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Cosserat gyro-birefringence. An introduction to nonsymmetrical photoelasticity

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Abstract. Mechanical couple stresses modify at a microscopic level optical properties of some materials so they can display gyro-birefringence phenomena.

The Optical Cosserat medium is defined and optical rotation tensor relative to couple stress is introduced. The generalized tensor of the dielectric permittivity is written for the Cosserat medium. Split of the plane-polarized light wave on passing through the Cosserat medium is shown and rotation of the azimuth of polarization is expressed by components of the couple stress tensor.

Key words: Cosserat medium, gyro-birefringence, photoelasticity.

1. Introduction

In this work the gyro-birefringence phenomena [1–2] is applied to describe the optical properties of the Cosserat medium.

According to the Cosserat theory (theory of the nonsymmetrical elasticity) [3–5] the transmission of mechanical action, through the surface dividing two neighboring unit cells of material, occurs not only via a force vector, but also by a couple vector. Therefore, in addition to force stresses one observes couple stresses. In the Cosserat material the stress tensor is nonsymmetrical.

Today the Cosserat theory does not have a complete experimental verification [6–9].

The noticed rigidity depends on the size in the Cosserat elastic material and it is possible to determine one or more of the Cosserat elasticity constants by the method of size effects [10]. Therefore, the theoretical solution to the Cosserat problems can be completely determined. However, in order to obtain the experimental solution to the nonsymmetrical elasticity, phenomena where couple stress is described by one physical parameter are searched. Phenomena applied in contemporary Experimental Mechanics allow us to determine the force stress only.

The analysis in a nano-scale [11–12] is taken under consideration, where the energetic state of the atom about a nonsymmetrical structure (with an optical active electron) is separate according to the mechanical quantum number $mech_S$ as follows (Appendix 1):

$$mech_s = \frac{1}{2}, \qquad E^+ = E_o + \frac{\hbar}{2}\omega^{couple}, \tag{1}$$

$$mech_s = -\frac{1}{2}, \qquad E^- = E_o - \frac{\hbar}{2}\omega^{couple}, \tag{2}$$

where: $\hbar = h/2\pi$, h is Planck constant, ω^{couple} is angular speed of disturbance of the optical electron in Cosserat medium, E_o is not perturbed part of the energy. For energy (1), (2) we obtain frequency:

$$\omega^+ = \omega_o + \frac{\omega^{couple}}{2}, \tag{3}$$

$$\omega^- = \omega_o - \frac{\omega^{couple}}{2}, \tag{4}$$

which creates a separation of the light wave travelling towards the Cosserat medium in two circular polarization waves, for $mech_S = +1/2$ right-handed, for $mech_S = -1/2$ left-handed. When two circular polarization light waves travel forward with different velocity of propagation as a result of interference, we obtain a rotation of the azimuth of polarization. The separation of the light wave in the Cosserat medium and the correlation of this phenomenon to the couple stresses are referred to as the Cosserat gyro-birefringence.

2. Cosserat gyro-birefringence medium

The elastic, transparent, dielectric medium with a property of the birefringence is studied. The density of the free charge and the electric conduction are zero. The transmission of the mechanical state is represented by two independent vectors: the force vector and the moment vector. The optical state is described by the tensor of the dielectric constant κ_{ij} and additionally by the optical rotation tensor g_{ij}.

3. Generalized dielectric constant

The tensor κ_{ij} is associated with the force stress tensor σ_{ij} when tensor g_{ij} is relative to the couple stress tensor μ_{ij}. The generalized tensor of the dielectric permittivity is written in the following formula [1–2]:

$$K_{ij} = \kappa_{ij} + i \in_{ijk} g_{kl} s_l, \tag{5}$$

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where $i = \sqrt{-1}$, $\in_{ijk}$ is permutation symbol, s_l is dimension of the unit victor s_l which is perpendicular to the front of the light wave.

4. Material equations

The basic equation of the gyro-birefringence [1–2]:

$$\boldsymbol{D}_k = \kappa_o \kappa_{kl} \boldsymbol{E}_l + i\kappa_o \left(g_{kl} \boldsymbol{s}_l \times \boldsymbol{E}\right)_k, \tag{6}$$

connect the vector of the electric induction D_k with vector of the intensity of the electric field E_l, through κ_{ij} and g_{ij}. We note κ_o as permittivity of the vacuum. The optical anisotropy generated by state of the force stress is described by formula [13–15]:

$$\kappa_{kl} = \kappa\delta_{kl} + C_1^\sigma \sigma_{kl} + C_2^\sigma \sum \sigma_{ii}\delta_{kl}, \tag{7}$$

where C_1^σ, C_2^σ are optical constants, κ is natural permittivity in the medium without the stresses, δ_{ij} is Kronecker symbol. The relation between rotation tensor g_{ij} and tensor of the couple stress μ_{ij} we propose as the following formula [16–17]:

$$g_{kl} = g\delta_{kl} + C_1^\mu \mu_{kl} + C_2^\mu \sum \mu_{ii}\delta_{kl}, \tag{8}$$

where g is the parameter of the natural rotation (without the stress), C_1^μ, C_2^μ are optical constants.

We define the optical rotation vector: $\boldsymbol{G}_k = g_{kl}\boldsymbol{s}_l$ and we write the general tensor of the dielectric constant in the coordinate system (x, y, z) in the form of the matrix:

$$T = \begin{bmatrix} \kappa_x & \kappa_{xy} - iG_z & \kappa_{xz} + iG_y \\ \kappa_{yx} + iG_z & \kappa_y & \kappa_{yz} - iG_x \\ \kappa_{zx} - iG_y & \kappa_{zy} + iG_x & \kappa_z \end{bmatrix}. \tag{9}$$

5. Light wave in Cosserat medium

Maxwell equations are grouped with material equations and are written in the first approximation as (Appendix 2):

$$\boldsymbol{D} = \kappa_o n^2 \left[\boldsymbol{E} - \boldsymbol{s}\left(\boldsymbol{E}\boldsymbol{s}\right)\right]. \tag{10}$$

Then we compare components of formulas (6), (10) adequately, and write the system of the equations [1–2]:

$$\begin{aligned} E_x\left[\kappa_x - \left(1 - s_x^2\right)n^2\right] + E_y\left(n^2 s_x s_y + \kappa_{xy} - iG_z\right) \\ + E_z\left(n^2 s_x s_z + \kappa_{xz} + iG_y\right) = 0, \end{aligned} \tag{11}$$

$$\begin{aligned} E_y\left[\kappa_y - \left(1 - s_y^2\right)n^2\right] + E_z\left(n^2 s_y s_z + \kappa_{yz} - iG_x\right) \\ + E_x\left(n^2 s_y s_x + \kappa_{yx} + iG_z\right) = 0, \end{aligned} \tag{12}$$

$$\begin{aligned} E_z\left[\kappa_z - \left(1 - s_z^2\right)n^2\right] + E_x\left(n^2 s_z s_x + \kappa_{zx} - iG_y\right) \\ + E_y\left(n^2 s_z s_y + \kappa_{zy} + iG_x\right) = 0. \end{aligned} \tag{13}$$

Formulas (11), (12), (13) present the light wave in the gyro-birefringence medium. We note (x) as direction of the light path and we write the unit vectors $s_x = 1$, $s_y = s_z = 0$ for the light path parallel to coordinate "x". The matrix (9) we write in the system of the quasi-principal directions $\left((x) = x = 1^{(x)}, 2^{(x)}, 3^{(x)}\right)$ in the form:

$$T^{(x)} = \begin{bmatrix} \kappa_x & \kappa_{x2}^{(x)} - iG_3^{(x)} & \kappa_{x3}^{(x)} + iG_2^{(x)} \\ \kappa_{2x}^{(x)} + iG_3^{(x)} & \kappa_2^{(x)} & \kappa_{23}^{(x)} - iG_x \\ \kappa_{3x}^{(x)} - iG_2^{(x)} & \kappa_{32}^{(x)} + iG_x & \kappa_3^{(x)} \end{bmatrix}, \tag{14}$$

For the chosen direction (x) Eqs. (12), (13) are written as:

$$E_2^{(x)}\left(\kappa_2^{(x)} - \left(n^{(x)}\right)^2\right) + E_3^{(x)}\left(\kappa_{23}^{(x)} - iG_x\right) = 0, \tag{15}$$

$$E_2^{(x)}\left(\kappa_{32}^{(x)} + iG_x\right) + E_3^{(x)}\left(\kappa_3^{(x)} - \left(n^{(x)}\right)^2\right) = 0. \tag{16}$$

We then write the non-zero condition of the solution:

$$\begin{vmatrix} \kappa_2^{(x)} - \left(n^{(x)}\right)^2 & \kappa_{23}^{(x)} - iG_x \\ \kappa_{32}^{(x)} + iG_x & \kappa_3^{(x)} - \left(n^{(x)}\right)^2 \end{vmatrix} = 0 \tag{17}$$

and we determine the roots:

$$\begin{aligned} \left(n^{(x)}\right)_r^2 = \frac{\kappa_2^{(x)} + \kappa_3^{(x)}}{2} \\ + \frac{1}{2}\sqrt{\left(\kappa_2^{(x)} - \kappa_3^{(x)}\right)^2 + 4\left(G_x^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right)}, \end{aligned} \tag{18}$$

$$\begin{aligned} \left(n^{(x)}\right)_l^2 = \frac{\kappa_2^{(x)} + \kappa_3^{(x)}}{2} \\ - \frac{1}{2}\sqrt{\left(\kappa_2^{(x)} - \kappa_3^{(x)}\right)^2 + 4\left(G_x^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right)}, \end{aligned} \tag{19}$$

where $\left(n^{(x)}\right)_r$, $\left(n^{(x)}\right)_l$ – are two refractive indexes of the light wave coming towards direction (x).

For roots (18), (19) the system (15), (16) have infinite number of solutions which are relative to following relations:

$$\frac{E_3^{(x)}}{E_2^{(x)}} = \frac{\left(n_r^{(x)}\right)^2 - \kappa_2^{(x)}}{\kappa_{23}^{(x)} - iG_x}, \tag{20}$$

$$\frac{E_3^{(x)}}{E_2^{(x)}} = \frac{\kappa_{32}^{(x)} + iG_x}{\left(n_r^{(x)}\right)^2 - \kappa_3^{(x)}}, \tag{21}$$

$$\frac{E_3^{(x)}}{E_2^{(x)}} = \frac{\left(n_l^{(x)}\right)^2 - \kappa_2^{(x)}}{\kappa_{23}^{(x)} - iG_x}, \tag{22}$$

$$\frac{E_3^{(x)}}{E_2^{(x)}} = \frac{\kappa_{32}^{(x)} + iG_x}{\left(n_l^{(x)}\right)^2 - \kappa_3^{(x)}}. \tag{23}$$

We write $E_2^{(x)} = E_o$ and on the basis of (20), (21), (22), (23) four light waves are obtained:

$$E_I = \left[0, E_o, \frac{\left(n_r^{(x)}\right)^2 - \kappa_2^{(x)}}{\kappa_{23}^{(x)} - iG_x} E_o\right] \exp\left[i\left(\omega t - \psi_r^{(x)}\right)\right], \tag{24}$$

$$E_{II} = \left[0, E_o, \frac{\kappa_{32}^{(x)} + iG_x}{\left(n_r^{(x)}\right)^2 - \kappa_3^{(x)}} E_o\right] \exp\left[i\left(\omega t - \psi_r^{(x)}\right)\right], \tag{25}$$

$$E_{III} = \left[0, E_o, \frac{\left(n_l^{(x)}\right)^2 - \kappa_2^{(x)}}{\kappa_{23}^{(x)} - iG_x} E_o\right] \exp\left[i\left(\omega t - \psi_l^{(x)}\right)\right], \quad (26)$$

$$E_{IV} = \left[0, E_o, \frac{\kappa_{32}^{(x)} + iG_x}{\left(n_l^{(x)}\right)^2 - \kappa_3^{(x)}} E_o\right] \exp\left[i\left(\omega t - \psi_l^{(x)}\right)\right]. \quad (27)$$

Having added the amplitudes of the components for the same phases, we obtain:

$$\mathrm{E}_I = \left[0, 2E_o, \left(\frac{\left(n_r^{(x)}\right)^2 - \kappa_2^{(x)}}{\kappa_{23}^{(x)} - iG_x} + \frac{\kappa_{32}^{(x)} + iG_x}{\left(n_r^{(x)}\right)^2 - \kappa_3^{(x)}}\right) E_o\right] \exp\left[i\left(\omega t - \psi_r^{(x)}\right)\right], \quad (28)$$

$$\mathrm{E}_{II} = \left[0, 2E_o, \left(\frac{\left(n_l^{(x)}\right)^2 - \kappa_2^{(x)}}{\kappa_{23}^{(x)} - iG_x} + \frac{\kappa_{32}^{(x)} + iG_x}{\left(n_l^{(x)}\right)^2 - \kappa_3^{(x)}}\right) E_o\right] \exp\left[i\left(\omega t - \psi_l^{(x)}\right)\right]. \quad (29)$$

Using Euler formula: $\exp\left[i\left(\omega t - \psi\right)\right] = \cos\left(\omega t - \psi\right) + i\sin\left(\omega t - \psi\right)$ we write real part of the component (28), (29):

$$E_{2I} = E_o \cos\left(\omega t - \psi_r^{(x)}\right), \quad (30)$$

$$E_{3I} = E_o A^{(x)} \cos\left(\omega t - \psi_r^{(x)} + \varphi^{(x)}\right), \quad (31)$$

$$E_{2II} = E_o \cos\left(\omega t - \psi_l^{(x)}\right), \quad (32)$$

$$E_{3II} = E_o B^{(x)} \cos\left(\omega t - \psi_l^{(x)} + \varphi^{(x)}\right), \quad (33)$$

where

$$A^{(x)} = \sqrt{A_1^2 + A_2^2},$$

$$B^{(x)} = \sqrt{B_1^2 + B_2^2},$$

$$\varphi^{(x)} = \arctan\frac{A_2}{A_1} = \arctan\frac{B_2}{B_1},$$

$$A_1 = \frac{\left(\kappa_3^{(x)} - \kappa_2^{(x)} + \sqrt{\left(\kappa_2^{(x)} - \kappa_3^{(x)}\right) + 4\left(G_x^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right)}\right)\left[\kappa_{23}^{(x)}\left(G_x^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right) + \kappa_{32}^{(x)}\left(G_x^2 + \left(\kappa_{23}^{(x)}\right)^2\right)\right]}{2\left(G_x^2 + \left(\kappa_{23}^{(x)}\right)^2\right)\left(G_x^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right)},$$

$$A_2 = \frac{\left(\kappa_3^{(x)} - \kappa_2^{(x)} + \sqrt{\left(\kappa_2^{(x)} - \kappa_3^{(x)}\right) + 4\left(G_x^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right)}\right)\left(2G_x^2 + \left(\kappa_{23}^{(x)}\right)^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right)}{2\left(G_x^2 + \left(\kappa_{23}^{(x)}\right)^2\right)\left(G_x^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right)},$$

$$B_1 = \frac{\left(\kappa_3^{(x)} - \kappa_2^{(x)} - \sqrt{\left(\kappa_2^{(x)} - \kappa_3^{(x)}\right) + 4\left(G_x^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right)}\right)\left[\kappa_{23}^{(x)}\left(G_x^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right) + \kappa_{32}^{(x)}\left(G_x^2 + \left(\kappa_{23}^{(x)}\right)^2\right)\right]}{2\left(G_x^2 + \left(\kappa_{23}^{(x)}\right)^2\right)\left(G_x^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right)},$$

$$B_2 = \frac{\left(\kappa_3^{(x)} - \kappa_2^{(x)} - \sqrt{\left(\kappa_2^{(x)} - \kappa_3^{(x)}\right) + 4\left(G_x^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right)}\right)\left(2G_x^2 + \left(\kappa_{23}^{(x)}\right)^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right)}{2\left(G_x^2 + \left(\kappa_{23}^{(x)}\right)^2\right)\left(G_x^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right)},$$

$$\varphi^{(x)} = \arctan\frac{G_x\left(2G_x^2 + \left(\kappa_{23}^{(x)}\right)^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right)}{G_x^2\left(\kappa_{23}^{(x)} + \kappa_{32}^{(x)}\right) + 2\left(\kappa_{23}^{(x)}\right)^2\kappa_{32}^{(x)}}.$$

We group formulas (30), (31), (32), (33) accordingly, and register two light waves travelling towards the Cosserat medium in the form:

$$\left(\frac{E_{2II}}{E_o}\right)^2 + \left(\frac{E_{3I}}{E_o A^{(x)}}\right)^2 - 2\frac{E_{2II}E_{3I}}{E_o^2 A^{(x)}} \cos\left[\left(\psi_r^{(x)} - \psi_l^{(x)}\right) - \varphi^{(x)}\right] = \sin^2\left[\left(\psi_r^{(x)} - \psi_l^{(x)}\right) - \varphi^{(x)}\right], \tag{34}$$

$$\left(\frac{E_{2I}}{E_o}\right)^2 + \left(\frac{E_{3II}}{E_o B^{(x)}}\right)^2 - 2\frac{E_{2I}E_{3II}}{E_o^2 B^{(x)}} \cos\left[\left(\psi_l^{(x)} - \psi_r^{(x)}\right) - \varphi^{(x)}\right] = \sin^2\left[\left(\psi_l^{(x)} - \psi_r^{(x)}\right) - \varphi^{(x)}\right]. \tag{35}$$

The formulas (34), (35) present right and left-handed elliptical polarization light waves which travel forward in an elliptical helical path.

The infinitesimal path retardation

$$d\Delta^{(x)} = \left[(n^{(x)})_r - (n^{(x)})_l\right] dx,$$

is written as:

$$d\Delta^{(x)} = C\sqrt{\left(\kappa_2^{(x)} - \kappa_3^{(x)}\right)^2 + 4\left(G_x^2 + \kappa_{23}^{(x)}\kappa_{32}^{(x)}\right)}dx, \tag{36}$$

where $C = \dfrac{1}{n_r^{(x)} + n_l^{(x)}} \cong \frac{1}{2n}$, $n_r^{(x)} + n_l^{(x)} \cong 2n$, (small optical anisotropy). When we arrive at the linear medium we ignore the vector of optical rotation, $G_x = 0$, and when the principal directions (1, 2, 3) are parallel to the directions of the coordinate system (x, y, z) we obtain formula:

$$d\Delta^{(x)} = C\left(\kappa_2 - \kappa_3\right)dx, \tag{37}$$

which is applied in linear birefringence [13–15].

Formulas (7) and (8) are substituted by formula (36) and the infinitesimal path retardation of the Cosserat birefringence medium is expressed by components of the force stress and couple stress tensor:

$$d\Delta^{(x)} = C\sqrt{a^*}\,dx, \tag{38}$$

where

$$a^* = (C_1^\sigma)^2\left[\left(\sigma_2^{(x)} - \sigma_3^{(x)}\right)^2 + 4\tau_{23}^{(x)}\tau_{32}^{(x)}\right] + 4\left[g + C_1^\mu\mu_x + C_2^\mu\left(\mu_x + \mu_2^{(x)} + \mu_3^{(x)}\right)\right]^2$$

For symmetrical elasticity without couple stresses formula (38) corresponding to linear photoelasticity [13–15] and we write for the principal directions ($(x) = 1, 2, 3$):

$$d\Delta^{(x)} = C_1\left(\sigma_2 - \sigma_3\right)dx, \tag{39}$$

where $C_1 = \dfrac{C_1^\sigma}{2n}$.

6. Cosserat gyro-birefringence formula

We write formula (14) in the form of the sum:

$$\begin{bmatrix} \kappa_x & \kappa_{x2}^{(x)} - iG_3^{(x)} & \kappa_{x3}^{(x)} + iG_2^{(x)} \\ \kappa_{2x}^{(x)} + iG_3^{(x)} & \kappa_2^{(x)} & \kappa_{23}^{(x)} - iG_x \\ \kappa_{3x}^{(x)} - iG_2^{(x)} & \kappa_{32}^{(x)} + iG_x & \kappa_3^{(x)} \end{bmatrix} = \begin{bmatrix} 0 & -iG_3^{(x)} & iG_2^{(x)} \\ iG_3^{(x)} & 0 & -iG_x \\ -iG_2^{(x)} & iG_x & 0 \end{bmatrix} + \begin{bmatrix} \kappa_x & \kappa_{x2}^{(x)} & \kappa_{x3}^{(x)} \\ \kappa_{2x}^{(x)} & \kappa_2^{(x)} & \kappa_{23}^{(x)} \\ \kappa_{3x}^{(x)} & \kappa_{32}^{(x)} & \kappa_3^{(x)} \end{bmatrix}. \tag{40}$$

Then we study only gyro-birefringence component when $\kappa_1^{(x)} = \kappa_2^{(x)} = \kappa_3^{(x)} = 0$, and $\kappa_{12}^{(x)} = \kappa_{21}^{(x)} = 0$, $\kappa_{23}^{(x)} = \kappa_{32}^{(x)} = 0$, $\kappa_{31}^{(x)} = \kappa_{13}^{(x)} = 0$. Equations (18), (19) are written as:

$$\left(n_r^{(x)}\right)^2 = G_x, \tag{41}$$

$$\left(n_l^{(x)}\right)^2 = -G_x. \tag{42}$$

We then substitute (41), (42) to (20), (21), (22), (23) in order to obtain two independent solutions:

$$\frac{E_3^{(x)}}{E_2^{(x)}} = i, \tag{43}$$

$$\frac{E_3^{(x)}}{E_2^{(x)}} = -i, \tag{44}$$

which allow us to describe two light waves:

$$\mathrm{E}_I = [0, 2E_o, i2E_o]\exp\left[i\left(\omega t - \psi_r^{(x)}\right)\right], \tag{45}$$

$$\mathrm{E}_{II} = [0, 2E_o, -i2E_o]\exp\left[i\left(\omega t - \psi_l^{(x)}\right)\right]. \tag{46}$$

We take the real part of the solutions (45), (46) and add mutual perpendicular waves leading us to obtain two pairs of the intensity component of the electric field:

$$\begin{cases} E_{2I} = E_o\cos\left(\omega t - \psi_l^{(x)}\right) \\ E_{3I} = E_o\cos\left(\omega t - \psi_l^{(x)} + \frac{\pi}{2}\right) \end{cases}, \tag{47}$$

$$\begin{cases} E_{2II} = E_o\cos\left(\omega t - \psi_r^{(x)}\right) \\ E_{3II} = E_o\cos\left(\omega t - \psi_r^{(x)} - \frac{\pi}{2}\right) \end{cases}, \tag{48}$$

expressed as:

$$E_{2I}^2 + E_{3I}^2 = E_o^2, \tag{49}$$

$$E_{2II}^2 + E_{3II}2 = E_o^2. \tag{50}$$

Solutions (47), (48) and (49), (50) mean that two right and left-handed light waves, travel forward in a circular helicon path.

The infinitesimal phase retardation for each of the waves on the *dx* way are written as:

$$d\psi_r^{(x)} = \frac{2\pi}{\lambda}\left(n_r^{(x)} - n\right)dx, \tag{51}$$

$$d\psi_l^{(x)} = \frac{2\pi}{\lambda}\left(n_l^{(x)} - n\right)dx. \tag{52}$$

The relative phase retardation (51), (52) of the right and left-handed circular polarization waves create the infinitesimal rotation of the azimuth of polarization:

$$\begin{gathered} d\Gamma_x^{couple} = \frac{1}{2}\left(d\psi_r^{(x)} - d\psi_l^{(x)}\right) \\ = \frac{\pi}{\lambda}\left(n_r^{(x)} - n_l^{(x)}\right)dx = \frac{\pi}{\lambda\sqrt{\kappa}}G_x dx. \end{gathered} \tag{53}$$

For geometrical way $\overline{AB}$, Fig. 1, we write:

$$\Gamma_{\mathrm{AB}}^{couple} = \frac{\pi}{\lambda n}\int\limits_A^B G_x dx, \tag{54}$$

where $\Gamma_{\mathrm{AB}}^{couple}$ is the angle of the rotation of the azimuth of polarization, $n \cong \sqrt{\kappa}$. We join (4) and (54), and we obtain the searched relation between $\Gamma_{\mathrm{AB}}^{couple}$ and components of the couple stresses:

$$\Gamma_{AB}^{couple} = \frac{\pi}{\lambda n}\int\limits_A^B \left[g + \left(C_1^\mu + C_2^\mu\right)\mu_x + C_2^\mu\left(\mu_2^{(x)} + \mu_3^{(x)}\right)\right]dx. \tag{55}$$

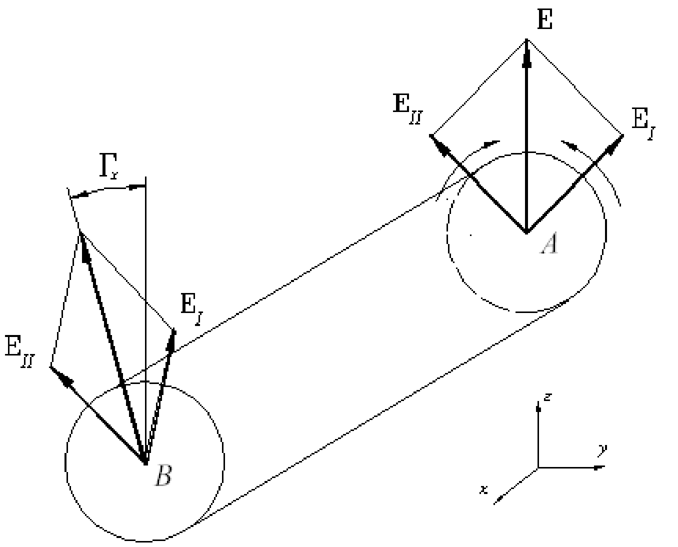

Fig. 1. Measurement model of the anticipate of the rotation of the azimuth of polarization in Cosserat gyro-birefringence material

7. Conclusions

As it is seen, a plane-polarized light wave on passing through the Cosserat medium is split into two right and left-handed elliptical polarization light waves which travel forward in an elliptical helical path. We divide elliptical light wave into two components: plane-polarized wave and circularly polarized wave. Circular polarized waves is right and left-handed and travels forward in an circular helical path. The linear polarization is associated with classical photoelasticity (force stresses) and circular polarization belongs to nonsymmetrical photoelasticity (couple stresses). Two circular helicon path travels forward with different speeds. As a result of interference we obtain a rotation of the azimuth of polarization proportional to couple stresses.

The work presented is an attempt to complement the birefringence theory in which the influence of the mechanical factor on gyro-birefringence is not known.

Appendix 1

We analyze an atom where the mass centre and the action centre of Coulomb forces is not lined up. This symmetry disturbance is created by an optical active electron.

The assumption is to eliminate the orbital angular moment of the optical active electron by the action of the crystal field. The energy contribution due to the electron spin $\boldsymbol{S}$ under a nonsymmetrical mechanical loading, can be written in this form [11]:

$$E^{couple} = -\boldsymbol{\omega}^{couple}\boldsymbol{S}. \tag{A1}$$

There $\boldsymbol{\omega}^{couple}$ is the precession vector of spin due to nonsymmetrical loading. Equation (A1) can be re-written in the operator form as follows:

$$\boldsymbol{\omega}^{couple}\widehat{\boldsymbol{S}}\Phi = E^{couple}\Phi, \tag{A2}$$

there Φ is the wave function, $\widehat{\boldsymbol{S}}$ is operator of spin. Equation (A1) corresponds now to the Schrödinger equation written in the component "z" Cartesian coordinate system:

$$\omega_z^{couple}\hat{S}_z\Phi_z = E_z^{couple}\Phi_z. \tag{A3}$$

By application the Pauli operators:

$$\hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A4}$$

and the spin wave functions:

$$\Phi_z = \begin{cases} \Phi_z^+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} & \text{for } mech_{S_z} = \dfrac{1}{2}, \\ \Phi_z^- = \begin{pmatrix} 0 \\ 1 \end{pmatrix} & \text{for } mech_{S_z} = -\dfrac{1}{2}, \end{cases} \tag{A5}$$

where $mech_{S_z}$ denotes the spin quantum number for the axes "z" the Eq. (A2) now read:

$$\widehat{S}_z\Phi_z = \hbar\, mech_{S_z}\Phi_z. \tag{A6}$$

Using analogy between (A3) and (A6) the eigenfunctions for both sets of equations can be presented in the form:

$$E_z^{couple} = \omega_z^{couple}\hbar\, mech_{S_z}. \tag{A7}$$

Appendix 2

Maxwell equations are grouped with material equations and are written in the first approximation [1–2]:

$$\left.\begin{aligned} rot\boldsymbol{H} - \dot{\boldsymbol{D}} = 0 \\ \boldsymbol{D} = \kappa\kappa_o\boldsymbol{E} \end{aligned}\right\} rot\dot{\boldsymbol{H}} - \kappa\kappa_o\ddot{\boldsymbol{E}} = 0, \tag{A1}$$

$$\left.\begin{aligned} rot\boldsymbol{E} + \dot{\boldsymbol{B}} = 0 \\ \boldsymbol{B} = \chi\chi_o\boldsymbol{H} \end{aligned}\right\} rotrot\boldsymbol{E} + \chi\chi_o rot\dot{\boldsymbol{H}} = 0, \tag{A2}$$

where $\boldsymbol{H}$ is the vector of the field of the magnetic intensity, $\boldsymbol{B}$ is the vector of the magnetic induction, χ_o is magnetic permeability of the vacuum, χ is magnetic permeability of the medium. On the basis of the identity:

$$rotrot\boldsymbol{E} = graddiv\boldsymbol{E} - \nabla^2\boldsymbol{E} = 0, \tag{A3}$$

we obtain the wave equation:

$$\nabla^2\boldsymbol{E} - \chi\chi_o\kappa\kappa_o\ddot{\boldsymbol{E}} = 0, \tag{A4}$$

and the solution (A3) as:

$$\boldsymbol{E} = \boldsymbol{E}_o \exp\left[i\omega\left(t - \frac{\boldsymbol{rs}}{c_n}\right)\right], \tag{A5}$$

where ω – angular frequency, t – time, $\boldsymbol{r}$ – radius vector, $c_n = 1\big/\sqrt{\kappa\kappa_o\chi\chi_o}$ – phase speed of the wave in the medium with refractive index n. We apply the rotation operator to (A4) and after the transmutation we write:

$$rot\boldsymbol{E} = \frac{i\omega}{c_n}\left(\boldsymbol{E}\times\boldsymbol{s}\right). \tag{A6}$$

Similarly, we obtain:

$$rot\boldsymbol{H} = \frac{i\omega}{c_n}\left(\boldsymbol{H}\times s\right). \tag{A7}$$

Then we group equations:

$$\left.\begin{aligned} &\boldsymbol{D} = \kappa\kappa_o\boldsymbol{E},\\ &\boldsymbol{E} = \boldsymbol{E}_o\exp\left[i\omega\left(t-\frac{\boldsymbol{rs}}{c_n}\right)\right]\end{aligned}\right\}\dot{\boldsymbol{D}} = i\omega\boldsymbol{D} \tag{A8}$$

and

$$\left.\begin{aligned} &\boldsymbol{B} = \chi\chi_o\boldsymbol{H},\\ &\boldsymbol{H} = \boldsymbol{H}_o\exp\left[i\omega\left(t-\frac{\boldsymbol{rs}}{c_n}\right)\right]\end{aligned}\right\}\dot{\boldsymbol{B}} = i\omega\boldsymbol{B}. \tag{A9}$$

After joining formulas written above we obtain:

$$rot\boldsymbol{H} - \dot{\boldsymbol{D}} = 0 \to \boldsymbol{H}\times\boldsymbol{s} = c_n\boldsymbol{D}, \tag{A10}$$

$$rot\boldsymbol{E} + \dot{\boldsymbol{B}} = 0 \xrightarrow{\boldsymbol{B}=\chi\chi_o\boldsymbol{H}} \boldsymbol{s}\times\boldsymbol{H} = c_n\chi\chi_o\boldsymbol{H}. \tag{A11}$$

Formulas (A10) are written in the form:

$$\sqrt{\frac{\kappa\kappa_o}{\chi\chi_o}}\left[\left(\boldsymbol{s}\times\boldsymbol{E}\right)\times\boldsymbol{s}\right] = c_n\boldsymbol{D}. \tag{A12}$$

Including: $n = \sqrt{\chi\kappa}$, $\chi = 1$, we obtain:

$$\boldsymbol{D} = -\kappa_o n^2\left[\left(\boldsymbol{E}\times\boldsymbol{s}\right)\times\boldsymbol{s}\right]. \tag{A13}$$

Finally, we write:

$$\boldsymbol{D} = \kappa_o n^2\left[\boldsymbol{E} - \boldsymbol{s}\left(\boldsymbol{Es}\right)\right]. \tag{A14}$$

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