An Improved LMI Approach for Static Output Feedback Fault-tolerant Control With Application to Flight Tracking Control

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***Abstract*—This paper proposes an improved Linear Matrix Inequality (LMI) approach for the synthesis of Static Output Feedback (SOF) Fault-Tolerant Control (FTC). A novel slack variable is introduced into the matrix inequalities, which provides an additional degree of freedom to compute the numerical solution. Subsequently, an improved iterative algorithm is developed to obtain an optimal SOF gain with less conservativeness. In this paper, designs of the SOF gain are shown in the framework of tracking control. The nonlinear simulations of the ADMIRE aircraft are included to demonstrate the effectiveness of the proposed method.**

***Index Terms*—Fault-Tolerant Control (FTC), Linear Matrix Inequality (LMI), Static Output Feedback (SOF), tracking control**

I. INTRODUCTION

Over last two decades, the growing demand for reliability, maintainability and survivability of dynamic systems has ignited enormous research activities in Fault-Tolerant Control (FTC). Generally speaking, the FTC design approaches can be categorized into two main classes: passive and active [1]. The Active FTC (AFTC) is too complicated to implement for flight control system. Therefore, Passive FTC (PFTC) is a popular method to design Fault-Tolerant Control System (FTCS) for aircraft. Some PFTC approaches have been proposed over the last decade [2]-[4]. However, these methods are all based on the state feedback.

It is well known that not all of states are available to feedback in practical systems. Based on this, Static Output Feedback (SOF) control becomes one of the most important open problems in control community. The survey paper [5] reviewed all kinds of early approaches to deal with the SOF problem of linear systems. Unfortunately, no efficient algorithmic solution can be obtained by using the approaches descried in [5]. With the development of numerical approaches for Linear Matrix Inequality (LMI) problem, the LMI-based methods [6-12] have become powerful tools for analysis and synthesis of the SOF control problem in recent years. Cao *et al.* [6] derived a sufficient and necessary condition for SOF stabilizability in the form of LMI. This method later is employed broadly to solve some problems of the SOF controller design. Apakarian *et al.* [8] described a new LMI characterization of Lyapunov stability and extended to robust and H_2 control problems. Liao *et al.* [9] investigated reliable H_2 tracking control against aircraft control surface impairment, and developed an Iterative LMI (ILMI) algorithm to compute the controller gain with less conservativeness. All of them employed some auxiliary variables to separate the Lyapunov function variables from the controller gain variable. However, it is difficult to obtain tractable numerical solution of the auxiliary variables in terms of LMI, especially for complex system such as aircraft.

In this paper, we study the SOF synthesis of FTC against actuator faults for a general class of uncertain system and its application to flight tracking control. By extending the result in [9], a novel slack variable is introduced into the Linear Matrix Inequalities (LMIs). This slack variable provides an additional degree of freedom to compute the auxiliary variables in [9]. Consequently, a sufficient condition is derived from the method above, which guarantees that the closed-loop system is robustly stable and satisfies H_2 constraint in both normal and fault cases. Then, we develop an improved ILMI algorithm via the sufficient condition to achieve optimal controller gain with less conservativeness. Numerical examples of flight tracking control are also given to demonstrate the effectiveness of the proposed method.

II. PRELIMINARIES AND PROBLEM FORMULATION

Consider an uncertain Linear Time Invariant (LTI) system of the form

$$\begin{cases} \dot{x}(t) = [A+\Delta A(t)]x(t) + [B+\Delta B(t)]u(t) + [G+\Delta G(t)]w(t) \\ y(t) = Cx(t) \\ z(t) = C_2 x(t) + D_2 u(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^h$ is the disturbance input, $y(t) \in \mathbb{R}^p$ is the measured output and $z(t)$ is the desired output. A, B, C, G, C_2 and D_2 are known real constant matrices with appropriate dimensions, which describe the nominal system. $\Delta A(t)$, $\Delta B(t)$ and $\Delta G(t)$ are real-valued time-varying matrix functions representing the norm-bounded parameter uncertainties.

$$\Delta A = E_a \Delta(t) F_a,\ \Delta B = E_b \Delta(t) F_b,\ \Delta G = E_g \Delta(t) F_g \quad (2)$$

where E_a, F_a, E_b, F_b, E_g and F_g are known constant matrices and $\Delta(t)$ is a given norm-bounded disturbance for all admissible uncertainties satisfying $\Delta^T(t)\Delta(t) \le I$ [13].

Actuator fault can be presented in relation to nominal control input $u(t)$ and control effectiveness factor ω_{Li}, as following:

$$u^F(t) = \omega_L u(t) \tag{3}$$

where ω_L satisfy

$$\begin{aligned} \omega_L \in \Theta \; \{ \omega_L = diag[\omega_{L1},\ \omega_{L2},\ \ldots,\ \omega_{Lm}], \\ \omega_{Li} \in [\underline{\omega}_{Li},\ \overline{\omega}_{Li}],\ \ i = 1, 2,\ \ldots,\ m \ \} \end{aligned} \tag{4}$$

For every fault mode, $\underline{\omega}_{Li}$ and $\overline{\omega}_{Li}$ represent the lower and upper bounds of ω_{Li}, respectively.

Remark 1: $0 < \omega_{Li} < 1$ describes partial loss in control effectiveness. $\omega_{Li} = 0$ means total outage of the ith actuator and $\omega_{Li} = 1$ denotes a healthy ith actuator.

Hence, the system (1) with actuator faults is given by

$$\begin{cases} \dot{x}(t) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)]\omega_L u(t) + [G + \Delta G(t)]w(t) \\ y(t) = Cx(t) \\ z(t) = C_2 x(t) + D_2 \omega_L u(t) \end{cases} \tag{5}$$

Based on the tracking problem setting, the selected output $Sy(t)$ tracks the reference signal $r(t) \in \mathbb{R}^l$ without steady-state error, that is

$$\lim_{t\to\infty} e(t) = 0, \qquad e(t) = r(t) - Sy(t) \tag{6}$$

where $e(t)$ is tracking error and S is a known constant matrix to select the output. Moreover, it is well known that integral action for the tracking error of a controller can effectively eliminate the steady-state tracking error [3]. In order to obtain a tracking controller with SOF plus the integral of the tracking error, denote

$$\bar{A} = \begin{bmatrix} 0 & -SC \\ 0 & A \end{bmatrix},\ \ \bar{B} = \begin{bmatrix} 0 \\ B \end{bmatrix},\ \ \bar{G} = \begin{bmatrix} I & 0 \\ 0 & G \end{bmatrix},$$

$$\bar{E}_a = \begin{bmatrix} 0 \\ E_a \end{bmatrix},\ \bar{F}_a = [0 \ \ F_a],\ \bar{E}_b = \begin{bmatrix} 0 \\ E_b \end{bmatrix},\ \bar{F}_b = F_b,$$

$$\bar{E}_g = \begin{bmatrix} 0 \\ E_g \end{bmatrix},\ \bar{F}_g = [0 \ \ F_g],\ \bar{\Delta}(t) = \Delta(t).$$

Then, the augmented system is represented as:

$$\begin{cases} \dot{\bar{x}}(t) = [\bar{A} + \bar{E}_a\bar{\Delta}(t)\bar{F}_a]\bar{x}(t) + [\bar{B} + \bar{E}_b\bar{\Delta}(t)\bar{F}_b]\omega_L u(t) + [\bar{G} + \bar{E}_g\bar{\Delta}(t)\bar{F}_g]v(t) \\ \bar{y}(t) = \bar{C}\bar{x}(t) \\ \bar{z}(t) = \bar{C}_2\bar{x}(t) + \bar{D}_2\omega_L u(t) \end{cases} \tag{7}$$

where $\bar{x}(t) = [(\int_0^t e(t)dt)^T,\ x^T(t)]^T$ is the augmented state, $\bar{y}(t) = [(\int_0^t e(t)dt)^T,\ y^T(t)]^T$ is the augmented measured output and $v(t) = [(r^T(t),\ w^T(t)]^T$ is the disturbance input.

III. SOF Synthesis With H_2 Constraint

In this section, we develop an improved LMI method to achieve a SOF fault-tolerant controller with H_2 constraint.

Without lost of generality, assume that pairs $(A,\ B\omega_L)$ and $(C,\ A)$ are stabilizable and detectable for all $L = 0, 1, \ldots, l_p$, respectively. Consider the following SOF controller for the augmented system (7):

$$u(t) = K\bar{y}(t) = K_e\int_0^t e(t)dt + K_y y(t) \tag{8}$$

where $K = [K_e \ \ K_y] \in \mathbb{R}^{m\times(l+p)}$ is the SOF controller gain to be determined.

The closed-loop augmented system is represented as follows by substituting (8) into (7)

$$\begin{cases} \dot{\bar{x}}(t) = \{[\bar{A} + \bar{E}_a\bar{\Delta}(t)\bar{F}_a] + [\bar{B} + \bar{E}_b\bar{\Delta}(t)\bar{F}_b]\omega_L K\bar{C}\}\bar{x}(t) + [\bar{G} + \bar{E}_g\bar{\Delta}(t)\bar{F}_g]v(t) \\ \bar{y}(t) = \bar{C}\bar{x}(t) \\ \bar{z}(t) = [\bar{C}_2 + \bar{D}_2\omega_L K\bar{C}]\bar{x}(t) \end{cases} \tag{9}$$

Let T_{zv} denote the transfer function from $v(t)$ to $\bar{z}(t)$. Then, the H_2 norm of T_{zv} is defined as the output energy of impulse response of $\bar{z}(t)$, i.e.,

$$\|T_{zv}\|_2 = \underset{\omega}{tr}[\frac{1}{2\pi}\int_{-\infty}^{\infty} T_{zv}(j\omega)T_{zv}^*(j\omega)d\omega]^{1/2} \tag{10}$$

Based on the presentations of H_2 norm, we first review the following useful lemmas in determining SOF gain.

Lemma 1 [8]: Consider the system (11) described in [8]. The following statements, involving symmetric positive definite matrix variables X, Z and general matrix variable V_1 are equivalent.

1): A is stable and $\|C(sI - A)^{-1}B\|_2^2 < \gamma$ (11)

2): $\exists X,\ Z$ such that

$$\begin{bmatrix} A^T X + XA & XB \\ B^T X & -I \end{bmatrix} < 0, \qquad \begin{bmatrix} X & C^T \\ C & Z \end{bmatrix} > 0, \qquad tr(Z) < 1 \tag{12}$$

3): $\exists X,\ Z,\ V_1$ such that

$$\begin{bmatrix} -(V_1 + V_1^T) & V_1^T A + X & V_1^T B & V_1^T \\ A^T V_1 + X & -X & 0 & 0 \\ B^T V_1 & 0 & -\gamma I & 0 \\ V_1 & 0 & 0 & -X \end{bmatrix} < 0, \qquad \begin{bmatrix} X & C^T \\ C & Z \end{bmatrix} > 0, \qquad tr(Z) < 1 \tag{13}$$

Lemma 2 [13]: Given matrices Y, E and F of appropriate dimensions where Y is symmetrical, and $\Delta^T(t)\Delta(t) \le I$. Then

$$Y + E\Delta(t)F + F^T\Delta^T(t)E^T < 0 \tag{15}$$

holds if and only if there exists a scalar $\varepsilon > 0$ such that

$$Y + \varepsilon EE^T + \varepsilon^{-1}F^TF < 0 \tag{16}$$

We will now present the main results of the paper which connect the synthesis of SOF FTC with H_2 constraint.

Theorem 1: Consider the uncertain closed-loop augmented system (9). For given positive constants λ_L, this system is robustly stabilized and satisfies the H_2 performance λ_L if there exist symmetric positive definite matrices P_L and Q_L, positive scalar ε, general matrices V and K satisfying

$$\begin{bmatrix} -(V+V^T) & * & * & * & * & * & * & * \\ (\bar{A}+\bar{B}\omega_L K\bar{C})^T V+P_L & -P_L+\varepsilon_1^{-1}\bar{F}_a^T\bar{F}_a & * & * & * & * & * & * \\ \bar{G}^T V & 0 & -I+\varepsilon_3^{-1}\bar{F}_g^T\bar{F}_g & * & * & * & * & * \\ V & 0 & 0 & -P_L & * & * & * & * \\ \bar{E}_a^T V & 0 & 0 & 0 & -\varepsilon_1^{-1}I & * & * & * \\ \bar{E}_b^T V & 0 & 0 & 0 & 0 & -\varepsilon_2^{-1}I & * & * \\ \bar{E}_g^T V & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3^{-1}I & * \\ 0 & \bar{F}_b\omega_L K\bar{C} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I \end{bmatrix}<0 \quad (17)$$

$$\begin{bmatrix} P_L & (\bar{C}_2+\bar{D}_2\omega_L K\bar{C})^T \\ \bar{C}_2+\bar{D}_2\omega_L K\bar{C} & Q_L \end{bmatrix}>0 \quad (18)$$

$$tr(Q_L)<\gamma_L \quad (19)$$

Proof: To the uncertain closed-loop augmented system (9), we can get the expression (18), (19) and following inequalities in terms of Lemma 1.

$$\begin{bmatrix} -(V+V^T) & * & * & * \\ \{[\bar{A}+\bar{E}_a\bar{\Delta}(t)\bar{F}_a]+[\bar{B}+\bar{E}_b\bar{\Delta}(t)\bar{F}_b]\omega_L K\bar{C}\}^T V+P_L & -P_L & * & * \\ [\bar{G}+\bar{E}_g\bar{\Delta}(t)\bar{F}_g]^T V & 0 & -I & * \\ V & 0 & 0 & -P_L \end{bmatrix}<0 \quad (20)$$

It is obvious that the inequality (20) is equivalent to

$$\begin{bmatrix} -(V+V^T) & * & * & * \\ (\bar{A}+\bar{B}\omega_L K\bar{C})^T V+P_L & -P_L & * & * \\ \bar{G}^T V & 0 & -I & * \\ V & 0 & 0 & -P_L \end{bmatrix} + \begin{bmatrix} V^T\bar{E}_a \\ 0 \\ 0 \\ 0 \end{bmatrix}\Delta(t)\begin{bmatrix} 0 & \bar{F}_a & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{F}_a^T \\ 0 \\ 0 \end{bmatrix}\Delta(t)\begin{bmatrix} \bar{E}_a^T V & 0 & 0 & 0 \end{bmatrix}$$
$$+\begin{bmatrix} V^T\bar{E}_b \\ 0 \\ 0 \\ 0 \end{bmatrix}\Delta(t)\begin{bmatrix} 0 & \bar{F}_b\omega_L K\bar{C} & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{C}^T K^T\omega_L\bar{F}_b^T \\ 0 \\ 0 \end{bmatrix}\Delta(t)\begin{bmatrix} \bar{E}_b^T V & 0 & 0 & 0 \end{bmatrix}$$
$$+\begin{bmatrix} V^T\bar{E}_g \\ 0 \\ 0 \\ 0 \end{bmatrix}\Delta(t)\begin{bmatrix} 0 & 0 & \bar{F}_g & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \bar{F}_g^T \\ 0 \end{bmatrix}\Delta(t)\begin{bmatrix} \bar{E}_g^T V & 0 & 0 & 0 \end{bmatrix}<0 \quad (21)$$

According to Lemma 2, the inequality (21) is equivalent to

$$\begin{bmatrix} -(V+V^T) & * & * & * \\ (\bar{A}+\bar{B}\omega_L K\bar{C})^T V+P_L & -P_L & * & * \\ \bar{G}^T V & 0 & -I & * \\ V & 0 & 0 & -P_L \end{bmatrix}$$
$$+\varepsilon_1\begin{bmatrix} V^T\bar{E}_a \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} \bar{E}_a^T V & 0 & 0 & 0 \end{bmatrix} + \varepsilon_1^{-1}\begin{bmatrix} 0 \\ \bar{F}_a^T \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 & \bar{F}_a & 0 & 0 \end{bmatrix}$$
$$+\varepsilon_2\begin{bmatrix} V^T\bar{E}_b \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} \bar{E}_b^T V & 0 & 0 & 0 \end{bmatrix} + \varepsilon_2^{-1}\begin{bmatrix} 0 \\ \bar{C}^T K^T\omega_L\bar{F}_b^T \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 & \bar{F}_b\omega_L K\bar{C} & 0 & 0 \end{bmatrix}$$
$$+\varepsilon_3\begin{bmatrix} V^T\bar{E}_g \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} \bar{E}_g^T V & 0 & 0 & 0 \end{bmatrix} + \varepsilon_3^{-1}\begin{bmatrix} 0 \\ 0 \\ \bar{F}_g^T \\ 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & \bar{F}_g & 0 \end{bmatrix}<0 \quad (22)$$

Subsequently, we obtain inequality (17) by using Schur complement theorem [14]. This completes the proof. ■

Remark 2: Theorem 1 guarantees the uncertain closed-loop augmented system (9) is robustly stable and satisfies H_2 performance in both normal and fault cases. Using the auxiliary variable V, the presumed actuator faults are introduced into matrix inequalities in Theorem 1 by the different Lyapunov function variables P_L. Thus, the conservativeness of Theorem 1 is smaller than other methods which use a common Lyapunov function matrix for all cases. However, the matrix inequalities described in Theorem 1 are not LMIs. Therefore, an important result is given as follows to solve this difficulty.

Theorem 2: For given positive constants λ_L, initial controller gain K_0 and initial auxiliary variables V_0, the system (9) is robustly stabilized and satisfies the H_2 performance λ_L if there exist symmetric positive definite matrices P_L and Q_L, positive scalars ε and λ, general matrices V and K satisfying inequalities (18), (19) and

$$
\begin{bmatrix}
M_L & * & * & * & * & * & * & * & * \\
\bar{A}^T V + P_L & N_L & * & * & * & * & * & * & * \\
\bar{G}^T V & 0 & -I+\varepsilon_3^{-1}\bar{F}_g^T\bar{F}_g & * & * & * & * & * & * \\
V & 0 & 0 & -P_L & * & * & * & * & * \\
\bar{E}_a^T V & 0 & 0 & 0 & -\varepsilon_1^{-1} I & * & * & * & * \\
\bar{E}_b^T V & 0 & 0 & 0 & 0 & -\varepsilon_2^{-1} I & * & * & * \\
\bar{E}_g^T V & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3^{-1} I & * & * \\
0 & \bar{F}_b \omega_L K \bar{C} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I & * \\
\lambda \bar{B}^T V & \lambda^{-1}\omega_L K \bar{C} & 0 & 0 & 0 & 0 & 0 & 0 & -I
\end{bmatrix} < 0 \quad (23)
$$

where

$$M_L = -(V+V^T) - \lambda^2 (V^T \bar{B}\bar{B}^T V_0 - V_0^T \bar{B}\bar{B}^T V + V_0^T \bar{B}\bar{B}^T V_0)$$

$$N_L = -P_L + \varepsilon_1^{-1}\bar{F}_a^T \bar{F}_a - \lambda^{-2}(\bar{C}^T K^T \omega_L \omega_L K_0 \bar{C} - \bar{C}^T K_0^T \omega_L \omega_L K \bar{C} + \bar{C}^T K_0^T \omega_L \omega_L K_0 \bar{C})$$

Proof: According to Schur complement theorem [14], (23) is equivalent to the following expression

$$(17) + \begin{bmatrix} (V-V_0)\bar{B} \\ 0 \\ 0 \end{bmatrix} \lambda^2 \begin{bmatrix} (V-V_0)\bar{B} \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} \bar{C}^T (K-K_0)\omega_L \\ 0 \\ 0 \end{bmatrix} \lambda^{-2} \begin{bmatrix} \bar{C}^T (K-K_0)\omega_L \\ 0 \\ 0 \end{bmatrix}^T < 0 \quad (24)$$

Since $\lambda^2 > 0$ and $\lambda^{-2} > 0$, it is obvious that (23) is satisfied if (24) is satisfied. The subsequent proof is easy to obtain via Theorem 1 and is omitted here. ■

Remark 3: The conservativeness of Theorem 2 lies in the differences between $K-K_0$ and $V-V_0$. Moreover, it is difficult to achieve an optimal numerical solution when the differences are too large. So we introduce a novel slack variable λ into LMIs, which provides an additional degree of freedom to eliminate the influence of the differences. As compared to the result in [9], this method effectively increases the tractability of numerical computation. Furthermore, the following improved iterative algorithm will be developed to minimize the conservativeness of the controller design.

Algorithm 1:

1) Compute the initial controller gain K_{ini} via the Algorithm 1 in [12]. If the initial controller can not be found, then stop.

2) Let $K_{opt}^1 = K_{ini}$, minimize$[tr(Q_0)]$ subject to $P_L > 0$, $Q_L > 0$ and inequalities (17), (18) and (19), then we get the initial auxiliary variables V_{opt}^1.

3) At the *i*th iteration ($i > 0$), let $V_0 = V_{opt}^{i-1}$ and $K_0 = K_{opt}^{i-1}$, minimize$[tr(Q_0)]$ subject to $P_L^i > 0$, $Q_L^i > 0$ and inequalities (18), (19) and (23), then we get the auxiliary variables V_{opt}^i and controller gain K_{opt}^i.

4) If $\left|tr(Q_0^i) - tr(Q_0^{i-1})\right| < \delta$ where δ is a given error tolerance, the obtained $K = K_{opt}^i$ is the optimal SOF fault-tolerant controller gain, stop. Otherwise, let $i = i+1$ and return to Step 3.

Remark 4: The sufficient condition described in Theorem 2 can be turned into a sufficient and necessary one under the ideal condition that iterative error of Algorithm 1 is zero.

IV. FLIGHT TRACKING CONTROL

In this section, the numerical examples of flight tracking control for the original nonlinear ADMIRE aircraft model are presented to demonstrate the advantage of our method.

A. Aircraft Model Representation

The ADMIRE model describes a single seated, single engine small fighter aircraft with a delta-canard configuration [15]. This model has been developed by the Swedish Defense Research Agency (FOI), as one of the Group of Aeronautical Research and Technology in EURope (GARTEUR) benchmark models in an Action Group project for flight control clearance investigation. Details of the ADMIRE can be found in [15]. The trimmed values of the ADMIRE model are obtained at Mach 0.22 and altitude 3000 m.

In this paper, the linear aircraft model is described by

$$\begin{cases} \dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))\omega_L u(t) + (G + \Delta G(t))w(t) \\ y(t) = Cx(t) \qquad\qquad L = 0,\ 1, \ldots, 7 \end{cases} \quad (25)$$

where $x(t) = [\alpha\ \beta\ p\ q\ r]^T$ is the state, $u(t) = [\delta_{rc}\ \delta_{lc}\ \delta_{roe}\ \delta_{rie}\ \delta_{lie}\ \delta_{loe}\ \delta_r]^T$ is the control surface deflection, $y(t) = [\alpha\ \beta\ \dot{\mu}_{rot}\ q\ r_{stab}]^T$ is the output and *W(t)* is disturbance. The tracking signal is $r(t) = [\alpha\ \beta\ \dot{\mu}_{rot}]^T$ because these variables are close to the maneuvers ability of aircraft.

For the considered flight case,

$$A = \begin{bmatrix} -0.5432 & 0.0137 & 0 & 0.9778 & 0 \\ 0 & -0.1179 & 0.2215 & 0 & -0.9661 \\ 0 & -10.5130 & -0.9967 & 0 & 0.6176 \\ 2.6221 & -0.0030 & 0 & -0.5057 & 0 \\ 0 & 0.7075 & -0.0939 & 0 & -0.2127 \end{bmatrix},$$

$$B=\begin{bmatrix} 0.0035 & 0.0035 & -0.0318 & -0.0548 & -0.0548 & -0.0318 & 0.0004 \\ -0.0063 & 0.0063 & 0.0024 & 0.0095 & -0.0095 & 0.0024 & 0.0287 \\ 0.6013 & -0.6013 & -2.2849 & -1.9574 & 1.9574 & 2.2849 & 1.4871 \\ 0.8266 & 0.8266 & -0.4628 & -0.8107 & -0.8107 & -0.4628 & 0.0024 \\ -0.2615 & 0.2615 & -0.0944 & -0.1861 & 0.1861 & 0.0944 & -0.8823 \end{bmatrix},$$

$$C=\begin{bmatrix} 57.2958 & 0 & 0 & 0 & 0 \\ 0 & 57.2239 & 0 & 0 & 0 \\ 0 & 0 & 57.2958 & 0 & 2.6415 \\ 0 & 0 & 0 & 57.274 & 0 \\ 0 & 0 & -2.6415 & 0 & 57.2958 \end{bmatrix},$$

$$G=\begin{bmatrix} 0.00541 & 0 & 0 & 0.00682 & 0 \end{bmatrix}^T.$$

The actuator used in the examples is simply a first order transfer function with limited angular deflection and maximum angular rate. Details of the actuator can be found in [15].

B. Controller Design

We first introduce seven kinds of actuator faults into the procedure of the controller design. Assume that the actuator is total outage, namely, the effectiveness factor of the actuator decreases to zero. Thus, every fault holds the following condition: one effectiveness factor $\omega_{Li}=0$ and other six effectiveness factors $\omega_{Lj}=1,\ j=1,\ 2\cdots 7, j\neq i$.

Then, select the following parameter perturbed matrices

$$E_a=\begin{bmatrix} 0.0050 & 0.0100 & 0.5000 & 0.1000 & 0.0500 \\ 0.0100 & 0.0100 & 1 & 0.1000 & 0.0500 \end{bmatrix}^T,$$

$$F_a=\begin{bmatrix} 0.0100 & 1 & 0.2000 & 0.1000 & 0.2000 \\ 0.0010 & 0.2000 & 0.2000 & 0.1000 & 0.2000 \end{bmatrix},$$

$$E_b=\begin{bmatrix} 0.0050 & 0.0100 & 0.5000 & 0.1000 & 0.0500 \\ 0.0100 & 0.0100 & 1 & 0.1000 & 0.0500 \end{bmatrix}^T,$$

$$F_b=\begin{bmatrix} 0.0005 & 0.0005 & 0.2500 & 0.2500 & 0.2500 & 0.2500 & 0.0100 \\ 0.0005 & 0.0005 & 0.5000 & 0.5000 & 0.5000 & 0.5000 & 0.0100 \end{bmatrix},$$

$$E_g=\begin{bmatrix} 0.1000 & 0 & 0 & 0.0020 & 0 \\ 0.0500 & 0 & 0 & 0.0100 & 0 \end{bmatrix}^T, \qquad F_g=\begin{bmatrix} 0.1000 \\ 0.0500 \end{bmatrix},$$

and weighting matrices

$$C_2=\begin{bmatrix} diag(4,2,2) & 0_{3\times 5} \\ 0_{7\times 6} & 0_{7\times 2} \end{bmatrix},\quad D_2=\begin{bmatrix} 0_{3\times 7} \\ 0.5*I_{7\times 7} \end{bmatrix}.$$

Then, the SOF fault-tolerant controller K_f can be designed to compensate the above actuator faults by using Algorithm 1. However, we cannot obtain a tractable numerical solution via the iterative algorithm described in [14] since the excessive differences between $K-K_0$ and $V-V_0$. At the same time, a standard SOF controller K_s is also carried out in this paper for comparison purpose.

C. Simulation Results

To demonstrate the effectiveness of our method, we carry out the simulations using the original nonlinear ADMIRE aircraft model. To be close to the real system, we set 40% parameter uncertainties in A matrix and 20% parameter uncertainties in B matrix and G matrix, and a vertical gust disturbance of 6 m/s in following simulations. For the convenience of comparison, we assume that all of the actuator faults occur at 40 seconds.

Fig. 1, Fig. 2 and Fig. 3 show the comparisons of the SOF fault-tolerant controller K_f and the standard SOF controller K_s in three actuator fault modes, respectively. When the right canard is outage, in Fig. 1, we can see that the transient behaviors of the controller K_f are similar to that of the controller K_s. In other words, the performance of the two controllers is close when the actuator failures are presumed faults. However, as the number of actuator failures increases, for example, right canard and left inner elevon actuators lose simultaneously, it is easy to see from Fig. 2 that the controller K_f results in superior system tracking performance than the controller K_s. Furthermore, in Fig. 3, when the actuators of right and left canards and left inner elevon are total outage, the controller K_s cannot stabilize the aircraft and the controller K_f just suffers from a slight performance degradation.

In summary, it is obvious that the SOF fault-tolerant controller designed by improved ILMI algorithm yields better performance than other controllers in the event of actuator faults, especially, for the case that the failures are not presumed.

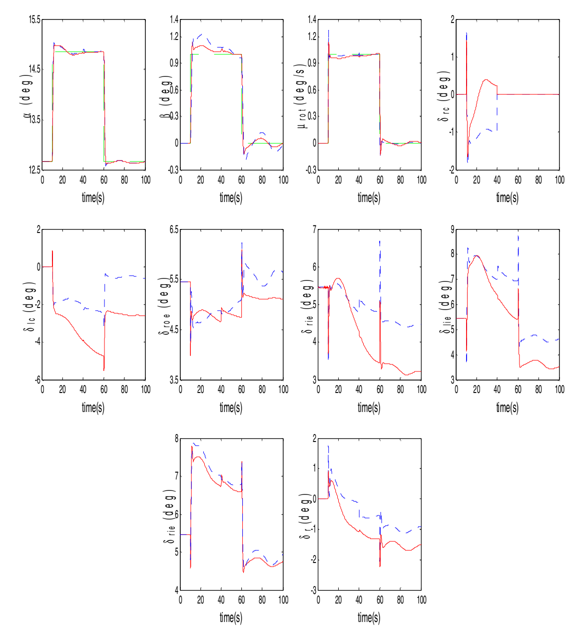

Fig. 1. Nonlinear simulation results with a single failure in right canard (solid line corresponds to controller K_f, dotted line corresponds to controller K_s, and the dashed line corresponds to the reference signal).

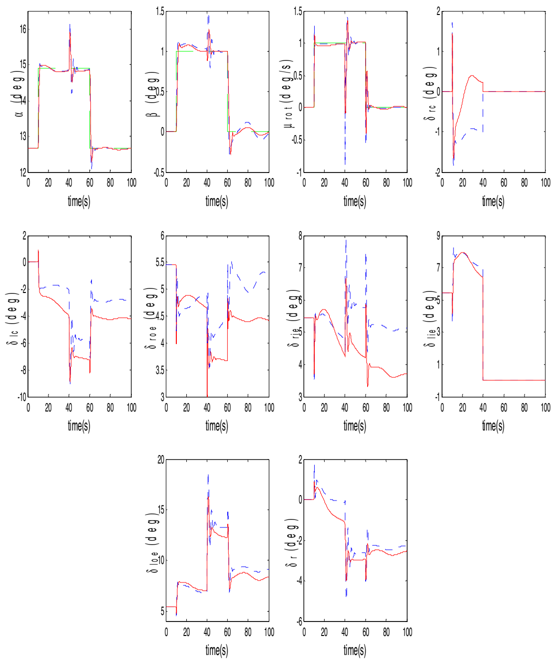

Fig. 2. Nonlinear simulation results with simultaneous failures in right canard and left inner elevon.

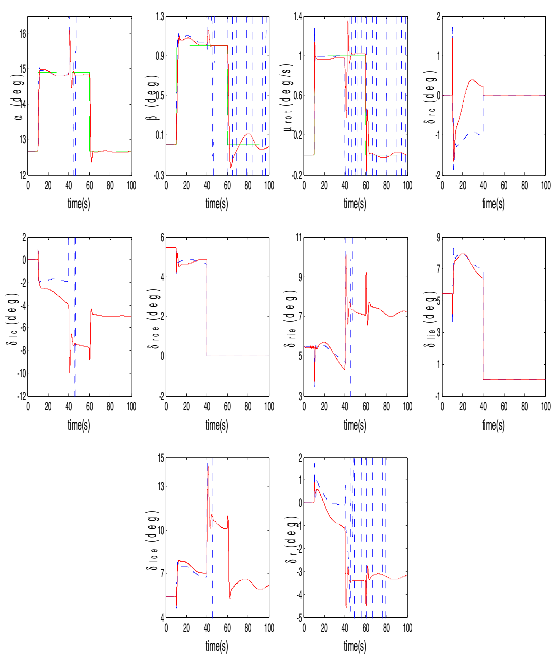

Fig. 3. Nonlinear simulation results with failures in right canard and right outer and left inner elevons.

V. CONCLUSION

In this paper, we discuss the SOF fault-tolerant controller design problem for a general class of uncertain systems and its application to flight tracking control. The additional slack variable increases the maneuverability of the algorithm effectively. The improved iterative algorithm derived from the proposed method produces less conservativeness than previous LMI methods. The simulation results of the nonlinear ADMIRE aircraft model illustrate the advantages of the proposed method.

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