

# Evaluation of the GRACE-Based Global Gravity Models in Canada

Jianliang Huang and Marc Véronneau

Geodetic Survey Division, CCRS, Natural Resources Canada, 615 Booth Street, Ottawa, Ontario, Canada, K1A 0E9, Email: [jianhuan@NRCan.gc.ca](mailto:jianhuan@NRCan.gc.ca); [marcv@NRCan.gc.ca](mailto:marcv@NRCan.gc.ca)

**Abstract.** Four GRACE-based gravity models, especially EGM08, have been evaluated using the surface gravity observations, GPS-levelling, deflections of the vertical, and the recent Canadian gravimetric geoid CGG05. The RMS of the differences between the EGM08-predicted gravity anomalies and the observed anomalies is smaller than 5 mGal on sea and lake surfaces, in contrast to about 14 mGal on land in Canada. The RMS increases with increasing elevation on land, exhibiting an evident height-dependent trend, while the RMS decreases with increasing depth on sea and lake surfaces, without a significant trend. The GPS-levelling comparisons suggest that EGM08 models the geoid with an accuracy of 10 cm or better in Canada. It is comparable with the Canadian Gravimetric Geoid 2005 (CGG05). Recent releases of GRACE models show noteworthy improvement over earlier ones. The comparisons between the EGM08-predicted and astronomical deflections of the vertical show the RMS of 1.8 arc-seconds in the north-south direction, and 2.1 arc-seconds in the east-west direction, which are significantly larger than the RMS of differences between the CGG05-predicted and astronomical deflections.

**Keywords.** GRACE, geoid, gravity, GPS-levelling, deflections of the vertical, EGM08

## 1 Introduction

A series of satellite-only and combined global gravity models have been developed by a number of scientific teams worldwide since the twin Gravity Recovery And Climate Experiment (GRACE) satellites were launched on March 17, 2002. Two representative GRACE-only models are GGM02S (Tapley et al., 2005) and GL04S1 (Forste et al., 2008) developed by the Center for Space Research, University of Texas, USA and jointly by

GFZ, Germany and GRGS, France, respectively. The most recent and revolutionary global model is the Earth Gravitational Model 2008 (EGM08) developed by the National Geospatial-Intelligence Agency (NGA) of USA, superseding its predecessor EGM96 (Lemoine et al., 1998). EGM08 combines satellite (GRACE), marine (satellite-altimetry-derived), and land gravity data to model the global gravity field with a geo-spatial resolution of 5 by 5 arcmin (Pavlis et al., 2008). It is complete to degree and order 2159 and contains additional spherical harmonic coefficients up to degree 2190, which account for the transform corrections from ellipsoidal to spherical coefficients. Its accuracy is largely dependent on the accuracy of GRACE, marine and land gravity data and their availability. The fact that the geo-spatial resolution of EGM08 approaches to that of Canadian regional geoid models will certainly have significant impact on the development of a geoid model with one-centimetre accuracy in Canada.

The objective of this study is to evaluate GRACE global models and EGM08, i.e., to examine improvement of the low-degree components and the accuracy of EGM08 using surface gravity, GPS-levelling, deflection of the vertical data and CGG05.

## 2 Gravity Comparisons

### 2.1 Surface Gravity Data

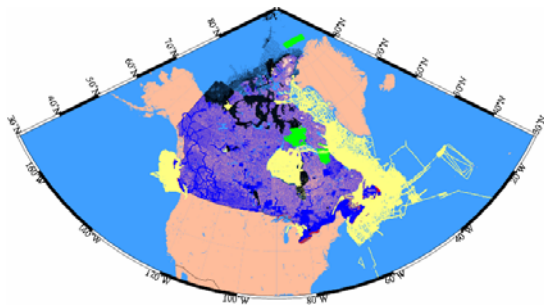
The Canadian Gravity Database (CGDB) is a collection of gravity observations on land, sea and lake surfaces, sea and lake bottoms, ice caps and over the air. About 98 percent of the observations were collected during the past 50 years covering entirely Canada and neighboring seas. The accuracy of these data varies from place to place depending on types of instruments, platforms, height reductions, etc. They have a mean error standard deviation of 1.88 mGal. They can be broken down into six different types in reference to observation platform as described in Tables 2.1 and 2.2, and

displayed in Figure 2.1. The free-air gravity anomalies are derived from the observations with reference to GRS80. The following formula was used to add the atmospheric corrections to these free-air anomalies in the gravity database.

$$Dg_A = 0.8658 - 9.727 \times 10^{-5} H + 3.482 \times 10^{-9} H^2 \quad (1)$$

where H is the orthometric height in meter. The atmospheric correction is in mGal.

**Figure 2.1** Distribution of gravity observations. Blue dots are on land; yellow dots on water surface; red dots on water bottom; black dots on ice surface; green dots for airborne data.



**Table 2.1** Types of the gravity data in Canada.

Code	Number	%	Description
Land	215,862	28.80	land
SLS	455,464	60.78	water surface
SLB	6,538	0.87	water bottom
SLI	36,487	4.87	ice surface
IC	802	0.11	ice cap
AIR	34,249	4.57	airborne
All	749,402	100.00	

**Table 2.2** Statistical description of the free-air gravity anomalies. Unit: mGal.

Code	Min	Max	Mean	Std	RMS
Land	-184	393	-10	29	31
SLS	-161	241	-6	36	36
SLB	-103	102	-18	26	32
SLI	-168	107	-23	30	38
IC	-98	218	55	43	70
AIR	-100	54	-27	30	40
All	-184	393	-9	34	35

## 2.2 Computation of the Gravity Anomaly from EGM

The gravity anomaly is computed from the following formula (Heiskanen and Moritz, 1967):

$$\Delta g = -\frac{\partial T}{\partial h} + \frac{1}{\gamma} \frac{\partial \gamma}{\partial h} T \quad (2)$$

where  $T$  is the disturbing potential,  $h$  is the ellipsoidal height,  $\gamma$  is the normal gravity.

A routine ('Harmonic\_synth.f', version 05/01/2006, isw=50, 3 and 7) provided by National Geospatial-Intelligence Agency (NGA) was used in computing point free-air gravity anomalies at the gravity stations on the Earth's surface. Input parameters were set to produce point values in the tide-free system with reference to GRS80 (Moritz, 1992). The gravity anomalies were synthesized from spherical harmonic degree 2 to 2190 of EGM08 as well as its preliminary model PGM07A at each gravity station. The zero-degree term due to the difference between the geopotential constants GM of EGM08 and GRS80 is 0.144 mGal in absolute value and was added to the synthesized anomalies.

## 2.3 Comparisons

The gravity anomalies synthesized from PGM07A and EGM08 were compared to the observed ones. The results are shown in Table 2.3.

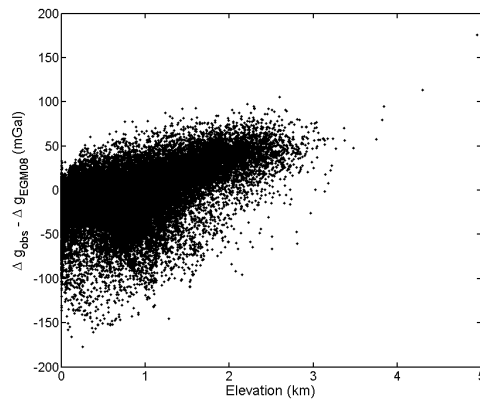
**Table 2.3** Differences between the observed gravity anomalies and the gravity anomalies predicted from (PGM)07A and (EGM)08. Unit: mGal.

Data	Model	Min	Max	Mean	Std
Land	07A	-176	174	-1.64	13.75
	08	-177	175	-1.69	13.71
SLS	07A	-202	48	-1.19	4.78
	08	-204	53	-1.14	4.61
SLB	07A	-43	29	-0.78	4.00
	08	-44	28	-0.68	4.17
SLI	07A	-81	50	-0.49	4.47
	08	-89	50	-0.56	4.14
IC	07A	-65	82	-1.30	19.61
	08	-61	82	-2.29	19.61
AIR	07A	-23	39	1.40	4.84
	08	-23	37	1.57	4.61
All	07A	-202	174	-1.16	8.45
	08	-204	175	-1.14	8.36

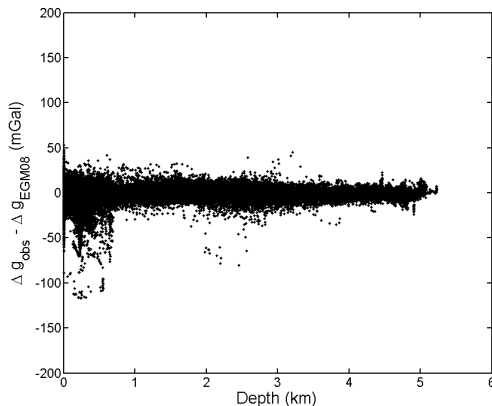
The RMS of the differences on land is about 14 mGal, which is caused by the land data errors, the

commission and omission errors of PGM07A and EGM08. Huang and Véronneau (2008) suggested that the land data contain a systematic error of about 1.4 in RMS. Pavlis et al. (2008) suggest that the commission error of EGM08 is about 1 - 2 mGal. These estimates indicate that the model omission errors are very likely dominant. This conclusion is consistent with the RMS estimate from the Tscherning-Rapp degree variance model (1974), that is 11.2 mGal for the spherical harmonic components higher than degree 2160. The differences in Figure 2.2 show a linear increase with respect to the elevations of the gravity stations at the rate of about 8.66 mGal/km, and a stronger trend associated with the elevations higher than 1 km.

**Figure 2.2** The differences between the gravity anomalies predicted from EGM08 and the gravity anomalies from observations vs. elevations on land.



**Figure 2.3** The differences between the gravity anomalies predicted from EGM08 and the gravity anomalies from observations vs. depth on sea and lake surfaces.



The comparison over sea and lake surfaces performs significantly better than the one on land. The RMS of the differences on sea and lake surfaces is smaller than 5 mGal which is about one third of the land one. It largely reflects the model omission level over seas where most marine data are located. Figure 2.3 indicates that the differences decrease with increasing depth without showing an evident dependence on depth. This reflects the fact that the separation between water surface and bottom serves as a natural low-pass filter to smooth the gravity field on the water surface: the greater the depth, the smoother the surface field. It can also be noticed that the differences for depths of less than 1 km show a similar magnitude to those on land. They mostly correspond to coastal areas with shallow water where only the altimetry-derived gravity data are used for EGM08.

A comparison between PGM07A and EGM08 suggests that the latter agrees notably better with the observed values than the former.

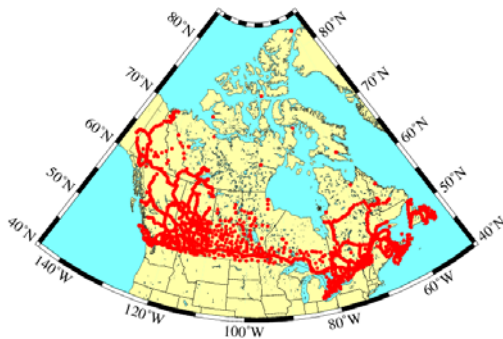
### 3 GPS-Levelling Comparisons

#### 3.1 GPS-Levelling Data

The geoid heights or height anomalies derived from GPS-levelling data are the highest quality and most reliable data in validating the gravimetric geoid. Two GPS-Levelling data sets are used in this evaluation. The first set is derived from the Canada-wide GPS and levelling adjustments of 2004 and includes 430 stations. The ellipsoidal heights at these stations were estimated from GPS observations collected after 1994, processed with precise IGS orbit products defined in ITRF97 and referred to the GRS80 reference ellipsoid. Their standard deviations vary from 0.2 cm and 7.6 cm with an average of 1.3 cm. The co-located orthometric heights were estimated from levelling measurements made after 1981 to minimize crustal motion effect. A single fixed station in Rimouski, Québec was chosen to define mean sea level for the adjustment, similar to the constraint for the North American Vertical Datum of 1988 (NAVD88). Their standard deviations increase with increasing distance from the fixed station in Rimouski ranging from 0 to 9 cm with a mean standard deviation of 5.4 cm.

The second set consists of all 2579 co-located GPS-levelling stations currently available in Canada. The early GPS observations date back to the late 1980s and the levelling data can trace back to the beginning of the 1990s. The standard deviations of the ellipsoidal heights vary from less than 1 cm to a few decimeters. Recent observations are generally more precise than earlier ones. On the other hand, the orthometric heights can be wrong by a few decimeters largely due to natural changes and human activity. Even though it is difficult to interpret validation results from this data set due to incomplete knowledge on its uncertainty, inclusion of more stations is considered to provide additional information on the quality of validated geoid models.

**Figure 3.1** Geographical distribution of 2579 GPS-levelling stations.



### 3.2 Computation of the Geoid Height from an EGM

The following equation is used to compute the geoid height with respect to GRS80:

$$N_L = \frac{G\delta M}{r_e \gamma_0} - \frac{\delta W}{\gamma_0} + \frac{GM}{r_e \gamma_0} \sum_{n=2}^L \left( \frac{a}{r_e} \right)^n \sum_{m=0}^n Y_{nm}(\phi, \lambda) + C_{TB} \quad (3)$$

The first two terms on the right side of Equation (3) are the zero-degree corrections. The second term represents the spherical harmonic expansion of the geoid. The last term is the so-called topographic bias that corrects for the analytical downward continuation error within the topography (see e. g. Rapp, 1997). The DTM is synthesized from a spherical harmonic topographic model provided by NGA complete to spherical harmonic degree and order 2190 over the land, while it is assigned as zero over the oceans. The geoid heights were

evaluated with respect to the reference ellipsoid defined by GRS80. The geoid potential is defined by  $W_0 = 62636856.88 \text{ m}^2\text{s}^{-2}$  which is the same as that for CGG05 (Véronneau and Huang, 2007).

### 3.3 Comparisons

One objective of the GPS-levelling comparisons is to check the improvement of the GRACE gravity models. GGM02S (Tapley et al., 2005) and EIGEN-GL04S1 (Förste et al., 2008) were chosen to represent early and recent releases of GRACE-only models, respectively. PGM07A uses a JPL GRACE model while EGM08 uses ITG-GRACE03 (Mayer-Gürr, 2007). Therefore, there are four different GRACE models included in the comparisons directly and indirectly. The comparisons were limited with a degree band spanning from spherical harmonic degrees 2 to 90 because the error of GRACE models increases rapidly beyond degree 90. In order to eliminate the omission error effect on the comparisons, GGM02S and GL04S1 were extended to degree and order 2159 with additional coefficients up to degree 2190 from EGM08. The validation results are shown in Tables 3.1 and 3.2.

**Table 3.1** Comparisons of EGM models against the GPS-levelling data at 430 stations. Unit: m.

Model	Min	Max	Mean	Std
GGM02S*	-0.672	-0.147	-0.351	0.102
GL04S1*	-0.661	-0.157	-0.354	0.100
PGM07A	-0.741	-0.069	-0.356	0.105
EGM08	-0.669	-0.149	-0.356	0.100
CGG05	-0.668	-0.120	-0.396	0.102

**Table 3.2** Comparisons of EGM models against the GPS-levelling data at 2579 stations. Unit: m.

Model	Min	Max	Mean	Std
GGM02S*	-0.923	0.088	-0.375	0.135
GL04S1*	-0.929	0.080	-0.375	0.132
PGM07A	-0.913	0.123	-0.367	0.136
EGM08	-0.922	0.090	-0.380	0.133
CGG05	-0.932	0.067	-0.420	0.134

It is evident that there exist biases of about 36 cm between the GPS-levelling derived geoid heights and the gravimetric geoid heights. These biases mainly represent the separation between the mean sea level at the fixed station in Rimouski and the adopted global sea level. Once determined, these constant biases can be corrected. Ignoring these

biases, the standard deviation gives a proper measure of the level of agreement between the geoid and GPS-levelling.

It can be seen that recent GL04S1 shows slight improvement over early GGM02S. EGM08 performs equally well with the extended GL04S1 in terms of standard deviation. This result suggests that the differences between the low degree parts (2 to 90) of EGM08 and GL04S1 are insignificant in term of the GPS-levelling accuracy. In the meantime, EGM08 shows slight improvement over its preliminary model PGM07A. It can partly be attributed to the improvement of ITG-GRACE03 over the early JPL GRACE model used for PGM07A. It is worth pointing out that a weighted scheme was used for EGM08 to combine ITG-GRACE03 and terrestrial gravity mainly within a degree band from degrees 70 to 120 using covariance information associated with both data sources. The direct truncation and extension used here for GGM02S and GL04S1 is not theoretically optimal, and may introduce additional errors.

The second objective is to estimate the accuracy of EGM08. The standard deviation of h-H-N results from errors in ellipsoidal, orthometric and geoid heights. The EGM08 results suggest it can predict the geoid in Canada with an accuracy of better than 10 cm when the errors in the ellipsoidal and orthometric heights are taken into consideration. That accuracy reflects the aggregate level of the EGM08 commission and omission error, and is comparable with that of CGG05 shown in Tables 3.1 and 3.2. The fact that EGM08 compares well with the regional geoid model in terms of precision represents an exceptional achievement.

## 4 Comparisons with Deflections of the Vertical

### 4.1 Deflections of the Vertical in Canada

**Figure 4.1** Geographical distribution of 939 deflection stations.



Similar to the gravity anomalies, deflections of the vertical are gradients of the anomalous disturbing potential but with respect to horizontal directions instead of the vertical. They are usually defined by their south-north and east-west components that represent geoid slopes in each direction. They are determined by astronomical and geodetic observations:

$$\begin{aligned}\xi &= \Phi - \phi \\ \eta &= (\Lambda - \lambda) \cos \phi\end{aligned}\quad (4)$$

where the pair  $(\Phi, \Lambda)$  are astronomical latitude and longitude, and the pair  $(\phi, \lambda)$  are geodetic latitude and longitude. In Canada, the astronomical observations were collected from 1910 to 1975, and estimated to be accurate to 0.3 to 0.5 arcsec. The geodetic observations are known in ITRF93 but with unknown accuracy. However, they should be more accurate than the astronomical ones in terms of observation technology. Figure 4.1 shows the distribution of 939 Canadian stations with deflections of the vertical. Table 1 gives a statistical description of these data.

**Table 4.1** Statistical description of the deflections of the vertical in Canada in arc-second.

	Min	Max	Mean	Std
$\xi$	-23.410	17.880	0.205	4.501
$\eta$	-22.390	24.350	-0.232	6.137

### 4.2 Computation

Deflections of the vertical on the Earth's surface can be computed by the following formulae (Heiskanen and Moritz, 1967):

$$\begin{aligned}\xi &= -\frac{1}{R} \frac{\partial \zeta}{\partial \phi} \\ \eta &= -\frac{1}{R \cos \phi} \frac{\partial \zeta}{\partial \lambda}\end{aligned}\quad (5)$$

where  $\zeta$  represents the height anomaly and can be computed from the geoid height by

$$\zeta = N - \frac{\Delta g_B}{\gamma} H \quad (6)$$

$H$  is the orthometric height, and  $\Delta g_B$  is the Bouguer gravity anomaly. The deflections at a point

were numerically computed from values at nine nodes closest to that point in an evenly-spaced grid of 2 by 2 arcmin.

### 4.3 Comparisons

Deflections of the vertical predicted from PGM07A and EGM08 were compared to the observations made at stations shown in Figure 4.1. The comparison results are described in Tables 4.2 and 4.3. It can be seen that the differences are significantly larger than the estimated 0.5 arcsec error level of the astronomical latitude and longitude. They are also significantly larger than the differences between the CGG05-derived and astronomical deflections reported in Tables 4.2 and 4.3. CGG05 is determined in the spacing of 2 by 2 arcmin while EGM08 has a spatial resolution of 5 by 5 arcmin. Thus, the omission error of CGG05 is smaller than that of EGM08. The omission error of PGM2007A and EGM08 are most likely a major source for those larger differences. This conclusion is consistent with the RMS estimate from the Tscherning-Rapp degree variance model (1974), that is 1.66 arcsec for the spherical harmonic components higher than degree 2160. One may argue that the performance of deflection tests is not as critical as the GPS-levelling one because of the predominant role of the geoid as height datum.

Like the gravity and GPS-levelling comparisons, the deflection comparisons in Tables 4.2 and 4.3 suggest a slightly better agreement of EGM08 than PGM2007A with the observations.

**Table 4.2** Differences between the predicted deflections of the vertical and the observations along the north-south ( $\xi$ ) direction in arc-second.

Models	Min	Max	Mean	Std
PGM07A	-12.364	9.946	-0.013	1.785
EGM08	-12.429	9.948	-0.014	1.760
CGG05	-5.331	9.835	0.051	1.215

**Table 4.3** Differences between the predicted deflections of the vertical and the observations along the east-west ( $\eta$ ) direction in arc-second.

Models	Min	Max	Mean	Std
PGM07A	-12.839	14.944	0.203	2.111
EGM08	-12.814	15.266	0.197	2.101
CGG05	-12.703	9.159	0.232	1.643

## 5. Summary

The two GRACE-only models GGM02S and GL04S1, and the two combined models PGM07A and EGM08 have been evaluated using the surface gravity observations, GPS-levelling, deflections of the vertical, and the recent Canadian geoid model CGG05. The main conclusions can be summarized as:

1. The accuracy of the EGM08-predicted geoid is better than 10 cm in Canada. It is comparable with that of the recent Canadian geoid CGG05.
2. The omission error of EGM08 is about two times larger on land (~14 mGal) than that on sea and lake surfaces (~5 mGal).
3. The recent GRACE-only models show noteworthy improvement over the earlier ones.
4. CGG05 significantly outperforms EGM08 in terms of their validation against deflections of the vertical on land mainly due to its higher spatial resolution.

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