Fatigue Life Prediction under Random Loading Conditions in 7475–T7351 Aluminum Alloy using the RMS Model

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ABSTRACT: This article is concerned with the fatigue life prediction in specimens of 7475–T7351 high strength aluminum alloy subjected to random fatigue loading. Fatigue life predictions are made using the root mean square model. This model is chosen because it has been defined as the most simple and effective one for fatigue life prediction in the components subjected to random loading by the authors of this article. The analysis procedure used in this study is relatively simple. The loading history for each specimen is analyzed to determine the root mean square maximum and minimum stresses and predictions are then made by assuming that the tests have been conducted under constant amplitude loading at the root mean square maximum and minimum stresses. The ratios of the predicted lives range from 3.22 to 1.52. These ratios are fairly good considering that the normal scatter in fatigue crack growth rates may range from a factor of two to four under identical load conditions. Moreover, an attempt has been made to improve prediction procedure using Forman's equation applied in the root mean square model. While using the improved prediction procedure, the ratios of the predicted lives range from 1.35 to 0.62 (e.g., error bound is reduced almost five times: from 222 to 48). Only relatively simple computer programs (Microsoft Excel for load history analysis and Mathematica for performing calculations) and a desktop computer are employed to make predictions. Improved prediction procedure allows more precise prediction of fatigue life as well as helps to obtain better prediction ratios but further experimental work should be performed to verify the validity of the attempt.

KEY WORDS: 7475–T7351 aluminum alloy, RMS model, fatigue crack growth prediction under random loading, fatigue of materials.

INTRODUCTION

F OR THE APPLICATION of damage tolerance concepts, it is necessary to make a reliable estimation of the number of load cycles required to propagate the crack from the minimum detectable (initial) size to the critical (final) size. In terms of fracture mechanics, this is referred to as fatigue crack growth prediction or fatigue life prediction. Inspection intervals thereafter have to be based on this estimate, or fatigue crack growth to the critical (final) size should take so much time that it covers the whole service life. The fatigue crack growth prediction or fatigue life prediction for the real structure is performed on the basis of relevant data, as for fatigue loads, crack propagation data, and structural geometry. Hence, fatigue crack growth prediction is obvious.

Usually, the fatigue strength of an alloy is measured using constant amplitude tests to determine the strain-life or stress-life curve of the materials. Then the fatigue life of the structure is predicted using the data from constant amplitude tests. However, the majority of finite-life engineering structures (e.g., aerospace, automotive, railway and bridge structures) experience random loading while in operation. Moreover, it has been determined by various researches (Christensen, 1959; Schijve and Broek, 1962; Hudson and Hardrath, 1963; Schijve, 1972; Stephens et al., 1976; Newman, 1982; Skopura, 1996; Newman, 1997; Wu et al., 1998; Narayanaswami, 2001) that the experimental fatigue lives for the specimens and components subjected to random amplitude loading can be well below the fatigue lives predicted using constant amplitude tests' results. Moreover, it has been determined that it is not possible to predict fatigue crack growth accurately without a thorough knowledge of the load–time history occurring in service (Rolfe and Barsom, 1975; Stephens et al., 1976). Therefore, it is of great importance for engineers to predict fatigue crack growth under random loading. And that is why the present study is so important. The majority of fatigue analyses rely on experimental data, however, such experimental data of fatigue tests under random loading are rarely available in the public literature (for instance Chang/Hudson, 1981) which makes the research in the area of random fatigue more complex.

Since aluminum alloys have been widely used as structural materials in various engineering applications, such as automotive, aerospace, shipbuilding and others, many studies concerning fatigue damage, and the fatigue crack growth prediction in aluminum alloys (Hudson and Hardrath, 1963; Schijve, 1972; Stephens et al., 1976; Chang/Hudson, 1981; Johnson, 1981; Hudson, 1982; Newman, 1997; Wu et al., 1998; Narayanaswami, 2001; Wei et al., 2002; Moreno et al., 2003) have been conducted on these materials recently.

As an example of previous studies, we may consider a round-robin analysis carried out by members of ASTM Task Group (NASA Langley Research Center, (Chang/Hudson, 1981). This analysis evaluated and compared different methods for predicting crack growth under random loading. Experiments showed that the root mean square (RMS) method usually bears an insignificant computation cost, and predictions obtained by using RMS approach showed fairly good results. In addition, it was shown that the linear analysis method was better than the methods that accounted for the load interaction effects. It was determined also, that the trade-off between cost and predictive accuracy is a factor worth considering when selecting an acceptable prediction method.

The authors of this article carried out a comparative study to evaluate models currently in practice for fatigue crack growth prediction under random loading. It was found out during the study that the RMS model is the most simple, reliable, and efficient method to predict fatigue crack growth in a component under random loading.

Nowadays, 7475 aluminum alloy having fine grain size, optimum dispersion, and highest toughness value among the aluminum alloys commercially available at high strength level as well as excellent corrosion fatigue behavior has better performance than that of many commercially available high strength aerospace aluminum alloys, such as 7050, 7075, and 2024. That is why the 7475–T7351 aluminum alloy has been chosen for fatigue crack growth prediction analysis performed to confirm the accuracy of the RMS model for fatigue crack growth prediction under random loading.

EXPERIMENTAL

Material and Specimen's Configuration

The 7475 (Al–Zn–Mg–Cu) alloy is a controlled toughness alloy developed for applications that require a combination of high strength, superior fracture toughness, and resistance to fatigue crack propagation both in air and aggressive environment. The 7475 aluminum alloy is basically a modified version of 7075 alloy. The properties of 7075 alloy are improved by reducing its iron and silicon contents, and altering quenching and aging conditions. These modifications in 7075 alloy result in the development of 7475 alloy having fine grain size, optimum dispersion, and highest toughness value among the aluminum alloys commercially available at high strength level. The 7475 alloy in the form of sheets is commercially available in T61 and T761 tempers, whereas the plates are usually tempered to T651, T7351,

Table 1. (a) Chemical composition of 7475–T7351 aluminum alloy.

Component	Al	Cr	Cu	Fe	Mg	Mn	Si	Ti	Zn
Content wt. %	90.3	0.18–0.25	1.2–1.9	max 0.12	1.9–2.6	max 0.06	max 0.1	max 0.06	5.2–6.2

Table 1. (b) Mechanical properties of 7475–T7351 aluminum alloy.

Property	Tensile strength (MPa)	Yield strength (MPa)	Percentage elongation, e (%)	Young's modulus, E (GPa)
Value	462	532	13	72

and T7651 conditions. The plate has the strength very close to some tempered 7075 alloy, whereas the fracture toughness values are up to 40% greater than that of 7075 alloy in the same temper conditions. The 7475 alloy, in the form of sheets and plates, is presently being specified for the critical components of high performance aircraft, where high fracture toughness is a major design consideration. Moreover, 7475 (under T7351 temper condition) exhibits superior fatigue life (Verma et al., 2001). The chemical composition and the mechanical properties of 7475–T7351 aluminum alloy are presented in Table 1(a) and (b).

The T7351 treatment involves solution treatment at 470°C, water quenching and controlled stretching from 1.5 to 3% followed by artificial aging in two stages: first at 121°C for 25 h and then at 163°C for a period of 24–30 h.

All test specimens used in the experimental study were surface crack specimens fabricated from 25.4 mm thick 7475–T7351 aluminum plates. Figure 1 illustrates the configuration of the specimen. The center notch in the specimens was fabricated by employing the electrical discharge machining (EDM), with the maximum width of the notch less than 0.254 mm.

All fatigue tests were performed using Servo-Hydraulic Instron-1332 fatigue testing machine with a maximum load capacity of 100 kN. The EDM slot in each specimen was fatigue precracked to produce a crack ≈2.54 mm in length including the EDM slot. Precracking was performed under constant-amplitude loading cycled at a maximum stress of 56 MPa. All tests were run at a cyclic rate of 5 Hz in ambient laboratory air at room temperature. Cyclic crack growth measurements were obtained using the traveling microscope with resolution of the crack length measurements ≈0.127 mm.
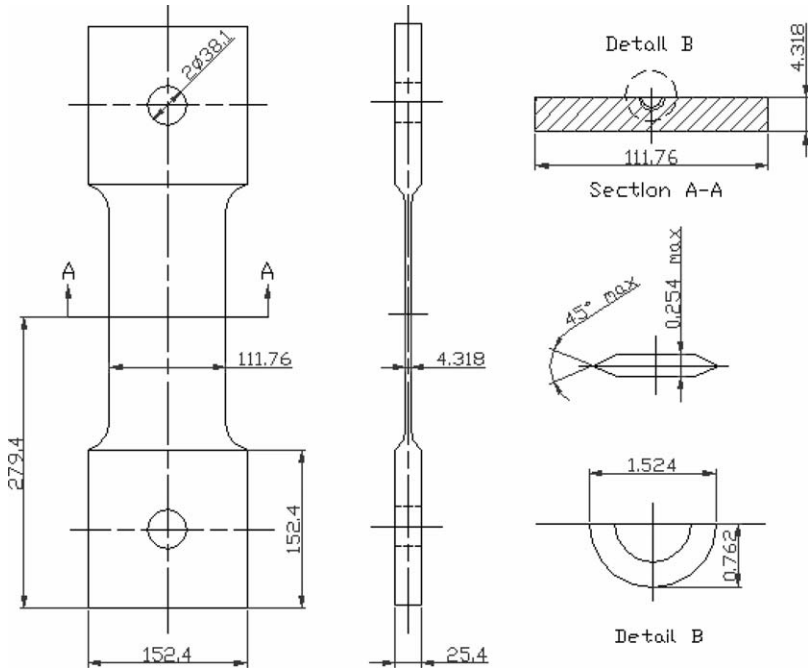
Figure 1. *Specimen's configuration (in millimeters).*

Random Fatigue Testing

Random loading spectrum, as shown in Figure 2, consisted of 123,586 load cycles. This load spectrum belongs to the sill longeron of the aircraft. The blocks were repeated until the failure of the specimen. The loading history for each specimen was analyzed to determine the RMS maximum and minimum stresses. Microsoft Excel program was used for load history analysis. All other calculations were performed using Mathematica software. Only a desktop computer and relatively simple computer programs were involved in the prediction procedure.

Prediction Procedure

The following relationships were used to find the RMS stresses:

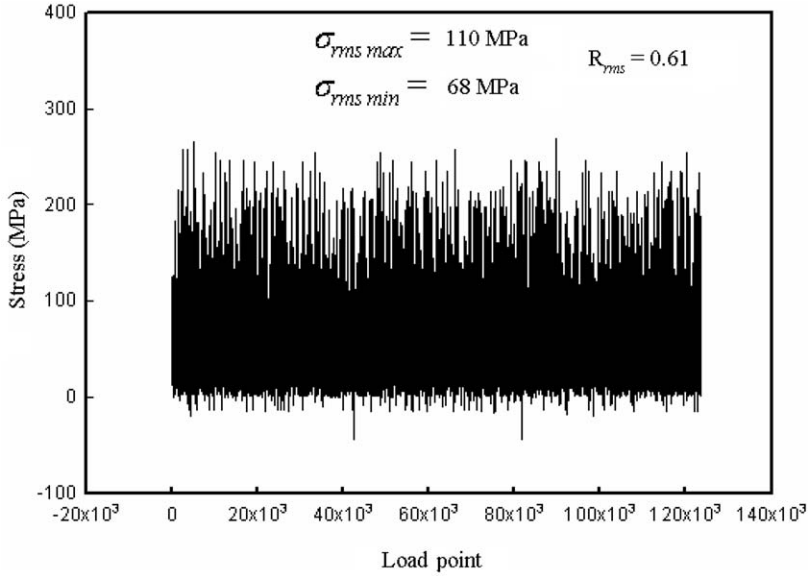$$\sigma_{\text{max rms}} = \left[\frac{1}{M} \sum_{i=1}^{m} (\sigma_{\text{max}})^2 \right]^{1/2} \tag{1}$$

Figure 2. *Random load spectrum for one block.*

and

$$\sigma_{\text{min rms}} = \left[\frac{1}{M} \sum_{i=1}^{m} (\sigma_{\text{min}})^2 \right]^{1/2} \tag{2}$$

where σ_{max} and σ_{min} are the maximum and minimum stresses derived from random stress history, and M is the total number of σ_{max} or σ_{min} values. When $\sigma_{\text{max rms}}$ and $\sigma_{\text{min rms}}$ are calculated, the RMS stress ratio, R_{rms} is found as:

$$R_{\text{rms}} = \frac{\sigma_{\text{min rms}}}{\sigma_{\text{max rms}}} \tag{3}$$

For surface-crack specimens used in experimental study, Newman's stress intensity solution (Newman, 1973) was used to calculate the RMS stress intensity factor range:

$$\Delta K_{\text{rms}} = (\sigma_{\text{max rms}} - \sigma_{\text{min rms}}) \cdot \sqrt{\frac{\pi a}{Q}} M_{\text{e}} \tag{4}$$

where,

$a =$ crack depth and $Q =$ elastic shape factor for an elliptical crack given by:

$$Q = 1 + 1.47\left(\frac{a}{c}\right)^{1.64} \tag{5}$$

$M_e =$ elastic magnification factor:

$$M_e = \left[M_1 + \left(\sqrt{\frac{Q \cdot c}{a}} - M_1 \right)\left(\frac{a}{t}\right)^p \right] \cdot \left[\sec\frac{\pi c a}{Wt} \right]^{1/2} \tag{6}$$

The front-face correction for elastic magnification factor M_1 is given by:

$$M_1 = 1.13 - 0.1\left(\frac{a}{c}\right) \tag{7}$$

and the exponent p for elastic magnification factor is given by:

$$p = 2 + 8\left(\frac{a}{c}\right)^3 \tag{8}$$

Since the same random fatigue spectrum (block) was used for all specimens, calculated values of $\sigma_{max\ rms}$, $\sigma_{min\ rms}$, and R_{rms} were the same for all the specimens. These values in tabular form are presented in Table 2. And the results of calculation performed for determining stress intensity factor using Newman's stress intensity solution are presented in Table 3.

The fatigue crack growth rate equation chosen for the analysis is the Forman's equation, which is a modified version of Paris' equation (Forman et al., 1967). The fatigue life of the specimens was predicted by numerically integrating Forman's equation from the initial crack size, c_i,

Table 2. RMS stresses and stress ratio for the test specimens analyzed.

Specimen	$\sigma_{max\ rms}$ (MPa)	$\sigma_{min\ rms}$ (MPa)	R_{rms}
A	110	67.4	0.612
C	110	67.4	0.612
D	110	67.4	0.612

Table 3. Results of the calculations for determining stress intensity factor.

Specimen	Initial crack depth a (mm)	Initial half-crack length c_i (mm)	a/c	Q	M_1	p	M_e
A	0.762	1.275	0.60	1.63	1.070	3.728	1.338
C	0.762	1.285	0.60	1.62	1.071	3.643	1.340
D	0.762	1.280	0.60	1.62	1.071	3.643	1.340

to the final crack size c_f. The forman equation used for fatigue crack growth prediction is given by:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \frac{C \Delta K_{\mathrm{rms}}^n}{(1 - R_{\mathrm{rms}})K_c - \Delta K_{\mathrm{rms}}} \tag{9}$$

This equation has been numerically integrated using the following input data:

(a) K_c: the experimental fracture toughness. It was determined using the relationships for plane stress and plane strain, which can be found in the literature that the specimens were subjected to plane stress conditions. Therefore, from Lang (2000), a value of 50.6 MPa $\sqrt{m}$ was selected for the specimens tested.

(b) C and n are the coefficient and exponent in the Forman's equation. C and n are equal to 4.006×10^{-9} and 3.18255 (in SI units) respectively for the 7475–T7351 aluminum alloy. These values were taken from the database of the Advanced Damage Analysis Modular System (ADAMSys) developed by the Lockheed Fort Worth Company as a part of the Aircraft Structural Integrity Program (Advanced Damage Analysis Modular System).

(c) ΔK_{rms} and R_{rms} – the RMS stress intensity factor range and stress ratio. These values were calculated by using the equations stated above.

When all the terms for Forman's equation had been defined, fatigue lives of the specimens subjected to random loading were determined by:

$$N_{\mathrm{pred}} = \int_{a_o}^{a_f} \frac{(1 - R_{\mathrm{rms}})K_c - \Delta K_{\mathrm{rms}}}{C \Delta K_{\mathrm{rms}}^n} = \frac{(1 - R_{\mathrm{rms}})K_c - \Delta K_{\mathrm{rms}}}{C \Delta K_{\mathrm{rms}}^n}(c_f - c_i) \tag{10}$$

The values of crack lengths used to predict the fatigue crack growth are presented in Table 4. The results of the fatigue crack growth prediction analysis using the RMS model are presented in Table 5. The fatigue crack growth under random loading is presented in the graphical form in Figure 3.

Table 4. Input crack length data to predict fatigue crack growth under random loading.

Specimen	Pre-crack length, $2c_i$ (mm)	Final crack length, $2c_f$ (mm)
A	2.54	40.97
C	2.56	28.98
D	2.57	36.13

Table 5. Results of the fatigue crack growth prediction analysis using the RMS model.

Specimen	N_{pred}	N_{test}	N_{pred}/N_{test}	Error bound (%)
	(cycles)			
A	6.97×10^6	4.57×10^6	1.52	52
C	4.72×10^6	1.60×10^6	2.95	195
D	5.99×10^6	1.86×10^6	3.22	222

To provide a better means in assessing predictive accuracies, ratios of the predicted life ('prediction ratios') were calculated for all the analytical predictions (see Table 5).

Prediction ratios smaller than one were considered to be conservative predictions and prediction ratios greater than one, were considered non-conservative (in the sense that the analytical prediction overestimates the specimen's test life).

RESULTS AND DISCUSSION

Observations

Fatigue crack growth predictions were made for three specimens subjected to random block loading. Predictions were made as to the number of cycles it would take to grow a crack from a given initial crack size to the final size in each test case. Predicted fatigue lives and the ratios of the predicted lives to the test lives (N_{pred}/N_{test}) for all the specimens are presented in Table 5. These ratios ranged from 3.22 to 1.52. These ratios are fairly good, considering that the normal scatter in fatigue crack growth rates may range from a factor of two to four under identical loading conditions. Hudson (1982) in his research on fatigue crack growth prediction under random loading using the RMS model obtained the ratios of the predicted fatigue life to the test life, which ranged from 2.13 to 0.82. Johnson (1981) in
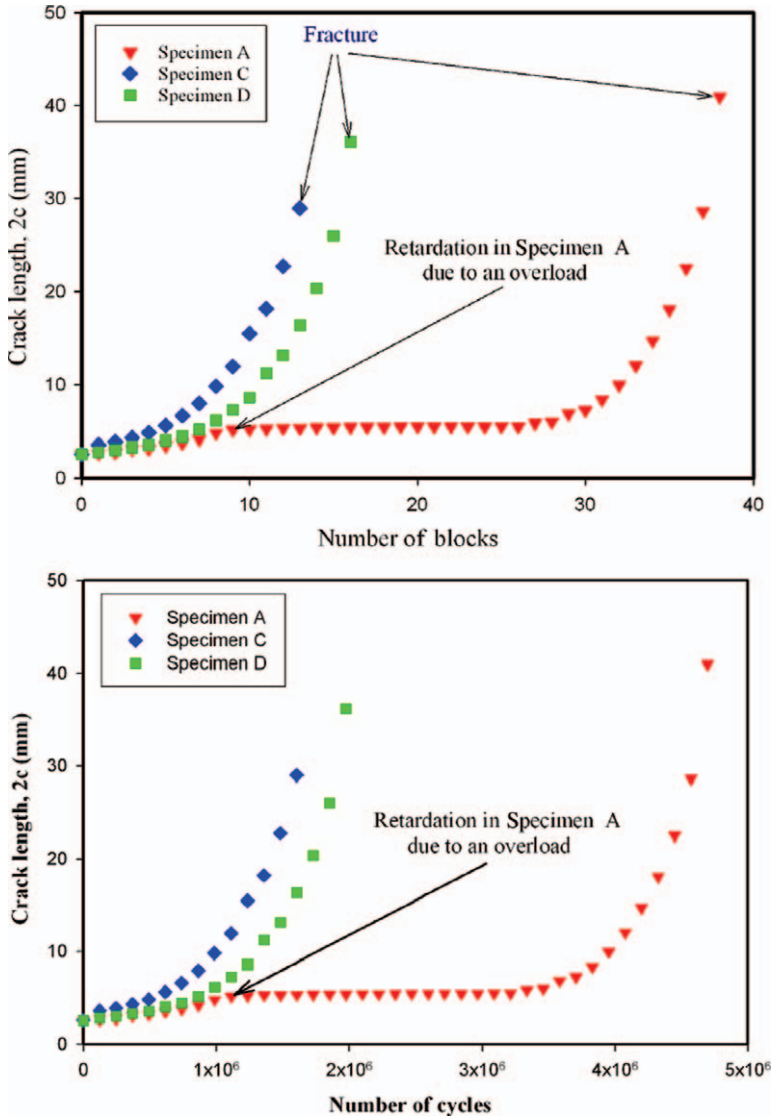
Figure 3. *Fatigue crack growth under random loading.*

his research on fatigue crack growth prediction under random loading using the Multi-Parameter Yield Zone Model obtained the scatter of fatigue crack growth results between 1.50 and 0.18 for load interaction case and 3.00 and 0.38 for non-load interaction case.

For all the specimens, prediction ratios were non-conservative (e.g., the ratios of predicted lives to the test lives were greater than 1). The following tendency was observed during the research: the larger the crack growth range (the more fatigue damage) the more precise the fatigue lives and consequently prediction ratios were obtained – in the case of Specimen A, the crack growth range was relatively large and a more accurate prediction ratio has been obtained while for Specimens C and D the crack growth ranges were small compared to that of Specimen A. Another trend appears in the data in Table 5. For a given random loading spectrum, the higher the fatigue lives, the more accurate the ratios of predicted lives to the test lives.

Effect of Different Variables on Fatigue Crack Growth Prediction

1. It has been observed during the research that the root mean square stress intensity range, ΔK_{rms} significantly affects fatigue life and the accuracy of the fatigue life prediction consequently;
2. Stress ratio, R_{rms}, has also a significant effect on fatigue crack growth prediction – the higher the stress ratio, the lower the fatigue life, e.g., the faster the fatigue crack growth under random loading;
3. Crack growth value, Δc, significantly affects fatigue life under random loading, the lesser the crack growth range, the more precise the fatigue life and consequently prediction ratios were obtained.
4. Fracture toughness value plays an important role in fatigue crack growth prediction.
5. It has been determined during the research that the C and n constants in Forman's equation have a significant effect on the accuracy of fatigue crack growth prediction.

Attempt to Improve Forman's Equation

An attempt has been made to improve the prediction procedure using Forman's equation applied in the RMS model to enable obtaining more precise predicted lives and consequently better prediction ratios. It has been revealed during the research, that using an average value of crack length, c_{avg}, in the Forman's equation instead of numerically integrating Forman's equation from the initial crack size, c_i, to the final crack size, c_f, allows more precise prediction of fatigue crack growth as well as helps to obtain better prediction ratios (e.g., lower error bound has been achieved during predicting procedure), but more experiments should be done to verify the

Table 6. Results of prediction analysis using modified Forman's equation.

Specimen	c_{avg} (mm)	N_{pred}	N_{test}	N_{pred}/N_{test}	Error bound (%)
		(cycles, 10^6)			
A	10.877	2.85	4.57	0.62	38
C	7.886	2.06	1.60	1.28	28
D	9.674	2.52	1.86	1.35	35

validity of our attempt. As shown in Table 6, incorporation of c_{avg} value into the Forman's equation allows for improved prediction ratios for all the three specimens.

The improved prediction procedure using Forman's equation to predict fatigue crack growth under random loading is given by:

$$N_{pred} = \frac{(1 - R_{rms})K_c - \Delta K_{rms}}{C \Delta K_{rms}^n} \cdot c_{avg} \quad \text{where} \quad c_{avg} = \frac{c_i + c_f}{2}$$

CONCLUSIONS

The accuracy of the RMS model for fatigue crack growth prediction under random loading was examined experimentally on the specimens of 7475–T7351 high strength aluminum alloy. Prediction ratios ranged from 3.22 to 1.52 (see Table 6). These ratios are fairly good considering that the normal scatter in fatigue crack growth rates may range from a factor of two to four under identical (random) loading conditions.

An attempt has been made to improve prediction procedure using Forman's equation applied in the RMS model to obtain more precise predicted lives and consequently better prediction ratios. When applying the modified prediction procedure, prediction ratios ranged from 1.35 to 0.62 (error bound was reduced from 222 to 38%). Employing of the modified prediction procedure allowed for better results and more accurate prediction ratios as shown in Table 6.

It may be inferred from the results of the research carried out, that the RMS model may be successfully applied to predict fatigue crack growth under random loading. It may be also assumed that the RMS model may be used for fatigue crack growth prediction in other high strength alloys (e.g., magnesium, titanium, nickel-based and cobalt-based alloys) under random

loading, but further investigation is needed to verify the accuracy of this model for these materials.

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