CHAPTER IV

Mathematics in the Secondary School

THEODORE E. KELLOGG and DONOVAN A. JOHNSON

During the period 1957-1960, a time of ferment in mathematics education, a variety of experiments with new curriculums were conducted at local, state, and national levels. Evaluation of programs is a project for the immediate future. In designing and implementing new curriculums and new approaches, it is important that answers from research be used.

This chapter includes descriptions of experimental curriculums, of the current status of teaching in secondary mathematics, and of controlled experimental studies.

Current Practices and Trends in Mathematics Curriculums

Shetler (1958), by questionnaire, surveyed aims, curriculum, methods, and evaluation in the teaching of mathematics to determine how closely they agreed with research results and proposals. Replies from a 10-percent random sample of secondary schools in 20 states showed disagreement between expert opinion and practices in classroom methods and evaluation. Many teachers reported inadequate equipment and poor library facilities. The teachers who consistently follow recommended practices are those who spend most of the school day teaching mathematics, and those who have professional assistance in teaching. In a related study Diebel (1959) considered the influence of the evaluative criteria of the Co-operative Study of Secondary School Standards in 57 Oregon schools evaluated during the seven-year period 1950-57. This study indicates that the recommendations resulting from these evaluations are sporadically implemented.

One way to study the mathematics curriculum is to analyze textbooks. Izzo (1957) analyzed 627 secondary mathematics texts to determine trends in the use of graphical material. He found increased attention to graphing, especially in connection with locus and analytic geometry in plane-geometry texts. Similarly, Rajaratnam (1957) studied 10 elementary-algebra texts to relate the development of the concepts of variable, function, equation, and equality with the work of mathematicians and logicians. She observed new ideas mixed with outworn and erroneous ideas and terminology. Wilson (1959), examining geometry texts published before 1900, saw authors generally competent mathematically, but not very familiar with good teaching techniques and the learning process.

Wales's (1958) appraisal of texts for purposes of building a course of study was a typical misuse of textbook analysis. From 20 commonly used general-mathematics texts she compiled a list of topics and submitted them to a selected panel of educators for ranking. On the basis of this appraisal, a recommended program for general mathematics was formulated.

One method of evaluating the effectiveness of a curriculum is to consider its contribution to success in college. Such an approach was used by Knights (1957), who analyzed a test given to entering college freshmen to determine their preparation for analytic geometry. Using these scores as predictors of success in analytic geometry, Knights showed that students need more skill in problem solving, more experience in applying learning, and more understanding of definitions, relationships, and symbols to be successful in analytic geometry.

A first step in a local curriculum reorganization is determination of the current status. In this kind of study, Bruns and Frazier (1957), surveying the variety of experimental programs in operation in Houston schools, found a fairly uniform sequence of algebra and geometry through grade 10 but considerable variation in including topics from analytic geometry and calculus in the upper grades. An interesting new course described was a laboratory geometry course for non-college-bound students.

Evaluating the Content of Mathematics Courses

A variety of new content is being brought into the mathematics curriculum, and the contributions of new topics need to be evaluated as well as those of traditional topics. We have little evidence with which to defend the old or the new. For example, it has been proposed that probability and statistical inference be a possible semester course in grade 12. The achievement of apt high-school juniors and seniors in such a course was studied by Bridges (1959). The experimental class of 19 members obtained significantly higher scores on the post-test as compared to the pretest and as compared to a control group of tenth-graders.

Another curriculum recommendation frequently made is that courses should be functional. Results of an investigation by Bush (1959) of two groups of students at one high school after two years of study do not favor a functional approach. The students with two years of formal mathematics were superior to the two-year functional mathematics students in every category tested. The technique of covariance was used to hold ability factors constant.

The success of curriculum proposals depends greatly on the attitudes of teachers toward the new ideas and their competence to teach them. Spillane (1959) studied the attitudes of Pennsylvania mathematics teachers toward the inclusion of analytic geometry, calculus, and statistics in the high-school program, and considered the teachers' competence to teach these subjects. On the basis of a 76-percent return of questionnaires to 500 randomly selected teachers, he found that teachers favor the inclusion of these topics in high school and feel generally competent to teach these subjects. As would be expected, younger teachers and those who felt competent to teach the subjects were more favorably inclined toward them. Similarly, Leissa and Fisher (1960) surveyed the attitudes of engineers and college and high-school mathematics teachers toward the recommendations of the Commission on Mathematics of the College Entrance Examination Board. This group was highly favorable toward the recommended changes, being much in favor of including inequalities, sets, vectors, and probability. The group was not so favorably inclined toward including calculus, field theory, group theory, or statistics.

Another frequent curriculum proposal is to eliminate solid geometry as a separate course and include the concepts of three-dimensional space in tenth-grade geometry. To resolve the problem of what topics should compose tenth-grade geometry, Small (1959) submitted a list of 109 solid-geometry concepts to 50 mathematics teachers. Topics approved on 75 percent of 31 returned questionnaires included line perpendicular to a plane, lines parallel to a plane, loci, perpendicular planes, polyhedral angles, area and volume of prisms, cylinders, pyramids, cones, spheres, distances, angles, and areas on a sphere. Topics to be omitted included spherical triangles and congruence of prisms.

The number of experimental mathematics curriculums continues to grow, and projects that have been under way some time continue experimentation. The content and operation of several programs were described by Allen (1958), Brumfiel, Eicholz, and Shanks (1960), Davis (1960), and Keedy (1959b). No experimental project has reported data adequate to permit evaluation of its effectiveness.

The evaluation of a new curriculum will not be easy. A conference sponsored by the American Association for the Advancement of Science, the American Association of School Administrators, and the Council of Chief State School Officers was reported by Hull and others (1958). Its purposes were (a) to develop guidelines for program appraisal and direction in the teaching of mathematics and the sciences, and (b) to evaluate proposals being made for changes in school programs. The report will be found useful by anyone evaluating new curriculums. Putnam and Frazier (1960) compiled an annotated bibliography of state curriculum guides.

Attitudes, Concept Formation, and Understanding in Mathematics

Experimental studies, although few in number, showed concern for the significance of attitudes, the nature of understanding, and the formation of concepts. The significance of maturity and other variables in relation to an understanding of the limit concept was studied by L. T. Smith (1959). Among 578 students in grades 7 through 12, some classes were given three hours of special instruction in limits, other classes equated in mental age were not. Data were collected on a limits test, chronological age, mental age, and grade-point averages in mathematics. Findings showed experience to be important; significant gains in conceptualizing limit occurred at all levels; chronological age is not a related variable; and mental age is positively correlated with limits test scores.

Troxel (1959) examined relationships within measures of reading eighth-grade expository mathematical materials, and between such measures and intelligence, arithmetic achievement, and general reading ability. He also studied difficulty and interest in relation to purpose as well as reading skills. Based on data from 45 students, sample conclusions were (a) reading purpose influences reading speed; (b) general reading ability is related to speed of reading expository mathematical material; and (c) difficulty is influenced by purposes. Johnson (1957b) analyzed readability of 25 samples of 100 words from each of 18 mathematics texts and proposed use of the modified Flesch formula for determining readability.

Poffenberger and Norton (1959) questioned 390 college freshmen to determine factors relating to attitudes toward mathematics. Factors having the greatest influence on attitude were previous teachers and parental expectations and attitudes toward mathematics. Renner (1957) tested functional competence among 237 Iowa high-school seniors who had taken one year of algebra or general mathematics. Using covariance to control initial difference on the *Iowa Tests of Educational Development,* he found a significant difference in favor of the algebra group over the general mathematics group. E. M. J. Ferguson (1957) developed an observational instrument for describing the algebra classroom in relation to selected aims such as (a) ability to think, (b) appreciation of mathematics, and (c) attitude of curiosity and initiative.

Approaches and Techniques in Teaching Mathematics

Studies of approach and technique emphasized individual differences, the use of visual materials, application, and specialized ways of teaching particular content. Zoll (1957) investigated the relative merits of varying amounts of application in plane geometry. Each of three teachers taught both experimental and control classes equated for intelligence, geometric aptitude, arithmetic and algebraic competency in one high school. Analysis-of-variance techniques showed no significant differences between classes with varying amounts of application and control classes nor among experimental classes in regard to ability to solve "originals," knowledge of facts and principles, or ability to apply facts and principles in practical problems. Ability to apply geometric facts and principles seemed to be associated with individual males of good ability and mathematical competence. With limited statistical and experimental controls, Miller (1959) compared a single-equation approach to solving verbal elementary-algebra problems with a combination of guessing specific solutions and a subsequent multi-equation approach. Thirteen classes split between the methods in one high school were used.

Shoemaker (1957) reviewed the effectiveness of teaching principles of mathematics and science in Ohio public-school trade and industrial programs and found the current plan ineffective. According to Mazzei (1959), teaching estimation to ninth-grade and tenth-grade students did not help significantly to reduce errors. Kenney and Stockton (1958) equated three groups of seventh-graders (with more than 100 in each group) and compared three approaches to teaching percentage: (a) drill emphasis, (b) emphasis on understanding and reasoning, and (c) a combination of the first two. Using a self-designed test after 19 days, they found progress in the upper three-quarters of all classes, and inconclusive evidence suggesting possible advantages for the composite and understanding approaches.

Among three heterogeneous interest groups totaling 79 plane-geometry students, Griff (1957) observed the effect of one-level and three-level assignments varied quarterly through the year. Using a test of functional competence and class quizzes, he found students doing more and better work in a one-level approach. The analysis and design were limited. Hines (1957) reported a limited study of the effect of homework on achievement in plane geometry. Although the matched groups were of restricted equivalence, achievement differences seemed to favor students doing out-of-class work, particularly on cumulative review tests as opposed to unit tests.

Crosby and Fremont (1960) found that small groups in algebra, with testing as appropriate, and freedom of topic choice, provided a better learning climate and effective opportunity for achievement. Although experimental and control groups were used, the study tended to be descriptive. An informal study, without controls, by Ivie, Fowler, and Graham (1958), indicated that use of small groups in algebra, geometry, and business mathematics provides a good learning situation. However, superior students showed limited desire to progress, and some students felt the need for class activities. Ilioff (1957) investigated the effect of systematic home-school co-operation on the achievement of eighth-grade students. Increased parental understanding had a consistently positive effect on pupil achievement.

Studies of Association and Prediction in Mathematics

Studies dealt with associations among mathematics achievement, school size, student ability and sex, teacher attitudes, curricular choices, and school policies. McCutcheon (1957) analyzed achievement in eighth-grade mathematics (and science) in relation to school organization, enrollment, and pupil-teacher factors in Minnesota public schools. A stratified random sample of 85 schools was used. Pretests and post-tests in mathematics, designed by the experimenter, and an intelligence test were administered. A total of 378 teachers answered questionnaires and 6471 students participated. Analysis-of-variance and covariance techniques led to such findings as the following: (a) there were no sex differences in final achievement with adjustments for initial differences; (b) girls scored higher on the pretests, post-tests, and intelligence tests; and (c) significant differences in achievement among the upper, middle, and lower 5 percent in intelligence were noted.

Pruett (1960) studied mathematics and science achievement of 44,649 ninth-grade pupils in 618 Indiana private and public schools. Girls did better than boys in mathematics, and the better mathematics students were found in schools with large enrollments.

Using 29 teachers of first-year algebra and 1643 pupils in 13 schools in a large midwestern city, McCardle (1959) related scores on the *Minnesota Teacher Attitude Inventory* (MTAI) to pupil achievement. He found pupils with teachers having high MTAI ratings profited most in quantitative thinking and functional competence; teacher attitudes were not significantly related to algebra achievement.

McKinley (1960) sought a relationship between achievement in a twelfth-grade probability and statistics unit and intelligence, reading comprehension, previous mathematics achievement, and previous mathematics experience. The study extended for only 13 class periods with 10 classes in nine schools. A maximum multiple-correlation coefficient of .68 was found with achievement and three intelligence factors, reading comprehension, and previous achievement in mathematics. McKinley believed such a unit desirable for college preparatory students and that aptitude for such work could be measured to a significant degree. Dinkel (1959) found a multiple correlation of .86 between algebra achievement and a series of seventh-grade and eighth-grade predictive variables, including previous grades, intelligence, and prognostic and achievement tests.

In an analysis of Wisconsin school policies as related to students' choice of high-school mathematics courses, Parkinson (1959) observed that (a) schools with college preparatory tracks enrolled a larger proportion of students in algebra at the ninth-grade level than those without; other schools enrolled larger proportions in grades 10 through 12; (b) 57.2 percent of the schools had college preparatory tracks and 70 percent of these required at least four semesters of mathematics for college preparatory students; (c) mathematics achievement, teacher reports, and IQ scores were influential in mathematics guidance; and (d) students indicated the importance of out-of-school factors, particularly parents, in the choice of mathematics courses. Stone (1959) saw an increase in enrollment in elective mathematics and science as possibly related to students' reaction toward teachers in introductory courses.

Secondary Enrollment, Teacher Characteristics and Preparation

Studies pertaining to enrollment and teacher factors include broad national studies, state-wide investigations, and proposed training programs. The most critical problem was the lack of properly trained teachers.

The National Education Association Research Division (1958) surveyed enrollment, curriculum revision, and facilities for mathematics and science in U.S. secondary schools. Of 1957 high-school graduates, 14.4 percent had four or more years of mathematics; 22.9 percent, three years; 35.2 percent, two years; 25.6 percent, one year; and 1.9 percent, none. Principals tended to select teachers on the basis of skill in instructional methods rather than subject-matter preparation. Curriculum revision was reported in about half the schools. Facilities were seen as lagging. The large comprehensive high school was singled out for its contribution to preparation in mathematics, teacher qualification, curriculum, and facilities. Maul (1958) reported on teacher supply and demand in mathematics and science. The supply of candidates for high-school mathematics teaching decreased each year over a five-year period; one out of every three newly qualified persons did not teach. Turnover was great in what was described as an improved but still critical situation.

Teaching load and qualifications were summarized by Brown (1960). Mathematics enrollment in grades 9 through 12 was 4.4 million in 1956 and 4.5 million in 1960. About 100,000 seniors were unable to get advanced mathematics in small high schools. In 1957 about 43 percent of schools had curriculum studies in progress. In that same year secondary teachers averaged 23 hours preparation in mathematics; 7.1 percent had no mathematics; one-third were mathematics majors; and one-third had the master's degree but not usually in mathematics. Ahrendt (1958) believed that student enrollment and interest in mathematics were greater than generally claimed but that the shortage of adequately trained mathematics teachers was more critical than generally realized.

Torrance (1958) analyzed the extent of change in Minnesota mathematics and science teaching, using a 50-percent random sample of public secondary-school principals and superintendents. Four-fifths reported recent or pending decisions to improve their programs. Summer institutes and training programs were strongly supported. Finding and retaining qualified teachers was a major problem. Lohela (1958) studied enrollment characteristics and teacher preparation in Michigan secondary-school mathematics. Enrollment in mathematics decreased from 1925 to 1950 but has increased since. Nonpublic schools in Michigan enrolled a higher proportion of students in mathematics than public schools. Large schools had better prepared and more experienced teachers than smaller schools. Enrollment in mathematics dropped off rapidly for each succeeding grade, especially among girls. Teacher questionnaires revealed interest in practical application, realistic student teaching, college topics geared to the secondary school, preparation for dealing with individual differences, and instruction in motivation and class management.

Small (1957) sent a 52-item questionnaire to 1465 members of the National Council of Teachers of Mathematics to discover recommended aspects of a fifth year of preparation for mathematics teachers. Responses proposed 50 percent mathematics and not more than 25 percent professional education, with some work in research, advanced teaching, and cultural areas. Topics of interest were number theory, mathematics history, mathematical statistics, modern algebra and geometry, theory of equations, mathematics of finance, adolescent growth and development, and measurement. Jorgensen (1958) outlined characteristics and advantages of an inservice institute.

Nelson (1959) received 100 usable questionnaires from 154 Nebraska secondary mathematics teachers designated as superior or above average by their administrators. Classes in 46 schools were visited and 2188 students were queried. These capable teachers entered the profession because they liked mathematics, selected their careers in college, participated in professional growth activities, taught upper-level courses, used a wide variety of methods, and were concerned about improving their teaching. The students of these teachers praised their explanations, helpfulness, and personalities.

Programs for the Gifted Student in Mathematics

Although much space was devoted to special programs for the gifted, few studies evaluated their effectiveness. Long (1957) examined an enrichment program in four classes of 98 randomly selected students. In two experimental classes talented students served as group leaders, presented new topics and materials, gave special reports and projects, and participated in contests. Activities for all of the experimental group included weekly review, extra-credit problems on the tests, and special projects. Both the control and the experimental groups received the same basic instruction from the same teacher and the same topics, assignments, and tests. On two achievement tests and an attitude inventory both the talented and the nontalented in the experimental group surpassed the control group in both achievement and attitude. As this study was carefully designed, used appropriate statistical tools, was extended over a school year, and applied principles of randomization and controls, considerable confidence can be placed in the implication of the contributions of an enrichment program.

Wells (1958) reported an informal experiment with a modified curriculum for capable students in one eighth-grade algebra class. Achievement of the 25 high-ability eighth-graders was comparable to that of ninth-graders completing a similar course. The former achieved as well as or better than the ninth-grade control group.

One of the problems in setting up a program is identification of superior students. Cherry (1958) selected 90 eighth-grade students out of 1600 on the basis of an aptitude test, an achievement test, a reading test, an intelligence test, and teachers' recommendations. Although this method was accepted by students and parents, Cherry urged continued study of individuals and flexibility of assignment. In a similar study, M. B. Jones (1959) found that Maryland schools use teacher recommendations, previous achievement records, intelligence test scores, and achievement test scores to select the rapid learners. Over 90 percent of the participating schools reported the practice of ability grouping. Almost 60 percent provided special courses for their high-ability students.

Various programs have been advocated. Devine (1960) described a seminar to provide an accelerated program for gifted senior-high-school students, and Elder (1957) described a seminar to provide an enriched program for gifted junior-high students. Summer seminars supported by National Science Foundation funds with emphasis on topics from contemporary mathematics were described by Nichols (1960), M. L. Ferguson (1960), and Nielsen and Gohman (1959). Ferguson's findings from a state-wide summer program at eight centers in Tennessee indicated significant gains in subject-matter achievement by the experimental group, but no significant gains in ability to use knowledge for problem solving.

The national contest sponsored by the Mathematical Association of America and the Society of Actuaries was described by Fagerstrom and Lloyd (1958). The test used in this contest emphasized mathematical insight rather than isolated facts or skills. Pruitt (1960) and Keaveny (1959) described programs for grades 8 through 12 which are essentially acceleration programs with content similar to that of traditional courses in algebra and geometry. Elementary algebra is taught in grade 8 and intermediate algebra in grade 9. Plane and solid geometry and trigonometry are completed in grade 11 so that an advanced course such as mathematical analysis can be given in grade 12.

Teaching Mathematics via Television

Even though interest in and support for educational television have increased in recent years, relatively few efforts have been made to use television to teach mathematics. Wells (1959) compared the effectiveness of television-correspondence study of first-year algebra with that of direct teaching. Students in 11 small Nebraska high schools apparently achieved as well with television-correspondence instruction as with direct instruction. In another experimental study involving three classes in each of nine schools, Jacobs and Bollenbacher (1960) compared the effectiveness of televised lessons in seventh-grade mathematics with the results of conventional instruction. The year-long experiment used 20 minutes of live telecasts three days a week followed by 30 minutes of discussion. Care was taken to insure randomness, replication, control of variables, acceptable evaluation instruments, and proper statistical tools. The television method was found to be superior for the average student, but the conventional approach proved better for superior students. Other television projects are described by Berger (1958) and Andrews (1960). Extensive experimentation at different levels with valid evaluation of all objectives needs to be made before we can know the extent of television's contribution to mathematics teaching.

Facilities and Equipment for the Mathematics Classroom

The National Defense Education Act has provided funds for improved facilities and equipment, and several surveys have been conducted to determine needs. The U.S. Office of Education study by Obourn and others (1960) surveyed with a questionnaire a random sample of 1207 high schools to obtain information on rooms, furniture, equipment, teaching aids, and library facilities, as well as on methods of purchasing and sources of money. Findings are that classroom facilities are usually inadequate and that fewer than half the mathematics teachers replying use commercial or improvised equipment even though models were rated the most valuable teaching aid. Another government survey by Martin (1960) reported requirements and recommendations of state departments of education in regard to facilities, equipment, and instructional material for teaching science and mathematics at the elementary-school and secondary-school levels.

College Preparation and Entrance

Despite considerable emphasis on mathematics preparation and curriculum changes, few investigations have reported implications for college entrance and preparation. G. B. Smith (1958), analyzing the preparation of 1124 freshmen entering the University of Kansas in 1956, found that 29 percent of the men and 5 percent of the women had four or more years of mathematics. Forty-seven percent of arts and science students, 81 percent of engineering students, and 28 percent of fine arts students had three years.

Keedy (1959a), using questionnaire returns from 134 engineering schools, learned that 38 required solid geometry for entrance; he concluded that solid geometry was not significant in relation to entrance to engineering. Brant (1960) followed up Keedy's study by asking 51 schools with some kind of solid-geometry requirement if they would accept a one-year course of plane, solid, and co-ordinate geometry. In the few instances where a solid-geometry requirement still existed and in the vast majority of remaining courses, a fused course would be accepted. Thus while three-dimensional concepts were still judged important, solid geometry, as such, was an uncommon requirement.

McLean (1960) surveyed the status of integrated algebra-geometry courses in California and sought to determine the acceptability of such courses to teachers and college directors of admission. Integrated courses were not commonly found, teachers disagreed as to the value of such courses, and colleges generally accepted such courses except for science majors. It was suggested that integrated algebra-geometry courses be offered only as a second track in the college-preparatory mathematics curriculum.

Comparative Mathematics Teaching

With increased interest in American education, more attention has been given to what is done elsewhere. Many teachers have visited other countries, and numerous informal observational studies describe a variety of mathematics programs and suggest comparisons.

Woodby (1957) found in French secondary schools emphasis on national examinations and attention to modern mathematics, statistics, and co-ordination of physics and mathematics. Reform objectives were (a) classes four hours a week with no more than 40 students, (b) one hour a week in directed study ("half classes"), (c) better co-ordination of secondary and vocational programs, and (d) addition of suitable modern mathematics.

Wood (1958) discussed the expansion of secondary education in South Australia, the problem of appropriate mathematics courses for less able students, the virtual elimination of solid geometry, and the shortage of adequately trained teachers. Vogeli (1960), describing the mathematics program in Soviet 10-year schools, reported three trends: (a) polytechnism, (b) effort to lighten students' academic load, and (c) effort to raise the scientific level of mathematics instruction. Rourke (1960) observed these proposed changes in Russian secondary mathematics teachings: (a) elimination of trigonometry as an independent subject, (b) inclusion of analytic geometry in function study, (c) addition of computational trigonometry to geometry, and (d) addition of differential calculus to the eleventh year. Soviet self-criticism included (a) lack of emphasis on understanding, (b) liberalism in grading, and (c) lack of uniform standards.

Wirszup (1958, 1959) discussed the mass problem-solving contests held for Polish secondary students and mathematics requirements for secondary students in the Soviet Union, Poland, Czechoslovakia, and Red China. According to Rollett (1960), because of examinations and government assistance, secondary mathematics teaching in England tends toward uniformity, even though schools are free to plan curriculums. Up to half of advanced secondary mathematics was devoted to mechanics and often included elementary statistics. Bodenman (1959) described mathematics requirements and content in the Federal Republic of Germany. Pólya (1960), discussing the teaching of mathematics in Switzerland, noted emphasis on specialization after grade 7, emphasis on subject matter in teacher training, the advanced nature of mathematics in university preparatory schools, and the lack of recent or likely change.

Doremus (1957) reported on an exchange of test data between British and New Jersey schools. Gattegno (1958), visiting U.S. schools, observed (a) an unusual lack of professional trust of teachers, (b) little productive research, (c) need for relating course content to grade level, and (d) emphasis on instruction rather than on learning. A. W. Jones (1958) also visited American classrooms and noted (a) failure to make mathematics interesting, (b) failure to integrate algebra and geometry, (c) overreliance on textbooks, (d) emphasis on time units rather than on understanding, (e) less emphasis on mental arithmetic than in Australia, (f) poor blackboard and student work, and (g) less time for individual help and work than in Australia.

Research Reviews and Proposals

Financial support by governmental agencies and foundations for research in mathematics education should result in improvement in quality as well as quantity of research in mathematics education. An illustration of this support in action is the U.S. Department of Health, Education, and Welfare, Office of Education (1960) report of a conference on "Psychological Problems and Research Methods in Mathematics Training." This conference brought together mathematicians and psychologists to discuss approaches to research in mathematics education, and the report provides a guide for investigation of mathematics education. It includes a review of completed research, formulation of problems in the learning of mathematics which should be investigated, and research methods and designs appropriate to the conduct of such studies.

The U.S. Office of Education continued its semi-annual survey of research in mathematics education. The latest surveys by Brown and Kinsella (1960) and Brown (1958) included a summary of completed studies during 1957-58 as well as a listing of questions which need solutions. Unfortunately, this survey is not a complete listing of all research in mathematics education in 1957-58, since many persons fail to submit information on the research completed.

Summary

Studies reviewed in this chapter point to these general conclusions: (a) there is urgent need for more well-trained secondary mathematics teachers; (b) facilities for the teaching of mathematics need substantial improvement; (c) although much curriculum development has occurred, there is yet need for co-ordination, better definition of goals, concepts, and understandings, and adequate evaluative research; (d) although teaching by television has attracted much interest, the appropriate uses of such instruction are not yet clear; (e) some evidence suggests that we have in many instances expected too little from our students; however, the precise implications for secondary mathematics and the secondary school as a whole are still a moot point; (f) much curriculum study has been fragmented with emphasis on one level or another rather than concentration on the total program; (g) opportunities for the talented student have been greatly extended, but again without adequate evaluation and without adequate attention to the total situation.

Research should make maximum use of the techniques, instruments, and conclusions of previous studies of related problems. Johnson's (1957a) summary of the implications of studies in the psychology of learning is indicative of the contributions to be found outside the field of mathematics.

In view of the increased cost and complexity of a well-designed study, Brown (1958) suggests that (a) key problems be identified by groups of teachers and schools; (b) problems be investigated by means of a team approach rather than by individuals; (c) results be published and distributed to avoid duplication and to suggest deeper studies. These are basic needs.

Bibliography

AHRENDT, MYRL H. "Facts, Fancy, Fabrications, and Figures." *Mathematics Teacher* 51: 236-39; April 1958.

ALLEN, FRANK B. "Report of the Secondary School Curriculum Committee." *Mathematics Teacher* 51: 146-50; February 1958.

ANDREWS, JOHN J. "High School Algebra via Television." *Mathematics Teacher* 53: 376-80; May 1960.

BERGER, EMIL. "Enriching Instruction via Television." *Mathematics Teacher* 51: 550-52; November 1958.

BODENMAN, PAUL S. "Secondary School Mathematics in the Federal Republic of Germany." *Mathematics Teacher* 52: 465-70; October 1959.

BRANT, VINCENT. "Is Solid Geometry an Entrance Requirement for Engineering Schools?" *Mathematics Teacher* 53: 564-70; November 1960.

BRIDGES, CHARLES MARTIN, JR. *The Application of Elementary Statistics in Analysis of Data by Selected Secondary School Students.* Doctor's thesis. Knoxville: University of Tennessee, 1959. 98 p. Abstract: *Dissertation Abstracts* 20: 1223; No. 4, 1959.

BROWN, KENNETH E. "Research in Teaching High School Mathematics." *Mathematics Teacher* 51: 593-95; December 1958.

BROWN, KENNETH E. "Teaching Load and Qualifications of Mathematics Teachers." *Mathematics Teacher* 53: 2-11; January 1960.

BROWN, KENNETH E., and KINSELLA, JOHN J. *Analysis of Research in the Teaching of Mathematics, 1957 and 1958.* U.S. Department of Health, Education, and Welfare, Office of Education, Bulletin 1960, No. 8. Washington, D.C.: Superintendent of Documents, Government Printing Office, 1960. 50 p.

BRUMFIEL, CHARLES; EICHOLZ, ROBERT; and SHANKS, MERRILL. "The Ball State Experimental Program." *Mathematics Teacher* 53: 75-84; February 1960.

BRUNS, RICHARD F., and FRAZIER, ALEXANDER. "Sequence and Range of High School Mathematics." *Mathematics Teacher* 50: 562-66; December 1957.

BUSH, WILLIAM JACK. *An Analysis of the Outcomes of Functional Mathematics and Formal Mathematics as Measured by Certain Objective Tests After Completion by the Students of Two Years of Study.* Doctor's thesis. Fayetteville: University of Arkansas, 1959. 162 p. Abstract: *Dissertation Abstracts* 20: 920-21; No. 3, 1959.

CHERRY, W. J. "Methods of Selecting Freshmen for Accelerated Work in Mathematics." *School Science and Mathematics* 58: 467-71; June 1958.

CROSBY, GWLADYS, and FREMONT, HERBERT I. "Individualized Algebra." *Mathematics Teacher* 53: 109-12; February 1960.

DAVIS, ROBERT B. "The 'Madison Project' of Syracuse University." *Mathematics Teacher* 53: 571-75; November 1960.

DEVINE, DONALD F. "A Seminar for Students with High Mathematical Aptitude." *Mathematics Teacher* 53: 263-65; April 1960.

DEWITT, NICHOLAS. "Soviet Science Education and Its Challenge." *Mathematics Teacher* 51: 66-73; February 1958.

DIEBEL, CLARENCE EDWARD. *Some Influences of Evaluations on the Science and Mathematics Programs in Oregon Secondary Schools.* Doctor's thesis. Eugene: University of Oregon, 1959. 106 p. Abstract: *Dissertation Abstracts* 19: 2266-67; No. 9, 1959.

DINKEL, ROBERT E. "Prognosis for Studying Algebra." *Arithmetic Teacher* 6: 317-19; December 1959.

DOREMUS, ALBERT F. "A Comparison of American- with European-Schooled Youngsters—A Challenge." *Mathematics Teacher* 50: 432-33; October 1957.
ELDER, FLORENCE L. "Providing for the Student with High Mathematical Potential." *Mathematics Teacher* 50: 502-506; November 1957.
FAGERSTROM, WILLIAM H., and LLOYD, DANIEL B. "The National High School Mathematics Contest." *Mathematics Teacher* 51: 434-39; October 1958.
FERGUSON, ELISABETH MURIEL JANE. *The Design of an Observational Instrument for the Description of the Algebra Classroom in the Light of Selected Aims—Indexed by Behaviors—of Secondary School Mathematics Teaching.* Doctor's thesis. St. Louis, Mo.: Washington University, 1957. 350 p. Abstract: *Dissertation Abstracts* 17: 2220-21; No. 10, 1957.
FERGUSON, MILTON LEWIS. *The Peabody-Public School Summer High School Program for Academically Talented Students in Mathematics and Science.* Doctor's thesis. Nashville, Tenn.: George Peabody College for Teachers, 1960. 268 p. Abstract: *Dissertation Abstracts* 21: 560; No. 3, 1960.
GATTEGNO, CALEB. "Observations on the Teaching of Mathematics in the United States." *Mathematics Teacher* 51: 194-96; March 1958.
GRIFF, ERNEST R. "The Comparative Effectiveness of One-Level and Three-Level Assignments in Plane Geometry." *Mathematics Teacher* 50: 214-16; March 1957.
HINES, VYNCE A. "Homework and Achievement in Plane Geometry." *Mathematics Teacher* 50: 27-29; January 1957.
HULL, J. DAN, and OTHERS. *Mathematics and Science Education in U.S. Public Schools.* U.S. Department of Health, Education, and Welfare, Office of Education, Circular 533. Washington, D.C.: Superintendent of Documents, Government Printing Office, 1958. 97 p.
ILIOFF, LOUIE B. *The Effect of Certain Teaching Practices Involving Systematic Home-School Cooperation upon the Achievement of Eighth Grade Pupils in Mathematics.* Doctor's thesis. University Park: Pennsylvania State University, 1957. 145 p. Abstract: *Dissertation Abstracts* 17: 2935; No. 12, 1957.
IVIE, CLAUDE; FOWLER, EUGENIA; and GRAHAM, VIRGINIA. "Grouping—In the Normal Mathematics Class." *Mathematics Teacher* 51: 450-52; October 1958.
IZZO, JOSEPH ANTHONY. *A History of the Use of Certain Types of Graphical Representation in Mathematics Education in the Secondary Schools of the United States.* Doctor's thesis. New York: Teachers College, Columbia University, 1957. 283 p. Abstract: *Dissertation Abstracts* 17: 1506-1507; No. 7, 1957.
JACOBS, JAMES N., and BOLLENBACHER, JOAN K. "Teaching Seventh Grade Mathematics by Television." *Mathematics Teacher* 53: 543-47; November 1960.
JOHNSON, DONOVAN A. "Implications of Research in the Psychology of Learning for Science and Mathematics Teaching." *Review of Educational Research* 27: 400-13; October 1957. (a)
JOHNSON, DONOVAN A. "The Readability of Mathematics Books." *Mathematics Teacher* 50: 105-10; February 1957. (b)
JONES, A. W. "Mathematics Teaching in American Classrooms." *Mathematics Teacher* 51: 344-49; May 1958.
JONES, MAJOR BOYD. *Techniques, Methods, Procedures and Provisions Used in Selected Maryland Public Secondary Schools in Teaching Mathematics to Rapid Learners.* Doctor's thesis. Ithaca, N.Y.: Cornell University, 1959. 368 p. Abstract: *Dissertation Abstracts* 20: 3663-64; No. 9, 1960.
JORGENSEN, PAUL S. "In-Service Institutes for Mathematics Teachers." *Mathematics Teacher* 51: 613-14; December 1958.
KEAVENY, WILLIAM P. "Mathematics Program Outline for Junior and Senior High Schools." *Mathematics Teacher* 52: 449-52; October 1959.
KEEDY, M. L. "Solid Geometry as an Entrance Requirement for Engineering Schools." *Mathematics Teacher* 52: 121-23; February 1959. (a)
KEEDY, M. L. "The University of Maryland Mathematics Project." *Mathematics Teacher* 52: 281-82; April 1959. (b)
KENNEY, RUSSELL A., and STOCKTON, JESSE D. "An Experimental Study in Teaching Percentage." *Arithmetic Teacher* 5: 294-303; December 1958.
KNIGHTS, FRANCES ELLURA. *The Development of an Instrument To Predict Success in Analytic Geometry of Entering College Freshmen in Engineering and the Indication of Some Possible Improvements Advisable in Their Secondary School Mathematics Courses.* Doctor's thesis. University Park: Pennsylvania State University, 1957. 132 p. Abstract: *Dissertation Abstracts* 18: 120-21; No. 1, 1958.

LEISSA, ARTHUR W., and FISHER, ROBERT C. "A Survey of Teachers' Opinions of a Revised Mathematics Curriculum." *Mathematics Teacher* 53: 113-18; February 1960.

LOHELA, ARVO EPHRAIM. *Enrollment Characteristics and Teacher Preparation in Michigan Secondary School Mathematics.* Doctor's thesis. Ann Arbor: University of Michigan, 1958. 116 p. Abstract: *Dissertation Abstracts* 19: 471-72; No. 3, 1958.

LONG, ROY GILBERT. *A Comparative Study of the Effects of an Enriched Program for the Talented in Advanced Algebra Classes.* Doctor's thesis. Bloomington: Indiana University, 1957. 238 p. Abstract: *Dissertation Abstracts* 18: 529-30; No. 2, 1958.

MCCARDLE, HUGH JOSEPH. *An Investigation of the Relationships Between Pupil Achievement in First-Year Algebra and Some Teacher Characteristics.* Doctor's thesis. Minneapolis: University of Minnesota, 1959. 209 p. Abstract: *Dissertation Abstracts* 20: 165; No. 1, 1959.

MCCUTCHEON, GEORGE JAMES. *An Analytical Study of Achievement in Grade Eight General Science and in Grade Eight General Mathematics in Minnesota Public Schools.* Doctor's thesis. Minneapolis: University of Minnesota, 1957. 507 p. Abstract: *Dissertation Abstracts* 18: 1306; No. 4, 1958.

MCKINLEY, JAMES ERNEST. *Relationship Between Selected Factors and Achievement in a Unit on Probability and Statistics for Twelfth Grade Students.* Doctor's thesis. Pittsburgh, Pa.: University of Pittsburgh, 1960. 67 p. Abstract: *Dissertation Abstracts* 21: 561-62; No. 3, 1960.

MCLEAN, ROBERT CLAY, JR. *The Establishment of Integrated Algebra-Geometry Courses in the Secondary Schools of California.* Doctor's thesis. Los Angeles: University of Southern California, 1960. 320 p. Abstract: *Dissertation Abstracts* 21: 135-36; No. 1, 1960.

MARTIN, W. EDGAR. *Facilities and Equipment for Science and Mathematics.* U.S. Department of Health, Education, and Welfare, Office of Education, Misc. No. 34. Washington, D.C.: Superintendent of Documents, Government Printing Office, 1960. 130 p.

MAUL, RAY C. "Let's Look at the New Mathematics and Science Teachers." *Mathematics Teacher* 51: 531-34; November 1958.

MAZZEI, RENATO. "A Technique for the Prevention of Errors in Arithmetic." *School Science and Mathematics* 59: 493-97; June 1959.

MILLER, HERBERT FRANCIS. *The Combination of the Guess-and-Check and Multi-Equation Methods for Deriving the Equations for Verbal Problems in Elementary Algebra.* Doctor's thesis. Columbus: Ohio State University, 1959. 238 p. Abstract: *Dissertation Abstracts* 20: 2180; No. 6, 1959.

NATIONAL EDUCATION ASSOCIATION, RESEARCH DIVISION. "Mathematics and Science." *NEA Research Bulletin* 36: 67-73; October 1958.

NELSON, THEODORA SOPHIA. *Factors Present in Effective Teaching of Secondary School Mathematics.* Doctor's thesis. Lincoln: University of Nebraska Teachers College, 1959. 393 p. Abstract: *Dissertation Abstracts* 20: 3207-3208; No. 8, 1960.

NICHOLS, EUGENE D. "A Summer Mathematics Program for the Mathematically Talented." *Mathematics Teacher* 53: 235-40; April 1960.

NIELSEN, ROSS A., and GOHMAN, WALTER J. "Junior High School Seminar in Science and Mathematics." *Mathematics Teacher* 52: 295-98; April 1959.

OBOURN, E. S., and OTHERS. *Science and Mathematics in Public High Schools, 1958.* U.S. Department of Health, Education, and Welfare, Office of Education, Bulletin 1960, No. 6. Washington, D.C.: Superintendent of Documents, Government Printing Office, 1960. Part 1, "General Facilities and Equipment," 70 p.

PARKINSON, DANIEL SMITH. *School Policy in Student Choices of Courses in High School Mathematics.* Doctor's thesis. Madison: University of Wisconsin, 1959. 300 p. Abstract: *Dissertation Abstracts* 20: 927-28; No. 3, 1959.

POFFENBERGER, THOMAS M., and NORTON, DONALD A. "Factors in the Formation of Attitudes Toward Mathematics." *Journal of Educational Research* 52: 171-76; March 1959.

PÓLYA, GYŐRGY. "Teaching of Mathematics in Switzerland." *Mathematics Teacher* 53: 552-58; November 1960.

PRUETT, ROLLA FRANCIS. *The Achievement in Mathematics and Science of Ninth-Grade Pupils in the Schools of Indiana.* Doctor's thesis. Bloomington: Indiana University, 1960. 178 p. Abstract: *Dissertation Abstracts* 21: 505; No. 3, 1960.

PRUITT, COY C. "An Experimental Program in Mathematics." *Mathematics Teacher* 53: 102-105; February 1960.

PUTNAM, E. ANNE, and FRAZIER, RALPH P. *State Curriculum Guides for Science, Mathematics, and Modern Foreign Languages.* U.S. Department of Health, Education, and Welfare, Office of Education, Circular 627. Washington, D.C.: Superintendent of Documents, Government Printing Office, 1960. 28 p.

RAJARATNAM, NAGESWARI. *A Study of Some Concepts in Algebra as Used by Writers of High-School Text-Books.* Doctor's thesis. Urbana: University of Illinois, 1957. 155 p. Abstract: *Dissertation Abstracts* 18: 532-33; No. 2, 1958.

RENNER, JOHN W. "Student Achievement of Functional Competence Three Years After Completing Algebra or General Mathematics." *Mathematics Teacher* 50: 160-61; February 1957.

ROLLETT, A. P. "Mathematics Teaching in Secondary Schools in England." *Mathematics Teacher* 53: 367-70; May 1960.

ROURKE, ROBERT E. "Some Observations on Mathematics Education in Russian Secondary Schools." *Mathematics Teacher* 53: 241-52; April 1960.

SHETLER, LUTHER LEROY. *Practices and Trends in the Teaching of Secondary School Mathematics.* Doctor's thesis. Bloomington: Indiana University, 1958. 321 p. Abstract: *Dissertation Abstracts* 19: 2033; No. 8, 1959.

SHOEMAKER, BYRL RAYMOND. *Adequacy of Related Technical Instruction in Vocational Trade and Industrial Education in Teaching Principles of Mathematics and Physical Science.* Doctor's thesis. Columbus: Ohio State University, 1957. 160 p. Abstract: *Dissertation Abstracts* 17: 1517; No. 7, 1957.

SMALL, DWAIN E. "The Fifth Year of Teacher Education for Teachers of Mathematics." *Mathematics Teacher* 50: 199-203; March 1957.

SMALL, DWAIN E. "Selection of Topics from Solid Geometry for a One-Year Course in Geometry." *Mathematics Teacher* 52: 546-48; November 1959.

SMITH, GEORGE B. *Let's Look at the Record.* Kansas Studies in Education, Vol. 9, No. 1. Lawrence: School of Education, University of Kansas, 1958. 71 p.

SMITH, LEHI TINGEN. *The Role of Maturity in Acquiring a Concept of Limit in Mathematics.* Doctor's thesis. Stanford, Calif.: Stanford University, 1959. 119 p. Abstract: *Dissertation Abstracts* 20: 1288-89; No. 4, 1959.

SPILLANE, DANIEL PAUL. *The Attitudes of Pennsylvania Secondary Mathematics Teachers Toward the Inclusion of Analytic Geometry, Calculus, and Statistics in the High School Program.* Doctor's thesis. Pittsburgh, Pa.: University of Pittsburgh, 1959. 72 p. Abstract: *Dissertation Abstracts* 20: 1646; No. 5, 1959.

STONE, VERNON W. "Student Interest in Science and Mathematics and the 'Introductory' Teacher." *School Science and Mathematics* 59: 249-53; April 1959.

TORRANCE, E. PAUL. *Minnesota Secondary School Administrators Look at Their Mathematics and Science Programs.* Research Memorandum No. 58-1. Minneapolis: Bureau of Educational Research, University of Minnesota, 1958. 13 p.

TROXEL, VERNON EARL. *Reading Eighth Grade Mathematical Materials for Selected Purposes.* Doctor's thesis. Urbana: University of Illinois, 1959. 200 p. Abstract: *Dissertation Abstracts* 20: 168-69; No. 1, 1959.

U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE, OFFICE OF EDUCATION. *Research Problems in Mathematics Education.* Cooperative Research Monograph No. 3. Washington, D.C.: Superintendent of Documents, Government Printing Office, 1960. 130 p.

VOGELI, BRUCE RAMON. *The Mathematics Program of the Soviet Secondary School: Its Status and Innovations.* Doctor's thesis. Ann Arbor: University of Michigan, 1960. 556 p. Abstract: *Dissertation Abstracts* 21: 305-306; No. 2, 1960.

WALES, LOIS TYLER. *A Recommended Program for High School General Mathematics as Determined by an Appraisal of Present Content and Placement of Subject Matter.* Doctor's thesis. Baton Rouge: Louisiana State University, 1958. 150 p. Abstract: *Dissertation Abstracts* 19: 745-46; No. 4, 1958.

WELLS, DAVID W. "A Modified Curriculum for Capable Students." *Mathematics Teacher* 51: 181-83; March 1958.

WELLS, DAVID WAYNE. *The Relative Effectiveness of Teaching First Year Algebra by Television-Correspondence Study and Teaching First Year Algebra by Conventional Methods.* Doctor's thesis. Lincoln: University of Nebraska Teachers College, 1959. 199 p. Abstract: *Dissertation Abstracts* 20: 3137; No. 8, 1960.

WILSON, JOHN DONALD. *An Analysis of the Plane Geometry Content of Geometry Textbooks Published in the United States Before 1900.* Doctor's thesis. Pittsburgh, Pa.: University of Pittsburgh, 1959. 259 p. Abstract: *Dissertation Abstracts* 20: 1648; No. 5, 1959.

WIRSZUP, IZAAK. "The Seventh Mathematical Olympiad for Secondary School Students in Poland." *Mathematics Teacher* 51: 585-89; December 1958.
WIRSZUP, IZAAK. "Current School Mathematics Curricula in the Soviet Union and Other Communist Countries." *Mathematics Teacher* 52: 334-46; May 1959.
WOOD, ALLEN. "Mathematics in South Australia." *Mathematics Teacher* 51: 18-22; January 1958.
WOODBY, LAUREN. "Mathematics in French Secondary Schools." *Mathematics Teacher* 50: 204-208; March 1957.
ZOLL, EDWARD JOSEPH. *The Relative Merits of Teaching Plane Geometry with Varying Amounts of Applications.* Doctor's thesis. New York: New York University, 1957. 518 p. Abstract: *Dissertation Abstracts* 18: 971-72; No. 3, 1958.