Cellular manufacturing systems design using Tabu search

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Abstract: Several techniques have appeared in the literature for the cell formation problem. However, most of these heuristics have the disadvantages that they cannot solve real-life problems typical of manufacturing firms in reasonable computational time, and deal with the part family formation and cell grouping problems separately. The Tabu search meta-heuristic is presented for resolving these problems. The minimization of intercellular movements as an objective function is demonstrated, although minimization of cell load variation and the multiobjective function option exist in the procedure developed. The results discussed for some problems taken from the literature show that the Tabu search is a promising technique for solving the cellular manufacturing systems design problem for both medium and large problem instances.

Keywords: cellular manufacturing, cell design, Tabu search, combinatorial optimization

1 INTRODUCTION

Cellular manufacturing is a manufacturing strategy for gaining advantage in the global competition by reducing manufacturing costs, improving quality and reducing the customer delivery lead time of products in a high-variety, moderate demand environment [**1, 2**]. To derive economic advantages, parts are divided into part families and existing machines are grouped accordingly to produce the parts by analysing the process routes information. A number of standard methods available for forming cells include the cluster analysis method based on similarity coefficient measures [**3, 4**], the integer programming method [**5**], the graph theoretic method [**6**] and the generalized cell formation model [**7**]. For more details on the cell formation methods, see Singh and Rajamani [**8**].

Some of the above cell formation approaches are essentially heuristic while integer programming and graph theoretic methods seek an optimal solution for the cell formation problem. All these methods have the disadvantage that they cannot be applied to large problem instances typically encountered in the industry, which are combinatorial in nature.

The Tabu search technique, together with genetic algorithms, and simulated annealing have become popular in solving combinatorial optimization problems. Since the cell design problem is combinatorial in nature, these techniques are successful in cell design. Venugopal and Narendran [**9**] have applied genetic algorithms to the cell formation problem. This paper provides a comprehensive treatment of various issues on the design of cellular manufacturing systems using the Tabu search meta-heuristic. The primary objective of this paper is therefore to provide application of the Tabu search [**10–12**] in cell design.

2 PROBLEM DESCRIPTION

Let $M = \{M_1, M_2, \ldots, M_n\}$ and $P = \{P_1, P_2, \ldots, P_n\}$ be, respectively, the set of machines and the set of parts, which have to be arranged into n cells C_1, $C_2, \ldots, C_p$. For any machine M_i, *cell* (M_i) denotes the cell to which it is assigned in the current solution. Similarly, *cell* (P_j) denotes the cell assignment for part P_j. Let $W = \{w_{ij}\}$ be the $m \times n$ matrix such that $w_{ij} > 0$ if and only if P_j must be processed by machine M_i; $w_{ij} = 0$ if otherwise. Here, w_{ij} denotes the workload on machine i induced by part j.

Every solution of the cell formation problem may be characterized by the set of machines and parts within each cell C_k, $k = 1, \ldots, p$. Let $x_{ik} = 1$ if machine M_i is assigned to cell C_k; $x_{ik} = 0$ if otherwise. In the same

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manner, let $y_{jk} = 1$ if part P_j is assigned to cell C_k; $y_{jk} = 0$ if otherwise. Then, the number m_k (respectively p_k) of machines (respectively parts) within cell C_k is given by

$$m_k = \sum_{i=1}^{m} x_{ik}, \qquad p_k = \sum_{j=1}^{n} y_{jk}$$

The total load of cell l induced by part j is given as

$$\sum_{i=1}^{m} x_{ik} w_{kj}$$

The average cell load is denoted by

$$m_{jk} = \frac{\sum_{i=1}^{m} x_{ik} w_{kj}}{\sum_{i=1}^{m} x_{ik}}$$

The number of exceptional elements is denoted by

$$z_{jk} = e_{ji} x_{ik} < z_{jk}^*$$

where $z_{ik}^* = \arg\max(z_{jk})$ means that part which is assigned to the cell where most of its operations are carried out. When exceptional elements are taken into consideration, it is necessary to define

$$e_{ij} = \begin{cases} 1 & \text{if } [w_{ij}]^{\mathrm{T}} > 0 \\ 0 & \text{if otherwise} \end{cases}$$

In the cellular design, the following objective functions f_1 and f_2 were minimized:

$$f_1 = \sum_{i=1}^{m} \sum_{k=1}^{p} \sum_{j=1}^{n} (w_{ij} - m_{kj})^2 \tag{1}$$

$$\text{s.t.} \sum_{k=1}^{p} x_{ik} = 1 \qquad \forall i \tag{2}$$

$$\sum_{i=1}^{m} x_{ik} \geqslant 1 \qquad \forall k \tag{3}$$

$$f_2 = \sum_{j=1}^{n} z_{jk} \tag{4}$$

$$\text{s.t.} \sum_{k=1}^{p} x_{ik} = 1 \qquad \forall i \tag{5}$$

$$\sum_{i=1}^{m} x_{ik} \geqslant 1 \qquad \forall k \tag{6}$$

3 TABU SEARCH

The Tabu search is an adaptive search procedure that may be employed for solving combinatorial optimization problems [**10–13**]. The remaining part of this section explains these and other features of the Tabu search in cellular manufacturing systems design.

3.1 Initialization

For a particular neighbourhood, machines are randomly assigned to cells while ensuring that all cell constraints such as minimum and maximum machines per cell, non-null cells, etc., are met.

3.2 Neighbourhood and moves

The *neighbourhood* of a solution is the set of all formations that can be arrived at by a *move*. A move is a feasible transfer of one machine from one cell to another in search of a better solution without violating cell cardinality constraints. The concept of neighbourhood can be clearly shown as follows:

Neighbour 1	1	2	2	1
Neighbour 2	2	1	2	1

The first neighbourhood is made up of machines $\{1, 4\}$ in cell 1 and machines $\{2, 3\}$ in cell 2. The second neighbourhood is made up of machines $\{2, 4\}$ in cell 1 and machines $\{1, 3\}$ in cell 2. The column positions show the machine number while the numbers $\{1, 2\}$ identify the cells.

3.3 Tabu list

In order to prevent the scheme cycling and returning to the same solutions it is necessary to introduce a condition to prevent this happening. Since storing and checking previous solutions is very expensive, especially where the function $f(\cdot)$ to be evaluated in every iteration is expensive, it is usual to carry this out by not allowing the reversal of moves for a certain number of iterations equal to the *Tabu length*. These non-admissible moves within the short interval are the class membership of a *Tabu list*. The length of the Tabu list is a parameter that needs to be decided. Tabu length with minimum 7 and maximum 11 has been suggested [**12**]. The Tabu length used is equal to 50 per cent in excess of the number of machines used.

3.4 Aspiration criterion

The *currentbest* values for each neighbourhood represent the local optima. The least of all these is found, and the best, known as *isbest*, yields the global minimum. The way that it is implemented in the cell formation problem allows no better solution to be found than the elite candidate for each neighbourhood.

3.5 Intensification

The mechanism for intensification enhances the search to focus on examining elite solutions in a neighbourhood.

It tends to move the search to a neighbouring position in the search space, and so could be considered a local search. The *intensification length* used for the work reported here is equal to $mc^2/4$, where m is the number of machines and c is the number of cells.

3.6 Diversification

The mechanism for diversification allows large jumps to be made in the solution space. This ensures that large areas of the space are searched and solutions do not get stuck in local minima. This mechanism is also referred to as the *restarting* procedure. The *diversification length* is defined as being 1.5 times the sum of the number of machines and cells. For each diversification process, a different initial cell formation is randomly generated. Therefore the search is able to explore a large solution space, thereby enhancing the possibility of finding the optimum solution in a very short time.

3.7 Stopping criteria

In the present implementation, the intensification and diversification lengths were used to terminate the solution search. For the work reported here, search is terminated after $(3k/8)(1+c)$ total iterations, where $k=(mc)^2$, m is the number of machines and c is the number of cells.

4 PERFORMANCE MEASURE

The performance measure used in this work is one of the two fully described in reference [**4**]. The grouping efficiency (e) has been adopted. Denoting the number of non-zero entries in the diagonal block (which forms the cells) in the machine–part incidence matrix as n_1, where M_i and P_i are the number of machines and parts in each cell i respectively, then

$$e_1 = \frac{n_1}{\sum_{i=1}^{c} M_i P_i}$$

where c is the number of cells in the matrix and e_1 is an indicator of the within-cell density of a cell. An intercellular transfer indicator is defined as

$$e_2 = 1 - \frac{n_1}{n_1 + n_0}$$

where n_0 is the number of non-zero entries in the machine–part incidence matrix outside the cells or intercell transfers. The grouping efficiency is given as the difference between e_1 and e_2:

$$e = e_1 - e_2, \qquad -1 \leqslant e \leqslant 1$$

5 COMPARING THE TABU SEARCH-BASED HEURISTIC WITH OTHER METHODS

In order to determine the effectiveness of the Tabu search-based heuristic for the cellular manufacturing systems design, it was necessary to compare its results with those obtained using other methods published in the literature. Table 1 shows the test problem set for eight problem instances from the literature. Table 2 compares the Tabu search-based heuristic results with the genetic algorithm–travelling sales person (GA–TSP) hybrid of Balakrishnan and Jog [**19**], SC–seed of Miltenburg and Zhang [**20**], the HPH method of Askin *et al.* [**21**], 0–1 programming of Wei and Gaither [**22**] and the similarity order clustering (SOC) of Onwubolu [**4**].

It can be observed that the Tabu search heuristic compares well with all the other methods that seek optimum solutions. The Tabu search heuristic also compares well with the GA–TSP, which is a hybrid meta-heuristic.

6 CONCLUSIONS

The Tabu search algorithm developed for the work reported in this paper was used to solve a set of published data previously solved using other methods. In terms of grouping quality, the Tabu search compared very well with the hybrid genetic algorithm, the SC–seed, the HPH method and the 0–1 programming for the problems solved. However, the SC–seed, the HPH method and the 0–1 programming solved only selected problems. The Tabu search-based heuristic and the hybrid genetic algorithm solved all the problems and compare very

Table 1 Matrix densities for published data set

Number	Problem	Parts	Machines	Matrix density
1	Chan and Milner [**14**]	10	15	0.31
2	De Witte [**15**]	19	12	0.33
3	Chandrasekharan and Rajagopalan [**16**]	20	8	0.38
4	Burbridge [**1**]	43	16	0.18
5	Carrie [**3**]	35	20	0.19
6	Chandrasekharan and Rajagopalan [**17**]	20	8	0.57
7	Vannelli and Kumar [**6**]	41	30	0.105
8	King [**18**]	24	14	0.175
9	Venugopal and Narendran [**9**]	15	30	0.34

Table 2 Clustering efficiency values for test experiments for the Tabu search and other algorithms

Number	Parts size	Machine size	Cell number	Tabu search [present]	SC–TSP **[19]**	SC–seed **[20]**	HPH method **[21]**	0–1 programming **[22]**	SOC **[4]**
1	10	15	3	0.92	0.92	0.93	—	—	0.92
2	19	12	2	0.41	0.42	—	—	—	0.29
			3	0.40	0.37	—	0.37	—	—
3	20	8	3	0.85	0.85	0.85	0.85	—	0.85
4	43	16	5	0.40	0.44	0.45	0.42	—	0.14
5	35	20	4	0.72	0.76	0.76	—	—	0.66
6	20	8	1	0.56	0.57	—	—	—	0.57
			2	0.49	0.58	—	—	—	0.40
7	41	30	4	0.27	0.31	—	—	0.16	0.26
8	24	14	2	0.31	0.33	—	—	0.33	0.33
			4	0.59	0.63	—	—	0.66	—
9	30	15	3	0.95	—	—	—	—	—

well. These two are meta-heuristics and deal with both small and large problem instances.

The objective functions considered in the work reported include minimization of exceptional elements, minimization of cell load variation and simultaneous minimization of both exceptional elements and cell load variation. Inclusion of several objective function options in the Tabu search procedure developed makes the approach flexible for designing cellular manufacturing systems. This approach for solving combinatorial optimization problems outperforms traditional techniques for cellular manufacturing systems design.

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