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# Passive control for uncertain stochastic time-delay systems

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**Key words:** uncertain stochastic systems; time-delay; passivity; linear matrix inequality (LMI)

**Abstract.** This paper investigates the problems of delay-dependent passive analysis and control for uncertain stochastic systems with time-varying delay and norm-bounded parameters uncertainties. Delay-dependent stochastic passive condition for the uncertain stochastic time-delay systems is obtained based on Laypunov-Krasovkii functional approach. On the basis of this condition, a delay-dependent passive controller is presented. Sufficient condition for the existence of desired controller is formulated in terms of linear matrix inequality. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

## Introduction

Stability of stochastic time-delay systems have received considerable research since stochastic phenomenon typically exhibits in branches of science and engineering applications in recent years[1-6]. Time-delays often appear in practical systems, such as manufacturing systems, telecommunication and economic systems, ect., oftentimes causing instability and poor performance. Several kinds of methods about ability problem for time-delay systems have been considered. These methods can be classified into two categories as delay- dependent methods and delay-independent ones. Since the introduction of the notion of positive realness in system and control theory, many results have been developed (see [7-10]). The objective of passive control is to design controllers such that the closed-loop system is stable and passive. In [11-12], delay-dependent sufficient conditions for passivity for a class of uncertain Markovian jump systems with multiple mode-dependent time-delays and a memoryless state-feedback controller are derived. However, up to now, the delay-dependent passive control problem for uncertain stochastic systems with time-delays has not been adequately addressed yet, which still remains an interesting research topic.

This paper is concerned with the problem of stochastically passive control for *Itô* 's stochastic system with time-varying norm-bounded parameter uncertainties. Using *Itô* 's differential formula and the Lyapunov stability, we derive a sufficient condition for passivity in terms of delay-dependent linear matrix inequality. We also derive a delay-dependent passive controller via linear matrix inequality. The proposed results can be easily checked by resorting to available software packages. A numerical example is exploited to demonstrate the effectiveness of the method.

## Problem formulation

Consider the following uncertain stochastic time-delay system ($\sum$) described by *Itô* 's differential equation

$$\begin{cases} dx(t) = [A(t)x(t) + A_1(t)x(t-h(t)) + B(t)u(t) + B_1(t)v(t)]dt + \\ \qquad [E(t)x(t) + E_1(t)x(t-h(t)) + B_2(t)v(t)]d\omega(t) \\ z(t) = Cx(t) + Dv(t) \\ x(t) = \varphi(t), \quad \forall t \in [-h,0] \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $v(t) \in R^p$ is the disturbance input which belongs to $L_2[0,\infty)$, $z(t) \in R^q$ is the controlled output, $\varphi(t)$ is a real-valued initial vector function that is continuous on the interval $[-h,0]$. $h(t)$ is nonnegative differential time-varying function which denotes the time delays and satisfies $0 \le h(t) \le h < \infty$, $\dot{h}(t) \le d < 1$. $\omega(t)$ is one-dimensional Brownian motion defined on a complete probability space $(\Omega, F, \mathrm{P})$, which satisfies $E\{dw(t)\} = 0$, and $E\{dw^2(t)\} = dt$. $A(t)$, $A_1(t)$, $B(t)$, $B_1(t)$, $B_2(t)$, $E(t)$ and $E_1(t)$ are matrix functions with time-varying uncertainties described as $A(t) = A + \Delta A(t)$, $A_1(t) = A_1 + \Delta A_1(t)$, $B(t) = B + \Delta B(t)$, $B_1(t) = B_1 + \Delta B_1(t)$, $E(t) = E + \Delta E(t)$, $E_1(t) = E_1 + \Delta E_1(t)$, $B_2(t) = B_2 + \Delta B_2(t)$, where $A$, $A_1$, $B$, $B_1$, $B_2$, $E$, and $E_1$ are known constant matrices while uncertainties $\Delta A(t), \Delta A_1(t), \Delta B(t), \Delta B_1(t), \Delta B_2(t), \Delta E(t)$, and $\Delta E_1(t)$ are assumed to be norm bounded, i.e.,

$$[\Delta A(t)\ \ \Delta A_1(t)\ \ \Delta B(t)\ \ \Delta E(t)\ \ \Delta E_1(t)\ \ \Delta B_1(t)\ \ \Delta B_2(t)] = MF(t)[N_1\ \ N_2\ \ N_3\ \ N_4\ \ N_5\ \ N_6\ \ N_7] \quad (2)$$

with known constant matrices $M, N_i (i = 1,\cdots,7)$, and unknown matrix function $F(t)$ having Lebesgue-measurable elements and satisfying $F^T(t) \cdot F(t) \le I$.

Throughout the paper we shall use the following definitions.

**Definition 1** For all admissible uncertainties (2), the uncertain stochastic time-delay system (1) $(u(t) = 0,\ v(t) = 0)$ is said to be stochastically mean-square stable if there exists $\delta(\varepsilon) > 0$, for any $\varepsilon > 0$, satisfying $\sup_{-h \le t \le 0} \{E \mid \varphi(t) \mid\} < \delta(\varepsilon)$, we have $E\{\| x(t) \|^2\} < \varepsilon$, and is said to be stochastically mean-square asymptotically stable if for any initial conditions, $\lim_{t \to \infty} E\{\| x(t) \|^2\} = 0$ holds.

**Definition 2** The uncertain stochastic system (1) with $u(t) = 0$ is said to be stochastically passive with dissipation rate $\gamma$ if for any $v(t) \in L_2[0,\infty)$, under zero initial state condition, there exists $\gamma > 0$ such that $E\{\int_0^t v^T(s)z(s)ds\} \ge -2\gamma E\{\int_0^t v^T(s)v(s)ds\}$, for all $t > 0$.

Given the system (1) and a prescribed dissipation rate $\gamma > 0$, determine the memoryless state feedback controller such that the corresponding closed-loop system is stochastically stable and stochastically passive with dissipation rate $\gamma$.

**Lemma 1([13])** For any constant symmetric matrix $R \in \mathrm{R}^{n \times n}$, $R = R^T > 0$, vector function $x(t)$: $[0,h] \to \mathrm{R}^n$ such that integrations in the following are well defined, then

$$-h\int_{t-h}^{t} x^T(s)Rx(s)ds \le -\int_{t-h}^{t} x^T(s)ds \cdot R \cdot \int_{t-h}^{t} x(s)ds.$$

**Lemma 2([12])** Given appropriately dimensioned matrices $\psi, H, G$ with $\psi = \psi^T$, then $\psi + HF(t)G + G^T F^T(t)H^T < 0$, holds for all $F(t)$ satisfying $F^T(t)F(t) \le I$ if and only if for some $\varepsilon > 0$, $\psi + \varepsilon HH^T + \varepsilon^{-1}G^T G < 0$.

**Passivity for uncertain stochastic systems with time-delay**

In the following theorem, a LMI method is used to solve the delay-dependent passivity problem for the uncertain stochastic system (1) with $u(t) = 0$, and a sufficient condition is derived ensuring the mean-square stochastic stability and stochastic passivity.

**Theorem 1** For given scalars $\gamma, h, \gamma \ge 0, h > 0$, the uncertain time-delay stochastic system (1) with $u(t) = 0$ is stochastically passive for all admissible uncertainties (2), if there exist symmetric positive definite matrices $X, Z, H \in \mathrm{R}^{n \times n}$ and scalars $\varepsilon_1 > 0, \varepsilon_2 > 0$, such that the following linear matrix inequality

$$\phi=\begin{bmatrix}\phi_{11} & A_1Z & 0 & \phi_{14} & XE^T & \phi_{16} & XN_1^T & XN_4^T & X\\ * & -(1-d)Z & 0 & 0 & ZE_1^T & hZA_1^T & ZN_2^T & ZN_5^T & 0\\ * & * & -H & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & \phi_{44} & B_2^T & hB_1^T & N_6^T & N_7^T & 0\\ * & * & * & * & \phi_{55} & 0 & 0 & 0 & 0\\ * & * & * & * & * & \phi_{66} & 0 & 0 & 0\\ * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0\\ * & * & * & * & * & * & * & -\varepsilon_2 I & 0\\ * & * & * & * & * & * & * & * & -Z\end{bmatrix}<0 \tag{3}$$

holds, where $\phi_{11}=AX+XA^T+\varepsilon_1 MM^T$ , $\phi_{14}=B_1-XC^T$ , $\phi_{16}=hXA^T+\varepsilon_1 hMM^T$ ,
$\phi_{44}=-\gamma I-D-D^T$ , $\phi_{55}=-X+\varepsilon_2 MM^T$ , $\phi_{66}=-H+\varepsilon_1 h^2 MM^T$ .

**Proof:** Choose a Lyapunov-Krasovskii functional candidate for system (1) as follows

$$V(t)=x^T(t)Px(t)+\int_{t-h(t)}^{t}x^T(s)Qx(s)ds+h\int_{-h}^{0}\int_{t+\theta}^{t}y^T(\alpha)Ry(\alpha)d\alpha d\beta \tag{4}$$

where $P, Q$ and $R$ are symmetric positive definite matrices to be chosen.

For convenience, let $y(t)=A(t)x(t)+A_1(t)x(t-h(t))$ , and

$$f(t)=A(t)x(t)+A_1(t)x(t-h(t))+B_1(t)v(t)\,,\ g(t)=E(t)x(t)+E_1(t)x(t-h(t))+B_2(t)v(t)$$

Then by using the *Itô* 's differential rule along the system (1) with $v(t)=0$ and $u(t)=0$ and lemma1, we have

$$LV(t)=2x^T(t)Py(t)+g^T(t)Pg(t)+x^T(t)Qx(t)-(1-d)x^T(t-h(t))Qx(t-h(t))$$
$$+h^2y^T(t)Ry(t)-\int_{t-h}^{t}y^T(s)ds\cdot R\cdot\int_{t-h}^{t}y(s)ds=\xi^T(t)\Sigma_1\xi(t) \tag{5}$$

where $\Sigma_1=\begin{bmatrix}\Sigma_{11} & \Sigma_{12} & 0\\ * & \Sigma_{22} & 0\\ 0 & 0 & -R\end{bmatrix}$, $\xi^T(t)=[x^T(t)\ \ x^T(t-h(t))\ \ \int_{t-h}^{t}y^T(\alpha)d\alpha)]$, and

$\Sigma_{11}=PA(t)+A^T(t)P+Q+E^T(t)PE(t)+h^2A^T(t)RA(t)$,

$\Sigma_{12}=PA_1(t)+E^T(t)PE_1(t)+h^2A^T(t)RA_1(t), \Sigma_{22}=-(1-d)Q+E_1^T(t)PE_1(t)+h^2A_1^T(t)RA_1(t)$

If $\Sigma_1<0$ , then $LV(t)<0$ . It remains to show that $\Sigma_1<0$ . Based on Schur complement formula, $\Sigma_1<0$ is equal to

$$\Sigma_2=\begin{bmatrix}PA(t)+A^T(t)P+Q & PA_1(t) & 0 & E^T(t)P & hA^T(t)R\\ * & -(1-d)Q & 0 & E_1^T(t)P & hA_1^T(t)R\\ * & * & -R & 0 & 0\\ * & * & * & -P & 0\\ * & * & * & * & -R\end{bmatrix}<0 \tag{6}$$

Define $P^{-1}=X, Q^{-1}=Z, R^{-1}=H$ . Now, premultiplying and postmultiplying (6) by $diag(P^{-1},Q^{-1},R^{-1},P^{-1},R^{-1})$ , respectively, then using lemma 2 and Schur complement formula, result in

$$\Sigma_3 = \begin{bmatrix} \phi_{11} & A_1 Z & 0 & XE^T & \phi_{16} & XN_1^T & XN_4^T & X \\ * & -(1-d)Z & 0 & ZE_1^T & hZA_1^T & ZN_2^T & ZN_5^T & 0 \\ * & * & -H & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & \phi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & -Z \end{bmatrix} < 0 \tag{7}$$

Obviously, according to (3), we can get $\Sigma_3 < 0$, which, by Definition 1, shows that the system (1) $(u(t)=0, v(t)=0)$ is stochastically mean-square asymptotically stable.

To prove the stochastic passivity for uncertain stochastic system with time-delay (1) $(u(t)=0)$, we modify the Lyapunov-Krasovskii functional candidate (4) as

$$V_1(t) = x^T(t)Px(t) + \int_{t-h(t)}^{t} x^T(s)Qx(s)ds + h\int_{-h}^{0}\int_{t+\theta}^{t} f^T(\alpha)Rf(\alpha)d\alpha d\beta \tag{8}$$

Similar to above progress, for $\eta^T(t) = [x^T(t) \;\; x^T(t-h(t)) \;\; \int_{t-h}^{t} f^T(\alpha)d\alpha \;\; v^T(t)]$, we have

$$LV_1(t) \le \eta^T(t)\Sigma_4\eta(t), \tag{9}$$

where

$$\Sigma_4 = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & 0 & PB_1(t) + E^T(t)PB_2(t) + h^2 A^T(t)RB_1(t) \\ * & \Sigma_{22} & 0 & h^2 A_1^T(t)RB_1(t) + E_1^T(t)PB_2(t) \\ * & * & -R & 0 \\ * & * & * & h^2 B_1^T(t)RB_1(t) + B_2^T(t)PB_2(t) \end{bmatrix}. \tag{10}$$

Considering

$$\begin{aligned} F(t) &= LV_1(t) - 2v^T(t)z(t) - \gamma v^T(t)v(t) \\ &= LV_1(t) - 2v^T(t)Cx(t) - 2v^T(t)Dv(t) - \gamma v^T(t)v(t) \le \eta^T(t)\Sigma_5\eta(t) \end{aligned} \tag{11}$$

where

$$\Sigma_5 = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & 0 & PB_1(t) - C^T + E^T(t)PB_2(t) + h^2 A^T(t)RB_1(t) \\ * & \Sigma_{22} & 0 & h^2 A_1^T(t)RB_1(t) + E_1^T(t)PB_2(t) \\ * & * & -R & 0 \\ * & * & * & -\gamma I - D - D^T + h^2 B_1^T(t)RB_1(t) + B_2^T(t)PB_2(t) \end{bmatrix} \tag{12}$$

Similarly, applying Schur complement and Lemma 2 show that $\Sigma_5 < 0$ is equivalent to $\phi < 0$. Therefore we have $F(t) < 0$. Then for zero initial state conditions, we can obtain

$$\begin{aligned} 2E\{\int_0^t v^T(s)z(s)ds\} &= E\{\int_0^t [LV_1(s) - F(s) - \gamma v^T(s)v(s)]ds\} \\ &\ge E\{\int_0^t LV_1(s)ds\} - \gamma E\{\int_0^t v^T(s)v(s)]ds\} \ge \gamma E\{\int_0^t v^T(s)v(s)ds\}, \end{aligned}$$

which show the stochastic passiveness of system (1) by Definition 2.

**Design of the passive controller for uncertain stochastic systems with time-delay**

Applying theorem 1 in this section, we aim to derive easy-to-test condition for the solvability of the state-feedback design problem for uncertain stochastic system with time-delay. Again, an LMI approach will be used in order to facilitate the design procedure.

We seek a state feedback memoryless controller of

$$u(t) = Kx(t) \tag{13}$$

which achieves passivity of the closed-loop system. The main result is given in Theorem 2.

**Theorem 2** Given scalars $\gamma \geq 0$ and $h > 0$. The uncertain time-delay stochastic system (1) is stochastically mean-square asymptotically stable for all admissible uncertainties (2), if there exist symmetric positive definite matrices $X, Z, H \in \mathrm{R}^{n \times n}$ , scalars $\varepsilon_1 > 0$ , $\varepsilon_2 > 0$ , and matrix $Y \in \mathrm{R}^{m \times n}$ such that the following linear matrix inequality

$$\Phi = \begin{bmatrix} \Phi_{11} & A_1 Z & 0 & \Phi_{14} & XE^T & \Phi_{15} & \Phi_{16} & XN_4^T & X \\ * & -(1-d)Z & 0 & 0 & ZE_1^T & hZA_1^T & ZN_2^T & ZN_5^T & 0 \\ * & * & -H & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & B_2^T & hB_1^T & N_6^T & N_7^T & 0 \\ * & * & * & * & \Phi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Phi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & * & -Z \end{bmatrix} < 0 \tag{14}$$

holds, where $\Phi_{11} = AX + XA^T + BY + Y^T B^T + \varepsilon_1 MM^T$ , $\Phi_{14} = B_1 - XC^T$ ,$\Phi_{16} = Y^T N_3^T + XN_1^T$ , $\Phi_{15} = hXA^T + \varepsilon_1 hMM^T$ ,$\Phi_{44} = -\gamma I - D - D^T$ , $\Phi_{55} = -X + \varepsilon_2 MM^T$ , $\Phi_{66} = -H + \varepsilon_1 h^2 MM^T$ .

Then, the stochastically passive controller of system (1) is given by $u(t) = Kx(t)$, and the gain matrix can be chosen as $K = YX^{-1}$.

**Proof** The result for system (1) with the passive control law of (13) follows immediately by Theorem1. Firstly, replacing $A$ by $A + BK$ and $\Delta A$ by $\Delta A + \Delta BK$ in $\Sigma_7$ , then using Schur complement and Lemma 2, we can obtain LMI (14). This completes the proof.

**A Numerical Example**

In this section, we shall give an example to demonstrate the effectiveness of the proposed method. Consider the stochastic time-delay system with norm-bounded parameter uncertainties as follows:

$$A = \begin{bmatrix} 0.2 & 0.5 \\ 1 & 3 \end{bmatrix},\ A_1 = \begin{bmatrix} -0.2 & 0.5 \\ 0.1 & 2 \end{bmatrix},\ B = \begin{bmatrix} -1 & 0.1 \\ 0 & 1 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.1 & 0.5 \\ 2 & -1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 0.2 \end{bmatrix},$$
$$E = \begin{bmatrix} 1 & 0.1 \\ 0.1 & -0.2 \end{bmatrix},\ E_1 = \begin{bmatrix} -0.2 & -0.5 \\ 0.1 & 2 \end{bmatrix},\ C = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & -0.8 \end{bmatrix},\ D = \begin{bmatrix} 1 & 0.1 \\ 0.3 & 2 \end{bmatrix},$$
$$M = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix},\ N_1 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix},\ N_2 = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix},\ N_3 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, N_4 = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix},$$
$$N_5 = \begin{bmatrix} 0.3 & 0.1 \end{bmatrix},\ N_6 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},\ N_7 = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}. \tag{15}$$

For the given data $d = 0.5$, $h = 0.9$, $\gamma = 1$, we use Matlab LMI Toolbox and solve the LMI (25), the corresponding state-feedback controller gain is obtained as $K = \begin{bmatrix} 8.3163 & 6.5909 \\ 7.9206 & -11.4873 \end{bmatrix}$.

That is to say, we can obtain the state feedback control law such that the corresponding closed-loop system (15) is stochastically stable.

## Conclusions

In this paper, we have studied the problems of stochastic passive analysis and state-feedback passive control of stochastic time-delay system with norm-bounded parameter uncertainties. The delay-dependent sufficient condition for guaranteeing the stochastic passivity of uncertain stochastic system with time-delay has been presented in terms of an LMI. Memoryless state feedback passive controller is designed such that the closed-loop system is stochastically stable and stochastically passive. Demonstrative example shows the effect of the proposed approach.

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