparison between $N = 2$ and $N = 1$ theories as transparent as possible, we shall use a formulation of $N = 2$ theories based on $N = 1$ superspace, where one of the supersymmetries is manifestly realized in terms of ordinary superfields, whereas the other is realized by a non-trivial transformation mixing different superfields. We will follow the approach of [13], and generalize it in such a way to reach a general and coordinate-covariant construction, which in components reproduces the rigid limit of the general $N = 2$ supergravity theory as formulated in [14, 15]. We will then use the same strategy as in previous supergravity studies and work out the metastability conditions by systematically computing the masses of all the scalar sGoldstini. We will also revisit the computation of the supertrace sum rule, since it represents a related information, and rederive in a more direct way the results that were indirectly deduced in [16] from a superspace evaluation of the quadratic divergence in the one-loop effective action. We shall use the conventions of [17].

The paper is organized as follows. In section 2 we present the simplest case of $N = 1$ theories with only chiral multiplets and discuss the rigid version of the results of [2, 3]. In section 3 we present the case of $N = 1$ theories with both chiral and vector multiplets, and describe the rigid version of the result of [5], generalized to non-Abelian gauge groups. In section 4 we consider the case of $N = 2$ theories with only hyper multiplets, formulated as particular cases of $N = 1$ theories with only chiral multiplets. We derive the analogue of the result of [8] in the rigid limit and clarify how its emerges when gravity is decoupled and which information is associated respectively to the minimal and to the additional supersymmetries. In section 5 we consider the case of $N = 2$ theories with only vector multiplets, formulated as particular cases of $N = 1$ theories with chiral and vector multiplets. After recovering the rigid limit of the result of [9] in the Abelian case, we study the non-Abelian case and derive a new result applying to this situation, discussing again carefully which information comes from the minimal supersymmetry and which from the additional one. In section 6, we finally consider the case of general $N = 2$

theories with both hyper and vector multiplets. We set up the logic of the study of the metastability condition, and discuss the form that it is expected to take. Finally, in section 7 we present our conclusions.

2   N=1 models with chiral multiplets

Let us start by considering the simplest case of $N = 1$ theories with n_C chiral multiplets Φ^i. The most general two-derivative Lagrangian is specified in terms of a real Kähler potential K and a holomorphic superpotential W, and reads:

$$\mathcal{L} = \int d^4\theta\, K(\Phi, \bar{\Phi}) + \int d^2\theta\, W(\Phi) + \text{h.c.}. \tag{2.1}$$

In components, this gives

$$\mathcal{L} = -g_{i\bar{j}}\,\partial_\mu\phi^i \partial^\mu \bar{\phi}^{\bar{j}} - ig_{i\bar{j}}\,\psi^i \big(\bar{\partial}\!\!\!/\,\bar{\psi}^{\bar{j}} + \Gamma^{\bar{j}}_{\bar{m}\bar{n}}\,\bar{\partial}\!\!\!/\,\bar{\phi}^{\bar{m}}\bar{\psi}^{\bar{n}} \big) - V_\text{S} - V_\text{F}\,, \tag{2.2}$$

where $g_{i\bar{j}} = K_{i\bar{j}}$ defines a Kähler geometry for the scalar manifold [18] and[2]

$$V_\text{S} = g^{i\bar{j}}\, W_i \bar{W}_{\bar{j}}\,, \tag{2.3}$$

$$V_\text{F} = \frac{1}{2}\nabla_i W_j\, \psi^i \psi^j + \text{h.c.} - \frac{1}{4}R_{i\bar{j}k\bar{l}}\,\psi^i\psi^k \bar{\psi}^{\bar{j}}\bar{\psi}^{\bar{l}}\,, \tag{2.4}$$

The supersymmetry transformation laws are defined by the action of the supercharges on the superfields and act as follows in components:

$$\delta\phi^i = \sqrt{2}\,\epsilon\,\psi^i\,, \tag{2.5}$$

$$\delta\psi^i = \sqrt{2}\,\epsilon\,F^i + \sqrt{2}i\,\partial\!\!\!/\phi^i\,\bar{\epsilon}\,. \tag{2.6}$$

The auxiliary fields F^i are given by

$$F^i = -g^{i\bar{j}}\,\bar{W}_{\bar{j}} + \frac{1}{2}\Gamma^i_{jk}\,\psi^j\psi^k\,. \tag{2.7}$$

The extension to supergravity is well known [19, 20] and does not present particularly subtle features. In particular, any model of the above type can be consistently coupled to gravity. The main new feature is that there appears a non-trivial $U(1)$ bundle over the scalar manifold, whose curvature is proportional to M_P^{-2}, and the manifold becomes Kähler-Hodge.

2.1   Supertrace

At a generic point in the scalar field space and for vanishing fermions, the auxiliary fields simplify to

$$F^i = -\bar{W}^i\,. \tag{2.8}$$

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[2]Our conventions for the curvature are such that the non-vanishing components of the Riemann tensor are given by $R_{i\bar{j}k\bar{l}} = K_{i\bar{j}k\bar{l}} - g^{\bar{s}r}K_{ik\bar{s}}K_{\bar{j}lr}$ and those of the Ricci tensor by $R_{i\bar{j}} = -g^{k\bar{l}}R_{i\bar{j}k\bar{l}}$.

The mass matrix of the scalar fields is given by the following two blocks:

$$(m_0^2)_{i\bar{j}} = \nabla_i W_k \nabla_{\bar{j}} \bar{W}^k - R_{i\bar{j}k\bar{l}} F^k \bar{F}^{\bar{l}} , \tag{2.9}$$

$$(m_0^2)_{ij} = -\nabla_i \nabla_j W_k F^k + \Gamma_{ij}^k V_{Sk} . \tag{2.10}$$

The mass matrix of the fermions is instead

$$(m_{1/2})_{ij} = \nabla_i W_j . \tag{2.11}$$

One easily computes

$$\mathrm{tr}[m_0^2] = 2 \nabla_i W_j \nabla^i \bar{W}^j + 2 R_{i\bar{j}} F^i \bar{F}^{\bar{j}} , \tag{2.12}$$

$$\mathrm{tr}[m_{1/2}^2] = \nabla_i W_j \nabla^i \bar{W}^j . \tag{2.13}$$

It follows that the supertrace of the mass matrix is given by [16]

$$\mathrm{str}[m^2] \equiv \mathrm{tr}[m_0^2] - 2\,\mathrm{tr}[m_{1/2}^2]$$

$$= 2 R_{i\bar{j}} F^i F^{\bar{j}} . \tag{2.14}$$

2.2 Metastability

The possible vacua of the theory correspond to points in the scalar manifold that satisfy the stationarity condition $V_{Si} = 0$, which reads:

$$\nabla_i W_j F^j = 0 . \tag{2.15}$$

On the vacuum $\delta\psi^i = \sqrt{2}\epsilon F^i$, and supersymmetry is spontaneously broken if some of the auxiliary fields F^i are non-vanishing. The order parameter is the norm of the vector of auxiliary fields, which defines the scalar potential energy $V_S = F^i \bar{F}_i$. In such a situation, there is then a massless Goldstino fermion given by:

$$\eta = \sqrt{2}\, \bar{F}_i \psi^i . \tag{2.16}$$

Indeed, the stationarity condition directly implies that this is a flat direction of the fermion mass matrix:

$$m_\eta = 0 . \tag{2.17}$$

The two would-be supersymmetric scalar partners of this fermionic mode, the sGoldstini, generically have non-zero masses, but these are controlled by the process of supersymmetry breaking, and cannot be affected by supersymmetric mass terms in the superpotential. These modes are then particularly dangerous for the metastability of the vacuum. From the form of the supersymmetry transformations, we see that they can be parametrized by the two independent real linear combinations that one can form with the complex Goldstino vector $\eta^i = \sqrt{2}F^i$, namely:

$$\varphi_+ = \bar{F}_i \phi^i + F_{\bar{i}}\bar{\phi}^{\bar{i}} , \quad \varphi_- = i\bar{F}_i \phi^i - iF_{\bar{i}}\bar{\phi}^{\bar{i}} . \tag{2.18}$$

The masses of these two scalar modes can now be computed by evaluating the scalar mass matrix along the directions $\varphi_+^I = (F^i, \bar{F}^{\bar{i}})$ and $\varphi_-^I = (iF^i, -i\bar{F}^{\bar{i}})$, and dividing by the

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length of these vectors, which is $2F^i\bar{F}_i$. After using the stationarity condition to simplify the results, one obtains:

$$m^2_{\varphi_\pm} = R\,F^i\bar{F}_i \pm \Delta\,. \tag{2.19}$$

The first term involving the quantity R comes from the contribution of the Hermitian block $(m_0^2)_{i\bar{j}}$ of the mass matrix, and it turns out that R is simply the holomorphic sectional curvature of the scalar manifold in the complex plane defined by the Goldstino direction F^i of supersymmetry breaking:

$$R = -\frac{R_{i\bar{j}m\bar{n}}F^i\bar{F}^{\bar{j}}F^m\bar{F}^{\bar{n}}}{(F^k\bar{F}_k)^2}\,. \tag{2.20}$$

The second term Δ corresponds instead to the contribution from the complex block $(m_0^2)_{ij}$, and has a more complicated expression, which depends also on second and third derivatives of the superpotential and is thus much more model-dependent. But happily, we see that the average of the two masses is independent of Δ, and one thus finds the following result, which defines an upper bound on the lowest mass eigenvalue:

$$m^2_\varphi \equiv \frac{1}{2}\big(m^2_{\varphi_+} + m^2_{\varphi_-}\big) = R\,F^i\bar{F}_i\,. \tag{2.21}$$

From this result, we conclude that a necessary condition for not having a tachyonic mode is that the holomorphic sectional curvature R be positive.[3]

The above result is the rigid limit of the result obtained in [2, 3] for the supergravity case. Introducing the gravitino mass $m_{3/2}$ and the Planck mass $M_{\rm P}$, the cosmological constant reads $V_{\rm S} = F^i\bar{F}_i - 3\,m^2_{3/2}M^2_{\rm P}$ and the average sGoldstino mass is given by the following formula in supergravity

$$m^2_\varphi = R\,F^i\bar{F}_i + 2\,m^2_{3/2}\,. \tag{2.22}$$

We see that the main feature of this result, namely the dependence on the curvature R, is also captured in the rigid limit, in which $m_{3/2} \to 0$ and $M_{\rm P} \to \infty$. Gravitational effects influence only quantitatively the result, and the metastability condition implies now that the holomorphic sectional curvature R be larger than the negative critical value $-2\,m^2_{3/2}/(V_{\rm S} + 3\,m^2_{3/2}M^2_{\rm P})$, which tends to 0 in the rigid limit.

The above necessary condition for metastability becomes also sufficient if for a given Kähler potential K one allows the superpotential W to be adjusted [6, 7]. Indeed, at the stationary point one may tune W_i to maximize the average sGoldstino mass, W_{ij} to make the other masses arbitrarily large, and W_{ijk} to set the splitting between the two sGoldstino masses to zero.

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[3]In the limiting case of models based on a flat geometry, for which R vanishes, one generically finds that one of the sGoldstini is tachyonic and the other not. The best thing that one may do is then to tune the superpotential to make both of them massless, with vanishing Δ. One can then show that in such a situation the sGoldstini are not only massless, but actually correspond to flat directions of the potential and are identified with the so-called pseudo-moduli arising in these models. See [21] for a recent discussion.

3   N=1 models with chiral and vector multiplets

Let us consider next the most general case of $N = 1$ theories with n_C chiral multiplets Φ^i and n_V vector multiplets V^a. The most general two-derivative Lagrangian is in this case specified by a real Kähler potential K, a holomorphic superpotential W, a holomorphic gauge kinetic function f_{ab}, some holomorphic Killing vectors X_a^i and some real Fayet-Iliopoulos constants ξ_a, and reads:

$$\mathcal{L} = \int d^4\theta \left[K(\Phi, \bar\Phi, V) + \xi_a V^a \right] + \int d^2\theta \left[W(\Phi) + \frac{1}{4} f_{ab}(\Phi) W^{a\alpha} W_\alpha^b \right] + \text{h.c.} . \quad (3.1)$$

The gauge transformations form a Lie group with structure constants $f_{ab}{}^c$, and act as follows on the superfields, with chiral multiplet parameters Λ^a:

$$\delta_g \Phi^i = \Lambda^a X_a^i(\Phi) , \quad (3.2)$$

$$\delta_g V^a = -\frac{i}{2}\big(\Lambda^a - \bar\Lambda^a\big) + \frac{1}{2} f_{bc}{}^a \big(\Lambda^b + \bar\Lambda^b\big) V^c + \mathcal{O}(V^2) . \quad (3.3)$$

Gauge invariance of the Lagrangian imposes that the variation of the non-holomorphic terms should be at most a Kähler transformation of the form $\Lambda^a f_a + \bar\Lambda^a \bar f_a$, where the f_a are some holomorphic functions, whereas the holomorphic terms should be strictly invariant. This implies the following conditions:

$$X_a^i K_i - \frac{i}{2} K_a = f_a , \quad (3.4)$$

$$X_a^i W_i = 0 , \quad (3.5)$$

$$X_a^i f_{bci} = -2 f_{a(b}{}^d f_{c)d} , \quad (3.6)$$

$$\xi_a = 0 \text{ whenever } f_{bc}{}^a \neq 0 . \quad (3.7)$$

These equations show that $-\frac{1}{2} K_a$ can be identified with the real Killing potential for the Killing vector X_a^i, and the Fayet-Iliopoulos constants ξ_a can be interpreted as coming from the freedom of adding a constant to this potential for Abelian generators:

$$X_a^i = \frac{i}{2} g^{i\bar j} \nabla_{\bar j} K_a , \quad (3.8)$$

One also has to impose the equivariance condition on the Killing vectors, i.e. that the operators $\delta_a = X_a^i \partial_i + \bar X_a^{\bar j} \partial_{\bar j}$ satisfy the group algebra $[\delta_a, \delta_b] = -f_{ab}{}^c \delta_c$. This guarantees that the Killing potentials can be chosen to transform in the adjoint representation, so that

$$g_{i\bar j} X_{[a}^i \bar X_{b]}^{\bar j} = \frac{i}{4} f_{ab}{}^c K_c . \quad (3.9)$$

In the Wess-Zumino gauge, the action simplifies to the following expression:

$$\mathcal{L} = \int d^4\theta \left[K(\Phi, \bar\Phi) + \big(K_a(\Phi, \bar\Phi) + \xi_a\big) V^a + 2 g_{i\bar j}(\Phi, \bar\Phi) X_a^i(\Phi) \bar X_b^{\bar j}(\bar\Phi) V^a V^b \right]$$

$$+ \int d^2\theta \left[W(\Phi) + \frac{1}{4} f_{ab}(\Phi) W^{a\alpha} W_\alpha^b \right] + \text{h.c.} . \quad (3.10)$$

In components, this gives

$$
\mathcal{L} = -g_{i\bar{j}} \, D_\mu \phi^i D^\mu \bar{\phi}^{\bar{j}} - \frac{1}{4} h_{ab} \, F^a_{\mu\nu} F^{b\mu\nu} + \frac{1}{4} k_{ab} \, F^a_{\mu\nu} \tilde{F}^{b\mu\nu} - i g_{i\bar{j}} \, \psi^i \big(\slashed{D} \bar{\psi}^{\bar{j}} + \Gamma^{\bar{j}}_{\bar{m}\bar{n}} \, \slashed{D} \bar{\phi}^{\bar{m}} \bar{\psi}^{\bar{n}} \big)
$$

$$
- \frac{i}{2} h_{ab} \, \lambda^a \slashed{D} \bar{\lambda}^b + \text{h.c.} + \frac{1}{\sqrt{2}} h_{abi} \, \lambda^a \sigma^{\mu\nu} \psi^i F^b_{\mu\nu} + \text{h.c.} - V_{\text{S}} - V_{\text{F}} \,, \tag{3.11}
$$

where:

$$
V_{\text{S}} = g^{i\bar{j}} \, W_i \bar{W}_{\bar{j}} + \frac{1}{8} h^{ab} (K_a + \xi_a)(K_b + \xi_b)
$$

$$
V_{\text{F}} = \frac{1}{2} \Big[\nabla_i W_j \, \psi^i \psi^j - g^{i\bar{j}} h_{abi} \bar{W}_{\bar{j}} \lambda^a \lambda^b + \sqrt{8} \big(g_{i\bar{j}} \bar{X}^{\bar{j}}_a + \frac{i}{4} h^{bc} h_{abi} (K_c + \xi_c) \big) \psi^i \lambda^a \Big] + \text{h.c.}
$$

$$
- \frac{1}{4} R_{i\bar{j}k\bar{l}} \, \psi^i \psi^k \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{l}} + \frac{1}{4} g^{i\bar{j}} h_{abi} h_{cd\bar{j}} \lambda^a \lambda^b \bar{\lambda}^{\bar{c}} \bar{\lambda}^{\bar{d}} + \frac{1}{2} h^{cd} h_{aci} h_{bd\bar{j}} \, \psi^i \lambda^a \bar{\psi}^{\bar{j}} \bar{\lambda}^b
$$

$$
- \frac{1}{4} \Big[\nabla_i h_{abj} \, \psi^i \psi^j \lambda^a \lambda^b + h^{cd} h_{aci} h_{bdj} \, \psi^i \lambda^a \psi^j \lambda^b \Big] + \text{h.c.} \,. \tag{3.12}
$$

In these formulae, D_μ is the gauge covariant derivative acting as $D_\mu \phi^i = \partial_\mu \phi^i + A^a_\mu X^i_a$, $D_\mu \psi^i = \partial_\mu \psi^i + A^a_\mu \partial_j X^i_a \, \psi^j$ and $D_\mu \lambda^a = \partial_\mu \lambda^a + f_{bc}{}^a A^b_\mu \lambda^c$, $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f_{bc}{}^a A^b_\mu A^c_\nu$ is the field-strength, whereas h_{ab} and k_{ab} are the real and imaginary parts of f_{ab}.

The supersymmetry transformation laws involve not only the usual action of the supercharges, but also a compensating gauge transformation needed to preserve the Wess-Zumino gauge choice, with superfield parameter given by $\Lambda^a = 2i\theta\sigma^\mu\bar{\epsilon} A^a_\mu + 2\theta^2 \bar{\epsilon}\bar{\lambda}^a$. The additional gauge transformation has no effect on the transformation laws of the components of V^a, but gives some additional terms in those of the components of Φ^i. In particular, it turns the ordinary derivative appearing in $\delta\psi^i$ into a gauge-covariant derivative. One finally finds

$$
\delta\phi^i = \sqrt{2} \, \epsilon \, \psi^i \,, \tag{3.13}
$$

$$
\delta\psi^i = \sqrt{2} \, \epsilon \, F^i + \sqrt{2} i \, \slashed{D} \phi^i \, \bar{\epsilon} \,, \tag{3.14}
$$

$$
\delta A^a_\mu = i\epsilon \, \sigma_\mu \bar{\lambda}^a - i\lambda^a \sigma_\mu \, \bar{\epsilon} \,, \tag{3.15}
$$

$$
\delta\lambda^a = i\epsilon \, D^a + \sigma^{\mu\nu} \epsilon \, F^a_{\mu\nu} \,. \tag{3.16}
$$

The auxiliary fields F^i and D^a are given by

$$
F^i = -g^{i\bar{j}} \bar{W}_{\bar{j}} + \frac{1}{2} \Gamma^i_{jk} \, \psi^j \psi^k + \frac{1}{2} g^{i\bar{j}} h_{ab\bar{j}} \, \bar{\lambda}^a \bar{\lambda}^b \,, \tag{3.17}
$$

$$
D^a = -\frac{1}{2} h^{ab} (K_b + \xi_b) - \frac{i}{\sqrt{2}} h^{ab} h_{bci} \, \psi^i \lambda^c + \text{h.c.} \,. \tag{3.18}
$$

The extension to supergravity is again well known [22, 23] and presents in this case a subtlety. It turns out that models of the above type can generically be coupled to gravity only in the absence of Fayet-Iliopoulos terms, i.e. when $\xi_a = 0$. This is due to the fact that the accidental gauge-invariance of this term in rigid supersymmetry is spoiled by gravitational effects. Similarly, there cannot be any non-trivial holomorphic function appearing in gauge transformations of the Kähler potential, once the superpotential is assumed to be gauge invariant, and one needs $f_a = 0$. A way out of this restriction arises only if the theory admits an R-symmetry, which can be gauged and for which a Fayet-Iliopoulos term is possible [24, 25]. For the rest, the main new feature is as before that there

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appears a non-trivial $U(1)$ bundle over the scalar manifold with curvature proportional to M_{P}^{-2}, and the manifold becomes Kähler-Hodge. From now on, we shall restrict to models that can emerge from a smooth rigid limit of the local case, although most of the results that we shall derive in the remainder of this section have a more general validity. We shall moreover not discuss the special possibility of gauging a $U(1)_R$ symmetry, and thus require for simplicity that

$$\xi_a = 0\,, \quad f_a = 0\,. \tag{3.19}$$

3.1 Supertrace

At a generic point in the scalar field space and for vanishing fermions and vector fields, the auxiliary fields simplify to

$$F^i = -\bar{W}^i\,, \tag{3.20}$$

$$D^a = -\frac{1}{2}h^{ab}K_b\,. \tag{3.21}$$

The mass matrix of the scalar fields is given by the following two blocks:

$$(m_0^2)_{i\bar{\jmath}} = \nabla_i W_k \nabla_{\bar{\jmath}} \bar{W}^k - R_{i\bar{\jmath}k\bar{l}} F^k \bar{F}^{\bar{l}} + h^{ab}\bar{X}_{ai}X_{b\bar{\jmath}} + h^{ab}h_{aci}h_{bd\bar{\jmath}} D^b D^c$$
$$+ \frac{i}{2}\big(\nabla_i X_{a\bar{\jmath}} - 2h^{bc}h_{abi}X_{c\bar{\jmath}}\big) D^a + \text{h.c.}\,, \tag{3.22}$$

$$(m_0^2)_{ij} = -\nabla_i \nabla_j W_k\, F^k - h^{ab}\bar{X}_{ai}\bar{X}_{bj} - \frac{1}{2}\big(\nabla_i h_{abj} - 2h^{cd}h_{aci}h_{bdj}\big) D^a D^b$$
$$+ 2i\, h^{bc}h_{ab(i}\bar{X}_{cj)}D^a + \Gamma_{ij}^k V_{\mathrm{S}k}\,. \tag{3.23}$$

The mass matrix of the fermions involves instead the following three blocks:

$$(m_{1/2})_{ij} = \nabla_i W_j\,, \tag{3.24}$$

$$(m_{1/2})_{ab} = h_{abi}\, F^i\,, \tag{3.25}$$

$$(m_{1/2})_{ia} = \sqrt{2}\,\bar{X}_{ai} - \frac{i}{\sqrt{2}}h_{abi}\, D^b\,. \tag{3.26}$$

Finally, the mass matrix of the vectors is

$$(m_1^2)_{ab} = 2\, X_{(a}^i \bar{X}_{b)i}\,. \tag{3.27}$$

A straightforward computation gives

$$\text{tr}[m_0^2] = 2\,\nabla_i W_j \nabla^i \bar{W}^j + 2\, R_{i\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}} + 2\, h^{ab}\bar{X}_{ai}X_b^i + 2h^{ab}h_{aci}h_{bd}{}^i\, D^b D^c$$
$$+ i\big(\nabla_i X_a^i - 2h^{bc}h_{abi}X_c^i\big)D^a + \text{h.c.}\,, \tag{3.28}$$

$$\text{tr}[m_{1/2}^2] = \nabla_i W_j \nabla^i \bar{W}^j + h^{ac}h^{bd}h_{abi}h_{cd\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}} + 4\, h^{ab}\bar{X}_{ai}X_b^i$$
$$+ h^{cd}h_{aci}h_{bd}{}^i\, D^a D^b - 2i\, h^{ab}h_{bci}X_a^i D^c + \text{h.c.}\,, \tag{3.29}$$

$$\text{tr}[m_1^2] = 2\, h^{ab}\bar{X}_{ai}X_b^i\,. \tag{3.30}$$

It follows that the supertrace of the mass matrix is given by [16]

$$\text{str}[m^2] \equiv \text{tr}[m_0^2] - 2\,\text{tr}[m_{1/2}^2] + 3\,\text{tr}[m_1^2]$$
$$= 2\big(R_{i\bar{\jmath}} - h^{ac}h^{bd}h_{abi}h_{cd\bar{\jmath}}\big)F^i \bar{F}^{\bar{\jmath}} + i\big(\nabla_i X_a^i + 2h^{bc}h_{abi}X_c^i\big)D^a + \text{h.c.}\,. \tag{3.31}$$

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Note that we did not need to fix a gauge for the ordinary gauge symmetry to perform this computation, thanks to the fact that the unphysical would-be Goldstone scalars that are eaten by the gauge fields correspond to flat directions of the scalar mass matrix. By tracing over the whole m_0^2, one does therefore not overcount these modes, since they come with a vanishing value of the mass.

3.2 Metastability

The possible vacua of the theory correspond to points in the scalar manifold that satisfy the stationarity condition $V_{\text{S}i} = 0$, which implies

$$\nabla_i W_j \, F^j + \frac{1}{2} h_{abi} D^a D^b + i \bar{X}_{ai} D^a = 0 \,. \tag{3.32}$$

By contracting this relation with the Killing vectors X_a^i and taking the imaginary part, and using (3.5) and its derivative as well as (3.9), one also finds the following relation between the values of the F^i and D^a auxiliary fields:

$$i \nabla_i X_{a\bar{j}} F^i \bar{F}^{\bar{j}} - X_{(a}^i X_{b)i} \, D^b + \frac{1}{2} f_{ab}{}^d k_{dc} \, D^b D^c = 0 \,. \tag{3.33}$$

By further contraction with D^a, this also implies $i \nabla_i X_{a\bar{j}} D^a F^i \bar{F}^{\bar{j}} - X_{(a}^i X_{b)i} \, D^a D^b = 0$. This formula shows in particular that if the F^i vanish then also the D^a vanish, under the assumption that there are neither Fayet-Iliopoulos terms nor non-trivial Kähler transformation functions associated to gauge transformations. Indeed, in such a situation the first term vanishes, and the equation implies that either D^a or X_a^i should vanish. But $X_a^i = 0$ implies also $D_a = 0$, whenever the total non-holomorphic term in the Lagrangian is strictly gauge invariant, since in that case $D_a = -i X_a^i K_i$.

On the vacuum one has $\delta \psi^i = \sqrt{2} \epsilon F^i$ and $\delta \lambda^a = i \epsilon D^a$, and supersymmetry is spontaneously broken if at least some of the auxiliary fields F^i or D^a are non-vanishing. The order parameter is the norm of the vector of auxiliary fields, which defines the scalar potential energy $V_{\text{S}} = F^i \bar{F}_i + \frac{1}{2} D^a D_a$. In such a situation, there is then a massless Goldstino given by

$$\eta = \sqrt{2} \bar{F}_i \psi^i + i D_a \lambda^a \,. \tag{3.34}$$

Indeed, the stationarity condition and the gauge invariance of the superpotential imply that this is a flat direction of the fermion mass matrix:

$$m_\eta = 0 \,. \tag{3.35}$$

As before, the would-be supersymmetric partners of this fermionic mode, the sGoldstini, have masses that are controlled by the process of supersymmetry breaking. They are then particularly dangerous for the metastability of the vacuum. From the form of the supersymmetry transformations, we see that in this case these modes are linear combinations of both scalars and vectors. However, since the vector components cannot get negative square masses, the relevant thing to look at is the projection onto the scalar field space. One then gets the same two independent real linear combinations as before, corresponding to the projection of the complex Goldstino vector $\eta^i = \sqrt{2} F^i$:

$$\varphi_+ = \bar{F}_i \phi^i + F_{\bar{\imath}} \bar{\phi}^{\bar{\imath}} \,, \quad \varphi_- = i \bar{F}_i \phi^i - i F_{\bar{\imath}} \bar{\phi}^{\bar{\imath}} \,. \tag{3.36}$$

The masses of these two scalar modes can now be computed as before, by evaluating the scalar mass matrix along the directions $\varphi_+^I = (F^i, \bar{F}^{\bar{i}})$ and $\varphi_-^I = (iF^i, -i\bar{F}^{\bar{i}})$, and dividing by the length of these vectors, which is $2F^i\bar{F}_i$. After using the stationarity condition as well as the various constraints imposed by gauge invariance to simplify the results, one obtains

$$m_{\varphi_\pm}^2 = R\,F^i\bar{F}_i + S\,D^aD_a + \frac{1}{4}\,T\,\frac{(D^aD_a)^2}{F^i\bar{F}_i} + M^2\frac{D^aD_a}{F^i\bar{F}_i} \pm \Delta\,. \tag{3.37}$$

The first four terms involving the quantities R, S, T and M^2 come from the contribution of the Hermitian block $(m_0^2)_{i\bar{j}}$ of the mass matrix. It turns out that R is as before the holomorphic sectional curvature in the complex plane defined by the Goldstino direction F^i, whereas S and T and similar objects defined out of the derivatives of h_{ab}, and M^2 is related to the mass of the vector fields:

$$R = -\frac{R_{i\bar{j}m\bar{n}}\,F^i\bar{F}^{\bar{j}}F^m\bar{F}^{\bar{n}}}{(F^k\bar{F}_k)^2}\,, \tag{3.38}$$

$$S = \frac{h_{aci}h^{cd}h_{db\bar{j}}\,F^i\bar{F}^{\bar{j}}D^aD^b}{(F^k\bar{F}_k)(D^cD_c)}\,, \tag{3.39}$$

$$T = \frac{h_{abi}h_{cb}{}^i\,D^aD^bD^cD^d}{(D^eD_e)^2}\,, \tag{3.40}$$

$$M^2 = \frac{2X_a^i\bar{X}_{bi}\,D^aD^b}{D^cD_c}\,. \tag{3.41}$$

The quantity Δ corresponds instead to the contribution from the complex block $(m_0^2)_{ij}$, and has again a more complicated and model-dependent expression. But as before, we see that the average of the two masses is independent of Δ, and one thus finds the following result, which defines an upper bound on the lowest mass eigenvalue:

$$m_\varphi^2 \equiv \frac{1}{2}\big(m_{\varphi_+}^2 + m_{\varphi_-}^2\big) = R\,F^i\bar{F}_i + S\,D^aD_a + \frac{1}{4}\,T\,\frac{(D^aD_a)^2}{F^i\bar{F}_i} + M^2\frac{D^aD_a}{F^i\bar{F}_i}\,. \tag{3.42}$$

From this, we conclude that a necessary condition for not having a tachyonic mode is that the holomorphic sectional curvature R be larger than a certain negative-definite value controlled by the data of the gauge sector.

In this case, there is an additional feature concerning scalar fields that has to be considered. Indeed, on the vacuum one has $\delta_g\phi^i = \lambda^a X_a^i$, and some of the gauge symmetries may be spontaneously broken if some of the components of X_a^i are non-vanishing. The order parameters are the eigenvalues of the matrix of scalar products of the Killing vectors, which defines the gauge boson mass matrix $(m_1^2)_{ab} = 2\,X_{(a}^i\bar{X}_{b)i}$. In such a situation, there are thus also other complex directions of special relevance, namely those defined by the Killing vectors X_a^i. These are related to the would-be Goldstone modes that are eaten by the massive vector fields when the gauge symmetry is spontaneously broken, which are given by the following real combinations:

$$\sigma_a = \bar{X}_{ai}\phi^i + X_{a\bar{i}}\bar{\phi}^{\bar{i}}\,. \tag{3.43}$$

Along these unphysical directions, the scalar mass matrix has vanishing value:

$$m_{\sigma_a}^2 = 0\,. \tag{3.44}$$

One may then wonder what happens along the conjugate directions defined by

$$\rho_a = i\bar{X}_{ai}\phi^i - iX_{a\bar{\imath}}\bar{\phi}^{\bar{\imath}}\,. \tag{3.45}$$

These generically have non-vanishing masses,

$$m_{\rho_a}^2 \neq 0\,. \tag{3.46}$$

These informations all directly follow from the gauge invariance of the scalar potential. This implies that $X_a^i\nabla_i V_{\mathrm{S}} + \bar{X}_a^{\bar{\jmath}}\nabla_{\bar{\jmath}} V_{\mathrm{S}} = 0$, and can be checked to be a consequence of gauge invariance conditions listed previously plus the equivariance condition. Taking then a further derivative and going to a stationary point, one immediately deduces that $(m_0^2)_{Ki}X_a^i + (m_0^2)_{K\bar{\jmath}}\bar{X}_a^{\bar{\jmath}} = 0$, which is the statement that the would-be Goldstone boson σ_a is massless.

At this point, one may wonder whether one could perhaps get some other relevant metastability conditions by looking at the complex partners ρ_a of the would-be Goldstone modes, which are a priory physical scalar fields. In the limit of unbroken supersymmetry, these modes have the same masses as the vector bosons. Upon supersymmetry breaking, they however split, and if the scale of supersymmetry breaking is much larger than that of gauge symmetry breaking, this splitting may become larger than the average mass of the multiplet and give rise to tachyons. A priori, there is no obstruction against making the gauge symmetry breaking scale much larger than the scale of supersymmetry breaking, thereby avoiding that some of these states become tachyonic. However, in such a limit the effect of the gauging on the sGoldstino masses gets suppressed, and the potential benefits from the presence of the vector multiplets disappear. A careful study may then perhaps unravel a limitation on how much one may increase the sGoldstino masses through a gauging, coming from the danger that these other states ρ_a become tachyonic. However, we have not been able to find any simple result along this line of reasoning. We thus refrain from reporting here the rather complicated expression for the mass matrix of the fields ρ_a, which consists of the mass matrix of the vectors plus a series of terms that involve various tensors built out of X_a^i and its derivatives contracted with the auxiliary fields F^i and D^a.

As a final remark on this issue, let us note that F^i is orthogonal to X_a^i, as a consequence of the gauge invariance of the superpotential. This means that the sGoldstini $\varphi_\pm$ and the above complex partners of the would-be Goldstones ρ_a actually probe the scalar mass matrix in two different sectors, the former orthogonal to X_a^i and the latter parallel to X_a^i. Moreover, in the absence of supersymmetry breaking, these two sector are disentangled: the former describes the light chiral multiplets and the latter the heavy vector multiplets. However, it should also be noted that there is no guarantee that the would-be Goldstone modes σ_a and their complex partners ρ_a represent independent modes. Indeed, the number of linearly independent vectors in each of the two sets of $\sigma_a^I = (X_a^i, \bar{X}_a^{\bar{\imath}})$ and $\rho_a^I = (iX_a^i, -i\bar{X}_a^{\bar{\imath}})$ equals the rank of the matrix of the scalar products within each set, which coincides with the symmetric gauge bosons mass matrix $2g_{i\bar{\jmath}}X_{(a}^i\bar{X}_{b)}^{\bar{\jmath}} = (m_1^2)_{ab}$. On the other hand, the total number of linearly independent vectors in the full set containing both the σ_a^I and the ρ_a^I may be lower, because some of the σ_a^I may be linear combinations of the ρ_a^I and vice versa. It is given by the rank of a twice

bigger matrix with diagonal blocks given by $2g_{i\bar{\jmath}}X^i_{(a}\bar{X}^{\bar{\jmath}}_{b)} = (m_1^2)_{ab}$ and off-diagonal blocks given by $2g_{i\bar{\jmath}}X^i_{[a}\bar{X}^{\bar{\jmath}}_{b]} = -if_{ab}{}^c D_c$, which is also equal to twice the rank of the Hermitian matrix $2g_{i\bar{\jmath}}X^i_a\bar{X}^{\bar{\jmath}}_b = (m_1^2)_{ab} - if_{ab}{}^c D_c$. Indeed, the existence of a complex null vector $\bar{v}^b$ for this matrix implies that $X^i_a v^a = \bar{X}^{\bar{\imath}}_a \bar{v}^a = 0$, and for each such null vector there are thus two linear relations between the ρ_a and the σ_a along the real directions $\operatorname{Re} v^a$ and $\operatorname{Im} v^a$: $\operatorname{Re} v^a \rho_a = -\operatorname{Im} v^a \sigma_a$ and $\operatorname{Im} v^a \rho_a = \operatorname{Re} v^a \sigma_a$. In such a situation, two combinations of the ρ_a are then unphysical would-be Goldstone modes too.

The result derived above for the average sGoldstino mass represents the rigid limit of the one derived in [5], generalized to arbitrary non-Abelian gauge groups. In terms of the gravitino mass $m_{3/2}$ and the Planck mass M_{P}, the cosmological constant reads $V_{\mathrm{S}} = F^i\bar{F}_i + \frac{1}{2}D^a D_a - 3\,m_{3/2}^2 M_{\mathrm{P}}^2$, and the averaged sGoldstino mass is

$$m_\varphi^2 = R\,F^i\bar{F}_i + \left(S + M_{\mathrm{P}}^{-2}\right)D^a D_a + \frac{1}{4}\,T\,\frac{(D^a D_a)^2}{F^i\bar{F}_i} + \left(M^2 - 4\,m_{3/2}^2\right)\frac{D^a D_a}{F^i\bar{F}_i}$$
$$+ 2\,m_{3/2}^2\,. \tag{3.47}$$

We again see that the main feature of this result, namely the dependence on the curvatures R, S, T and on the mass M^2, is also captured in the rigid limit, in which $m_{3/2} \to 0$ and $M_{\mathrm{P}} \to \infty$. As before, gravitational effects influence only quantitatively the result.

In this case the necessary condition for metastability does not become sufficient even if for a given Kähler potential K one allows the superpotential W to be adjusted. Indeed, the restriction of gauge invariance of W implies that $W_i X^i_a = 0$, $W_{ij}X^j_a = -\partial_i X^j_a W_j$ and $W_{ijk}X^k_a = -2\partial_{(i}X^k_a W_{j)k} - \partial_i\partial_j X^k_a W_k$. This shows that at the stationary point W_i, W_{ij} and W_{ijk} cannot be freely tuned along the complex directions associated to the Killing vectors X^i_a. The real modes corresponding to these directions are the would-be Goldstone modes σ_a and their complex partners ρ_a. The masses of the latter can therefore not be adjusted through their F-term part depending on W and represent a left-over danger, whenever they are physical. These masses have however also a D-term part depending on the Killing potentials K_a, and tend to the vector bosons masses in the supersymmetric limit. This suggests that if one could somehow also allow the Killing potential K_a to be adjusted, the metastability condition would become once again effectively sufficient. The extent to which one can imagine to do that is however clearly restricted, since K_a, on the contrary of W, does have some relation to the geometry defined by K.

4   N=2 models with hyper multiplets

Let us now consider the simplest case of $N = 2$ theories with $n_{\mathcal{H}}$ hyper multiplets $\mathcal{H}^k$. This is a particular case of $N = 1$ theory with $n_{\mathrm{C}} = 2n_{\mathcal{H}}$ chiral multiplets Q^u, with the particularity that it admits a second supersymmetry. The most general two-derivative Lagrangian is specified by a real Kähler potential K and a holomorphic superpotential W, and in $N = 1$ superspace it takes the usual form

$$\mathcal{L} = \int d^4\theta\, K(Q,\bar{Q}) + \int d^2\theta\, W(Q) + \text{h.c.}\,. \tag{4.1}$$

The existence of a second supersymmetry mixing different $N = 1$ superfields implies strong additional restrictions on K and W. To derive these restrictions, we shall follow [13] and

construct systematically the most general form of the second supersymmetry.

The general form of the second non-manifest supersymmetry transformation can be parametrized as follows with a general complex function $\bar{N}^u$, a holomorphic function X^u and a phase s [13, 26, 27]:

$$\hat{\delta} Q^u = \frac{1}{2} \bar{D}^2 \Big(\bar{N}^u(Q, \bar{Q})(\hat{\epsilon}\theta + \hat{\bar{\epsilon}}\bar{\theta}) \Big) - 2i(s + \bar{s}) X^u(Q) \hat{\epsilon}\theta \,. \tag{4.2}$$

In order for this to correctly satisfy an $N = 1$ supersymmetry subalgebra, more precisely $[\hat{\delta}_1, \hat{\delta}_2] Q^u = -2i(\hat{\epsilon}_1 \sigma^\mu \hat{\bar{\epsilon}}_2 - \hat{\epsilon}_2 \sigma^\mu \hat{\bar{\epsilon}}_1) \partial_\mu Q^u$, one needs to impose some restrictions on the functions $\bar{N}^u$ and X^u. A straightforward computation shows that the required conditions are the following:

$$\partial_{\bar{w}} \bar{N}^u \partial_v N^{\bar{w}} = -\delta^u_v \,, \quad \partial_{\bar{s}} \partial_v \bar{N}^u \partial_{\bar{t}} \bar{N}^v - \partial_{\bar{t}} \partial_v \bar{N}^u \partial_{\bar{s}} \bar{N}^v = 0 \tag{4.3}$$

$$\partial_w X^u \partial_{\bar{v}} \bar{N}^w - \partial_{\bar{v}} (\partial_{\bar{w}} \bar{N}^u \bar{X}^{\bar{w}}) - \partial_{\bar{v}} \partial_w \bar{N}^u X^w = 0 \,, \tag{4.4}$$

Let us now check under what circumstances the Lagrangian (4.1) is left invariant by a second supersymmetry of this general allowed form. One finds that this is the case provided that

$$\nabla_u N_v + \nabla_v N_u = 0 \,, \quad \nabla_w(\nabla_u N_v) = 0 \,, \quad \nabla_{\bar{w}}(\nabla_u N_v) = 0 \,, \tag{4.5}$$

$$X^u = i\bar{s} \nabla^u \bar{N}^v W_v \,, \quad K_u X^u + K_{\bar{u}} X^{\bar{u}} = f + \bar{f} \,. \tag{4.6}$$

In these equations, we have used the Kähler metric to raise and lower indices, and f denotes an arbitrary holomorphic function of the chiral multiplets.

In order to clarify the geometrical meaning of the above restrictions, let us introduce the following notation:

$$\Omega_{uv} = \nabla_u N_v \,. \tag{4.7}$$

In terms of this quantity, the constraints (4.5) for the invariance of the action imply that Ω_{uv} should be antisymmetric, covariantly constant and holomorphic. Moreover, the first constraint (4.3) from the closure of the algebra implies a further constraint on the contraction of Ω_{uv} with its conjugate, while the second of (4.3) is automatically satisfied as a consequence of the holomorphicity of Ω_{uv}. One thus finds:

$$\Omega_{uv} = -\Omega_{vu} \,, \quad \nabla_w \Omega_{uv} = 0 \,, \quad \nabla_{\bar{w}} \Omega_{uv} = 0 \,, \tag{4.8}$$

$$\bar{\Omega}^u{}_{\bar{w}} \Omega^{\bar{w}}{}_v = -\delta^u_v \,. \tag{4.9}$$

It then follows that the Kähler manifold admits three complex structures, constructed out of Ω_{uv} as

$$(J^1)^U{}_V = \begin{pmatrix} 0 & \bar{\Omega}^u{}_{\bar{v}} \\ \Omega^{\bar{u}}{}_v & 0 \end{pmatrix} , \quad (J^2)^U{}_V = \begin{pmatrix} 0 & i\bar{\Omega}^u{}_{\bar{v}} \\ -i\Omega^{\bar{u}}{}_v & 0 \end{pmatrix} , \quad (J^3)^U{}_V = \begin{pmatrix} i\delta^u_v & 0 \\ 0 & -i\delta^{\bar{u}}_{\bar{v}} \end{pmatrix} , \tag{4.10}$$

which are covariantly constant and satisfy the quaternions algebra:

$$\nabla_U (J^x)^V{}_W = 0 \,, \tag{4.11}$$

$$(J^x)^U{}_W (J^y)^W{}_V = -\delta^U_V \delta^{xy} + \epsilon^{xyz} (J^z)^U{}_V \,. \tag{4.12}$$

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This means that the Kähler manifold must actually be Hyper-Kähler [28, 29].

Notice that the transformation functions $\bar{N}^u$ are implicitly determined by the quantity Ω_{uv} specifying the quaternionic structure. Indeed, compatibly with all the properties listed above, one can write:

$$\bar{N}^u = -\bar{\Omega}^{uv}(K_v + f_v) \,, \quad K_u \bar{N}^u = K_u g^u \,. \tag{4.13}$$

The arbitrary holomorphic functions f_v and $g^u = -\bar{\Omega}^{uv} f_v$ reflect the ambiguities related to Kähler transformations of K and in the definition of $\bar{N}^u$.

Concerning the superpotential, we see that the basic object controlling its structure is the holomorphic vector defined by (4.6):

$$X^u = i\bar{s}\bar{\Omega}^{uv} W_v \,. \tag{4.14}$$

The constraints (4.6) from the invariance of the action imply, upon taking some derivatives, that X^u is holomorphic and satisfies the Killing equation, whereas the condition (4.4) coming from the closure of the algebra implies that it also satisfies a further Killing-like equation involving $\bar{\Omega}^u_{\bar{v}}$:

$$\nabla_{\bar{w}} X^u = 0 \,, \quad \nabla_u X_{\bar{v}} + \nabla_{\bar{v}} \bar{X}_u = 0 \,, \tag{4.15}$$

$$\bar{\Omega}^u_{\bar{w}} \nabla_{\bar{v}} \bar{X}^{\bar{w}} - \bar{\Omega}^w_{\bar{v}} \nabla_w X^u = 0 \,. \tag{4.16}$$

This shows that X^u must actually be a triholomorphic Killing vector of the Hyper-Kähler manifold, meaning that the Lie derivative along it of any of the three complex structures J^x must vanish:

$$(\mathcal{L}_{X+\bar{X}} J^x)^U_V = 0 \,. \tag{4.17}$$

For $x = 3$, this is simply the statement in the first of the relations (4.15) that it is holomorphic with respect to the complex structure that is already manifest from the beginning, whereas for $x = 1, 2$ it amounts to the additional relation (4.16), which guarantees that it is also holomorphic with respect to the two additional complex structures.

Since X^U is a triholomorphic Killing vector, it admits three different real Killing potentials P^x, one for each complex structure J^x (no sum over x):

$$X^U = (J^x)^U_V \nabla^V P^x \,. \tag{4.18}$$

Notice that the Killing potentials P^x are only defined modulo constants, which are here irrelevant. In complex coordinates one then finds $X^u = \Omega^u_{\bar{v}} \nabla^{\bar{v}} P^1 = i\Omega^u_{\bar{v}} \nabla^{\bar{v}} P^2 = i\nabla^u P^3$. We see that $-P^3$ corresponds to the standard real Killing potential for X^u seen as holomorphic with respect to J^3. In addition, one may however also use P^1 and P^2 to form a complex Killing potential

$$P = -\frac{i}{2}(P^1 + iP^2) \,, \tag{4.19}$$

which has the property of being holomorphic with respect to J^3:

$$\nabla_{\bar{u}} P = 0 \,. \tag{4.20}$$

We may then write $X^u = i\nabla^u P^3$ but also $X^u = i\bar{\Omega}^{uv} P_v$. Comparing with (4.14), we see that the superpotential can be identified with this holomorphic Killing potential [30], times the phase s:

$$W = sP. \tag{4.21}$$

Note that the phase s cannot be trivially eliminated by rescaling X^u and P, because only a real rescaling of these quantities preserves their defining properties.

Having constructed the most general model that is invariant under both the usual and the extra supersymmetries, we may now compute the commutator of such transformations and check that it closes only on-shell and with a non-trivial central charge related to the Killing vector X^u. Indeed, the superfield equations of motion read $\bar{D}^2 K_u - 4W_u = 0$ and thanks to the properties of the tensor Ω^{uv} they imply that $D^2 \bar{N}^u + 4is X^u = 0$. Using this equation, one then easily verifies that $[\delta_1, \hat{\delta}_2] Q^u = -2i(\bar{s}\epsilon_1 \hat{\epsilon}_2 - s\bar{\epsilon}_1 \hat{\bar{\epsilon}}_2) X^u$, whose right-hand side is of the form

$$\delta_c Q^u = \alpha X^u(\Phi). \tag{4.22}$$

This central charge transformation corresponds to a global symmetry of the theory. Indeed, $\delta_c K = X^u K_u + \bar{X}^{\bar{u}} K_{\bar{u}} = f + f$ and $\delta_c W = X^u W_u = is X^u \Omega_{uv} X^v = 0$, as a consequence of the second of (4.6) and the first of (4.8). It follows that the Lagrangian (4.1) is invariant.

It is worth emphasizing that it is possible to consider alternative versions of the second supersymmetry transformations, which look different but yield the same on-shell transformations. For instance, as explained in [26] one may add to the transformation (4.2) the trivial transformation $\delta_t Q^u = \frac{1}{2}\bar{\Omega}^{uv}(\bar{D}^2 K_v - 4sP_v)\hat{\epsilon}\theta$, which is a symmetry of the on-shell theory since $\bar{\Omega}^{uv}$ is antisymmetric and the parenthesis is proportional to the equations of motion of Q^u. One then obtains $\hat{\delta} Q^u = \frac{1}{2}\bar{D}^2(\bar{N}^u \hat{\epsilon}\theta) - 2is X^u \hat{\epsilon}\theta$.

Before going on, let us summarize some important features of Hyper-Kähler manifolds that will be relevant in the following. First, notice that the properties (4.8) imply that $\bar{\Omega}^u_{\bar{w}}\partial_{[s}\Omega^{\bar{w}}_{t]} = 0$ and that the Christoffel symbols are entirely determined in terms of $\Omega^u_{\bar{v}}$ and its conjugate:

$$\Gamma^u_{st} = -\bar{\Omega}^u_{\bar{w}}\partial_{(s}\Omega^{\bar{w}}_{t)}. \tag{4.23}$$

From this expression one may compute the Ricci tensor and show that it identically vanishes, due to the above properties of $\bar{\Omega}^u_{\bar{v}}$:

$$R_{u\bar{v}} = 0. \tag{4.24}$$

Finally, the integrability condition associated to the differential constraint (4.8) implies that the Riemann tensor, which is also completely determined by $\Omega^u_{\bar{v}}$ and its conjugate, satisfies the following algebraic constraint:

$$\Omega^{\bar{w}}_{[u} R_{v]\bar{w}s\bar{t}} = 0. \tag{4.25}$$

Using (4.9), this also implies

$$R_{u\bar{v}s\bar{t}} = -\Omega^{\bar{n}}_u \bar{\Omega}^m_{\bar{v}} R_{s\bar{n}m\bar{t}} = \Omega^{\bar{n}}_u \bar{\Omega}^m_{\bar{v}} \Omega^{\bar{q}}_s \bar{\Omega}^p_{\bar{t}} R_{p\bar{n}m\bar{q}}. \tag{4.26}$$

Let us also quote for later reference the following important property of the triholomorphic Killing vector X^u, which follows from eqs. (4.15) and (4.16):

$$\nabla_u X^u = 0\,. \tag{4.27}$$

To sum up, we see that in order to get an $N = 2$ model, the geometry must be Hyper-Kähler and the superpotential must be given by the holomorphic Killing potential defining a triholomorphic Killing vector associated to a central charge:

$$\mathcal{L} = \int d^4\theta\, K(Q, \bar{Q}) + \int d^2\theta\, s P(Q) + \text{h.c.}\,. \tag{4.28}$$

The component Lagrangian reads

$$\mathcal{L} = -g_{u\bar{v}}\,\partial_\mu q^u \partial^\mu \bar{q}^{\bar{v}} - i g_{u\bar{v}}\, \chi^u \big(\slashed{\partial}\bar{\chi}^{\bar{v}} + \Gamma^{\bar{v}}_{\bar{s}\bar{t}}\, \slashed{\partial}\bar{q}^{\bar{s}}\bar{\chi}^{\bar{t}} \big) - V_{\text{S}} - V_{\text{F}}\,, \tag{4.29}$$

where:

$$V_{\text{S}} = g_{u\bar{v}} X^u \bar{X}^{\bar{v}} \tag{4.30}$$

$$V_{\text{F}} = \frac{i}{2} s\, \Omega_{uw} \nabla_v X^w\, \chi^u \chi^v + \text{h.c.} - \frac{1}{4} R_{u\bar{v}s\bar{t}}\, \chi^u \chi^s \bar{\chi}^{\bar{v}} \bar{\chi}^{\bar{t}}\,, \tag{4.31}$$

The first supersymmetry transformations are specified by the usual action of the supercharges on the superfields and act as follows on component fields:

$$\delta q^u = \sqrt{2}\,\epsilon\,\chi^u\,, \tag{4.32}$$

$$\delta \chi^u = \sqrt{2}\,\epsilon\, F^u + \sqrt{2}i\,\slashed{\partial} q^u \bar{\epsilon}\,. \tag{4.33}$$

The value of the auxiliary fields F^u is

$$F^u = i\bar{s}\,\bar{\Omega}^u{}_{\bar{v}} \bar{X}^{\bar{v}} + \frac{1}{2}\Gamma^u_{st}\,\chi^s \chi^t\,. \tag{4.34}$$

The action of the second supersymmetry is obtained by computing the components of the superfield expression (4.2). One finds:

$$\hat{\delta} q^u = -\sqrt{2}\,\bar{\Omega}^u{}_{\bar{v}}\,\hat{\bar{\epsilon}}\,\bar{\chi}^{\bar{v}}\,, \tag{4.35}$$

$$\hat{\delta} \chi^u = \sqrt{2}\,\hat{\epsilon}\,\hat{F}^u + \sqrt{2}\,\Gamma^u_{st}\bar{\Omega}^s{}_{\bar{v}}\,\hat{\bar{\epsilon}}\,\bar{\chi}^{\bar{v}}\chi^t + \sqrt{2}i\,\bar{\Omega}^u{}_{\bar{v}}\slashed{\partial}\bar{q}^{\bar{v}}\hat{\bar{\epsilon}}\,. \tag{4.36}$$

The quantity $\hat{F}^u$ is found to be given by

$$\hat{F}^u = \bar{\Omega}^u{}_{\bar{v}}\big(\bar{F}^{\bar{v}} + (s + \bar{s})P^{\bar{v}} - \frac{1}{2}\,\Gamma^{\bar{v}}_{\bar{s}\bar{t}}\,\bar{\chi}^{\bar{s}}\bar{\chi}^{\bar{t}} \big) = -i\bar{s}X^u\,. \tag{4.37}$$

The extension to supergravity is described in [14, 15]. It turns out that there is no obstruction in coupling a model of the above type to gravity. The main new feature is that there appears a non-trivial $SU(2)$ bundle over the scalar manifold with curvature proportional to M_{P}^{-2}, and the manifold becomes Quaternionic-Kähler. In this setting, the fact that the scalar potential depends on a Killing vector can be understood as coming from a gauging, of the type described in [31] and involving the graviphoton A^0_μ. To see how it works, it is convenient to rewrite X^u in terms of some new X^u_0 with dimension 1 rather than 2, by introducing some mass scale μ and defining $X^u = \sqrt{2}\mu X_0$. One may

further promote the mass scale μ to a complex mass parameter including the arbitrary phase s appearing in the supersymmetry transformations laws, $L^0 = -is\mu$, and write:

$$X^u = \sqrt{2}i\bar{s}\,X_0^u L^0\,. \tag{4.38}$$

Correspondingly, one may rewrite the Killing potentials as $P^x = \sqrt{2}i\bar{s}\,P_0^x L^0$, in such a way that $P = \sqrt{2}i\bar{s}\,P_0 L^0$. For simplicity, we set from now on $s = i$, corresponding to L^0 real, but it is clear that an arbitrary s and a complex L^0 can be easily restored. We then see that the Lagrangian obtained above coincides with the one that emerges by taking a suitable rigid limit of $N = 2$ supergravity coupled to hyper multiplets with a gauging of the central charge by the graviphoton A_μ^0, whose action involves X_0^u. The non-trivial superpotential of the rigid theory, which is the generalization of the mass terms for the hyper multiplets that are allowed already in renormalizable theories, is obtained in the double scaling limit in which the Planck scale is sent to infinity and the graviphoton coupling to zero, but in such a way that their product gives rise to a finite mass scale.

Notice finally that the scalar potential can be rewritten in a more familiar way by switching to general real coordinates: $2g_{u\bar{v}}X_0^u\bar{X}_0^{\bar{v}} = g_{UV}X_0^U X_0^V$. One gets then the same result as in [14, 15], namely:

$$V_{\text{S}} = g_{UV}X_0^U\bar{L}^0 X_0^V L^0\,. \tag{4.39}$$

4.1 Supertrace

At a generic point in the scalar field space and for vanishing fermions, the auxiliary fields simplify to

$$F^u = \bar{\Omega}^u{}_{\bar{v}}\bar{X}^{\bar{v}}\,. \tag{4.40}$$

The corresponding hatted quantities similarly simplify to

$$\hat{F}^u = \bar{\Omega}^u{}_{\bar{v}}\bar{F}^{\bar{v}} = -X^u\,. \tag{4.41}$$

The mass matrix of the scalar fields is given by

$$(m_0^2)_{u\bar{v}} = \nabla_u X^w \nabla_{\bar{v}}\bar{X}_w - R_{u\bar{v}s\bar{t}}F^s\bar{F}^{\bar{t}}\,, \tag{4.42}$$

$$(m_0^2)_{uv} = -R_{u\bar{s}v\bar{t}}\Omega^{\bar{s}}{}_m\Omega^{\bar{t}}{}_n F^m F^n + \Gamma^t_{uv}V_{\text{S}t}\,. \tag{4.43}$$

The mass matrix of the fermions is instead

$$(m_{1/2})_{uv} = -\Omega_{(uw}\nabla_{v)}X^w\,. \tag{4.44}$$

Recalling that Hyper-Kähler manifolds are Ricci-flat, one easily computes

$$\text{tr}[m_0^2] = 2\,\nabla_u X^v \nabla^u \bar{X}_v\,, \tag{4.45}$$

$$\text{tr}[m_{1/2}^2] = \nabla_u X^v \nabla^u \bar{X}_v\,. \tag{4.46}$$

It follows that the supertrace of the mass matrix vanishes [13]:

$$\text{str}[m^2] \equiv \text{tr}[m_0^2] - 2\,\text{tr}[m_{1/2}^2]$$
$$= 0\,. \tag{4.47}$$

This result also follows directly from (2.14) and the fact that Hyper-Kähler manifolds are Ricci-flat.

4.2 Metastability

The possible vacua of the theory correspond to points in the scalar manifold that satisfy the stationarity condition $V_{Su} = 0$. This reads:

$$\Omega_{uw}\nabla_v X^w F^v = 0 \,. \tag{4.48}$$

On the vacuum one has $\delta\psi^u = \sqrt{2}\epsilon F^u$ and $\hat{\delta}\psi^u = \sqrt{2}\hat{\epsilon}\hat{F}^u$, and the first and second supersymmetries are spontaneously broken respectively if some of the auxiliary fields F^u or some of the $\hat{F}^u$ are non-vanishing. The order parameters are the norms of the two vectors formed out of these two types of quantities. Since $\hat{F}^u\hat{\bar{F}}_u = F^u\bar{F}_u$, these two norms actually coincide and both define the scalar potential energy, in two equivalent ways emphasizing the two supersymmetries: $V_S = F^u\bar{F}_u = \hat{F}^u\hat{\bar{F}}_u$. In such a situation, there are then two massless Goldstini given by:

$$\eta = \sqrt{2}\,\bar{F}_u\chi^u \,, \quad \hat{\eta} = \sqrt{2}\,\hat{\bar{F}}_u\chi^u \,. \tag{4.49}$$

Indeed, the stationarity condition implies that these are both flat directions of the fermion mass matrix:

$$m_\eta = 0 \,, \quad m_{\hat{\eta}} = 0 \,. \tag{4.50}$$

In this case the two supersymmetries can only be broken simultaneously. This is due to the fact that the conditions that F^u and $\hat{F}^u$ vanish are equivalent, since they are related by the invertible relation (4.41). From the structure of the supersymmetry transformations, we see that the four would-be supersymmetric scalar partners of these fermionic modes, the sGoldstini, can be parametrized by the four independent real linear combinations that one can form with the two complex Goldstino vectors $\eta^u = \sqrt{2}F^u$ and $\hat{\eta}^u = \sqrt{2}\,\hat{F}^u$:

$$\varphi_+ = \bar{F}_u q^u + F_{\bar{u}}\bar{q}^{\bar{u}} \,, \quad \varphi_- = i\bar{F}_u q^u - iF_{\bar{u}}\bar{q}^{\bar{u}} \,, \tag{4.51}$$
$$\hat{\varphi}_+ = \hat{\bar{F}}_u q^u + \hat{F}_{\bar{u}}\bar{q}^{\bar{u}} \,, \quad \hat{\varphi}_- = i\hat{\bar{F}}_u q^u - i\hat{F}_{\bar{u}}\bar{q}^{\bar{u}} \,. \tag{4.52}$$

The masses of these four scalar modes can now be computed by evaluating the scalar mass matrix along the directions $\varphi_+^U = (F^u, \bar{F}^{\bar{u}})$, $\varphi_-^U = (iF^u, -i\bar{F}^{\bar{u}})$, $\hat{\varphi}_+^U = (\hat{F}^u, \hat{\bar{F}}^{\bar{u}})$ and $\hat{\varphi}_-^U = (i\hat{F}^u, -i\hat{\bar{F}}^{\bar{u}})$, and dividing by the length of these vectors, which is $2F^u\bar{F}_u = 2\hat{F}^u\hat{\bar{F}}_u$. Notice that F^u and $\hat{F}^u$ are orthogonal, $\hat{F}^u\bar{F}_u = 0$, and should thus lead to two independent informations.

Viewing the theory as an $N = 1$ theory with F breaking, the first pair of masses is given by eq. (2.19), with R given by (2.20). The constraints imposed by the fact that the geometry is Hyper-Kähler do not substantially simplify neither the stationarity condition nor the form of the curvature at a stationary point, and one still has:

$$R = \text{generically non-zero} \,. \tag{4.53}$$

Coming back to the $N = 2$ picture, one may compute more explicitly all the four masses. After using the stationarity condition to simplify the results, one obtains:

$$m^2_{\varphi_\pm} = (R \pm R_\Delta)\, F^u\bar{F}_u \,, \tag{4.54}$$
$$m^2_{\hat{\varphi}_\pm} = (\hat{R} \pm \hat{R}_\Delta)\, F^u\bar{F}_u \,. \tag{4.55}$$

The terms involving the quantity R and $\hat{R}$ come from the contributions of the Hermitian block $(m_0^2)_{i\bar{j}}$ of the mass matrix, whereas the terms involving R_Δ and $\hat{R}_\Delta$ correspond to the contributions from the complex block $(m_0^2)_{ij}$. In this case, these quantities are all related to sectional curvatures, and one finds

$$R = -\frac{R_{u\bar{v}s\bar{t}}F^u\bar{F}^{\bar{v}}F^s\bar{F}^{\bar{t}}}{(F^w\bar{F}_w)^2}\,,\tag{4.56}$$

$$R_\Delta = -\frac{R_{u\bar{v}s\bar{t}}F^u\hat{\bar{F}}^{\bar{v}}F^s\hat{\bar{F}}^{\bar{t}}}{2(F^w\bar{F}_w)^2} + \text{h.c.}\,,\tag{4.57}$$

$$\hat{R} = -\frac{R_{u\bar{v}s\bar{t}}\hat{F}^u\hat{\bar{F}}^{\bar{v}}F^s\bar{F}^{\bar{t}}}{(F^w\bar{F}_w)^2}\,,\tag{4.58}$$

$$\hat{R}_\Delta = -\frac{R_{u\bar{v}s\bar{t}}\hat{F}^u\hat{\bar{F}}^{\bar{v}}\hat{F}^s\hat{\bar{F}}^{\bar{t}}}{(F^w\bar{F}_w)^2}\,.\tag{4.59}$$

It then follows that

$$m_\varphi^2 \equiv \frac{1}{2}\big(m_{\varphi_+}^2 + m_{\varphi_-}^2\big) = R\,F^u\bar{F}_u\,,\tag{4.60}$$

$$\hat{m}_{\hat{\varphi}}^2 \equiv \frac{1}{2}\big(m_{\hat{\varphi}_+}^2 + m_{\hat{\varphi}_-}^2\big) = \hat{R}\,F^u\bar{F}_u\,.\tag{4.61}$$

This represents exactly the same type of information as in the case of $N=1$ theories with chiral multiplets, but once for each supersymmetry.

The crucial sharpening in the necessary conditions for metastability comes now when one takes into account that the scalar manifold is not only Kähler but actually Hyper-Kähler. From (4.26) it follows indeed that:

$$\hat{R} = -\hat{R}_\Delta = -R\,.\tag{4.62}$$

The four sGoldstino masses then simplify to:

$$m_{\varphi_\pm}^2 = (R \pm R_\Delta)\,F^u\bar{F}_u\,,\tag{4.63}$$

$$m_{\hat{\varphi}_+}^2 = 0\,,\quad m_{\hat{\varphi}_-}^2 = -2\,R\,F^u\bar{F}_u\,.\tag{4.64}$$

This finally leads to the following results:

$$m_{\varphi\,\text{sing}}^2 \equiv m_{\hat{\varphi}_+}^2 = 0\,,\tag{4.65}$$

$$m_{\varphi\,\text{trip}}^2 \equiv \frac{1}{3}\big(m_{\varphi_+}^2 + m_{\varphi_-}^2 + m_{\hat{\varphi}_-}^2\big) = 0\,.\tag{4.66}$$

The first of these implies that there is always a massless mode, which can be interpreted as the Goldstone boson of the spontaneously broken central charge symmetry. The second implies instead that there generically occurs at least one tachyonic mode.

The above results can be made more transparent by switching to more general real coordinates and exploiting the $SU(2)$ symmetry rotating the three complex structures $(J^x)^U{}_V$. More precisely, the four sGoldstini can be organized as a singlet $\varphi_0^U = X^U$ plus a triplet $\varphi_x^U = (J^x)^U{}_V X^V$, so that modulo irrelevant factors $\varphi_0^U = \hat{\varphi}_+^U$ and $\varphi_1^U = \varphi_+^U$, $\varphi_2^U = \varphi_-^U$, $\varphi_3^U = \hat{\varphi}_-^U$. One then has $m_{\varphi_0}^2 = 0$, corresponding again to the Goldstone mode of the spontaneously broken central charge symmetry, and $\sum_x m_{\varphi_x}^2 = 0$, corresponding to

an $SU(2)$ invariant sum rule on the masses of the remaining triplet of sGoldstini. More precisely, one finds:

$$m^2_{\varphi_0} = 0 \,, \tag{4.67}$$

$$m^2_{\varphi_x} = -2 \, R_x \, F^u F_u \,, \tag{4.68}$$

where R_x denotes the holomorphic sectional curvature defined by the complex structure $(J^x)^U{}_V$ and the direction X^U:

$$R_x = \frac{R_{UVMN} X^U (J^x X)^V X^M (J^x X)^N}{(X^K \bar{X}_K)^2} \,. \tag{4.69}$$

Indeed, one easily verifies that $R_1 = -\frac{1}{2}(R + R_\Delta)$, $R_2 = -\frac{1}{2}(R - R_\Delta)$ and $R_3 = R$. Moreover, the result (4.66) is now seen to descend directly from the integrability condition of the covariant constancy of the three complex structures, which reads $\Omega^{\bar{w}}{}_{[u} R_{v]\bar{w}s\bar{t}} = 0$ and implies the following sum rule:

$$\sum_x R_x = 0 \,. \tag{4.70}$$

Summarizing, besides the $N = 1$ information on two of the sGoldstini, which implies that $m^2_{\varphi_1} + m^2_{\varphi_2} = 2 \, R \, F^u \bar{F}_u$, there is a further information on the other two sGoldstini coming from the second supersymmetry and which implies that $m^2_{\varphi_0} = 0$, corresponding to the Goldstone mode associated to the spontaneous breaking of the central charge symmetry, and $m_{\varphi_3} = -2 \, R \, F^u \bar{F}_u$. It follows that one of the sGoldstini always has a non-positive square mass, independently of the sign of R. It should be emphasized that the $N = 1$ metastability condition is recovered through the average of the sGoldstino masses associated to the first and second non-canonical complex structures, and not through the sGoldstino mass associated to the third canonical complex structure, which has instead the opposite sign.

The above results are the rigid limit of the results obtained in [8] for the supergravity case. The cosmological constant reads $V_{\rm S} = F^u \bar{F}_u - 3 \, m^2_{3/2} M^2_{\rm P}$ and the relevant combination of sGoldstino masses is

$$m^2_{\varphi \, {\rm trip}} = -2 \, M^{-2}_{\rm P} \, F^u \bar{F}_u + \frac{16}{3} \, m^2_{3/2} \,. \tag{4.71}$$

We see again that the main features of this result are also captured in the rigid limit, in which $m_{3/2} \to 0$ and $M_{\rm P} \to \infty$. Gravitational effects influence only quantitatively the result. For the triplet sGoldstino, the first term partly arises from the fact that in the local case the scalar manifold is Quaternionic-Kähler, rather than Hyper-Kähler, and the sum rule (4.70) is deformed due to the $SU(2)$ curvature of order $M^{-2}_{\rm P}$ characterizing these manifolds. The singlet sGoldstino, on the other hand, is unphysical in the local case, the corresponding degree of freedom being eaten by the graviphoton. But in the limit defined by the double scaling in which $M_{\rm P} \to \infty$ and $g \to 0$ with $g M_{\rm P} \to$ finite, this becomes the physical massless Goldstone boson of the spontaneously broken central charge global symmetry. This clarifies the rigid limit interpretation of the result of [8]. It also allows to check their structure and their normalization by comparing them with the corresponding result found here. By doing so, one verifies in particular that the sectional curvatures

must appear with opposite signs in the $N = 1$ and the $N = 2$ sGoldstino masses. This is related to the sum rule $R_1 + R_2 = -R_3 + \mathcal{O}(M_{\mathrm{P}}^{-2})$ holding on the three holomorphic sectional curvatures. One however also sees that the $N = 2$ result of [8] must be wrong by a factor of 2 in its dependence on the curvature, whereas the sign is correct. We believe it may simply miss an overall factor of 2 in its normalization, which we have included in (4.71).

In this case it is not clear to what extent the necessary condition for metastability could be made sufficient by allowing a tuning. Indeed, from the $N = 1$ perspective the superpotential W is not arbitrary but rather related to an isometry of the geometry defined by K. This substantially restricts the freedom to adjust it.

5 N=2 models with vector multiplets

Let us continue by considering the case of $N = 2$ theories with $n_{\mathcal{V}}$ vector multiplets $\mathcal{V}^a$. This is a particular case of $N = 1$ theory with $n_{\mathrm{C}} = n_{\mathcal{V}}$ chiral multiplets Φ^i plus $n_{\mathrm{V}} = n_{\mathcal{V}}$ vector multiplets V^a. The most general two-derivative Lagrangian is specified by a real Kähler potential K, a holomorphic superpotential W, a holomorphic gauge kinetic function f_{ab}, some holomorphic Killing vectors X_a^i and some real Fayet-Iliopoulos constants ξ_a, and in $N = 1$ superspace it reads

$$\mathcal{L} = \int d^4\theta \left[K(\Phi, \bar{\Phi}, V) + \xi_a V^a \right] + \int d^2\theta \left[W(\Phi) + \frac{1}{4} f_{ab}(\Phi) W^{a\alpha} W_\alpha^b \right] + \text{h.c.} . \quad (5.1)$$

The existence of a second supersymmetry mixing different $N = 1$ superfields implies further strong restrictions on K, W, f_{ab} and X_a^i. To work out these restrictions, we follow again the logic of [13], with some additional ingredients taken from [32] (see also [33]) to obtain the most general allowed superpotential, and also some generalization to make the formulation covariant under general field reparametrizations.

The general form of the second supersymmetry can be parametrized in terms of two holomorphic functions f_a^i and L^a plus some complex constants m^a, and takes the following form:[4]

$$\hat{\delta}\Phi^i = \sqrt{2}i f_a^i(\Phi)\,\hat{\epsilon}\,W^a , \quad (5.2)$$

$$\hat{\delta}V^a = -\sqrt{2}i\big(\bar{L}^a(\bar{\Phi}) - i f_{bc}{}^a \bar{L}^b(\bar{\Phi})V^c + \mathcal{O}(V^2) + \sqrt{8}i\,m^a\bar{\theta}^2\big)\hat{\epsilon}\,\theta + \text{h.c.} . \quad (5.3)$$

In order for this to correctly satisfy an $N = 1$ supersymmetry subalgebra, more precisely $[\hat{\delta}_1, \hat{\delta}_2]\Phi^i = -2i(\hat{\epsilon}_1\sigma^\mu\hat{\bar{\epsilon}}_2 - \hat{\epsilon}_2\sigma^\mu\hat{\bar{\epsilon}}_1)\partial_\mu\Phi^i$ and $[\hat{\delta}_1, \hat{\delta}_2]V^a = -2i(\hat{\epsilon}_1\sigma^\mu\hat{\bar{\epsilon}}_2 - \hat{\epsilon}_2\sigma^\mu\hat{\bar{\epsilon}}_1)\partial_\mu V^a$, one needs to impose some relation between the functions f_a^i and L^a. A straightforward computation shows that one just needs to require that:

$$f_a^i \partial_i L^b = \delta_a^b , \quad f_a^i \partial_j L^a = \delta_j^i . \quad (5.4)$$

The invariance of the action defined by (5.1) under this second supersymmetry is instead guaranteed by the following constraints, where M_a and f_a denote arbitrary holo-

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[4]The transformation (5.3) implies that $\hat{\delta}W^a = \frac{i}{\sqrt{8}}\hat{\epsilon}\,\bar{D}^2\bar{L}^a(\bar{\Phi}) + \sqrt{2}\,\partial\!\!\!/L^a(\Phi)\,\hat{\epsilon} + \mathcal{O}(V) + 4\,m^a\,\hat{\epsilon}$

morphic functions and e_a some complex constants:

$$f_{ab} = -i f_a^i \, \partial_i M_b = -i f_b^i \, \partial_i M_a \,, \quad f_a^i K_i \big|_{V=0} - \frac{1}{2} f_{ab} \bar{L}^b - \frac{i}{2} \bar{M}_a = f_a \,, \tag{5.5}$$

$$W_i f_a^i = \sqrt{2} \big(e_a + i f_{ab} m^b \big) \,, \quad X_a^i = f_b^i f_{ac}{}^b L^c \,, \tag{5.6}$$

$$\xi_a, e_a, m^b = 0 \text{ whenever } f_{bc}{}^a \neq 0 \,, \quad i f_{ad} f_{bc}{}^d L^c = - f_{ba}{}^c M_c \,. \tag{5.7}$$

To find out the geometrical meaning of the above constraints, we need first of all to interpret the meaning of the holomorphic functions L^a appearing in the transformation laws and the holomorphic functions M_a parametrizing the constraints put by the invariance of the action. Concerning L^a, it is natural to think of them as representing a general reparametrization of the original fields Φ^i. One can then define the Jacobian matrix of this transformation:

$$f_i^a = \nabla_i L^a \,. \tag{5.8}$$

The constraints (5.4) from the closure of the algebra then imply that this Jacobian matrix is invertible and that the functions f_a^i are given by the inverse of this matrix:

$$f_a^i = (f^{-1})_a^i \,. \tag{5.9}$$

Concerning M_a, we may similarly introduce the matrix

$$h_{ai} = \nabla_i M_a \,, \tag{5.10}$$

and denote its inverse by

$$h^{ai} = (h^{-1})^{ai} \,. \tag{5.11}$$

The two constraints (5.5) coming from the invariance of the action then imply the following relations for the gauge kinetic function f_{ab} and the Kähler metric $g_{i\bar{j}}$, where h_{ab} denotes the real part of f_{ab}:

$$f_{ab} = -i f_a^i h_{ib} = -i f_b^i h_{ia} \,, \tag{5.12}$$

$$g_{i\bar{j}} = h_{ab} f_i^a \bar{f}_{\bar{j}}^b \,. \tag{5.13}$$

We now observe that the first of the relations (5.5) can be rewritten in terms of L^a and M_a as $f_{ab} = -i \partial M_b / \partial L^a = -i \partial M_a / \partial L^b$, and implies thus that modulo some irrelevant constants the functions M_a must be the gradients with respect to the functions L^a of some holomorphic function M:

$$M = \text{holomorphic prepotential} \,. \tag{5.14}$$

In other words, this means that the index a in M_a can be interpreted as the derivative with respect to L^a. It finally follows that the Kähler potential and the gauge kinetic function are both determined by the prepotential M and read:

$$K = \frac{i}{2} \big(\bar{M}_a L^a - \bar{L}^a M_a \big) + \mathcal{O}(V) = \frac{i}{2} \big((\bar{M} \, e^{-2V})_a L^a - (\bar{L} \, e^{-2V})^a M_a \big) \,, \tag{5.15}$$

$$f_{ab} = -i M_{ab} \,. \tag{5.16}$$

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This is the statement that the geometry is Special-Kähler [34, 9, 35, 36], with L^a and M_a playing the roles of the electric and magnetic components of the symplectic sections.

Concerning the superpotential, the constraints (5.6) and (5.7) from the invariance of the action imply that it is restricted to be a linear combination of the electric and magnetic sections L^a and M_a corresponding to Abelian factors, with complex coefficients e_a and m^a:

$$W = \sqrt{2}\big(e_a L^a + m^a M_a\big). \tag{5.17}$$

This superpotential for the $N = 1$ chiral superfields Φ^i, which is linear in the sections, represents the $N = 2$ completion of the possibility of having a linear Fayet-Iliopoulos term for the $N = 1$ vector superfields V^a. More precisely, the term linear in L^a is trivially invariant on its own, thanks to the fact that the natural partners of the vector superfields V^a under the second supersymmetry are the sections L^a, in the sense that $\hat{\delta} L^a = \sqrt{2} i \hat{\epsilon} W^a$. On the other hand, the term in M_a is non-trivially invariant, and its variation $\hat{\delta} M_a = -\sqrt{2} \hat{\epsilon} f_{ab} W^b$ is canceled by the extra variation of the vector kinetic term induced by the explicit shift in $\hat{\delta} W^a$ proportional to the coefficients m^a.

We see that the well-known symplectic structure of $N = 2$ theories with only vector multiplets emerges quite naturally from this framework. Moreover, one automatically finds a coordinate-covariant formulation, along the lines of [37, 38]. For vanishing non-Abelian gauge couplings and vanishing Fayet-Iliopoulos parameters, the theory is invariant under a duality symmetry acting as symplectic transformations on the sections (L^a, M_a).

At this point, one may check that the two supersymmetries commute, meaning that there is no central charge in this case: $[\delta_1, \hat{\delta}_2] \Phi^i = 0$, $[\delta_1, \hat{\delta}_2] V^a = 0$. This means that the full supersymmetry algebra closes off-shell.

The form of the gauge transformations leaving the action invariant is fixed by the expression (5.6) that the Killing vector must take:[5]

$$\delta_{\mathrm{g}} \Phi^i = f_a^i f_{bc}{}^a \Lambda^b L^c, \tag{5.18}$$

$$\delta_{\mathrm{g}} V^a = -\frac{i}{2}\big(\Lambda^a - \bar{\Lambda}^a\big) + \frac{1}{2} f_{bc}{}^a \big(\Lambda^b + \bar{\Lambda}^b\big) V^c + \mathcal{O}(V^2). \tag{5.19}$$

This means that the sections L^a must transform in the adjoint representation of the gauge group: $\delta L^a = f_{bc}{}^a \Lambda^b L^c$. The properties (5.7) then guarantee that the Lagrangian is gauge invariant. Indeed, the invariance of the Kähler potential requires that $\delta M_a = -f_{ba}{}^c \Lambda^b M_c$. But since $\delta M_a = M_{ad} \delta L^d$, this implies the constraint $M_{ad} f_{bc}{}^d L^c = -f_{ba}{}^c M_c$, which coincides with the second of (5.7). The invariance of the gauge kinetic term further requires that $\delta f_{ab} = 2i f_{c(a}{}^d M_{b)d} \Lambda^c$. But since $\delta f_{ab} = -i M_{abe} \delta L^e$, this implies that $M_{abe} f_{cd}{}^e L^d = -2 f_{c(a}{}^d M_{b)d}$. It is however straightforward to check that this relation automatically follows from the former constraint, by taking a further derivative. Finally, the invariance of the superpotential implies that it should vanish in the non-Abelian directions, corresponding to the first condition in (5.7).

Before going on, let us summarize some important results concerning Special-Kähler geometry. The basic objects characterizing such a geometry are the sections L^a and the

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[5]One also has $\delta_{\mathrm{g}} W^a = f_{bc}{}^a \Lambda^b W^c$.

following holomorphic symmetric tensor, which is related to the third derivative of the prepotential M [39, 40]:

$$C_{ijk} = \frac{1}{2} M_{abc} f_i^a f_j^b f_k^c \,. \tag{5.20}$$

Indeed, the Christoffel symbols and the Riemann tensor are found to be given by the following expressions:

$$\Gamma^i_{jk} = \partial_j f_k^a f_a^i - i C_{jkl} \bar{f}^{ia} f_a^l \,, \tag{5.21}$$

$$R_{i\bar{j}p\bar{q}} = -C_{ipk} \bar{C}^k_{\bar{j}\bar{q}} \,. \tag{5.22}$$

From (5.21) one then deduces the following basic relation underlying Special-Kähler geometry, out of which the expression (5.22) for the Riemann tensor emerges as the integrability condition:

$$\nabla_i f_j^a = i C_{ijk} \bar{f}^{ka} \,. \tag{5.23}$$

From this it also follows that:

$$\nabla_{[i} C_{j]kl} = 0 \,. \tag{5.24}$$

One also easily finds

$$h_{abi} = -i C_{ijk} f_a^j f_b^k \,, \tag{5.25}$$

$$\nabla_i h_{abj} - 2 h_{aci} h^{cd} h_{bdj} = -i \nabla_i C_{jkl} f_a^k f_b^l \,. \tag{5.26}$$

In addition to the above restrictions posed by the geometry, there are also a number of relations descending from the fact that the sections describing the scalar fields transform in the adjoint representation and the Killing vectors X_a^i are rigidly fixed and given by the second of eq. (5.6). Since the Kähler potential is strictly invariant, the real Killing potentials associated to these Killing vectors are determined by $K_a = -2i X_a^i K_i = 2i \bar{X}_a^{\bar{j}} K_{\bar{j}}$, in such a way that $X_a^i = \frac{i}{2} g^{i\bar{j}} \nabla_{\bar{j}} K_a$. Using the second of (5.7) and its derivative, one then finds the following two equivalent expressions:

$$\begin{aligned} K_a &= f_{ab}{}^c \big(L^b \bar{M}_c + \bar{L}^b M_c \big) \\ &= 2i \, h_{ad} f_{bc}{}^d \bar{L}^b L^c \,. \end{aligned} \tag{5.27}$$

From the expressions (5.6) and (5.27) one then derives the following identities:

$$X_a^i L^a = 0 \,, \quad X_a^i \bar{L}^a = -\frac{i}{2} \bar{f}^{ia} K_a \,, \quad K_a L^a = 0 \,, \quad K_a \bar{L}^a = 0 \,. \tag{5.28}$$

The equivariance condition reads

$$g_{i\bar{j}} X_{[a}^i \bar{X}_{b]}^{\bar{j}} = \frac{i}{4} f_{ab}{}^c K_c \,. \tag{5.29}$$

Moreover, as a consequence of the identity $M_{abe} f_{cd}{}^e L^d = -2 f_{c(a}{}^d M_{b)d}$ implied by the transformation properties of the gauge kinetic function, one finds the following cyclic identity:

$$X_a^i h_{bci} + X_b^i h_{cai} + X_c^i h_{abi} = 0 \,. \tag{5.30}$$

Notice finally that using (5.23) one deduces that $\nabla_i X_{a\bar{j}} = f_i^b \bar{f}_{\bar{j}}^c (f_{ab}{}^d h_{dc} + X_a^k h_{bck})$, and using then (5.30) and the fact that $f_{ab}{}^b = 0$, one arrives at the following identity:

$$\nabla_i X_a^i = -2 X_b^k h^{bc} h_{cak} \,. \tag{5.31}$$

To summarize, the Lagrangian takes the following general form, after choosing the Wess-Zumino gauge:

$$\mathcal{L} = \int d^4\theta \left[K(\Phi, \bar{\Phi}) + (K_a(\Phi, \bar{\Phi}) + \xi_a) V^a + 2 g_{i\bar{j}}(\Phi, \bar{\Phi}) X_a^i(\Phi) \bar{X}_b^{\bar{j}}(\bar{\Phi}) V^a V^b \right]$$
$$+ \int d^2\theta \left[\sqrt{2}(e_a L^a(\Phi) + m^a M_a(\Phi)) - \frac{i}{4} M_{ab}(\Phi) W^{a\alpha} W_\alpha^b \right] + \text{h.c.}\,. \tag{5.32}$$

One may now verify more explicitly that this is invariant under the second supersymmetry, by retaining only terms at most linear in the vector multiplets in eqs. (5.2) and (5.3). In components, this gives

$$\mathcal{L} = -g_{i\bar{j}} D_\mu \phi^i D^\mu \bar{\phi}^{\bar{j}} - \frac{1}{4} h_{ab} F_{\mu\nu}^a F^{b\mu\nu} + \frac{1}{4} k_{ab} F_{\mu\nu}^a \tilde{F}^{b\mu\nu} - i g_{i\bar{j}} \psi^i \left(\slashed{D} \bar{\psi}^{\bar{j}} + \Gamma_{\bar{m}\bar{n}}^{\bar{j}} \slashed{D} \bar{\phi}^{\bar{m}} \bar{\psi}^{\bar{n}} \right)$$
$$- \frac{i}{2} h_{ab} \lambda^a \slashed{D} \bar{\lambda}^b + \text{h.c.} - \frac{i}{\sqrt{2}} C_{ijk} f_a^j f_b^k \lambda^a \sigma^{\mu\nu} \psi^i F_{\mu\nu}^b + \text{h.c.} - V_{\text{S}} - V_{\text{F}}\,, \tag{5.33}$$

where:

$$V_{\text{S}} = 2 h^{ab} (e_a + i f_{ac} m^c)(\bar{e}_b - i \bar{f}_{bd} \bar{m}^d) + \frac{1}{8} h^{ab} (K_a + \xi_a)(K_b + \xi_b)\,, \tag{5.34}$$

$$V_{\text{F}} = \frac{1}{2} \Big[\sqrt{2} i \, C_{ijk} \bar{f}^{ka} \big((e_a - i \bar{f}_{ab} m^b) \psi^i \psi^j + (\bar{e}_a - i \bar{f}_{ab} \bar{m}^b) f_b^i f_c^j \lambda^b \lambda^c \big)$$
$$+ \sqrt{8} \big(\bar{X}_{ai} + \frac{1}{4} C_{ijk} f_a^j \bar{f}^{kb} (K_b + \xi_b) \big) \psi^i \lambda^a \Big] + \text{h.c.}$$
$$- \frac{1}{4} R_{i\bar{j}k\bar{l}} \big(\psi^i \psi^k \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{l}} + f_a^i f_b^k \bar{f}_c^{\bar{j}} \bar{f}_d^{\bar{l}} \lambda^a \lambda^b \bar{\lambda}^c \bar{\lambda}^d + 2 f_a^k \bar{f}_b^{\bar{l}} \psi^i \lambda^a \bar{\psi}^{\bar{j}} \bar{\lambda}^b \big)$$
$$+ \frac{1}{4} \Big[\big(i \nabla_i C_{jkl} + 2 C_{ikm} C_{jln} f_c^m \bar{f}^{nc} \big) f_a^k f_b^l \psi^i \psi^j \lambda^a \lambda^b$$
$$+ C_{ikm} C_{jln} f_c^m \bar{f}^{nc} f_a^k f_b^l \psi^i \lambda^a \psi^j \lambda^b \Big] + \text{h.c.}\,. \tag{5.35}$$

The first supersymmetry transformation laws involve not only the usual action of the supercharge, but also a compensating gauge transformation with superfield parameter $\Lambda^a = 2i\theta\sigma^\mu\bar{\epsilon}A_\mu^a + 2\theta^2\bar{\epsilon}\bar{\lambda}^a$ needed to preserve the Wess-Zumino gauge choice. The additional gauge transformation turns the ordinary derivative appearing in $\delta\psi^i$ into a gauge-covariant derivative, and one finds

$$\delta\phi^i = \sqrt{2}\,\epsilon\,\psi^i\,, \tag{5.36}$$
$$\delta\psi^i = \sqrt{2}\,\epsilon\,F^i + \sqrt{2}i\,\slashed{D}\phi^i\,\bar{\epsilon}\,, \tag{5.37}$$
$$\delta A_\mu^a = i\epsilon\,\sigma_\mu\bar{\lambda}^a - i\lambda^a\sigma_\mu\,\bar{\epsilon}\,, \tag{5.38}$$
$$\delta\lambda^a = i\epsilon\,D^a + \sigma^{\mu\nu}\epsilon\,F_{\mu\nu}^a\,. \tag{5.39}$$

The auxiliary fields F^i and D^a are given by

$$F^i = -\sqrt{2}\,\bar{f}^{ia}(\bar{e}_a - i\bar{f}_{ab}\bar{m}^b) + \frac{1}{2}\Gamma_{mn}^i \psi^m \psi^n + \frac{i}{2}\bar{C}^i{}_{\bar{m}\bar{n}} \bar{f}_a^{\bar{m}} \bar{f}_b^{\bar{n}} \bar{\lambda}^a \bar{\lambda}^b\,, \tag{5.40}$$

$$D^a = -\frac{1}{2}h^{ab}(K_b + \xi_b) - \frac{1}{\sqrt{2}} C_{ijk} \bar{f}^{ja} f_b^k \psi^i \lambda^b + \text{h.c.}\,. \tag{5.41}$$

The second supersymmetry transformation laws similarly involve not only (5.2), (5.3), but also a gauge transformation with superfield parameter $\hat{\Lambda}^a = -\sqrt{8}\theta\hat{\epsilon}\bar{L}^a - 2\theta^2 f_i^a \hat{\epsilon}\psi^i$, needed to preserve the Wess-Zumino gauge. The extra gauge transformation shifts the D^a auxiliary field appearing in $\hat{\delta}\psi^i$ by $h^{ab}K_b$, and one finds

$$\hat{\delta}\phi^i = \sqrt{2}\,\hat{\epsilon}\,f_a^i \lambda^a\,, \tag{5.42}$$

$$\hat{\delta}\psi^i = \sqrt{2}\,\hat{\epsilon}\,\hat{F}^i + \sqrt{2}\,\partial_j f_a^i \psi^j(\hat{\epsilon}\lambda^a) + \sigma^{\mu\nu}\hat{\epsilon}\,f_a^i F_{\mu\nu}^a\,, \tag{5.43}$$

$$\hat{\delta}A_\mu^a = -i\,\hat{\epsilon}\,\sigma_\mu \bar{f}_{\bar{\imath}}^a \bar{\psi}^{\bar{\imath}} + i f_i^a \psi^i \sigma_\mu \hat{\bar{\epsilon}}\,, \tag{5.44}$$

$$\hat{\delta}\lambda^a = i\hat{\epsilon}\,\hat{D}^a + \sqrt{2}i\,f_i^a \slashed{D}\phi^i\,\hat{\bar{\epsilon}}\,. \tag{5.45}$$

The quantities $\hat{F}^i$ and $\hat{D}^a$ appearing in these expressions are found to be given by

$$\hat{F}^i = \frac{i}{\sqrt{2}}f_a^i\big(D^a + h^{ab}K_b\big) = \frac{i}{\sqrt{8}}\bar{f}^{ia}(K_a - \xi_a) + \text{ferm.}\,, \tag{5.46}$$

$$\hat{D}^a = -\sqrt{2}i\left(\bar{f}_{\bar{\imath}}^a \bar{F}^{\bar{\imath}} + \sqrt{8}i\,m^a - \frac{1}{2}\partial_{\bar{\imath}}f_{\bar{\jmath}}^a \bar{\psi}^{\bar{\imath}}\bar{\psi}^{\bar{\jmath}}\right) = 2i\,h^{ab}(e_b - i\bar{f}_{bc}m^c) + \text{ferm.}\,. \tag{5.47}$$

It is clear from the form of these expressions that the vectors $(\lambda^a, f_i^a \psi^i)$ are doublets of the $SU(2)_R$ automorphism group of the $N = 2$ supersymmetry algebra. In particular, the second supersymmetry transformation can be obtained by supplementing the first supersymmetry transformation with the non-trivial element of the center Z_2 of $SU(2)_R$, acting as $(\lambda^a, f_i^a \psi^i) \to (-f_i^a \psi^i, \lambda^a)$. The above transformation laws, derived by using an $N = 1$ superfield approach, agree with those derived in a component approach in [41, 42, 43, 44] by imposing the above Z_2 invariance, in the special case where $f_i^a = \delta_i^a$.

The extension to supergravity was developed in [34, 35, 14, 15]. It presents again some subtleties related to those terms in the action that were not genuinely but accidentally invariant. More precisely, it turns out that models of the above type can be consistently coupled to gravity only if the coefficients of the Fayet-Iliopoulos terms and the electric and magnetic linear superpotentials satisfy some restrictions. Again, this is due to the fact that the trivial invariance of such terms in the rigid limit is spoiled by gravitational effects. The main new feature is that there appears a non-trivial $U(1)$ bundle over the scalar manifold with curvature proportional to M_P^{-2}, and the manifold becomes Special-Kähler-Hodge. To spell out more precisely the restrictions that need to be imposed on the $N = 2$ Fayet-Iliopoulos terms, let us set the complex magnetic constants to 0:[6]

$$m^a = 0\,. \tag{5.48}$$

Let us furthermore parametrize the real Fayet-Iliopoulos constants ξ_a and the complex electric constants e_a in terms of a triplet of real constants P_a^x:

$$P_a^1 = 2\,\mathrm{Re}(e_a)\,,\quad P_a^2 = 2\,\mathrm{Im}(e_a)\,,\quad P_a^3 = \frac{1}{2}\xi_a\,. \tag{5.49}$$

It is quite common to introduce also a similar notation for the non-Abelian part of the Killing potential, which is however not a constant but a real function of the scalar fields, and behaves as a singlet:

$$P_a^0 = -\frac{1}{2}K_a\,. \tag{5.50}$$

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[6]For the inclusion of magnetic gaugings, see [45, 46, 47].

The statement is then that in supergravity the triplet of constants P_a^x must satisfy a non-trivial equivariance condition, and are thus constrained. More precisely, there is a non-trivial effect coming from an $SU(2)$ curvature, which is of order $M_{\rm P}^{-2}$ and is thus a genuine supergravity effect. For Abelian factors, however, this is the only term that arises, and one then obtains a constraint that is independent of $M_{\rm P}^{-2}$ and survives in the rigid limit. This constraint on $N = 2$ theories is the analogue of the constraint on $N = 1$ theories that the Fayet-Iliopoulos term can arise only under the very special circumstance that it is associated to a gauged $U(1)_R$ symmetry, and it reads

$$\epsilon^{xyz} P_a^y P_b^z = 0 \,. \tag{5.51}$$

This means that when interpreted as trivectors, the P_a^x for the various values of a must all be parallel. The general solution to this equivariance condition is then parametrized in terms of a single trivector P^x, whose direction defines a definite $U(1)_R$ subgroup of $SU(2)_R$, and some real coefficients p_a:

$$P_a^x = p_a P^x \,. \tag{5.52}$$

Notice that in terms of the original coefficients, this restriction implies that besides having the ξ_a real, one needs also the e_a to have all the same phase z. We shall here allow for non-zero ξ_a, contrarily to what we did in the $N = 1$ case, since as soon as P^x is not zero, we are in the peculiar situation where a $U(1)_R$ symmetry is gauged when gravity is switched on. From now on, we will then restrict to theories of this type, admitting a consistent coupling to gravity, whereas we shall discard to other more peculiar possibility of gauging the whole $SU(2)_R$. In this situation, the superpotential takes the form $W = \sqrt{2}\, z |e_a| L^a$ and as a result it satisfies the following relation, descending from (5.23):

$$\nabla_i W_j = iz^2 C_{ijk} \bar{W}^k \,. \tag{5.53}$$

Notice finally that one can reshuffle the scalar potential (5.34) as follows. For the F-term part, we get $2h^{ab} e_a \bar{e}_b = \frac{1}{2} h^{ab}(P_a^1 P_b^1 + P_a^2 P_b^2)$. For the D-term part, three types of terms arise. First, we see from (5.28) that $\frac{1}{8} h^{ab} K_a K_b = \frac{1}{2} g_{i\bar{\jmath}} X_a^i \bar{L}^a \bar{X}_b^{\bar{\jmath}} L^b = \frac{1}{2} h^{ab} P_a^0 P_b^0$. Next, $\frac{1}{8} h^{ab} \xi_a \xi_b = \frac{1}{2} h^{ab} P_a^3 P_b^3$. Finally, from the second of (5.27) and the fact that ξ_a is non-vanishing only for Abelian factors, it follows that $\frac{1}{4} h^{ab} K_a \xi_b = 0$. The scalar potential can then be rewritten in the following form, which reproduces that of [14, 15]:

$$\begin{aligned} V_{\rm S} &= \frac{1}{2} g_{i\bar{\jmath}} X_a^i \bar{L}^a \bar{X}_b^{\bar{\jmath}} L^b + \frac{1}{2} h^{ab} P_a^x P_b^x \\ &= \frac{1}{2} h^{ab} \big(P_a^0 P_b^0 + P_a^x P_b^x \big) \,. \end{aligned} \tag{5.54}$$

5.1 Supertrace

At a generic point in the scalar field space and for vanishing fermions and vector fields, the auxiliary fields simplify to

$$F^i = -\sqrt{2}\, \bar{f}^{ia} \bar{e}_a \,, \tag{5.55}$$

$$D^a = -\frac{1}{2} h^{ab} (K_b + \xi_b) \,. \tag{5.56}$$

The corresponding hatted quantities similarly simplify to

$$\hat{F}^i = \frac{i}{\sqrt{2}} f_a^i \big(D^a + h^{ab} K_b \big) = \frac{i}{\sqrt{8}} \bar{f}^{ia} (K_a - \xi_a) = \hat{F}_\perp^i + \hat{F}_\parallel^i \,, \tag{5.57}$$

$$\hat{D}^a = -\sqrt{2} i \, \bar{f}_{\bar{i}}^a \bar{F}^{\bar{i}} = 2i \, h^{ab} e_b \,. \tag{5.58}$$

The mass matrix of the scalar fields is given by

$$(m_0^2)_{i\bar{j}} = -R_{i\bar{j}k\bar{l}} \big(2 \, F^k \bar{F}^{\bar{l}} + f_a^k \bar{f}_b^{\bar{l}} D^a D^b \big) + h^{ab} \bar{X}_{ai} X_{b\bar{j}}$$

$$+ \frac{i}{2} \big(\nabla_i X_{a\bar{j}} - 2 h^{bc} h_{abi} X_{c\bar{j}} \big) D^a + \text{h.c.} \,, \tag{5.59}$$

$$(m_0^2)_{ij} = \frac{i}{2} \nabla_i C_{jkl} \big(2 \, z^2 F^k F^l + f_a^k f_b^l D^a D^b \big) - h^{ab} \bar{X}_{ai} \bar{X}_{bj}$$

$$+ 2i h^{bc} h_{ab(i} \bar{X}_{cj)} D^a + \Gamma_{ij}^k V_{\text{S}k} \,, \tag{5.60}$$

The mass matrix of the fermions reads instead

$$(m_{1/2})_{ij} = -i z^2 C_{ijk} F^k \,, \tag{5.61}$$

$$(m_{1/2})_{ab} = -i C_{ijk} f_a^i f_b^j F^k \,, \tag{5.62}$$

$$(m_{1/2})_{ia} = \sqrt{2} \, \bar{X}_{ai} - \frac{1}{\sqrt{2}} C_{ijk} f_a^j f_b^k D^b \,. \tag{5.63}$$

Finally, the mass matrix of the vectors is

$$(m_1^2)_{ab} = 2 \, X_{(a}^i \bar{X}_{b)i} \,. \tag{5.64}$$

A straightforward computation gives

$$\text{tr}[m_0^2] = 2 \, R_{i\bar{j}} \big(2 \, F^i \bar{F}^{\bar{j}} + f_a^i \bar{f}_b^{\bar{j}} D^a D^b \big) + 2 \, h^{ab} \bar{X}_{ai} X_b^i - 4i \, h^{bc} h_{abi} X_c^i D^a + \text{h.c.} \tag{5.65}$$

$$\text{tr}[m_{1/2}^2] = R_{i\bar{j}} \big(2 \, F^i \bar{F}^{\bar{j}} + f_a^i \bar{f}_b^{\bar{j}} D^a D^b \big) + 4 \, h^{ab} \bar{X}_{ai} X_b^i - 2i \, h^{ab} h_{bci} X_a^i D^c + \text{h.c.} \,, \tag{5.66}$$

$$\text{tr}[m_1^2] = 2 \, h^{ab} \bar{X}_{ai} X_b^i \,. \tag{5.67}$$

It follows that the supertrace of the mass matrix vanishes [13]:

$$\text{str}[m^2] \equiv \text{tr}[m_0^2] - 2 \, \text{tr}[m_{1/2}^2] + 3 \, \text{tr}[m_1^2]$$

$$= 0 \,. \tag{5.68}$$

This result also follows directly from (3.31) and the properties that the Christoffel symbols are related to the derivative of the gauge kinetic function, the Ricci tensor to the contraction between two of these, and finally that the trace of the charge matrix satisfies the property (5.31).

5.2 Metastability

The possible vacua of the theory correspond to points in the scalar manifold that satisfy the stationarity condition $V_{\text{S}i} = 0$, which implies

$$-\frac{i}{2} C_{ijk} \big(2 \, z^2 F^j F^k + f_a^j f_b^k \, D^a D^b \big) + i \bar{X}_{ai} D^a = 0 \tag{5.69}$$

The relation (3.33) between the values of the F^i and D^a auxiliary fields can be simplified a bit by using the fact that $f_a^i \bar{F}_i$ vanishes for non-Abelian generators. One finds

$$i X_a^k h_{bck} f_i^b \bar{f}_{\bar{j}}^c F^i \bar{F}^{\bar{j}} - X_{(a}^i \bar{X}_{b)i} D^b + \frac{1}{2} f_{ab}{}^d k_{dc} D^b D^c = 0 \,. \tag{5.70}$$

On the vacuum, one has $\delta \psi^i = \sqrt{2} \epsilon F^i$, $\delta \lambda^a = i \epsilon D^a$, $\hat{\delta} \psi^i = \sqrt{2} \hat{\epsilon} \hat{F}^i$, $\hat{\delta} \lambda^a = i \hat{\epsilon} \hat{D}^a$, and the first and second supersymmetries are spontaneously broken respectively if some of the auxiliary fields F^i, D^a or some of the $\hat{F}^i$, $\hat{D}^a$ are non-vanishing. The order parameters are given by the norms of the two vectors built out of these two sets of quantities. Since $\hat{F}^i \hat{\bar{F}}_i = \frac{1}{2} \hat{D}^a \hat{D}_a$ and $\frac{1}{2} \hat{D}^a \hat{D}_a = F^i \bar{F}_i$, these two norms actually coincide and define again in two equivalent ways, emphasizing the two supersymmetries, the scalar potential energy: $V_{\rm S} = F^i \bar{F}_i + \frac{1}{2} D^a D_a = \hat{F}^i \hat{\bar{F}}_i + \frac{1}{2} \hat{D}^a \hat{D}_a$. In such a situation, there are then two massless Goldstini, associated to the two independent supersymmetries and given by:

$$\eta = \sqrt{2} \bar{F}_i \psi^i + i D_a \lambda^a \,, \quad \hat{\eta} = \sqrt{2} \hat{\bar{F}}_i \psi^i + i \hat{D}_a \lambda^a \,, \tag{5.71}$$

In fact, one can verify that the stationarity condition and the gauge invariance of the superpotential imply that these are always flat directions of the fermion mass matrix:

$$m_\eta = 0 \,, \quad m_{\hat{\eta}} = 0 \,. \tag{5.72}$$

In the situation under consideration, the two supersymmetries can only be broken simultaneously.[7] The sGoldstini are in this case linear combinations of scalars and vectors, but the relevant thing to look at is the projection along the scalar field space. One then gets four independent real linear combinations, corresponding to the projection of the complex Goldstino vectors $\eta^i = \sqrt{2} F^i$ and $\hat{\eta}^i = \sqrt{2} \hat{F}^i$:

$$\varphi_+ = \bar{F}_i \phi^i + F_{\bar{\imath}} \bar{\phi}^{\bar{\imath}} \,, \quad \varphi_- = i \bar{F}_i \phi^i - i F_{\bar{\imath}} \bar{\phi}^{\bar{\imath}} \,, \tag{5.73}$$

$$\hat{\varphi}_+ = \hat{\bar{F}}_i \phi^i + \hat{F}_{\bar{\imath}} \bar{\phi}^{\bar{\imath}} \,, \quad \hat{\varphi}_- = i \hat{\bar{F}}_i \phi^i - i \hat{F}_{\bar{\imath}} \bar{\phi}^{\bar{\imath}} \,. \tag{5.74}$$

The masses of these four scalar modes can now be computed by evaluating the scalar mass matrix along the directions $\varphi_+^I = (F^i, \bar{F}^{\bar{\imath}})$, $\varphi_-^I = (i F^i, -i \bar{F}^{\bar{\imath}})$, $\hat{\varphi}_+^I = (\hat{F}^i, \hat{\bar{F}}^{\bar{\imath}})$ and $\hat{\varphi}_-^I = (i \hat{F}^i, -i \hat{\bar{F}}^{\bar{\imath}})$, and dividing by the length of these vectors, which is $2 F^i \bar{F}_i$ for the first two and $2 \hat{F}^i \hat{\bar{F}}_i$ for the last two, with $F^i \bar{F}_i \neq \hat{F}^i \hat{\bar{F}}_i$. Notice however that F^i and $\hat{F}^i$ are in general not orthogonal, and do thus not necessarily lead to two independent informations. More precisely, one has $\hat{F}^i = \hat{F}_\perp^i + \hat{F}_\parallel^i$, where $\hat{F}_\perp^i$ is non-vanishing only in the non-Abelian case and orthogonal to F^i, whereas $\hat{F}_\parallel^i$ is non-vanishing whenever there are $N = 1$ Fayet-Iliopoulos terms for some Abelian factors and is parallel to F^i whenever the alignment condition on the $N = 2$ Fayet-Iliopoulos terms is satisfied.

Viewing the theory as an $N = 1$ theory with F and D breaking, the first pair of masses is given by eq. (3.37), with R, S, T and M^2 given by eqs (3.38), (3.39), (3.40) and (3.41).

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[7]The result (5.72) actually holds true even in more general situations where the Fayet-Iliopoulos terms are not aligned and magnetic superpotentials are considered. In such a situation, partial supersymmetry breaking is known to be possible [48] (see also [49, 50]). But in that case F^i and $\hat{F}^i$ turn out to be parallel on the vacuum, and there is thus only one independent massless Goldstino. In models compatible with gravity, on the other hand, partial supersymmetry breaking requires also the presence of hyper multiplets, whose presence can modify the alignment consistency condition [51, 52].

But the constraints imposed by the fact that the geometry is Special-Kähler do in this case substantially simplify both the stationarity condition and the form of the curvatures, and there emerges a relation between the quantities R, S, T and M^2 evaluated at a stationary point. This relations can be derived by solving for $C_{ijk}F^jF^k$ in the stationarity condition (5.69) and taking its square norm. One then sees that the mixed terms drop out thanks to the properties implied by gauge invariance on the prepotential, and one deduces that

$$R\, F^i\bar{F}_i = \frac{1}{4}\, T\, \frac{(D^aD_a)^2}{F^i\bar{F}_i} + \frac{1}{2}\, M^2 \frac{D^aD_a}{F^i\bar{F}_i}\,. \tag{5.75}$$

Coming back to the $N = 2$ picture, one may compute more explicitly all the four masses and simplify them by using the stationarity condition. To emphasize the important aspects of the results, we shall study separately the Abelian and non-Abelian cases.

Abelian case

Consider first Abelian gauge groups. In this case $X^i_a = 0$ and $K_a = 0$. One then has $\bar{F}_i = -\frac{1}{\sqrt{2}}f^a_i(P^1_a + iP^2_a)$ and $\hat{\bar{F}}_i = \frac{i}{\sqrt{2}}f^a_i P^3_a$, so that $F^i\bar{F}_i = \frac{1}{2}h^{ab}(P^1_aP^1_b + P^2_aP^2_b)$ and $\hat{F}^i\hat{\bar{F}}_i = \frac{1}{2}h^{ab}P^3_aP^3_b$.

For simplicity, let us first study the situation where all the parallel Fayet-Iliopoulos parameters are rotated in the plane where $e_a \neq 0$ but $\xi_a = 0$. This implies that $F^i \neq 0$ but $\hat{F}^i = 0$. As a consequence, only the first pair of sGoldstino directions is well defined, whereas the second pair is not. The first two sGoldstino masses are easily found to be given by:

$$m^2_{\varphi\pm} = R\, F^i\bar{F}_i \pm \Delta\,. \tag{5.76}$$

In this expression, the quantity R originates from the contribution from the Hermitian block $(m^2_0)_{i\bar{j}}$ of the mass matrix, whereas Δ encodes the contribution coming from the off-diagonal block $(m^2_0)_{ij}$. The former corresponds to a sectional curvature:

$$R = -\frac{R_{i\bar{j}m\bar{n}}\, F^i\bar{F}^{\bar{j}}F^m\bar{F}^{\bar{n}}}{(F^k\bar{F}_k)^2}\,. \tag{5.77}$$

It then follows that

$$m^2_\varphi \equiv \frac{1}{2}\big(m^2_{\varphi+} + m^2_{\varphi-}\big) = R\, F^i\bar{F}_i\,. \tag{5.78}$$

This result represents the informations associated to the first supersymmetry, to which a non-degenerate sGoldstino can be associated.

At this point, a sharp simplification does however occur when taking into account the form (5.22) implied for the Riemann tensor by the fact that the geometry is not only Kähler but actually Special-Kähler. Indeed, we see that at a stationary point satisfying the stationarity condition $C_{ijk}F^jF^k = 0$, the sectional curvature R actually vanishes. This corresponds to eq. (5.75) applied to the present case:

$$R = 0\,. \tag{5.79}$$

The two sGoldstino masses then simplify to

$$m^2_{\varphi\pm} = \pm\Delta\,. \tag{5.80}$$

It finally follows that

$$m_\varphi^2 = 0\,. \tag{5.81}$$

Let us now consider the more general situation where $e_a \neq 0$ and $\xi_a \neq 0$, where $F^i \neq 0$ and $\hat{F}^i \neq 0$. In this more general situation, both pairs of sGoldstini are well defined. However, we do not expect to get any additional information, since all the ξ_a can be set to zero by an overall $SU(2)$ transformation, and we known that V_{S} is $SU(2)$ invariant. Nevertheless, it is instructive to see how it works in this case. The four sGoldstino masses are found to be of the following form:

$$m_{\varphi_\pm}^2 = 2\,R\,F^i\bar{F}_i + 2\,R'\,\hat{F}^i\hat{\bar{F}}_i \pm \Delta\,, \tag{5.82}$$

$$m_{\hat{\varphi}_\pm}^2 = 2\,\hat{R}\,\hat{F}^i\hat{\bar{F}}_i + 2\,R'\,F^i\bar{F}_i \pm \hat{\Delta}\,. \tag{5.83}$$

In these expressions, the quantities R, R' and $\hat{R}$ originate from the Hermitian block $(m_0^2)_{i\bar{j}}$ of the mass matrix, whereas Δ and $\hat{\Delta}$ encode the contributions coming from the off-diagonal blocks $(m_0^2)_{ij}$. As usual, only the former have simple expressions, which are

$$R = -\frac{R_{i\bar{j}m\bar{n}}\,F^i\bar{F}^{\bar{j}}F^m\bar{F}^{\bar{n}}}{(F^k\bar{F}_k)^2}\,, \tag{5.84}$$

$$R' = -\frac{R_{i\bar{j}m\bar{n}}\,F^i\bar{F}^{\bar{j}}\hat{F}^m\hat{\bar{F}}^{\bar{n}}}{(F^k\bar{F}_k)(\hat{F}^l\hat{\bar{F}}_l)}\,, \tag{5.85}$$

$$\hat{R} = -\frac{R_{i\bar{j}m\bar{n}}\,\hat{F}^i\hat{\bar{F}}^{\bar{j}}\hat{F}^m\hat{\bar{F}}^{\bar{n}}}{(\hat{F}^k\hat{\bar{F}}_k)^2}\,. \tag{5.86}$$

Note that compared to the treatment of $N = 1$ theories with F and D breaking of section 3, the quantities R, R' and $\hat{R}$ introduced here correspond to the quantities R, S and T, whereas $F^i\bar{F}_i$ and $\hat{F}^i\hat{\bar{F}}_i$ correspond to $F^i\bar{F}_i$ and $\frac{1}{2}D^aD_a$. Using the relation (5.75), we then see that the terms $R\,F^i\bar{F}_i$, $S\,D^aD_a$ and $T\,(D^aD_a)^2/(4\,F^i\bar{F}_i)$ in eq. (3.37) become respectively $R\,F^i\bar{F}_i$, $2R'\hat{F}^i\hat{\bar{F}}_i$ and $R\,F^i\bar{F}_i$, and there is some simplification in the masses of the first pair of sGoldstini, whereas the mass of the new second pair of sGoldstini takes a similar expression with hatted and unhatted quantities exchanged. For the average of each pair of masses, one finds

$$m_\varphi^2 \equiv \frac{1}{2}\big(m_{\varphi_+}^2 + m_{\varphi_-}^2\big) = 2\,R\,F^i\bar{F}_i + 2R'\,\hat{F}^i\hat{\bar{F}}_i\,, \tag{5.87}$$

$$m_{\hat{\varphi}}^2 \equiv \frac{1}{2}\big(m_{\hat{\varphi}_+}^2 + m_{\hat{\varphi}_-}^2\big) = 2\,\hat{R}\,\hat{F}^i\hat{\bar{F}}_i + 2\,R'\,F^i\bar{F}_i\,. \tag{5.88}$$

These results represent the informations associated to the two supersymmetries. In the case of aligned Fayet-Iliopoulos terms, however, these two expressions should coincide and represent the same information, since F^i and $\hat{F}^i$ are proportional to each other: $\hat{F}^i = iz(p_3/\sqrt{p_1^2 + p_2^2})F^i$.

The crucial simplification comes again from the form (5.22) of the Riemann tensor in Special-Kähler geometry. First, the stationarity condition reads $C_{ijk}F^jF^k = \bar{z}^2C_{ijk}\hat{F}^j\hat{F}^k$ and leads to a relation between R and $\hat{R}$, which is just eq. (5.75) applied to the present

case. In addition, the alignment condition implies that $F^i \hat{\bar{F}}^{\bar{j}} = -\bar{z}^2 \hat{F}^i \bar{F}^{\bar{j}}$ and leads to a relation between R' and R or $\hat{R}$. The two relations are:

$$R\,(F^i\bar{F}_i)^2 = \hat{R}\,(\hat{F}^i\hat{\bar{F}}_i)^2 = -R'(F^i\bar{F}_i)(\hat{F}^i\hat{\bar{F}}_i)\,. \tag{5.89}$$

The expressions for the four sGoldstino masses then simplify to

$$m^2_{\varphi_\pm} = \pm\Delta\,, \tag{5.90}$$

$$m^2_{\hat{\varphi}_\pm} = \pm\hat{\Delta}\,. \tag{5.91}$$

It finally follows that

$$m^2_\varphi = 0\,, \tag{5.92}$$

$$m^2_{\hat{\varphi}} = 0\,. \tag{5.93}$$

As expected, these two results coincide and it is clear that they represent the same information, since they are defined out of the two complex directions F^i and $\hat{F}^i$, which are parallel. There is thus really only one $SU(2)$-invariant information, stating that:

$$m^2_{\varphi\,\text{inv}} = 0\,. \tag{5.94}$$

The above result represents the rigid limit of the result obtained in [9] for the supergravity case (see also [10] for a derivation of the same result in the language of [15]). The cosmological constant reads $V_\mathrm{S} = F^i\bar{F}_i + \frac{1}{2}D^a D_a - 3\,m^2_{3/2}M^2_\mathrm{P}$ and the average sGoldstino mass is

$$m^2_{\varphi\,\text{inv}} = -2\,M^{-2}_\mathrm{P}\big(F^i\bar{F}_i + \frac{1}{2}D^a D_a\big) + 6\,m^2_{3/2}\,. \tag{5.95}$$

Again, we see that the main feature of this result, namely the fact that it is independent of the curvature, is also captured in the rigid limit, in which $m_{3/2} \to 0$ and $M_\mathrm{P} \to \infty$. Gravitational effects influence only quantitatively the result, making it negative instead of zero in the case of positive cosmological constant.

Non-Abelian case

Consider next non-Abelian gauge groups. In this case $X^i_a \neq 0$ and $K_a \neq 0$. Then $\bar{F}_i = -\frac{1}{\sqrt{2}}f^a_i(P^1_a + iP^2_a)$ and $\hat{\bar{F}}_i = \frac{i}{\sqrt{2}}f^a_i(P^3_a + P^0_a)$, so that $F^i\bar{F}_i = \frac{1}{2}h^{ab}(P^1_a P^1_b + P^2_a P^2_b)$ and $\hat{F}^i\hat{\bar{F}}_i = \frac{1}{2}h^{ab}(P^3_a P^3_b + P^0_a P^0_b)$.

As before, let us consider first the case where all the parallel Fayet-Iliopoulos terms are in the plane corresponding to $e_a \neq 0$ and $\xi_a = 0$. One then has $F^i \neq 0$ and $\hat{F}^i \neq 0$, but whereas the first is truly generic the second is in fact related to the Killing vectors, $\hat{F}^i = -\frac{1}{\sqrt{2}}X^i_a\bar{L}^a$, and this brings up some substantial simplifications. In such a situation, all the four sGoldstini are well defined and their masses are found to be given by the following expressions, after using the stationarity conditions and all the relations descending from gauge invariance:

$$m^2_{\varphi_\pm} = 2\,R\,F^i\bar{F}_i + 2\,R'\,\hat{F}^i\hat{\bar{F}}_i + M^2\frac{\hat{F}^i\hat{\bar{F}}_i}{F^j\bar{F}_j} \pm \Delta\,, \tag{5.96}$$

$$m^2_{\hat{\varphi}_\pm} = 0\,. \tag{5.97}$$

In these expressions, the quantities R, R' and M^2 emerge from the contribution of the diagonal block $(m_0^2)_{i\bar{\jmath}}$ of the mass matrix, whereas Δ encodes the contribution from the off-diagonal block $(m_0^2)_{ij}$. The quantities R, R' and M^2, together with the quantity $\hat{R}$ introduced for later use, are given by:

$$R = -\frac{R_{i\bar{\jmath}m\bar{n}}\, F^i \bar{F}^{\bar{\jmath}} F^m \bar{F}^{\bar{n}}}{(F^k \bar{F}_k)^2}\,, \tag{5.98}$$

$$R' = -\frac{R_{i\bar{\jmath}m\bar{n}}\, F^i \bar{F}^{\bar{\jmath}} \hat{F}^m \hat{\bar{F}}^{\bar{n}}}{(F^k \bar{F}_k)(\hat{F}^l \hat{\bar{F}}_l)}\,, \tag{5.99}$$

$$\hat{R} = -\frac{R_{i\bar{\jmath}m\bar{n}}\, \hat{F}^i \hat{\bar{F}}^{\bar{\jmath}} \hat{F}^m \hat{\bar{F}}^{\bar{n}}}{(\hat{F}^k \hat{\bar{F}}_k)^2}\,, \tag{5.100}$$

$$M^2 = \frac{2 X_a^k \bar{X}_{bk}\, f_i^a \bar{f}_{\bar{\jmath}}^b \hat{F}^i \hat{\bar{F}}^{\bar{\jmath}}}{\hat{F}^l \hat{\bar{F}}_l}\,. \tag{5.101}$$

Note that compared to the treatment of $N=1$ theories with F and D breaking of section 3, the quantities R, R', $\hat{R}$ and M^2 correspond to the quantities R, S, T and M^2, whereas $F^i \bar{F}_i$ and $\hat{F}^i \hat{\bar{F}}_i$ correspond to $F^i \bar{F}_i$ and $\frac{1}{2}D^a D_a$. Using the relation (5.75), we then see that the terms $R\, F^i \bar{F}_i$, $S\, D^a D_a$, $T\,(D^a D_a)^2/(4\, F^i \bar{F}_i)$ and $M^2 D^a D_a / F^i \bar{F}_i$ in eq. (3.37) become respectively $R\, F^i \bar{F}_i$, $2R' \hat{F}^i \hat{\bar{F}}_i$, $R\, F^i \bar{F}_i - M^2 \hat{F}^i \hat{\bar{F}}_i / F^j \bar{F}_j$ and $2 M^2 \hat{F}^i \hat{\bar{F}}_i / F^j \bar{F}_j$, and there is some simplification in the masses of the first pair of sGoldstini. Concerning the second pair of sGoldstini, we now observe that they can actually be identified with particular real linear combinations of the would-be Goldstone modes $\sigma_a = \bar{X}_{ai}\phi^i + X_{a\bar{\jmath}}\phi^{\bar{\jmath}}$, and their conjugates $\rho_a = i\bar{X}_{ai}\phi^i - iX_{a\bar{\jmath}}\phi^{\bar{\jmath}}$. Indeed, since $L^a X_a^i = 0$ and $L^a \bar{X}_a^{\bar{\jmath}} = -\sqrt{2}\hat{\bar{F}}^{\bar{\jmath}}$, one has $\hat{\varphi}_+ = -\sqrt{2}\,\mathrm{Re}\,L^a \sigma_a = -\sqrt{2}\,\mathrm{Im}\,L^a \rho_a$ and $\hat{\varphi}_- = -\sqrt{2}\,\mathrm{Re}\,L^a \rho_a = \sqrt{2}\,\mathrm{Im}\,L^a \sigma_a$. We moreover see that due to the fact that $X_a^i L^a = 0$, we are in the situation where, as explained at the end of section 3, the Goldstone modes in the directions $\mathrm{Re}\,L^a$ and $\mathrm{Im}\,L^a$ are linearly related to their conjugates in these directions. As a result, both $\hat{\varphi}_+$ and $\hat{\varphi}_-$ correspond to unphysical would-be Goldstone modes σ_+ and σ_-. This explains why they have vanishing masses, and also tells us that this information should be discarded. Taking the average of the first pair of sGoldstino masses, one is finally left with the following information:

$$m_\varphi^2 \equiv \frac{1}{2}\big(m_{\varphi_+}^2 + m_{\varphi_-}^2\big) = 2\,R\, F^i \bar{F}_i + 2\,R'\, \hat{F}^i \hat{\bar{F}}_i + M^2 \frac{\hat{F}^i \hat{\bar{F}}_i}{F^j \bar{F}_j}\,. \tag{5.102}$$

Once again, the special form (5.22) taken by the Riemann tensor implies some relations among the quantities R, R', $\hat{R}$ and M^2. More precisely, the stationarity condition implies that $C_{ijk}F^j F^k = \bar{z}^2 C_{ijk}\hat{F}^j \hat{F}^k + \sqrt{2}i\bar{z}^2 \bar{X}_{ai}f_j^a \hat{F}^j$ and leads to a relation between R, $\hat{R}$ and M^2, which is just eq. (5.75) applied to the present case:

$$R\,(F^i \bar{F}_i)^2 = \hat{R}\,(\hat{F}^i \hat{\bar{F}}_i)^2 + M^2 \hat{F}^i \hat{\bar{F}}_i\,. \tag{5.103}$$

The expressions of the masses of the first pair of sGoldstini can then be recast in the following form:

$$m_{\varphi_\pm}^2 = 2\,R'\, \hat{F}^i \hat{\bar{F}}_i + 2\,\hat{R}\,\frac{(\hat{F}^i \hat{\bar{F}}_i)^2}{F^j \bar{F}_j} + 3\,M^2 \frac{\hat{F}^i \hat{\bar{F}}_i}{F^j \bar{F}_j} \pm \Delta\,. \tag{5.104}$$

This finally yields:

$$m_\varphi^2 \equiv \frac{1}{2}\big(m_{\varphi_+}^2 + m_{\varphi_-}^2\big) = 2\,R'\,\hat{F}^i\hat{\bar{F}}_i + 2\,\hat{R}\,\frac{(\hat{F}^i\hat{\bar{F}}_i)^2}{F^j\bar{F}_j} + 3\,M^2\,\frac{\hat{F}^i\hat{\bar{F}}_i}{F^j\bar{F}_j}\,. \tag{5.105}$$

This result corresponds to the information related to the first supersymmetry. We have seen that it can be obtained by simplifying the corresponding expression obtained in section 3 for $N=1$ theories with F and D breaking. There is instead no useful information related to the second supersymmetry, because the corresponding sGoldstini coincide with unphysical would-be Goldstone modes. Notice that in the limiting situations where $F^i \neq 0$ but $\hat{F}^i = 0$, the above positive-definite result for the average masses goes to zero. One is then back to a situation that is similar to the one arising in the Abelian case.

As before, one may now consider the more general situation with $e_a \neq 0$ and $\xi_a \neq 0$, where $F^i \neq 0$ and $\hat{F}^i \neq 0$. As for the Abelian case, we do not expect to get any new information with this generalization, because all the ξ_a can be set to zero through an overall $SU(2)$ transformation, provided the $N=2$ Fayet-Iliopoulos terms are aligned. It is nevertheless instructive to work out the results also in this more general situation. In this case, we shall however not redo a detailed comparison with the $N=1$ perspective, and rather work out the results in a manifestly $SU(2)$ invariant way, in order to gain insight on how the information behaves under $SU(2)$. Using the notation (5.49) and (5.50), the four sGoldstino masses are found to be given by:

$$m_{\varphi_\pm}^2 = \mathcal{R}'\,P^{a0}P_a^0 + \hat{\mathcal{R}}\,\frac{(P^{a0}P_a^0)^2}{P^{bx}P_b^x} + 3\mathcal{M}^2\,\frac{P^{a0}P_a^0}{P^{bx}P_b^x} \pm \Delta\,, \tag{5.106}$$

$$m_{\hat{\varphi}_\pm}^2 = \frac{P^{a3}P_a^3}{P^{a3}P_a^3 + P^{a0}P_a^0}\,m_{\varphi_\pm}^2\,, \tag{5.107}$$

where

$$\mathcal{R}' = -\frac{R_{i\bar{j}p\bar{q}}f_a^i\bar{f}_b^{\bar{j}}f_c^p\bar{f}_d^{\bar{q}}\,P^{ax}P^{bx}P^{c0}P^{d0}}{(P^{ey}P_e^y)(P^{f0}P_f^0)}\,, \tag{5.108}$$

$$\hat{\mathcal{R}} = -\frac{R_{i\bar{j}p\bar{q}}f_a^i\bar{f}_b^{\bar{j}}f_c^p\bar{f}_d^{\bar{q}}\,P^{a0}P^{b0}P^{c0}P^{d0}}{(P^{e0}P_e^0)^2}\,, \tag{5.109}$$

$$\mathcal{M}^2 = \frac{2\,X_a^i\bar{X}_{bi}\,P^{a0}P^{b0}}{P^{c0}P_c^0}\,. \tag{5.110}$$

We see that (5.106) is simply the $SU(2)$ invariant completion of (5.104), and therefore represents the correct generalization of the information. On the other hand, (5.107) is not $SU(2)$ invariant and does not represent any additional information. The reason is that when $\xi_a \neq 0$, the two directions F^i and $\hat{F}^i$ are no-longer orthogonal. The most appropriate way to proceed is then to subtract from $\hat{F}^i$ its projection $F_\parallel^i$ along F^i, and look at the direction $\hat{F}_\perp^i$. But this direction is nothing but the complex would-be Goldstone direction $X_a^i\bar{L}^a$, corresponding to the unphysical modes $\sigma_+ = -\sqrt{2}\,\mathrm{Re}\,L^a\sigma_a = -\sqrt{2}\,\mathrm{Im}\,L^a\rho_a$ and $\sigma_- = -\sqrt{2}\,\mathrm{Re}\,L^a\rho_a = \sqrt{2}\,\mathrm{Im}\,L^a\sigma_a$, which lead to vanishing masses. This shows that (5.107) represents in fact the same information as (5.106), but diluted along an unphysical direction. So once again the only useful information comes from the first pair of sGoldstini,

and reads:

$$m_\varphi^2 \equiv \frac{1}{2}\left(m_{\varphi_+}^2 + m_{\varphi_-}^2\right) = \mathcal{R}' \, P^{a0} P_a^0 + \hat{\mathcal{R}} \, \frac{(P^{a0} P_a^0)^2}{P^{bx} P_b^x} + 3\,\mathcal{M}^2 \, \frac{P^{a0} P_a^0}{P^{bx} P_b^x} \,. \tag{5.111}$$

One may wonder whether it is possible to get this $SU(2)$-invariant information in a more transparent way, by somehow reorganizing the four sGoldstini according to their $SU(2)$ transformation properties, as in the case of the hyper multiplets. To answer this question, notice first that in this case, contrarily to the case involving only hypers, the Lagrangian is not $SU(2)$-invariant, unless one promotes the Fayet-Iliopoulos constants P_a^x to triplet spurions. The transformation properties of the sGoldstini are then determined by the dependence of the Goldstino directions on the singlets P_a^0 and the triplets P_a^x. Notice in this respect that we have defined the two Goldstino directions in terms of $F^i \propto \bar{f}^{ia}(P_a^1 - iP_a^2)$ and $\hat{F}^i \propto \bar{f}^{ia}(P_a^0 + P_a^3)$. But one could have equivalently used also the other two quantities $f_a^i D^a \propto \bar{f}^{ia}(P_a^0 - P_a^3)$ and $f_a^i \hat{D}^a \propto \bar{f}^{ia}(P_a^1 + iP_a^2)$; these would have given the same information in the above analysis, as a consequence of the alignment of the triplets P_a^x and the relation of the singlets P_a^0 to would-be Goldstone modes. Then, considering all these four complex directions on equal footing one might equally well switch to the linear combinations $\bar{f}^{ia} P_a^0$ and $\bar{f}^{ia} P_a^x$, which are clearly a singlet and a triplet of $SU(2)$. Notice however that due to the alignment condition $P_a^x = p_a P^x$, the latter three vectors differ only by their normalization, and define thus the same direction. In this way one recovers just two independent complex directions, which are both $SU(2)$ invariant, and the masses of the corresponding pairs of real sGoldstini are respectively given by 0 ± 0 and $m_\varphi^2 \pm \Delta$, with m_φ^2 given by eq. (5.111).

The above result is new. It shows that the situation improves when generalizing the gauging from Abelian to non-Abelian. Tachyons do no longer necessarily appear, because those states that were giving rise to them in the Abelian case receive an additional positive definite contribution to their mass in the non-Abelian case. Note however that when $P_a^x = 0$ one gets $P_a^0 = 0$ at stationary points, by the reasoning after (3.33). It is thus necessary to switch on at least some of the P_a^x to achieve metastability. Another case where the result (5.111) vanishes identically is when the prepotential is quadratic, since in that case $\mathcal{R}'$ and $\hat{\mathcal{R}}$ vanish due to the vanishing of the curvature and $\mathcal{M}^2$ vanishes due to eq. (5.70) contracted with D^a and the constancy of the gauge kinetic function. This is compatible with what happens in the rigid limit of the examples constructed in [10], where for $M_\mathrm{P} \to \infty$ the geometry becomes flat and the scalar masses tend to zero.

We expect that to obtain the generalization of this result to supergravity, one should proceed exactly along the same lines and compute the average mass of the first pair of sGoldstini. But as usual, the supergravity result can differ from the rigid one derived here only by quantitative effects, suppressed by inverse powers of the Planck scale. One should then be left with some freedom to keep the value of the average mass positive also in the presence of gravity. Concerning the second pair of sGoldstini, we believe that they are again associated to two would-be Goldstone modes, and do therefore not yield any further information. Indeed, the relevant direction in group space is changed from L^a to L^A, with $A = 0, a$ and involves now also the graviphoton direction, but the crucial property $X_a^i L^a = 0$ simply generalizes to $X_A^i L^A = 0$. As a result, it remains true also in supergravity that these two modes are both massless but unphysical. We have verified

this statement in the explicit examples constructed in [10], where there is always a pair of would-be Goldstone modes forming a complex scalar field.

In this case too it is unclear to what extent the necessary condition for metastability could be made sufficient by allowing a tuning. Indeed, for a given geometry associated to K the only things one may change are the Killing potentials defining the gauge symmetries. But these are not arbitrary functions, and can therefore be adjusted only in a limited way.

6 N=2 models with hyper and vector multiplets

Let us finally consider the most general case of $N = 2$ theories with $n_{\mathcal{H}}$ hyper multiplets $\mathcal{H}^k$ and $n_{\mathcal{V}}$ vector multiplets $\mathcal{V}^a$. This is a particular case of $N = 1$ theory containing $n_C = 2n_{\mathcal{H}} + n_{\mathcal{V}}$ chiral multiplets Q^u and Φ^i plus $n_V = n_{\mathcal{V}}$ vector multiplets V^a. The most general two-derivative Lagrangian is specified by a real Kähler potential K, a holomorphic superpotential W, a holomorphic gauge kinetic function f_{ab}, some triholomorphic and holomorphic Killing vectors X_a^u and X_a^i, and some real Fayet-Iliopoulos constants ξ_a, all subject to strong restrictions required for the existence of a second supersymmetry. We shall not derive in full detail these restrictions, because they emerge essentially in the same way as in the cases involving only hyper and vector multiplets, discussed in sections 4 and 5. Moreover we shall restrict from the beginning to theories where the superpotential involves only an electric term and no magnetic term. In $N = 1$ superspace, the Lagrangian is then found to take the following form:

$$\mathcal{L} = \int d^4\theta \left[K^H(Q, \bar{Q}, V) + K^V(\Phi, \bar{\Phi}, V) + \xi_a V^a \right] \tag{6.1}$$

$$+ \int d^2\theta \left[sP(Q) + \sqrt{2}\, e_a L^a(\Phi) + \sqrt{2}i\, P_a(Q) L^a(\Phi) - \frac{i}{4}\, M_{ab}(\Phi)\, W^{a\alpha} W_\alpha^b \right] + \text{h.c.} .$$

Besides the normal coupling between hyper and vector multiplets, which involves the real Killing potentials $-\frac{1}{2} K_a^H$ associated to the Killing vectors X_a^u, there is also an additional coupling which involves the holomorphic Killing potentials P_a admitted by the X_a^u due to the fact that they are triholomorphic. These extra couplings are required by the second supersymmetry, and generalize the well-known couplings arising already in the minimal theory based on a flat geometry between the pair of chiral multiplets forming each hyper multiplet and the adjoint scalar contained in each vector multiplet. The self-interaction of hyper multiplets, which represents the generalization of the hyper multiplet mass terms in the flat case, are again described by a triholomorphic Killing vector $X^u = \sqrt{2}i\bar{s}\, X_0^u L^0$, and the associated holomorphic Killing potential $P = \sqrt{2}i\bar{s}\, P_0 L^0$.

The above Lagrangian is invariant under a second supersymmetry, which acts on the $N = 1$ superfields in the following way:

$$\hat{\delta}Q^u = -\frac{1}{2}\bar{\Omega}^{uv}\bar{D}^2 \left[\left(K_v(Q, \bar{Q}) + 2i\bar{X}_{av}(Q, \bar{Q})V^a + \mathcal{O}(V^2) \right)(\hat{\epsilon}\theta + \hat{\bar{\epsilon}}\bar{\theta}) \right]$$
$$- 2i \left[(s + \bar{s})\, X^u(Q) + \sqrt{2}i\, X_a^u(Q, \bar{Q})L^a(\Phi) \right]\hat{\epsilon}\theta , \tag{6.2}$$

$$\hat{\delta}\Phi^i = \sqrt{2}i f_a^i(\Phi)\hat{\epsilon}W^a , \tag{6.3}$$

$$\hat{\delta}V^a = -\sqrt{2}i \left(\bar{L}^a(\bar{\Phi}) - i\, f_{bc}{}^a \bar{L}^b(\bar{\Phi})V^c + \mathcal{O}(V^2) \right)\hat{\epsilon}\theta + \text{h.c.} . \tag{6.4}$$

The full $N = 2$ supersymmetry algebra closes only on-shell, by using the equations of motion of the superfields Q^u describing the hyper multiplets, and there is a central charged acting on the latter:

$$\delta_{\rm c} Q^u = \alpha X^u(Q) \,, \tag{6.5}$$

$$\delta_{\rm c} \Phi^i = 0 \,, \tag{6.6}$$

$$\delta_{\rm c} V^a = 0 \,. \tag{6.7}$$

One may again use alternative forms of the supersymmetry transformations, which are equivalent on-shell for the Q^u. For instance, one may add to (6.2) the trivial transformation $\delta_{\rm t} Q^u = \frac{1}{2}\Omega^{uv}[\bar{D}^2(K_v + 2i\bar{X}_{av}V^a + \mathcal{O}(V^2)) - 4sP_v - 4\sqrt{2}iP_{av}L^a]\hat{\epsilon}\theta$, which is a symmetry of the on-shell theory since the parenthesis is proportional to the equations of motion of Q^u. This gives $\hat{\delta}Q^u = -\frac{1}{2}\bar{\Omega}^{uv}\bar{D}^2[(K_v + 2i\bar{X}_{av}V^a + \mathcal{O}(V^2))\hat{\bar{\epsilon}}\bar{\theta}] - 2i\bar{s}X^u\hat{\epsilon}\theta$.

The gauge transformations are defined by the triholomorphic Killing vector X_a^u for Q^u, and take the same fixed form as before for Φ^i and V^a, corresponding to the adjoint representation:

$$\delta_{\rm g} Q^u = \Lambda^a X_a^u \,, \tag{6.8}$$

$$\delta_{\rm g} \Phi^i = f_a^i f_{bc}{}^a \Lambda^b L^c \,, \tag{6.9}$$

$$\delta_{\rm g} V^a = -\frac{i}{2}(\Lambda^a - \bar{\Lambda}^a) + \frac{1}{2}f_{bc}{}^a(\Lambda^b + \bar{\Lambda}^b)V^c + \mathcal{O}(V^2) \,. \tag{6.10}$$

The Killing vectors are related to the Killing potentials in the usual way, both in the hyper and in the vector multiplet sectors:

$$X_a^i = \frac{i}{2}g^{i\bar{j}}\nabla_{\bar{j}}K_a^V \,, \quad X_a^u = \frac{i}{2}g^{u\bar{v}}\nabla_{\bar{v}}K_a^H \,. \tag{6.11}$$

The equivariance conditions following from the fact that these Killing vectors X_a^i and X_a^u are holomorphic take the usual form:

$$g_{i\bar{j}}X_{[a}^i\bar{X}_{b]}^{\bar{j}} = \frac{i}{4}f_{ab}{}^c K_c^V \,, \quad g_{u\bar{v}}X_{[a}^u\bar{X}_{b]}^{\bar{v}} = \frac{i}{4}f_{ab}{}^c K_c^H \,. \tag{6.12}$$

In addition, there is an other equivariance condition emerging in the hyper multiplet sector, due to the fact that X_a^u is actually triholomorphic. More precisely, exploiting the fact that it is also holomorphic with respect to the two extra complex structures yields the following extra complex condition, involving the holomorphic Killing potential P_c:

$$\Omega_{uv}X_a^u X_b^v = if_{ab}{}^c P_c \,. \tag{6.13}$$

We see that this condition is actually crucial to guarantee the gauge invariance of the term in the superpotential that mixes hyper and vector multiplets. Finally, global central charge invariance of the minimal gauge coupling $K_a V^a$ and gauge invariance of the superpotential $P = \sqrt{2}i\bar{s}P_0 L^0$ for hyper multiplets impose two further constraints, one real and one complex, which read:

$$g_{u\bar{v}}X_{[0}^u\bar{X}_{a]}^{\bar{v}} = 0 \,, \quad \Omega_{uv}X_0^u X_a^v = 0 \,. \tag{6.14}$$

These conditions ensure the compatibility between the local gauge symmetry and the global central charge symmetry, which are independent.

In the Wess-Zumino gauge, the action can be expanded at quadratic order in the vector superfields and simplifies to the following expression:

$$\mathcal{L} = \int d^4\theta \left[K^H(Q,\bar{Q}) + K^V(\Phi,\bar{\Phi}) + \left(K_a^H(Q,\bar{Q}) + K_a^V(\Phi,\bar{\Phi}) + \xi_a \right) V^a \right.$$
$$\left. + 2 \left(g_{u\bar{v}}(Q,\bar{Q}) X_a^u(Q) \bar{X}_b^{\bar{v}}(\bar{Q}) + g_{i\bar{j}}(\Phi,\bar{\Phi}) X_a^i(\Phi) \bar{X}_b^{\bar{j}}(\bar{\Phi}) \right) V^a V^b \right]$$
$$+ \int d^2\theta \left[sP(Q) + \sqrt{2} \left(iP_a(Q) + e_a \right) L^a(\Phi) - \frac{i}{4} M_{ab}(\Phi) W^{a\alpha} W_\alpha^b \right] + \text{h.c.}. \quad (6.15)$$

We see now that much as the real constants ξ_a correspond to the ambiguity in the real Killing potentials K_a^H, the complex constants e_a correspond to the ambiguity in the holomorphic Killing potentials P_a, for Abelian factors. Moreover, one may now verify more explicitly the invariance of the couplings between hyper and vector multiplets, by keeping terms with up to one vector multiplet in eqs. (6.2)-(6.4). In components, one finds:

$$\mathcal{L} = -g_{u\bar{v}} D_\mu q^u D^\mu \bar{q}^{\bar{v}} - g_{i\bar{j}} D_\mu \phi^i D^\mu \bar{\phi}^{\bar{j}} - \frac{1}{4} h_{ab} F_{\mu\nu}^a F^{b\mu\nu} + \frac{1}{4} k_{ab} F_{\mu\nu}^a \tilde{F}^{b\mu\nu}$$
$$- ig_{u\bar{v}} \chi^u \left(\slashed{D} \bar{\chi}^{\bar{v}} + \Gamma_{\bar{s}\bar{t}}^{\bar{v}} \slashed{D} \bar{q}^{\bar{s}} \bar{\chi}^{\bar{t}} \right) - ig_{i\bar{j}} \psi^i \left(\slashed{D} \bar{\psi}^{\bar{j}} + \Gamma_{\bar{m}\bar{n}}^{\bar{j}} \slashed{D} \bar{\phi}^{\bar{m}} \bar{\psi}^{\bar{n}} \right) - \frac{i}{2} h_{ab} \lambda^a \slashed{D} \bar{\lambda}^b + \text{h.c.}$$
$$- \frac{i}{\sqrt{2}} C_{ijk} f_a^j f_b^k \lambda^a \sigma^{\mu\nu} \psi^i F_{\mu\nu}^b + \text{h.c.} - V_S - V_F, \quad (6.16)$$

where:

$$V_S = g_{u\bar{v}} (sX^u + \sqrt{2}i X_a^u L^a)(\bar{s}\bar{X}^{\bar{v}} - \sqrt{2}i \bar{X}_a^{\bar{v}} \bar{L}^a) + 2 h^{ab} \left(P_a - ie_a \right) \left(\bar{P}_b + i\bar{e}_b \right)$$
$$+ \frac{1}{8} h^{ab} (K_a^H + K_a^V + \xi_a)(K_b^H + K_b^V + \xi_b), \quad (6.17)$$

$$V_F = \frac{1}{2} \Big[i\,\Omega_{uw} \nabla_v (sX^w + \sqrt{2}i X_a^w L^a) \chi^u \chi^v - \sqrt{8}\,\Omega_{uv} X_a^v f_i^a \chi^u \psi^i + \sqrt{8}\,\bar{X}_{au} \chi^u \lambda^a$$
$$- \sqrt{2}\, C_{ijk} \bar{f}^{ka} \big((P_a - ie_a)\, \psi^i \psi^j - (\bar{P}_a + i\bar{e}_a) f_b^i f_c^j \lambda^b \lambda^c \big)$$
$$+ \sqrt{8} \big(\bar{X}_{ai} + \frac{1}{4} C_{ijk} f_a^j \bar{f}^{kb} (K_b^H + K_b^V + \xi_b) \big) \psi^i \lambda^a \Big] + \text{h.c.}$$
$$- \frac{1}{4} R_{u\bar{v}s\bar{t}}\, \chi^u \chi^s \bar{\chi}^{\bar{v}} \bar{\chi}^{\bar{t}} - \frac{1}{4} R_{i\bar{j}k\bar{l}} \big(\psi^i \psi^k \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{l}} + f_a^i f_b^k \bar{f}_c^{\bar{j}} \bar{f}_d^{\bar{l}}\, \lambda^a \lambda^b \bar{\lambda}^c \bar{\lambda}^d + 2 f_a^k \bar{f}_b^{\bar{l}}\, \psi^i \lambda^a \bar{\psi}^{\bar{j}} \bar{\lambda}^b \big)$$
$$+ \frac{1}{4} \Big[\big(i\nabla_i C_{jkl} + 2\, C_{ikm} C_{jln} f_c^m \bar{f}^{nc} \big) f_a^k f_b^l\, \psi^i \psi^j \lambda^a \lambda^b$$
$$+ C_{ikm} C_{jln} f_c^m \bar{f}^{nc} f_a^k f_b^l\, \psi^i \lambda^a \psi^j \lambda^b \Big] + \text{h.c.}. \quad (6.18)$$

To determine the first supersymmetry transformation laws in components, one has as usual to take into account the need for a compensating gauge transformation to stay in the Wess-Zumino gauge, with parameter $\Lambda^a = 2i\theta\sigma^\mu\bar{\epsilon} A_\mu^a + 2\theta^2 \bar{\epsilon}\bar{\lambda}^a$. The additional gauge transformation turns the ordinary derivatives appearing in $\delta\chi^u$ and $\delta\psi^i$ into gauge-covariant derivatives, and one finds

$$\delta q^u = \sqrt{2}\,\epsilon\,\chi^u, \quad (6.19)$$
$$\delta\chi^u = \sqrt{2}\,\epsilon\,F^u + \sqrt{2}i\,\slashed{D}q^u \bar{\epsilon}, \quad (6.20)$$
$$\delta\phi^i = \sqrt{2}\,\epsilon\,\psi^i, \quad (6.21)$$
$$\delta\psi^i = \sqrt{2}\,\epsilon\,F^i + \sqrt{2}i\,\slashed{D}\phi^i\,\bar{\epsilon}, \quad (6.22)$$
$$\delta A_\mu^a = i\epsilon\,\sigma_\mu \bar{\lambda}^a - i\lambda^a \sigma_\mu \bar{\epsilon}, \quad (6.23)$$
$$\delta\lambda^a = i\epsilon\,D^a + \sigma^{\mu\nu}\epsilon\,F_{\mu\nu}^a, \quad (6.24)$$

The auxiliary fields F^u, F^i and D^a are given by

$$F^u = i\bar{\Omega}^u_{\bar{v}}(\bar{s}\,\bar{X}^{\bar{v}} - i\sqrt{2}\bar{X}^{\bar{v}}_a\bar{L}^a) + \frac{1}{2}\Gamma^u_{st}\chi^s\chi^t\,, \tag{6.25}$$

$$F^i = \sqrt{2}i\,\bar{f}^{ia}(\bar{P}_a + i\bar{e}_a) + \frac{1}{2}\Gamma^i_{mn}\psi^m\psi^n + \frac{i}{2}\bar{C}^i_{\bar{m}\bar{n}}\bar{f}^{\bar{m}}_a\bar{f}^{\bar{n}}_b\,\bar{\lambda}^a\bar{\lambda}^b\,, \tag{6.26}$$

$$D^a = -\frac{1}{2}h^{ab}(K^H_b + K^V_b + \xi_b) - \frac{1}{\sqrt{2}}\,C_{ijk}\bar{f}^{ja}f^k_b\,\psi^i\lambda^b + \text{h.c.}\,. \tag{6.27}$$

To determine the second supersymmetry transformation laws, one has to similarly supplement the transformations (6.2)–(6.4) with a compensating gauge transformation to stay in the Wess-Zumino gauge, with parameter $\hat{\Lambda}^a = -\sqrt{8}\theta\hat{\epsilon}\bar{L}^a - 2\theta^2 f^a_i\hat{\epsilon}\psi^i$. The additional gauge transformation shifts the $\bar{F}^{\bar{v}}$ auxiliary field appearing in $\hat{\delta}\chi^u$ by $-\sqrt{2}iP^{\bar{v}}_a\bar{L}^a$ and the D^a auxiliary field appearing in $\hat{\delta}\psi^i$ by $h^{ab}K_b$, and one finds

$$\hat{\delta}q^u = -\sqrt{2}\,\bar{\Omega}^u_{\bar{v}}\,\hat{\bar{\epsilon}}\,\bar{\chi}^v\,, \tag{6.28}$$

$$\hat{\delta}\chi^u = \sqrt{2}\,\hat{\epsilon}\,\hat{F}^u + \sqrt{2}\,\Gamma^u_{st}\bar{\Omega}^s_{\bar{v}}\,\hat{\bar{\epsilon}}\,\bar{\chi}^{\bar{v}}\chi^t + \sqrt{2}i\,\bar{\Omega}^u_{\bar{v}}\,D\!\!\!/\bar{q}^{\bar{v}}\hat{\bar{\epsilon}}\,, \tag{6.29}$$

$$\hat{\delta}\phi^i = \sqrt{2}\,\hat{\epsilon}\,f^i_a\lambda^a\,, \tag{6.30}$$

$$\hat{\delta}\psi^i = \sqrt{2}\hat{\epsilon}\,\hat{F}^i + \sqrt{2}\,\partial_j f^i_a\psi^j(\hat{\epsilon}\lambda^a) + \sigma^{\mu\nu}\hat{\epsilon}\,f^i_a F^a_{\mu\nu}\,, \tag{6.31}$$

$$\hat{\delta}A^a_\mu = -i\,\hat{\epsilon}\,\sigma_\mu\bar{f}^a_{\bar{i}}\bar{\psi}^{\bar{i}} + if^a_i\psi^i\sigma_\mu\hat{\bar{\epsilon}}\,, \tag{6.32}$$

$$\hat{\delta}\lambda^a = i\hat{\epsilon}\,\hat{D}^a + \sqrt{2}i\,f^a_i\,D\!\!\!/\phi^i\,\hat{\epsilon}\,. \tag{6.33}$$

The quantities $\hat{F}^u$, $\hat{F}^i$ and $\hat{D}^a$ are found to be given by

$$\hat{F}^u = \bar{\Omega}^u_{\bar{v}}\big(\bar{F}^{\bar{v}} + (s+\bar{s})P^{\bar{v}} + \sqrt{2}i(L^a-\bar{L}^a)P^{\bar{v}}_a - \frac{1}{2}\,\Gamma^{\bar{v}}_{\bar{s}\bar{t}}\,\bar{\chi}^{\bar{s}}\bar{\chi}^{\bar{t}}\big) = -i\bar{s}X^u - \sqrt{2}X^u_a\bar{L}^a\,, \tag{6.34}$$

$$\hat{F}^i = \frac{i}{\sqrt{2}}f^i_a\big(D^a + h^{ab}K^V_b\big) = \frac{i}{\sqrt{8}}\,\bar{f}^{ia}(K^V_a - K^H_a - \xi_a) + \text{ferm.}\,, \tag{6.35}$$

$$\hat{D}^a = -\sqrt{2}i\big(\bar{f}^a_{\bar{i}}\bar{F}^{\bar{i}} - \frac{1}{2}\,\partial_{\bar{j}}f^a_{\bar{i}}\bar{\psi}^{\bar{i}}\bar{\psi}^{\bar{j}}\big) = -2\,h^{ab}(P_b - ie_b) + \text{ferm.}\,. \tag{6.36}$$

The extension to supergravity can be found in [34, 35, 14, 15]. It turns again out that models of the above type can be consistently coupled to gravity only if the coefficients of the Fayet-Iliopoulos terms and the part of the superpotential linear in the sections satisfy some restrictions. The main new feature is that there appears a non-trivial $SU(2)$ bundle over the hyper multiplet scalar manifold and a non-trivial $U(1)$ bundle over the vector multiplet scalar manifold, with curvatures proportional to $M_{\rm P}^{-2}$, and the full scalar manifold becomes the product of a Quaternionic-Kähler manifold and a Special-Kähler-Hodge manifold. To spell out more precisely the restrictions that need to be imposed on the $N = 2$ Fayet-Iliopoulos terms, we proceed as before and relabel the various Killing potentials in a more appropriate way, by defining a triplet of new potentials P^x_a as follows:

$$P^1_a = -2\,\text{Im}(P_a - ie_a)\,,\quad P^2_a = 2\,\text{Re}(P_a - ie_a)\,,\quad P^3_a = \frac{1}{2}\big(K^H_a + \xi_a\big)\,, \tag{6.37}$$

One may also introduce as before the notation

$$P^0_a = -\frac{1}{2}K^V_a\,. \tag{6.38}$$

The triplet of functions P^x_a must satisfy a non-trivial equivariance condition, and are thus constrained. As before, there is a non-trivial effect coming from the curvature of the

$SU(2)$, which is of order $M_{\rm P}^{-2}$ and is thus a genuine supergravity effect. For Abelian factors under which no hyper multiplet is charged, however, this is the only term that arises, and one then obtains a constraint that is independent of $M_{\rm P}^{-2}$, and survives thus in the rigid limit. This constraint takes the form (5.51), whose solution is (5.52). For non-Abelian factors, on the other hand, the gravitational deformation of the equivariance condition is smooth and can be safely discarded in the rigid limit. One is then left with the equivariance conditions (6.12)–(6.14). Notice finally that the superpotential does no longer display the special property (5.53), because it now involves also the non-Abelian sections.

Notice finally that it is possible to reshuffle the scalar potential (6.17) by proceeding in the following way, with manipulations that are similar to those used in [53, 54] to discuss truncations of $N = 2$ to $N = 1$ supergravity theories. From now on we set again $s = i$ for simplicity. For the F-term part from the hyper multiplets, we start by rewriting it as $2g_{u\bar{v}}X_A^u\bar{X}_B^{\bar{v}}L^A\bar{L}^B$, with a new index $A = 0, a$ comprising both the Killing vector defining the central charge global symmetry and the Killing vectors defining the gauge symmetry. Since this ranges over at least two values, both the symmetric and the antisymmetric parts of $g_{u\bar{v}}X_A^u\bar{X}_B^{\bar{v}}$ contribute. For the symmetric part, we may proceed as in the case with only hypers, and switch to general real coordinates by rewriting $g_{u\bar{v}}X_{(A}^u\bar{X}_{B)}^{\bar{v}} = \frac{1}{2}g_{UV}X_A^U X_B^V$, which gives $2g_{u\bar{v}}X_{(A}^u\bar{X}_{B)}^{\bar{v}}L^A\bar{L}^B = g_{UV}X_A^U X_B^V L^A\bar{L}^B$. For the antisymmetric part, the equivariance relations (6.12) and (6.14) imply $g_{u\bar{v}}X_{[a}^u\bar{X}_{b]}^{\bar{v}} = \frac{i}{4}f_{ab}{}^c K_c^H$ and $g_{u\bar{v}}X_{[0}^u\bar{X}_{a]}^{\bar{v}} = 0$, and therefore $2g_{u\bar{v}}X_{[A}^u\bar{X}_{B]}^{\bar{v}}L^A\bar{L}^B = \frac{i}{2}f_{ab}{}^c K_c^H L^a\bar{L}^b = -\frac{1}{4}h^{ab}K_a^V K_b^H$. Putting everything together, we see that the F-term part coming from the hyper multiplets finally gives $2g_{u\bar{v}}X_A^u\bar{X}_B^{\bar{v}}L^A\bar{L}^B = g_{UV}X_A^U X_B^V L^A\bar{L}^B - \frac{1}{4}h^{ab}K_a^V K_b^H$. For the F-term part of the vectors, we get instead $2h^{ab}(P_a - ie_a)(\bar{P}_b + i\bar{e}_b) = \frac{1}{2}h^{ab}(P_a^1 P_b^1 + P_a^2 P_b^2)$. Finally for the D-term part it is convenient to consider separately the three types of terms that arise respectively from hyper multiplets, from vector multiplets and from their interference. For the vector multiplet part, we have as before $\frac{1}{8}h^{ab}K_a^V\bar{K}_b^V = \frac{1}{2}g_{i\bar{j}}X_a^i\bar{L}^a\bar{X}_b^{\bar{j}}L^b = \frac{1}{2}h^{ab}P_a^0 P_b^0$. For the hyper multiplet part, we get $\frac{1}{8}h^{ab}(K_a^H + \xi_a)(K_b^H + \xi_b) = \frac{1}{2}h^{ab}P_a^3 P_b^3$. Finally, for the mixed part we get $\frac{1}{4}h^{ab}K_a^V(K_b^H + \xi_b) = \frac{1}{4}h^{ab}K_a^V K_b^H$. Collecting the above results for the three terms in (6.17), we see that the interference terms involving $h^{ab}K_a^V K_b^H$ cancel out, and the scalar potential can finally be rewritten in the following form:

$$
\begin{aligned}
V_{\rm S} &= g_{UV}X_A^U\bar{L}^A X_B^V L^B + \frac{1}{2}g_{i\bar{j}}X_a^i\bar{L}^a\bar{X}_b^{\bar{j}}L^b + \frac{1}{2}h^{ab}P_a^x P_b^x \\
&= g_{UV}X_A^U\bar{L}^A X_B^V L^B + \frac{1}{2}h^{ab}\bigl(P_a^0 P_b^0 + P_a^x P_b^x\bigr).
\end{aligned} \tag{6.39}
$$

Notice also that the equivariance conditions (6.12)–(6.14) can be rewritten in the following more compact form:

$$
ig_{i\bar{j}}X_{[a}^i\bar{X}_{b]}^{\bar{j}} = \frac{1}{2}f_{ab}{}^c P_c^0, \quad J_{UV}^x X_{[A}^U X_{B]}^V = f_{AB}{}^C P_C^x. \tag{6.40}
$$

Here $f_{AB}{}^C$ denote the structure constants of the group $G \times U(1)$ defined by the gauge group G and the $U(1)$ central charge symmetry, such that $f_{ab}{}^c$ are the structure constants of the gauge group and $f_{0a}{}^b = f_{ab}{}^0 = 0$. This rewriting reflects once again the fact that the superpotential for the hyper multiplets comes in supergravity from a gauging of the central charge by the graviphoton A_μ^0, which is then treated on equal footing with the

other gauge fields A^a_μ. It also shows that in order for the graviphoton gauging to leave a remnant in the rigid limit, it must be associated to a factorized $U(1)$.

6.1 Supertrace

At a generic point in the scalar field space and for vanishing fermions and vector fields, the auxiliary fields simplify to

$$F^u = \sqrt{2}\,\bar\Omega^u_{\bar v}\bar X^{\bar v}_A\bar L^A\,, \tag{6.41}$$

$$F^i = \sqrt{2}i\,\bar f^{ia}(\bar P_a + i\bar e_a)\,, \tag{6.42}$$

$$D^a = -\frac{1}{2}h^{ab}(K^H_a + K^V_a + \xi_b)\,. \tag{6.43}$$

The corresponding hatted quantities similarly simplify to

$$\hat F^u = \bar\Omega^u_{\bar v}\big(\bar F^{\bar v} + \sqrt{2}i(L^A - \bar L^A)P^{\bar v}_A\big) = -\sqrt{2}\,X^u_A\bar L^A\,, \tag{6.44}$$

$$\hat F^i = \frac{i}{\sqrt{2}}f^i_a\big(D^a + h^{ab}K^V_b\big) = \frac{i}{\sqrt{8}}\bar f^{ia}(K^V_a - K^H_a - \xi_a)\,, \tag{6.45}$$

$$\hat D^a = -\sqrt{2}i\,\bar f^a_{\bar\imath}\bar F^{\bar\imath} = -2\,h^{ab}(P_b - ie_b)\,. \tag{6.46}$$

The mass matrix of the scalar fields is given by

$$\begin{aligned}(m^2_0)_{u\bar v} &= 2\,\nabla_u X^w_A L^A\nabla_{\bar v}\bar X_{Bw}\bar L^B + 2\,\Omega_{us}\bar\Omega_{\bar v\bar t}h^{ab}X^s_a\bar X^{\bar t}_b \\ &\quad - R_{u\bar v s\bar t}F^s\bar F^{\bar t} + h^{ab}\bar X_{au}X_{b\bar v} + \frac{i}{2}\nabla_u X_{a\bar v}D^a + \text{h.c.}\,,\end{aligned} \tag{6.47}$$

$$\begin{aligned}(m^2_0)_{i\bar\jmath} &= -R_{i\bar\jmath k\bar l}\big(F^k\bar F^{\bar l} + f^k_a\bar f^a_{\bar m}\bar f^{\bar l}_b f^b_n F^n\bar F^{\bar m} + f^k_a\bar f^{\bar l}_b D^a D^b\big) + 2\,f^a_i\bar f^b_{\bar\jmath}X^s_a\bar X_{bs} \\ &\quad + h^{ab}\bar X_{ai}X_{b\bar\jmath} + \frac{i}{2}\big(\nabla_i X_{a\bar\jmath} - 2h^{bc}h_{abi}X_{c\bar\jmath}\big)D^a + \text{h.c.}\,,\end{aligned} \tag{6.48}$$

$$\begin{aligned}(m^2_0)_{u\bar\imath} &= 2\,\nabla_u X_{A\bar v}L^A\bar X^{\bar v}_b\bar f^b_{\bar\imath} - \sqrt{2}i\,\Omega_{us}X^s_a\bar C_{\bar\imath\bar\jmath\bar k}f^{\bar\jmath a}\bar f^{\bar k}_b f^b_l F^l \\ &\quad + h^{ab}\bar X_{au}X_{b\bar\imath} + ih^{bc}h_{ab\bar\imath}\bar X_{cu}D^a\,,\end{aligned} \tag{6.49}$$

$$(m^2_0)_{uv} = -2\,R_{u\bar s v\bar t}\bar X^{\bar s}_A L^A\bar X^{\bar t}_B\bar L^B + \sqrt{2}\,\Omega_{us}\nabla_v X^s_a f^a_i F^i - h^{ab}\bar X_{au}\bar X_{bv} + \Gamma^k_{uv}V_{Sk}\,, \tag{6.50}$$

$$\begin{aligned}(m^2_0)_{ij} &= \frac{i}{2}\nabla_i C_{jkl}\big(2\,f^k_a\bar f^a_{\bar q}F^l\bar F^{\bar q} + f^k_a f^l_b D^a D^b\big) + \sqrt{2}i\,\Omega_{us}X^s_a C_{ijk}\bar f^{ka}F^u \\ &\quad - h^{ab}\bar X_{ai}\bar X_{bj} + 2ih^{bc}h_{ab(i}\bar X_{cj)}D^a + \Gamma^k_{ij}V_{Sk}\,,\end{aligned} \tag{6.51}$$

$$(m^2_0)_{ui} = \sqrt{2}\,\Omega_{us}\big(\nabla_v X^s_a f^a_i F^v + iX^s_a C_{ijk}\bar f^{ja}F^k\big) - h^{ab}\bar X_{au}\bar X_{bi} + i\,h^{bc}h_{abi}\bar X_{cu}D^a\,. \tag{6.52}$$

The mass matrix of the fermions is instead

$$(m_{1/2})_{uv} = -\sqrt{2}\,\Omega_{(uw}\nabla_{v)}X^w_A L^A\,, \tag{6.53}$$

$$(m_{1/2})_{ij} = -iC_{ijk}f^k_a\bar f^a_{\bar l}\bar F^{\bar l}\,, \tag{6.54}$$

$$(m_{1/2})_{ab} = -iC_{ijk}f^i_a f^j_b F^k\,, \tag{6.55}$$

$$(m_{1/2})_{ia} = \sqrt{2}\,\bar X_{ai} - \frac{1}{\sqrt{2}}C_{ijk}f^j_a f^k_b D^b\,, \tag{6.56}$$

$$(m_{1/2})_{ui} = -\sqrt{2}\,\Omega_{uv}X^v_a f^a_i\,, \tag{6.57}$$

$$(m_{1/2})_{ua} = \sqrt{2}\,\bar X_{au}\,. \tag{6.58}$$

Finally, the mass matrix of the vectors is

$$(m_1^2)_{ab} = 2 \left(X_{(a}^u \bar{X}_{b)u} + X_{(a}^i \bar{X}_{b)i} \right). \tag{6.59}$$

A straightforward computation gives:

$$\mathrm{tr}[m_0^2] = 4\,\nabla_u X_A^v L^A \nabla^u \bar{X}_{Bv} \bar{L}^B + 2\,R_{i\bar{\jmath}} \left(F^i \bar{F}^{\bar{\jmath}} + f_a^i \bar{f}_{\bar{q}}^a \bar{f}_b^{\bar{\jmath}} f_p^b F^p \bar{F}^{\bar{q}} + f_a^i \bar{f}_b^{\bar{\jmath}} D^a D^b \right)$$
$$+ 2\,h^{ab} \left(5\,\bar{X}_{au} X_b^u + \bar{X}_{ai} X_b^i \right) - 4ih^{bc} h_{abi} X_c^i D^a + \text{h.c.}\,, \tag{6.60}$$

$$\mathrm{tr}[m_{1/2}^2] = 2\,\nabla_u X_A^v L^A \nabla^u \bar{X}_{Bv} \bar{L}^B + R_{i\bar{\jmath}} \left(F^i \bar{F}^{\bar{\jmath}} + f_a^i \bar{f}_{\bar{q}}^a \bar{f}_b^{\bar{\jmath}} f_p^b F^p \bar{F}^{\bar{q}} + f_a^i \bar{f}_b^{\bar{\jmath}} D^a D^b \right)$$
$$+ 4\,h^{ab} \left(2\,\bar{X}_{au} X_b^u + \bar{X}_{ai} X_b^i \right) - 2i\,h^{ab} h_{bci} X_a^i D^c + \text{h.c.}\,, \tag{6.61}$$

$$\mathrm{tr}[m_1^2] = 2\,h^{ab} \left(\bar{X}_{au} X_b^u + \bar{X}_{ai} X_b^i \right). \tag{6.62}$$

It follows that the supertrace of the mass matrix vanishes [13]:

$$\mathrm{str}[m^2] \equiv \mathrm{tr}[m_0^2] - 2\,\mathrm{tr}[m_{1/2}^2] + 3\,\mathrm{tr}[m_1^2]$$
$$= 0\,. \tag{6.63}$$

This result also follows directly from (3.31) and the properties that the Christoffel symbols are related to the derivative of the gauge kinetic function, the special form of the Ricci tensor and finally that the trace of the charge matrix satisfies the generalization $\nabla_u X_a^u = 0$ of (4.27) in the hyper multiplet sector and (5.31) in the vector multiplet sector.

6.2   Metastability

The possible vacua of the theory correspond to points in the scalar manifold that satisfy the stationarity conditions $V_{\mathrm{S}u} = V_{\mathrm{S}i} = 0$, which imply

$$-\sqrt{2}\,\Omega_{uw} \left(\nabla_v X_A^w L^A F^v + X_a^w f_i^a F^i \right) + i\bar{X}_{au} D^a = 0\,, \tag{6.64}$$

$$-\frac{i}{2} C_{ijk} \left(2\,f_a^j \bar{f}_{\bar{q}}^a F^k \bar{F}^{\bar{q}} + f_a^j f_b^k\,D^a D^b \right) - \sqrt{2}\,\Omega_{uw} X_a^w f_i^a F^u + i\bar{X}_{ai} D^a = 0 \tag{6.65}$$

The relation (3.33) between the values of the F^i, F^u and D^a auxiliary fields, becomes

$$i\left(\nabla_u X_{a\bar{v}} F^u \bar{F}^{\bar{v}} + \left(X_a^k h_{bck} f_i^b \bar{f}_{\bar{\jmath}}^c + f_{ac}{}^b f_i^c f_{\bar{\jmath}b} \right) F^i \bar{F}^{\bar{\jmath}} \right)$$
$$- \left(X_{(a}^u \bar{X}_{b)u} + X_{(a}^i \bar{X}_{b)i} \right) D^b + \frac{1}{2} f_{ab}{}^d k_{dc}\,D^b D^c = 0\,. \tag{6.66}$$

On the vacuum, one has $\delta\chi^u = \sqrt{2}\epsilon F^u$, $\delta\psi^i = \sqrt{2}\epsilon F^i$, $\delta\lambda^a = i\epsilon D^a$ and similarly $\hat{\delta}\chi^u = \sqrt{2}\hat{\epsilon}\hat{F}^u$, $\hat{\delta}\psi^i = \sqrt{2}\hat{\epsilon}\hat{F}^i$, $\hat{\delta}\lambda^a = i\hat{\epsilon}\hat{D}^a$, and the first and second supersymmetries are spontaneously broken respectively if some of the auxiliary fields F^u, F^i, D^a or some of the $\hat{F}^u$, $\hat{F}^i$, $\hat{D}^a$ are non-vanishing. The order parameters are the norms of the vectors defined by these two sets of quantities. Thanks to the identities $\hat{F}^u \hat{\bar{F}}_u = F^u \bar{F}_u + \frac{1}{2} K^{aV} K_a^H$, $\hat{F}^i \hat{\bar{F}}_i = \frac{1}{2} D^a D_a - \frac{1}{2} K^{aV} K_a^H$ and $\frac{1}{2} \hat{D}^a \hat{D}_a = F^i \bar{F}_i$, these two norms do actually as before coincide, and define in two equivalent ways related to the two supersymmetries the scalar potential energy: $V_{\mathrm{S}} = F^u \bar{F}_u + F^i \bar{F}_i + \frac{1}{2} D^a D_a = \hat{F}^u \hat{\bar{F}}_u + \hat{F}^i \hat{\bar{F}}_i + \frac{1}{2} \hat{D}^a \hat{D}_a$. In such a

situation, there are then as usual two massless Goldstini, associated to the two independent supersymmetries and given by:

$$\eta = \sqrt{2}\bar{F}_u\chi^u + \sqrt{2}\bar{F}_i\psi^i + iD_a\lambda^a\,, \quad \hat{\eta} = \sqrt{2}\hat{\bar{F}}_u\chi^u + \sqrt{2}\hat{\bar{F}}_i\psi^i + i\hat{D}_a\lambda^a\,, \tag{6.67}$$

With a bit of work one can verify that the stationarity conditions and the various identities related to gauge invariance imply that these are always flat directions of the fermion mass matrix:

$$m_\eta = 0\,, \quad m_{\hat{\eta}} = 0\,. \tag{6.68}$$

In the situation under consideration, the two supersymmetries may again only be broken simultaneously.[8] As before, the sGoldstini are linear combinations of scalars and vectors, but what is relevant is their projection along the scalar field space. One then gets four independent real linear combinations, corresponding to the projection of the complex Goldstino vectors $\eta^\theta = (\sqrt{2}F^u, \sqrt{2}F^i)$ and $\hat{\eta}^\theta = (\sqrt{2}\hat{F}^u, \sqrt{2}\hat{F}^i)$:

$$\varphi_+ = \bar{F}_u q^u + \bar{F}_i \phi^i + F_{\bar{u}}\bar{q}^{\bar{u}} + F_{\bar{\imath}}\bar{\phi}^{\bar{\imath}}\,, \quad \varphi_- = i\bar{F}_u q^u + i\bar{F}_i\phi^i - iF_{\bar{u}}\bar{q}^{\bar{u}} - iF_{\bar{\imath}}\bar{\phi}^{\bar{\imath}}\,, \tag{6.69}$$

$$\hat{\varphi}_+ = \hat{\bar{F}}_u q^u + \hat{\bar{F}}_i \phi^i + \hat{F}_{\bar{u}}\bar{q}^{\bar{u}} + \hat{F}_{\bar{\imath}}\bar{\phi}^{\bar{\imath}}\,, \quad \hat{\varphi}_- = i\hat{\bar{F}}_u q^u + i\hat{\bar{F}}_i\phi^i - i\hat{F}_{\bar{u}}\bar{q}^{\bar{u}} - i\hat{F}_{\bar{\imath}}\bar{\phi}^{\bar{\imath}}\,. \tag{6.70}$$

The masses of these four scalar modes can now be computed by evaluating the scalar mass matrix along the directions $\varphi_+^\Theta = (F^u, F^i, \bar{F}^{\bar{u}}, \bar{F}^{\bar{\imath}})$, $\varphi_-^\Theta = (iF^u, iF^i, -i\bar{F}^{\bar{u}}, -i\bar{F}^{\bar{\imath}})$, $\hat{\varphi}_+^\Theta = (\hat{F}^u, \hat{F}^i, \hat{\bar{F}}^{\bar{u}}, \hat{\bar{F}}^{\bar{\imath}})$ and $\hat{\varphi}_-^\Theta = (i\hat{F}^u, i\hat{F}^i, -i\hat{\bar{F}}^{\bar{u}}, -i\hat{\bar{F}}^{\bar{\imath}})$, and dividing by the length of these vectors, which is $2(F^u\bar{F}_u + F^i\bar{F}_i)$ for the first two and $2(\hat{F}^u\hat{\bar{F}}_u + \hat{F}^i\hat{\bar{F}}_i)$ for the last two.

One may at this point proceed in computing more explicitly the above sGoldstino masses and trying to simplify them as much as possible, in order to extract some information that has a simple-enough form to be useful. We will not attempt to do this here, but hope to examine this problem elsewhere, now that it has been set up in full detail within $N = 2$ rigid supersymmetry. We again expect only one $SU(2)$-invariant information, generalizing those found for situations involving only hyper multiplets or only vector multiplets. It is however not entirely obvious how to proceed to extract such an information within the $N = 1$ superspace formalism used in this paper, where the $SU(2)$ symmetry is not manifest. In particular, the way the four sGoldstini must be combined to yield this $SU(2)$-invariant information cannot be easily determined a priori, and as a matter of fact it looks different in the two subcases involving respectively only hyper or only vector multiplets. We believe that to clarify this issue it might be useful to compare with a manifestly $SU(2)$-covariant formalism, like for instance the on-shell approach of [14, 15].

7   Conclusion

In this work, we have performed a general study of the conditions under which vacua breaking spontaneously supersymmetry may be at least metastable, in the context of

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[8]See [55] for a recent systematic discussion on the conditions under which one may have partial supersymmetry breaking. As explained in previous section, at stationary points with such a partial supersymmetry breaking, the two Goldstini must become degenerate and represent only one massless Goldstone fermion.

general $N = 2$ non-linear sigma-models. To do so we have relied on a construction of these models based on $N = 1$ superspace, which allows to emphasize their peculiarities as special cases of $N = 1$ non-linear sigma-models. We have then systematically applied to these models the strategy of looking at the masses of the scalar modes belonging to the Goldstino would-be multiplet, which are the most dangerous modes for metastability.

We have been able to reproduce the two known no-go theorems available in the supergravity context, concerning theories with only hyper multiplets [8] and only Abelian vector multiplets [9]. We have then clarified the origin of these sharp results, taking the perspective that such theories are particular cases of $N = 1$ theories involving only chiral multiplets, where supersymmetry breaking is controlled only by F auxiliary fields. We have then studied in quite some detail the case of theories with only vector multiplets but with general non-Abelian gaugings, giving evidence that no obstruction against achieving metastability subsists in this case. From the $N = 1$ perspective, these are special classes of theories involving chiral multiplets in the adjoint representation and vector multiplets, where supersymmetry breaking is controlled not only by F auxiliary fields but also by D auxiliary fields. Finally, we have set up the study of general theories involving both hyper and vector multiplets, although we did not present any simple general result in this case. From the $N = 1$ point of view, these are particular cases of theories involving chiral multiplets both in the adjoint representation and in more general representation, as well as vector multiplet, where the process of supersymmetry breaking is controlled both by F and D auxiliary fields. We think that the effect of the latter should generically allow for metastable supersymmetry breaking vacua, since for general $N = 1$ theories it is known to systematically improve the situation compared to the effect of the former.

We believe that the results derived in this paper should be useful to address the general question of what are the mandatory ingredients to obtain metastable de Sitter vacua in $N = 2$ supergravity theories. The results that we have obtained in the analysis of the corresponding problem in the rigid limit suggest that the only necessary ingredient is that from the $N = 1$ perspective supersymmetry breaking should receive not only F-type but also D-type contributions. This requires either non-Abelian gauge groups, or charged hyper multiplets, or both of these ingredients.

Concerning the implications of the necessary conditions for metastable supersymmetry breaking for potentially realistic string models, one should keep in mind that these are described by $N = 1$ effective theories, but with a hidden sector that displays many features of $N = 2$ or even $N = 4$ models. As a result, applying the $N = 1$ constraints is too optimistic, whereas applying $N = 2$ or even $N = 4$ constraints is too restrictive. One may then try to consider the intermediate framework of $N = 1$ theories obtained by truncations of $N = 2$ or $N = 4$ supersymmetries. In this kind of truncations, the projection getting rid of the additional supersymmetries also eliminates the corresponding additional sGoldstini and the resulting implications on metastability. As a result, one should get conditions that are stronger but have the same form as those for general $N = 1$ theories. These should account for the possibility of starting from an unstable supersymmetry breaking vacuum and getting a metastable one by a truncation, where the tachyonic sGoldstini are projected out. For instance, it has been recently shown in [56] that the metastable $N = 2$ de Sitter vacua of [10] can be obtained by truncations of the unstable $N = 4$ de

Sitter vacua of [57, 58], which can themselves be related to truncations of the unstable $N = 8$ de Sitter vacua discussed in [59]. It should be similarly possible to construct stable $N = 1$ de Sitter vacua by truncating unstable $N = 2$ de Sitter vacua. Since a detailed general description of this kind of truncations is available [53, 54], it would be interesting to perform a general study of the metastability conditions in this case.

During the completion of this work, the interesting paper [60] appeared, which explores the possibility of constructing a low-energy effective description of $N = 2$ theories below the supersymmetry breaking scale in terms of constrained superfields, containing only the two Goldstini and no other light state. It was found that under the assumption of an $SU(2)_R$ symmetry, such an effective theory does not exist. This fact was interpreted as signaling the impossibility of achieving metastable supersymmetry breaking in such $N = 2$ theories. This is compatible with what we found in this paper for theories involving only hyper multiplets or only vector multiplets without $N = 2$ Fayet-Iliopoulos terms, where some of the sGoldstini are unavoidably massless or tachyonic. We believe that the algebraic obstruction uncovered in [60] should correspond to the physical obstruction studied here against achieving a positive mass squared for all the sGoldstini, since whenever one of the sGoldstini is tachyonic one clearly cannot define a sensible low-energy effective theory for just the Goldstini. It would be very interesting to make this connection more precise and try to exploit it to study more efficiently the most general case of $N = 2$ theories involving both hyper and vector multiplets as well as $SU(2)_R$-breaking Fayet-Iliopoulos terms.

Acknowledgements

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