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Robust Nonlinear Feedback Control of Discrete-Time Nonlinear Systems with Mixed Performance Criteria

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***Abstract*— A novel nonlinear state feedback control design is presented for discrete-time nonlinear systems and mixed performance criteria. The purpose behind this new approach is to convert a nonlinear system control design into a convex optimization problem involving state dependent linear matrix inequality solutions. By solving the inequalities at each time step, the optimal control solution is found to satisfy mixed performance criteria guaranteeing quadratic optimality with inherent stability property in combination with H_∞ or a passivity type of disturbance reduction. The effectiveness of the proposed technique is demonstrated by simulations involving the control of a benchmark mechanical system.**

I. Introduction

In this paper, we aim to address nonlinear state feedback control design of discrete-time nonlinear control systems using the state-dependent Linear Matrix Inequalities (LMI) approach. We characterize the solution of the nonlinear discrete-time control system with a state dependent LMI, which are essentially equivalent to the discrete-time version of classical Hamilton-Jacobi Inequalities (HJI) [1]-[3]. As a precursor to this approach, few examples of state dependent Riccati equation control approach can be found in [4]-[6]. A preliminary investigation into the state dependent LMI approach to nonlinear systems can be found in [7], [8]. The purpose behind this novel approach is to convert a nonlinear system control problem into a convex optimization problem which is solved by LMI at each time step. The recent development in numerical algorithms for solving convex optimization provides very efficient means for solving LMI. If a solution can be expressed in LMI form, then there exist efficient algorithms providing global numerical solutions [9]. Therefore if the design LMI are feasible, then state-dependent LMI control technique provides global optimal solutions for nonlinear control systems. *Mixed performance criteria are used to design the controller in order to guarantee quadratic optimality with inherent stability property in combination with H_∞ or a passivity type of disturbance attenuation.*

In the following section, we introduce the discrete system model and the performance index satisfying mixed performance criteria. Then, LMI control solution derivation is presented, which characterizes the optimal and robust control of nonlinear systems. We further examine the properties of this powerful alternative to the HJI technique. Extensive simulations have been used to examine the effectiveness of the new state dependent LMI control technique. Only the inverted pendulum control problem is used in this paper as an illustrative example due to space limitations.

The following notation is used in this work: $x \in \Re^n$ denotes n-dimensional real vector with norm $\|x\| = (x^T x)^{1/2}$ where $(\cdot)^T$ indicates transpose. $A \geq 0$ for a symmetric matrix denotes a positive semi-definite matrix. L_2 is the space of infinite sequences of finite dimensional vectors with finite energy: $\sum_{k=0}^{\infty} \|x_k\|^2 < \infty$.

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II. System Model And Performance Index

Consider the input-affine discrete time nonlinear system represented by the following difference equation:

$$x_{k+1} = f(x_k) + B(x_k)\cdot u_k + F(x_k)\cdot w_k = A_k \cdot x_k + B_k \cdot u_k + F_k \cdot w_k \tag{1}$$

where

$x_k \in \Re^n$: state vector

$u_k \in \Re^m$: applied input

$w_k \in \Re^q$: L_2 type of disturbance

A_k, B_k, F_k: known coefficient matrices of appropriate dimensions, which can be functions of x_k

Note that the simplified notation for time varying matrices A_k, B_k, etc. is used to denote the state dependent matrices. The performance output $z_k \in \Re^p$ is

$$z_k = C(x_k)\cdot x_k + D(x_k)\cdot w_k = C_k \cdot x_k + D_k \cdot w_k \tag{2}$$

where C_k, D_k are, in general, state dependent coefficient matrices of appropriate dimensions.

It is assumed that the state feedback is available and the nonlinear state feedback control input is given by

$$u_k = K(x_k)\cdot x_k = K_k \cdot x_k \tag{3}$$

Consider the quadratic energy function

$$V_k = x_k^T P_k x_k > 0$$

(4)

for the following difference inequality

$$V_{k+1} - V_k + x_k^T Q_k x_k + u_k^T R_k u_k + \alpha \cdot z_k^T z_k - \beta \cdot z_k^T w_k + \gamma \cdot w_k^T w_k \le 0 \quad (5)$$

with $Q_k > 0, R_k > 0$ being functions of x_k, in general, and the model (1)-(3).

Note that upon summation over k, (5) yields

$$V_N + \sum_{k=0}^{N-1} \left[x_k^T Q_k x_k + u_k^T R_k u_k + \alpha \cdot z_k^T z_k - \beta \cdot z_k^T w_k + \gamma \cdot w_k^T w_k \right] \le V_0 \quad (6)$$

By properly specifying the value of the weighing matrices Q_k, R_k, C_k, D_k and α, β, γ, mixed performance criteria can be used in nonlinear control design, which yields a mixed Nonlinear Quadratic Regulator (NLQR) in combination with H_∞ or passivity performance index. For example, if we take $\alpha = 1, \beta = 0, \gamma < 0$, (6) yields

$$V_N + \sum_{k=0}^{N-1} \left[x_k^T Q_k x_k + u_k^T R_k u_k + z_k^T z_k \right] \le V_0 - \gamma \cdot \sum_{k=0}^{N-1} w_k^T w_k \quad (7)$$

which is mixed suboptimal NLQR- H_∞ design [10].

The possible performance criteria which can be used in this framework with different design parameters α, β, γ are given in Table.1. *Note also that for* $\alpha = 0, \beta = 0, \gamma = 0$, *it follows from Eqn. (7) that* $\sum_{k=0}^{\infty} \|x_k\|^2 < \infty$ *and therefore, the controlled system is exponentially asymptotically stable* [10] *for all of the criteria given in the table.*

Table.1. Various performance criteria in a general framework

α	β	γ	Performance criteria
1	0	<0	Suboptimal NLQR- H_∞ Design
0	1	0	NLQR-Passivity Design
0	1	>0	NLQR-Input Strict Passivity Design
>0	1	0	NLQR-Output Strict Passivity Design
>0	1	>0	NLQR-Very Strict Passivity

III. MAIN RESULTS

The following theorem summarizes the main results of the paper:

Theorem—Given the system Eqn.(1), performance output Eqn.(2), control equation (3) and performance index (6), if there exist matrices $M_k = P_k^{-1} > 0$ and Y_k for all $k \ge 0$, such that the following state dependent LMI hold:

$$\begin{bmatrix} M_k & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} \\ * & \Xi_{22} & \Xi_{23} & 0 & 0 & 0 \\ * & * & M_k & 0 & 0 & 0 \\ * & * & * & I_n & 0 & 0 \\ * & * & * & * & I_m & 0 \\ * & * & * & * & * & I_p \end{bmatrix} \ge 0 \quad (8)$$

where

$$\Xi_{12} = -\alpha M_k C_k^T D_k + 0.5 \cdot \beta M_k C_k^T$$
$$\Xi_{13} = M_k A_k + Y_k^T B_k^T$$
$$\Xi_{14} = M_k Q_k^{T/2}$$
$$\Xi_{15} = Y_k^T R_k^{T/2}$$
$$\Xi_{16} = \alpha^{1/2} M_k C_k^T$$
$$\Xi_{22} = -\gamma I - \alpha D_k^T D_k + 0.5 \cdot \beta \left(D_k + D_k^T \right)$$
$$\Xi_{23} = F_k^T \quad (9)$$

and

$$M_{k+1} \ge M_k \quad (10)$$

then inequality (6) is satisfied. The nonlinear feedback gain of the controller is given by

$$K_k = Y_k \cdot P_k \quad (11)$$

□

Proof

By applying system Eqn.(1), performance output Eqn.(2) and state feedback input Eqn.(3), the inequality (5) becomes

$$\begin{aligned} &\left(A_k \cdot x_k + B_k \cdot u_k + F_k \cdot w_k\right)^T P_{k+1} \left(A_k \cdot x_k + B_k \cdot u_k + F_k \cdot w_k\right) \\ &- x_k^T P_k x_k + x_k^T Q_k x_k + u_k^T R_k u_k + \\ &\alpha \cdot \left(C_k \cdot x_k + D_k \cdot w_k\right)^T \left(C_k \cdot x_k + D_k \cdot w_k\right) \\ &- \beta \cdot \left(C_k \cdot x_k + D_k \cdot w_k\right)^T w_k + \gamma \cdot w_k^T w_k \le 0 \end{aligned} \quad (12)$$

Equivalently,

$$[x_k^T \quad w_k^T] \Psi [x_k \quad w_k]^T = [x_k^T \quad w_k^T] \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ * & \Psi_{22} \end{bmatrix} \begin{bmatrix} x_k \\ w_k \end{bmatrix} \le 0 \quad (13)$$

where

$$\begin{aligned} \Psi_{11} &= \left(A_k + B_k K_k\right)^T P_{k+1} \left(A_k + B_k K_k\right) - P_k + Q_k \\ &\quad + K_k^T R_k K_k + \alpha C_k^T C_k \\ \Psi_{12} &= \left(A_k + B_k K_k\right)^T P_{k+1} F_k + \alpha C_k^T D_k - 0.5 \cdot \beta C_k^T \\ \Psi_{22} &= F_k^T P_{k+1} F_k + \alpha D_k^T D_k + \gamma I - 0.5 \cdot \beta \left(D_k + D_k^T\right) \end{aligned} \quad (14)$$

Therefore, (6) is equivalent to matrix $\Psi \le 0$, which is equivalent to the inequality

$$\begin{bmatrix} P_k - Q_k - K_k^T R_k K_k - \alpha C_k^T C_k & -\alpha C_k^T D_k + 0.5 \cdot \beta C_k^T \\ -\alpha D_k^T C_k + 0.5 \cdot \beta C_k & -\gamma I + 0.5 \cdot \beta \left(D_k + D_k^T \right) - \alpha D_k^T D_k \end{bmatrix}$$

$$-\begin{bmatrix} (A_k + B_k K_k)^T \\ F_k^T \end{bmatrix} P_{k+1} \left[(A_k + B_k K_k) \quad F_k \right] \geq 0 \tag{15}$$

By adding and subtracting the same term in (15), the following inequality results

$$\begin{bmatrix} P_k - Q_k - K_k^T R_k K_k - \alpha C_k^T C_k & -\alpha C_k^T D_k + 0.5 \cdot \beta C_k^T \\ -\alpha D_k^T C_k + 0.5 \cdot \beta C_k & -\gamma I + 0.5 \cdot \beta \left(D_k + D_k^T \right) - \alpha D_k^T D_k \end{bmatrix}$$

$$-\begin{bmatrix} (A_k + B_k K_k)^T \\ F_k^T \end{bmatrix} (P_{k+1} - P_k) \left[(A_k + B_k K_k) \quad F_k \right] -$$

$$\begin{bmatrix} (A_k + B_k K_k)^T \\ F_k^T \end{bmatrix} P_k \left[(A_k + B_k K_k) \quad F_k \right] \geq 0 \tag{16}$$

Therefore, subject to $P_{k+1} \leq P_k$, (16) can be rewritten as

$$\begin{bmatrix} P_k - Q_k - K_k^T R_k K_k - \alpha C_k^T C_k & -\alpha C_k^T D_k + 0.5 \cdot \beta C_k^T \\ -\alpha D_k^T C_k + 0.5 \cdot \beta C_k & -\gamma I + 0.5 \cdot \beta \left(D_k + D_k^T \right) - \alpha D_k^T D_k \end{bmatrix}$$

$$-\begin{bmatrix} (A_k + B_k K_k)^T \\ F_k^T \end{bmatrix} P_k \left[(A_k + B_k K_k) \quad F_k \right] \geq 0 \tag{17}$$

By applying Schur complement result [9], we obtain

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ * & \Gamma_{22} & \Gamma_{23} \\ * & * & \Gamma_{33} \end{bmatrix} \geq 0 \tag{18}$$

where

$$\begin{aligned} \Gamma_{11} &= P_k - Q_k - K_k^T R_k K_k - \alpha C_k^T C_k \\ \Gamma_{12} &= -\alpha C_k^T D_k + 0.5 \cdot \beta C_k^T \\ \Gamma_{13} &= (A_k + B_k K_k)^T P_k \\ \Gamma_{22} &= -\gamma I + 0.5 \cdot \beta \left(D_k + D_k^T \right) - \alpha D_k^T D_k \\ \Gamma_{23} &= F_k^T P_k \\ \Gamma_{33} &= P_k \end{aligned} \tag{19}$$

By pre-multiplying and post-multiplying the matrix with block diagonal matrix $diag\{M_k, I, M_k\}$, where $M_k = P_k^{-1}$, the following inequality follows

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ * & \Theta_{22} & \Theta_{23} \\ * & * & \Theta_{33} \end{bmatrix} \geq 0 \tag{20}$$

where

$$\begin{aligned} \Theta_{11} &= M_k - M_k \left(Q_k + K_k^T R_k K_k + \alpha C_k^T C_k \right) M_k \\ \Theta_{12} &= -\alpha M_k C_k^T D_k + 0.5 \cdot \beta M_k C_k^T \\ \Theta_{13} &= M_k (A_k + B_k K_k)^T \\ \Theta_{22} &= -\gamma I + 0.5 \cdot \beta \left(D_k + D_k^T \right) - \alpha D_k^T D_k \\ \Theta_{23} &= F_k^T \\ \Theta_{33} &= M_k \end{aligned} \tag{21}$$

Equivalently,

$$\begin{bmatrix} M_k & \Theta_{12} & \Theta_{13} \\ * & \Theta_{22} & \Theta_{23} \\ * & * & M_k \end{bmatrix} - \begin{bmatrix} M_k Q_k^{T/2} & M_k K_k^T R_k^{T/2} & \alpha^{1/2} M_k C_k^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot$$

$$I_{n+m+p}^{-1} \cdot \begin{bmatrix} Q_k^{1/2} M_k & 0 & 0 \\ R_k^{1/2} K_k M_k & 0 & 0 \\ \alpha^{1/2} C_k M_k & 0 & 0 \end{bmatrix} \geq 0 \tag{22}$$

Finally, by applying Schur complement [9] again, the following LMI result is obtained

$$\begin{bmatrix} M_k & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} \\ * & \Xi_{22} & \Xi_{23} & 0 & 0 & 0 \\ * & * & M_k & 0 & 0 & 0 \\ * & * & * & I_n & 0 & 0 \\ * & * & * & * & I_m & 0 \\ * & * & * & * & * & I_p \end{bmatrix} \geq 0 \tag{23}$$

where

$$\begin{aligned} \Xi_{12} &= -\alpha M_k C_k^T D_k + 0.5 \cdot \beta M_k C_k^T \\ \Xi_{13} &= M_k A_k + Y_k^T B_k^T \\ \Xi_{14} &= M_k Q_k^{T/2} \\ \Xi_{15} &= Y_k^T R_k^{T/2} \\ \Xi_{16} &= \alpha^{1/2} M_k C_k^T \\ \Xi_{22} &= -\gamma I - \alpha D_k^T D_k + 0.5 \cdot \beta \left(D_k + D_k^T \right) \\ \Xi_{23} &= F_k^T \end{aligned} \tag{24}$$

Hence, if the LMI (8) and (10) hold, inequality (6) is satisfied. This concludes the proof of Theorem 1. ■

Remark: For the chosen performance criterion among those in Table 1, the LMI (8) and (10) need to be solved at each time step and the state feedback gain (11) needs to be applied to control system (1) to achieve desired performance.

IV. SIMULATION STUDIES

The inverted pendulum on a cart problem is a classical control problem used widely as a benchmark for testing control algorithms. The pendulum mass is above the pivot point which is mounted on a horizontally moving cart. The problem is related to rocket or missile guidance, where the actuator is operating at the bottom of a long vehicle. It is also used herein to demostrate the effectiveness of the state dependent LMI control approach.

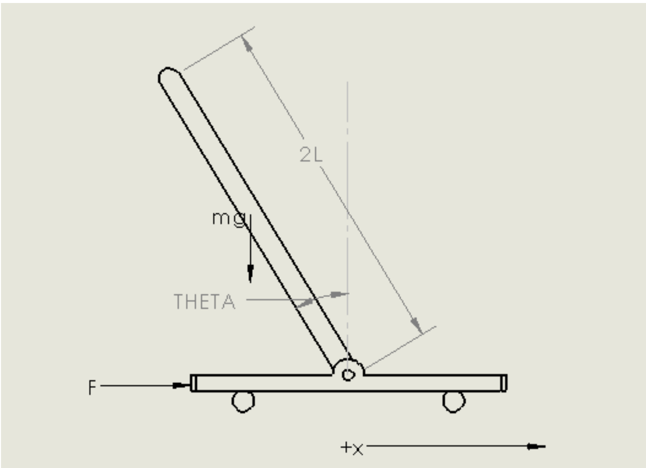

Fig.1. Inverted pendulum system diagram

Fig.1. shows the physical representation of the inverted pendulum system. A beam attached to the cart can rotate freely in the vertical 2-dimensional plane. The angle of the beam with respect to the vertical is denoted by angle θ. The cart moves in the 1-dimensional track, with position x. The external force F, the control input acting on the cart, is used to stabilize this highly nonlinear system while satisfying the mixed performance criteria.

The control objective is to find the state dependent LMI control to set cart position x, velocity of the cart $\dot{x}$, angle of the beam θ and angular velocity $\dot{\theta}$ all to zero while satisfying some chosen optimality criteria.

Traditional nonlinear control techniques assume that θ is a very small angle, $\cos(\theta) \cong 1$ and $\sin(\theta) \cong 0$, then linearize the system equation around its equlibrium point afterwards. Other nonlinear control methods have also been applied [10]. However, it can be shown that the control is not guaranteed to be globally optimal or stable. *In this paper, we will not resort to the usual linearization approach. That is why a detailed account of the system modeling is provided.*

A model of the inverted pendulum problem can be derived using standard techniques [10]:

$$\begin{cases}(M+m)\ddot{x}+b\dot{x}+mL\ddot{\theta}\cos(\theta)-mL\dot{\theta}^2\sin(\theta)=F\\(I+mL^2)\ddot{\theta}+mgL\sin(\theta)+mL\ddot{x}\cos(\theta)=0\end{cases} \tag{25}$$

where

M	mass of the cart
m	mass of the pendulum
b	friction coefficient between cart and ground
L	length to the pendulum center of mass (length of the pendulum equals $2L$)
$I=\frac{1}{3}m(2L)^2$	inertia of the pendulum
F	external force, input ot the system

Denote the following state variables:

$x_{1,k}=x(kT)$, $x_{2,k}=\dot{x}(kT)$, $x_{3,k}=\theta(kT)$, $x_{4,k}=\dot{\theta}(kT)$

By applying Euler discretization method with sampling period T, and using the notation

$$\Omega_1 = I + mL^2 - \frac{m^2L^2\cos^2(x_{3,k})}{M+m}$$

$$\Omega_2 = M + m - \frac{m^2L^2\cos^2(x_{3,k})}{I+mL^2} \tag{26}$$

the discrete-time system equation can be written as

$$\begin{bmatrix}x_{1,k+1}\\x_{2,k+1}\\x_{3,k+1}\\x_{4,k+1}\end{bmatrix}=\begin{bmatrix}1 & T & 0 & 0\\0 & a_{22} & a_{23} & a_{24}\\0 & 0 & 1 & T\\0 & a_{42} & a_{43} & a_{44}\end{bmatrix}\begin{bmatrix}x_{1,k}\\x_{2,k}\\x_{3,k}\\x_{4,k}\end{bmatrix}+\begin{bmatrix}0\\b_2\\0\\b_4\end{bmatrix}u_k \tag{27}$$

where

u_k is the k^{th} sampling instant value of the input force F and

$$a_{22}=1+T\frac{-b}{\Omega_2}$$

$$a_{23}=T\frac{m^2L^2g\cos(x_{3,k})}{\Omega_2(I+mL^2)}\frac{\sin(x_{3,k})}{x_{3,k}}$$

$$a_{24}=T\frac{mL\sin(x_{3,k})}{\Omega_2}x_{4,k}$$

$$a_{42}=T\frac{mLb\cos(x_{3,k})}{(M+m)\Omega_1}$$

$$a_{43}=-T\frac{mgL}{\Omega_1}\frac{\sin(x_{3,k})}{x_{3,k}}$$

$$a_{44}=1-T\frac{m^2L^2\cos(x_{3,k})\sin(x_{3,k})x_{4,k}}{(M+m)\Omega_1}$$

$$b_2=\frac{T}{\Omega_2}$$

$$b_4=-T\frac{mL\cos(x_{3,k})}{(M+m)\Omega_1} \tag{28}$$

It should be noted that this state space formulation does not involve a process of linearization, but a process of state-dependent parameterization. To avoid the division by zero, the term $\frac{\sin(x_{3,k})}{x_{3,k}}$ is substituted for $x_{3,k}=0$ by the limit

$$\lim_{x_{3,k}\to 0}\frac{\sin(x_{3,k})}{x_{3,k}}=1 \quad (29)$$

The following system parameters are assumed

$$M=0.5kg, m=0.5kg, b=0.1N\cdot\frac{\sec}{m}, L=0.3m,$$
$$I=0.06kg\cdot m^2$$

The following design parameters are chosen to satisfy different mixed criteria:

Mixed NLQR- H_∞ Design (Predominant NLQR)

$C=[0.01\quad 0.01\quad 0.01\quad 0.01]$, $D=[0.01]$, $Q=I_4$, $R=1$, $\alpha=1$, $\beta=0$, $\gamma=-5$

Mixed NLQR- H_∞ Design (Predominant H_∞)

$C=[1\quad 1\quad 1\quad 1]$, $D=[1]$, $Q=0.01\times I_4$, $R=0.01$, $\alpha=1$, $\beta=0$, $\gamma=-5$

NLQR-Very Strict Passivity

$C=[1\quad 1\quad 1\quad 1]$, $D=[1]$, $Q=I_4$, $R=1$, $\alpha=0.01$, $\beta=1$, $\gamma=0.01$

The following initial conditions are assumed:

$$x_1=1, x_2=0, x_3=\pi/4, x_4=0$$

All of the above mixed criteria control performance results are shown in the Fig.2-6, in comparison with the traditional Linear Quadratic Regulator (LQR) technique based on linearization [11]. From these figures, we find that the novel state dependent LMI control has better performance compared with the traditional LQR technique based on linearization. Especially, Figs.2, 4 and 5 show that the traditional LQR technique loses control of the state variables. It should also be noted that predominant NLQR and predominant H_∞ control techniques lead to faster response times than the NLQR-passivity technique. Fig.6 shows that the highest magnitude of control is needed by the predominant H_∞ control and the lowest control magnitude is needed by the linearization based LQR technique.

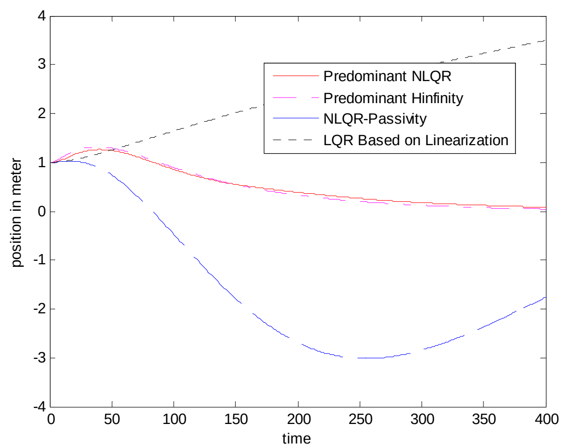

Fig.2. Position trajectory of the inverted pendulum

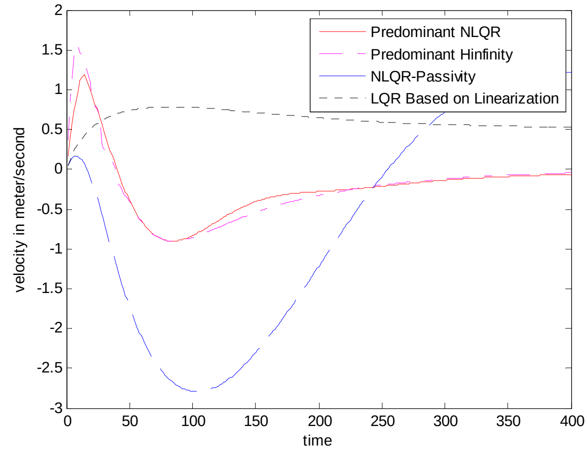

Fig.3. Velocity trajectory of the inverted pendulum

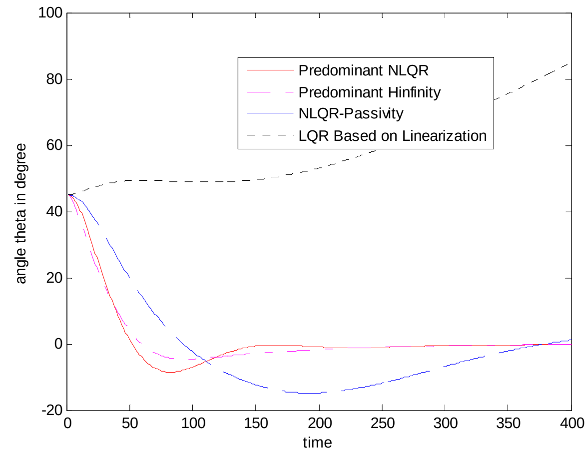

Fig.4. Angle "theta" trajectory of the inverted pendulum

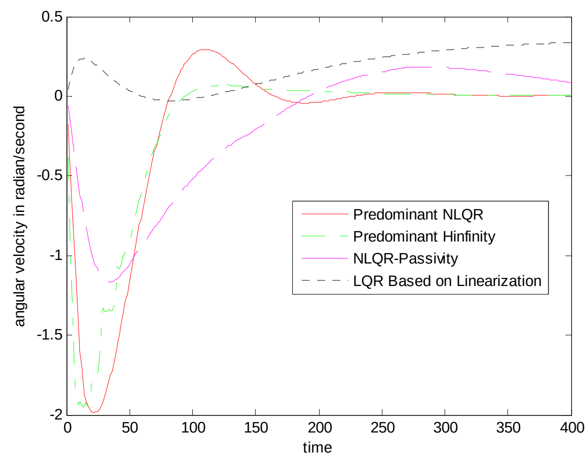

Fig.5. Angular velocity trajectory of the inverted pendulum

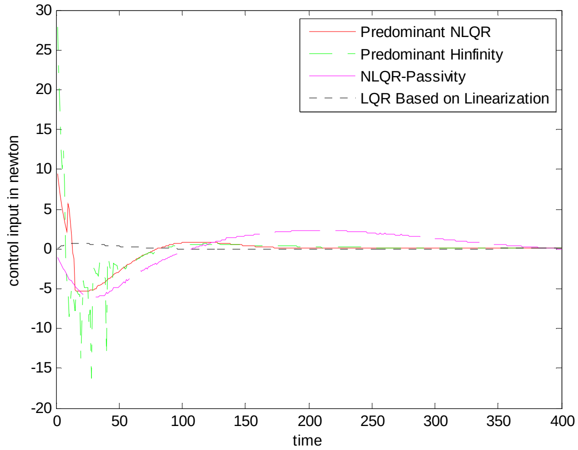

Fig.6. Control input

V. CONCLUSION

This paper has presented a novel discrete time nonlinear system control approach based on the state dependent LMI solutions. Mixed performance criteria are used to design the controller and relative weighting of these criteria can be achieved by choosing different coefficient matrices. The benchmark inverted pendulum on a cart control problem is used as an example to demonstrate its effectiveness. The proposed method provides us a powerful alternative to the existing nonlinear control approaches.

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