Cascading Defaults and Systemic Risk of a Banking Network*

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Abstract

Systemic risk of a banking system arises from cascading defaults due to interbank linkages. We propose a model which distinguishes systemic risk from its drivers – systematic and idiosyncratic risks. Systemic risk is characterised by *systemic exposure* and *systemic fragility*, corresponding to the expected losses and pervasiveness of defaults respectively (under a stress scenario). The model takes into account the banking network, asset-liability dynamics, interbank exposures and netting. Using actual data for 15 British banks, we find that systematic shocks are more likely to drive systemic risk, as opposed to banks idiosyncratic elements. We also demonstrate a method for ranking banks according to systemic importance.

Keywords: Systematic risk, systemic exposure, systemic fragility, credit risk, operational risk, netting, stress testing, bridge sampling, and SIFI.

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1 Introduction

The objective of this paper is to measure systemic risk, which enables the study of various banking network configurations. Specifically, systemic risk of a banking system arises from cascading defaults due to interbank linkages. Any large negative external shock can in principle trigger cascading defaults, but shocks to systematic risk factors, as opposed to banks' idiosyncratic elements, are more likely to drive cascading defaults and hence to cause higher systemic risk.

To put to rest any confusion over *systemic risk* and *systematic risk*, we begin by distinguishing the two. Schwarcz (2008) defined systemic risk as "an economic shock such as market or institutional failure (that) triggers the failure of a *chain of markets or institutions, or a chain of significant losses to financial institutions...*" Noting that the concept of systemic risk is sometimes conflated with systematic risk, he aptly distinguished the two. Systematic risk is commonly known in the finance literature and refers to the type of risk that cannot be diversified away and therefore affects most, if not all, market participants.

Caruana (2010)[1] noted that an exogenous (systematic) shock can become systemic because of direct common exposures. He also points out that the "financial system is a network of interconnected balance sheets. As a result, an increasingly complex web of daily transactions means that a shock hitting one institution can spread to the other institutions that are connected to it and become systemic". In short, he meant that systemic risk is more likely caused by a systematic shock, and such a shock propagates through the banking system with stronger knock-on effects.

In a similar manner, Bandt and Hartmann (2000) disentangled "systematic shock" from "systemic event". Furthermore, they opined that a "key element in systemic events... is the mechanism through which shocks propagate from one financial institution or market to the other. In our view, this is the very core of the systemic risk concept."

In summary, we state the concept simply as follows: *systematic risk* arises from exposures to common risk factors, and *systemic risk* is purely due to interconnections. Although large systematic risk may lead to systemic risk, they are *not* synonymous. We clearly distinguish and measure the relative impact of these two effects by a structural model in which individual banks' asset values react to systematic risk factors in addition to their individual idiosyncratic risks. When a bank's asset value falls below a solvency threshold, moderated by, say, a liquidity consideration, the bank fails, and its failure may knock out other banks through its inability to honor debt obligations to others in the banking network. Our model is able to show that a banking system's total risk is attributable to systemic linkages, systematic factors and idiosyncratic shocks facing individual members of the system.

Our work contrasts with the existing literature. By modelling the banking network, we are able to formally and structurally distinguish systemic risk from systematic risk. Acharya, *et al* (2010) defined "systemic expected shortfall" (SES) as the propensity of a financial institution

[1] General Manager of Bank for International Settlements at the time of making the observation.

being undercapitalised when the system as a whole is undercapitalised. Their measurement relies on the historical response of individual equity returns to market returns (adjusted for tail risk). Brownlees and Engle (2012) built on the above and employed dynamic volatility models to come up with a systemic measure known as SRISK, which is the extent of expected capital shortfall in the economy experiencing a significant drop in the stock market. In essence, these approaches touch upon the source of systemic risk as being systematic risk but without providing explicit channels. In other words, their systemic risk measure is a reflection of correlations which could be due to systematic risk and/or interbank linkages.

Hautsch, *et al* (2012) defined systemic risk beta as the marginal effect of a bank's VaR on the VaR of the entire financial system, whereas Adrian and Brunnermeier (2011) proposed CoVaR as the VaR of the financial system conditional on institutions being under distress. Again, these are simply measures of correlation of sort. In another paper, Billio *et al* (2011) used a principal component analysis to decompose the returns of hedge funds, banks, brokers, and insurance companies. They did not model the network effect directly, but viewed the extent of Granger causality in their network as a proxy for return-spillover effects among market participants. Huang, *et al* (2010) viewed systemic risk as the premium required to insure against systemic financial distress. Banks' assets are associated with common risk factors, but bank interactions in a default event are otherwise not modelled.

All in all, these approaches, albeit useful, rely on correlations revealed through the past data, and their lack of structure in terms of the interplay between exogenous shocks and the bankruptcy trigger makes them unsuitable for policy analysis such as asking what if the banking network is altered in some way, say, through consolidation and netting arrangements.

Another line of research adopts network-based models, which can potentially capture the cascade effect explicitly, and hence measure systemic risk. Semi-analytic solutions for network models are obtained by a few authors. Giesecke and Weber (2005) adopted a model of interacting particle systems used in physics while Horst (2007) used a "mean-field" model for firms' credit ratings. While interesting results are obtained, there are four obvious limitations with these analytic solutions. Firstly, the strong assumption of homogeneity is required. Secondly, the linkages based, for example, on mutual debt obligations, are an inadequate characterization of linkages because the banks' asset sizes and riskiness have not been explicitly accounted for. Thirdly, the source of exogenous shocks are not explicit and their interplay with the functioning of the banking network is not explicit. Lastly, the abstraction of financial systems into physical systems do not lend easily to interpretation. Our structural model naturally accommodates heterogeneity using individual balance sheets in a banking network and explicitly models systematic and idiosyncratic shocks that generate endogenous cascading defaults through individual solvency conditions.

Still within the context of a network, other papers model the default of banks more explicitly. Nier, *et al* (2007) relied on the adequacy of total assets, triggering a default when net worth is inadequate. In the same vein, Marquez and Martinez (2009) compared exposures against bank-specific thresholds. In comparison, our model employs a soft insolvency default trigger with the occurrence probability as a function of the distance-to-default measure, which has been shown to be consistently and significantly related to likelihood of default. This becomes possible because we explicitly model each bank's assets and liabilities dynamics by introducing common risk factors. Their natural dynamic evolutions in response to common risk factors and idiosyncratic shocks provides an explicit and more realistic model of causes and reactions of a banking system.

While not squarely the focus of this paper, we note that several authors have explored the existence of a post-liquidation equilibrium in a static setting; that is, to show that an initial shock to a banking network can indeed be resolved through the balance sheets of the banks and there is an end-state after all defaults have taken place. Eisenberg and Noe (2001) showed the existence of such an equilibrium. Gourieroux, Heam and Monfort (2012) provided an extension and used it to explore contagion risk.

In our framework, the balance sheet serves as the main channel by which banks may default (or survive). The balance sheet may be weakened for several reasons. Losses occur as a result of systematic or idiosyncratic shocks, or caused by a bank's counterparties. A defaulting bank's assets are also subject to fire-sale discounts which may exacerbate the losses. This approach is generally consistent with the literature, although we note that several other possible channels have also been mentioned. Allen and Gale (2000) proposed a microeconomic approach that characterizes financial crises as forced liquidation of assets due to excess liquidity demand by consumers. Rosenthal (2011) viewed post-bankruptcy rehedging as a game, where some surviving banks may engage in predatory actions and force others into bankruptcy. Nier, *et al* (2007) briefly described four mechanisms by which multiple banks may fail: (1) direct bilateral exposures between banks, (2) correlated exposures to a common source of risk, (3) feedback effects from endogenous fire-sale of assets by distressed institutions, and (4) informational contagion. Roughly speaking, our approach covers the first three described by Nier, *et al* (2007).

Due to their complexity, the systemic risk measures in our model can only be computed by Monte Carlo simulations. In this regard, our model bears resemblance to that of Anand, *et al* (2013), which sets up an extensive network involving domestic banks, international banks, and corporate firms. They relied on some prescribed heuristics to model the contagion dynamics. Because defaults in their model are not triggered by an explicit solvency condition, their model is more of a reduced-form type as opposed to our structural approach. Another key difference is the way that banks' assets and liabilities are shocked. The model of Anand, *et al* (2013) does not allow for dynamic behavior, whereas in our model the assets and liabilities follow stochastic processes and are structured into systematic and idiosyncratic risk components. Cascading defaults in our model can therefore be more meaningfully analyzed, and a clear separation of systematic and systemic risks becomes possible.

Systemic risk would not become particularly interesting if we view it as *unconditional* expected incremental losses due to interbank linkages. Because the likelihood of cascading defaults is supposed to be quite low, any significant incremental losses in a crisis environment will be masked by its small probability of occurrence, *ex ante*. A meaningful systemic risk measure should therefore be expected losses conditional on some crisis scenario; for example, a stock market decline by 40% or more over six months adopted by Brownlees and Engle (2012). Our estimation of systemic risk critically needs an efficient simulation technique that can effectively conduct simulations of dynamic systems by concentrating on rare future events. For this, we devise a novel bridge-sampling technique that can efficiently compute various quantities of interest in our model.

We demonstrate the use of our structural model with a network of 15 British banks that was analyzed previously by Anand, *et al* (2013). The sample period is quarterly from Q1 of 2004 to Q4 of 2012. Our data such as the implied asset value and distance-to-default are obtained from the Credit Research Initiative database of the Risk Management Institute, National University of Singapore. Due to data confidentiality, we do not know the actual interbank exposures of these 15 banks at different points of time over our sample period. But we are able to obtain from the authors of Anand, *et al* (2013) the interbank exposures distribution of these banks and use that distribution to generate interbank exposures.

Our measurement is responsive to the evolution of the credit crisis, peaking in 2008-2009 and abating thereafter. This may have been brought about by the consolidation of banks' capital structure through a series of mergers and the UK bank rescue package.

A deeper analysis reveals two key findings. Firstly and as expected, we find that the level of systematic risk (exposure or fraility) is directly and positively related to the performance of market-wide risk factors. A large negative common shock is typically associated with large magnitudes of systemic risk. More interestingly, we find that systemic risk becomes significant *only when* systematic risk is large. In a hypothetical setting in which bank assets were driven purely by idiosyncratic risk, systemic risk would be low. On the other hand, if bank assets move strongly in the common direction, the impact of systemic risk is considerable. In principle, idiosyncratic shocks may trigger cascading defaults, but shocks to systematic risk factors, as opposed to banks' idiosyncratic elements, are more likely to drive cascading defaults and hence to cause higher systemic risk. As expected, interbank losses do exhibit a decreasing relationship with the level of interbank netting. Based on the British interbank exposure data available to us, however, interbank netting does not significantly reduce systemic risk.

Finally, we consider marginal systemic risk measures which are computed as the increase in systemic risk due to a particular bank between (i) using the true banking configuration and (ii) employing a hypothetical banking configuration under which only this particular bank faces no interbank exposures. "Interconnectedness" has been identified as a primary source of systemic disruption, and our model is particularly apt, because this is measured directly. We show how marginal systemic risk measures can be used to rank banks according to their individual contributions to systemic risk. Thus, our method can potentially identify systemically important financial institutions (SIFIs), i.e., those which are either "too-big-to-fail" or "too-connected-to-fail".

2 Asset and liability dynamics before a bank default

Consider a banking network that comprises M banks. Denote the first time that a bank defaults by τ_1, the second by τ_2, and so forth. The random times of bank default occurrences are increasing; that is, $0 \leq \tau_1 \leq \tau_2 \leq \cdots \leq \tau_M$. The exact default definition will be given in the next section.

Between two consecutive random default times τ_j and τ_{j+1}, the i-th bank that has survived beyond τ_j is assumed to have the asset and liability dynamics: for $\tau_j \leq t < \tau_{j+1}$ ($\tau_0 = 0$),

$$\frac{dV_{it}}{V_{it_-}} = \mu_i dt + \sum_{k=1}^{K} \beta_{ik} df_{kt} + \sigma_i dW_{it} + Y_i^{(c)} dN_t^{(c)} + Y_i dN_{it} + \frac{dL_{it}}{V_{it_-}} \tag{1}$$

$$\frac{dL_{it}}{L_{it_-}} = \varphi_i dt + \sum_{k=1}^{K} \gamma_{ik} df_{kt} + \nu_i dB_{it}. \tag{2}$$

Common risk factors such as stock market index, interest rate, exchange rate and so on are captured generically by f_{kt} ($k = 1, \cdots, K$). The dynamics for three common risk factors (stock market index, interest rate term spread and exchange rate) will be specified later when we implement the model. Selecting interest rate, exchange rate and stock market index as common risk factors are backed by well-established empirical findings. For example, Choi, Elyasiani and Kopecky (1992) concluded that these three factors are statistically significant for banks. Flannery and James (1984) found that the co-movement of stock returns and interest rate changes is positively related to the size of the maturity difference between the firm's nominal assets and liabilities, whereas Kwan (1991) arrived at the same conclusion using an alternative specification. It should be noted that we will use the LIBOR spread (12-month rate minus 1-month rate) as the interest rate factor in the application to a British banking network later to better reflect the fact that the term spread as opposed to the interest rate level serves as a better indicator of interest rate risk.

W_{it} is a Wiener process independent of all common risk factors, and it is used to reflect the idiosyncratic risk that is unique to the i-th bank, i.e., also independent of the idiosyncratic shocks facing other banks.

The operational/credit risk is captured by the two terms: $Y_i^{(c)} dN_t^{(c)}$ and $Y_i dN_{it}$. The interbank credit risk is excluded from this term because we will model it separately. We refer to these two terms as common and idiosyncratic operational/credit risks. $N_t^{(c)}$ and N_{it} are the Poisson processes with intensity parameters λ^c and λ_i, respectively. These Poisson terms are well suited for capturing low-frequency but high-impact operational/credit risk events. High-frequency, low-impact operational/credit risks may be regarded as parts of the diffusive idiosyncratic risk term, W_{it} described earlier.

$N_t^{(c)}$ captures the common operational/credit risk event. Enron's bankruptcy is a good example. It generates direct loan losses to banks as well as creates subsequent ligation settlements between

Enron's creditors and major international banks.[2] A common operational/credit risk event need not induce identical losses to different banks, and thus the loss percentage ($Y_i^{(c)}$) is bank-specific. In addition to common operational/credit risk, one can expect bank-specific risk events, which are captured by N_{it} with the loss percentage being Y_i in the event of its occurrence.[3]

$Y_i^{(c)}$ and Y_i are independent of each other and also independent of $N_t^{(c)}$ and N_{it}, and they follow some appropriate distributions. In the example provided later, we assume they are normally distributed with means $\mu_{Y_i}^{(c)}$ and μ_{Y_i} and standard deviations $\sigma_{Y_i}^{(c)}$ and σ_{Y_i}. Moreover, N_{it}, Y_i and $Y_i^{(c)}$ are independent across banks.

A typical operational/credit risk event will bring down a bank's asset value; that is, both $Y_i^{(c)}$ and Y_i are to have negative means. However, they need not always take on negative values; for example, an initial litigation loss estimate may be revised downward due to the emergence of new and favorable information so that it is actually a positive operational risk event. Wahlen (1994), for example, noted that bank managers include a discretionary component into loan loss provisions, over and above expected future losses.

Together, the operational/credit risk dynamics is a sum of two compound Poisson processes that are assumed to be independent of all other risk factors. Because the asset value process is discontinuous, we need to use the left time limit of the value process, i.e., V_{it_-}, to describe the asset value dynamics.

We have incorporated the nominal-value liability process (L_{it}) into the asset value dynamic to reflect the fact that an asset value change occurs due to two reasons: (1) the value of the assets in place moves in response to changes in market conditions, and (2) the asset value's increase/decrease corresponds to a change in a bank's deposits and/or other financing vehicles. The second dimension is often left out in the option-based credit and/or deposit insurance pricing models, because liabilities have been typically treated as fixed over the horizon of interest. However, it is both conceptually and practically important to let asset value reflect liability changes as in equation (1). Otherwise, banks in the model would be more likely to default because it had failed to reflect a simple accounting reality.

[2]The Enron accounting scandal began to unravel in late 2001 and only resolved in 2008. Financial institutions were sued for helping to hide Enron's true financial condition. Citigroup paid out US$2 billion to Enron investors in 2005, and US$1.66 billion to the creditors of the bankrupt company in 2008. Similarly, Royal Bank of Scotland, Deutsche Bank, J.P. Morgan Chase and CIBC paid settlements.

[3]Examples of idiosyncratic operational losses abound. Reuters reported on December 9, 2005 that Mizuho Financial Group of Japan lost an estimated US$224 million in a trader's blunder by keying in a wrong order. In January 2008, Societe Generale lost EUR 4.9 billion due to fictitious trades done by a rouge trader. In September 2011, UBS lost over US$2 billion to unauthorized trading in its Global Synthetic Equities Trading desk. Over April-May 2012, JP Morgan suffered large trading losses from massive positions on CDS derivatives.

3 Interbank credit links and cascading defaults

The extent to which interbank exposures result in higher systemic risk is the focus of our analysis. These exposures can be a result of derivatives trades such as foreign currency forwards, non-collateralized interbank lending and/or simple credit derivatives in reference to other entities. We will use the term "nominal value" to denote the present value of an interbank claim under no counterparty default. The i-th bank's nominal claim against the j-th bank at time t is denoted by π_{ijt}. By definition, its nominal-value liability to the j-th bank is π_{jit} because it is Bank j's nominal claim against Bank i. The overall interbank exposures are summarized in Π_t, an $M \times M$ matrix with π_{ijt} being its (i,j)-th element. The diagonal elements of Π_t are naturally zeros. The nominal value of interbank exposures are contained in the bank's assets V_{it} and liabilities L_{it}. A bank default will result in adjustments to the assets and liabilities of other banks in the banking network. The involuntary contraction of balance sheets for other banks is the main cause for cascading defaults. The specific adjustments will be described in a following section.

One can model the evolution of interbank exposures directly by a system of stochastic processes. To make the model more manageable, we choose to link them to the liability processes of the respective banks. Specifically, we assume a fixed percentage of Bank i's liabilities is its obligation to Bank j. We can express Π_t using a constant $M \times M$ matrix Q in combination with a time-varying L_t, a diagonal matrix with its elements being the total liabilities of different banks at time t. Let q_{ij} be the (i,j)-th element of Q where q_{ij} is the fraction of Bank i's total liabilities being its specific obligation to Bank j. Note that $q_{ii} = 0$. Thus, $\Pi_t = Q' L_t$. We will refer to Q as the **banking network configuration matrix**.

3.1 Bilateral netting reduces interbank exposures

Bilateral netting has often been discussed in the context of reducing counterparty exposures and systemic risk. Bilateral netting takes two forms: payment netting and close-out netting. Payment netting refers to offsetting of cash flows for regular settlements. We are interested in close-out netting, which applies to transactions between a defaulting firm and non-defaulting firms. Close-out netting terminates obligations with the defaulting party and combines the replacement values of multiple transactions into a single net payable or receivable. Mengle (2010) reported in his ISDA research paper that close-out netting reduced credit exposures by more than \$500 billion worldwide.[4]

The specifics of bilateral netting are as follows. Interbank exposures typically comprise multiple transactions, some of which are payables and others receivables. Due to the nature of banking business, two banks, say i and j, often have mutual interbank exposures, i.e., $\pi_{ijt} > 0$ and $\pi_{jit} > 0$. If either one defaults, the outcome depends on the enforceability of bilateral netting. In the worst case, the judicial manager of the defaulting bank may cherry pick obligations in the sense that it demands payments owed by other banks while it defaults on its own obligations (i.e. zero netting).

[4] OTC derivatives are typically operated under a bilateral netting master agreement entered into by two parties.

With bilateral netting enforceable, however, both parties will first net out their mutual interbank positions.

Generally, there are legal opinions validating the enforceability of bilateral netting in major jurisdictions including the US and UK. However, bilateral netting has not been legally tested fully in all jurisdictions, as there is the risk that in actual bankruptcy proceedings, the court may impose stays in preference of third-party creditors' interests, unbundle netted transactions, or impose claw-backs. The Basel Accord provides a ballpark parameter of 60% for the extent of netting to be recognized for regulatory capital purposes, and we will use this value in our later implementation.

It is important to note that close-out netting does not take effect until one bank defaults. Thus, one cannot net out interbank positions prior to a default. Upon default, both defaulting and non-defaulting banks must face asset-liability revisions. Let ψ represent the extent to which netting can be recognized. As stated earlier, we will set $\psi = 60\%$ per the Basel Accord. When Bank i defaults, its assets and liabilities are closed-out by the amount of $\psi \min(\pi_{ijt}, \pi_{jit})$ for Bank j ($j \neq i$), which is a bank that has not yet defaulted, but bilateral netting clears out its counterparty exposure with the defaulting bank. Bilateral netting poses to Bank j a net exposure of ω_{jit} where

$$\omega_{jit} = \pi_{jit} - \psi \min(\pi_{ijt}, \pi_{jit}) \tag{3}$$

Bank j must then compete with other creditors for a share of the remaining assets of Bank i. Any irrecoverable loss will be charged to Bank j's capital.

3.2 The bank default clearing process

Let D_{it} denote the default indicator process for Bank i with 1 implying default and 0 otherwise. Obviously, D_{it} is a non-decreasing process. The default indicator process cannot, however, be determined individually without being influenced by other banks in the banking network. In other words, it is endogenous to a banking network.

Starting from time 0, the asset and liability evolve according to equations (1) and (2) until some bank defaults. We define, for $\tau_j \leq t$,

$$\begin{aligned} V_{it}^* &= V_{i\tau_j} + \int_{\tau_j}^{t} V_{is_-}\left(\mu_i ds + \sum_{k=1}^{K} \beta_{ik} df_{ks} + \sigma_i dW_{is} + Y_i^{(c)} dN_s^{(c)} + Y_i dN_{is}\right) \\ &\quad + L_{it}^* - L_{i\tau_j} \end{aligned} \tag{4}$$

$$L_{it}^* = L_{i\tau_j} + \int_{\tau_j}^{t} L_{is_-}\left(\kappa_i ds + \sum_{k=1}^{K} \gamma_{ik} df_{ks} + \nu_i dB_{is}\right) \tag{5}$$

Let $\tau_0 = 0$ and define $\tau_1 = \inf\{0 \leq t : \xi_{it} = 1 \text{ for some } i\}$. The default probability of Bank i at any time t is assumed to be a logistic function: $P(\xi_{it} = 1 | \xi_{it-} = 0) = \frac{e^{\alpha_0 + \alpha_1 h(V_{it}, L_{it})}}{1+e^{\alpha_0 + \alpha_1 h(V_{it}, L_{it})}}$, with the logistic function relating the probability of default to its solvency condition, $h(V_{it}, L_{it})$. Lower assets or higher liabilities would increase the likelihood of default. In our implementation, we use a distance-to-default (DTD) measure for $h(V_{it}, L_{it})$. Defaults in our model are thus mainly triggered by insolvency but moderated by liquidity and other factors implicit in our later use of an empirically estimated probability of default function to govern the endogenous default behavior. In short, we have employed a *soft* insolvency default trigger as opposed to a *hard* one. The default indicator process is defined by setting $\{D_{it} = 1 \text{ for } t \geq \tau_1\}$ if Bank i is the one that defaults. If multiple banks, say j banks, default at the same time, set $\tau_j = \tau_{j-1} = \cdots = \tau_1$.

A default will call for interbank clearing. Clearing may generate sufficient credit losses to some other banks and drag them into defaults. Continuing from time τ_{k-1}, the asset and liability dynamics evolve as in equations (4) and (5). Once an additional bank default occurs, i.e., at time τ_k, one must make asset and liability adjustments to properly reflect the interbank credit losses. This turns out to be a fixed-point problem that can be solved by an iterative scheme with a finite number of iterations. Let $0 \leq \phi_j \leq 1$ be the bankruptcy cost adjustment factor which multiplies the pre-bankruptcy asset value of Bank j to yield the post-bankruptcy value at the point of its default. The following steps describe the asset and liability adjustments and the value assignments for the default indicator processes:

1. Let $L^*_{\tau_k}$ be the diagonal matrix with its i-th element being $L^*_{i\tau_k}$ and compute $\Pi_{\tau_k} = Q' L^*_{\tau_k}$ with its (i,j)-th element being denoted by $\pi_{ij\tau_k}$.

2. Start with $D^{(0)}_{i\tau_k} = D_{i\tau_{k-}}$, $V^{(1)}_{i\tau_k} = V^*_{i\tau_k}$ and $L^{(1)}_{i\tau_k} = L^*_{i\tau_k}$. Update the default indicator process as well as the asset and liability values for the banks that have not yet defaulted up to the l-th iteration; that is,

$$D^{(l)}_{i\tau_k} = \begin{cases} 1 & \text{if } D^{(l-1)}_{i\tau_k} = 1 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

$$\hat{V}^{(l)}_{i\tau_k} = \max\left[0, V^{(l)}_{i\tau_k} - 1_{\{D^{(l-1)}_{i\tau_k}=0 \ \& \ D^{(l)}_{i\tau_k}=1\}} \sum_{j\neq i}^{M} 1_{\{D^{(l-1)}_{j\tau_k}=0\}} \psi \min\left(\pi_{ji\tau_k}, \pi_{ij\tau_k}\right)\right] \tag{7}$$

$$\hat{L}^{(l)}_{i\tau_k} = L^{(l)}_{i\tau_k} - 1_{\{D^{(l-1)}_{i\tau_k}=0 \ \& \ D^{(l)}_{i\tau_k}=1\}} \sum_{j\neq i}^{M} 1_{\{D^{(l-1)}_{j\tau_k}=0\}} \psi \min\left(\pi_{ji\tau_k}, \pi_{ij\tau_k}\right) \tag{8}$$

$$\hat{V}^{(l+1)}_{i\tau_k} = \begin{cases} V^{(l)}_{i\tau_k} - \sum_{j\neq i}^{M} 1_{\{D^{(l-1)}_{j\tau_k}=0 \ \& \ D^{(l)}_{j\tau_k}=1\}} \left[\pi_{ji\tau_k} + \omega_{ij\tau_k}\left(1 - \frac{\phi_j \hat{V}^{(l)}_{j\tau_k}}{\hat{L}^{(l)}_{j\tau_k}}\right)\right] & \text{if } D^{(l)}_{i\tau_k} = 0 \\ \hat{V}^{(l)}_{i\tau_k} & \text{otherwise} \end{cases} \tag{9}$$

$$V^{(l+1)}_{i\tau_k} = \max\left(0, \hat{V}^{(l+1)}_{i\tau_k}\right) \tag{10}$$

$$L^{(l+1)}_{i\tau_k} = \begin{cases} L^{(l)}_{i\tau_k} - \sum_{j\neq i}^{M} 1_{\{D^{(l-1)}_{j\tau_k}=0 \ \& \ D^{(l)}_{j\tau_k}=1\}} \pi_{ji\tau_k} & \text{if } D^{(l)}_{i\tau_k} = 0 \\ \hat{L}^{(l)}_{i\tau_k} & \text{otherwise} \end{cases} \tag{11}$$

Note that Bank i's asset value loss due to Bank j's default ($i \neq j$) is $\omega_{ij\tau_k}\left(1 - \frac{\phi_j \hat{V}^{(l)}_{j\tau_k}}{\hat{L}^{(l)}_{j\tau_k}}\right)$ which reflects its partial claim recovery from the post-bankruptcy asset value (after bilateral netting) on a proportional basis. In addition, Bank i's assets and liabilities must be reduced by $\pi_{ji\tau_k}$ as a result of shrinking balance sheet due to Bank j's default. Since Bank i is a non-defaulting party, this balance sheet adjustment should be treated on a full-recovery basis.

Repeat the above iterative system until convergence to a fixed-point. Set $D_{i\tau_k}$, $V_{i\tau_k}$ and $L_{i\tau_k}$ equal to the fixed-point values.

3. Let $n(\tau_k) = \sum_{i=1}^{M} D_{i\tau_k}$, which is the number of defaulted banks up to and including time τ_k. Note that τ_k is the time that a new round of bank defaults starts and previously $k-1$ banks had already defaulted. In general, $n(\tau_k) \geq k$ due to cascading defaults. If $n(\tau_k) > k$, set $\tau_{n(\tau_k)} = \tau_{n(\tau_k)-1} = \cdots = \tau_k$ to reflect multiple defaults at the same time point. Note that in such a case, $V_{i\tau_{n(\tau_k)}} = V_{i\tau_{n(\tau_k)-1}} = \cdots = V_{i\tau_k}$, $L_{i\tau_{n(\tau_k)}} = L_{i\tau_{n(\tau_k)-1}} = \cdots = L_{i\tau_k}$ and $D_{i\tau_{n(\tau_k)}} = D_{i\tau_{n(\tau_k)-1}} = \cdots = D_{i\tau_k}$ because $n(\tau_k) - k + 1$ banks end up defaulting at the same time.

It should be noted that multiple banks may default concurrently even in the absence of any interbank credit links. As emphasized earlier, that type of dependence may, however, just be a reflection of the fact that bank assets and liabilities are all subject to the market-wide systematic risk factors.

Cascading defaults are explicitly modelled via the iterative system in equations (6)-(11). Every additional iteration (asset and liability value revision) may cause more defaults, which in turn calls for another iteration and may then cause further defaults. The default indicator processes are right-continuous stochastic processes. D_{it} ($i = 1, \cdots, M$) are clearly dependent default indicator processes, since one bank's asset and liability values depend on other bank defaults. A numerical example of a system of five banks showing how cascading defaults take place (with and without bilateral netting) is given in Appendix A.

4 Measures of systemic risk

Here, we re-emphasize the distinction between systemic and systematic risks. Market-wide risk factors such as interest rates, exchange rates, broad-based stock market indices affect all banks' assets and liabilities. The risks arise from these common risk factors are systematic in nature. Common operational/credit risk events is another form of such systematic risk. Systematic risk has much to do with the nature of a bank's assets and liabilities but has little to do with how a banking network is organized. Therefore, the effects of systematic and systemic risks may not be empirically distinguishable unless a model such as ours is deployed.

Systemic risk strictly arises from how the interbank exposures are structured. In the current context, systemic risk is the incremental risk that is attributable to the credit risk links in a banking network. In short, systematic risk may be a key driver of systemic risk, but it is not the systemic risk, however.

Our model naturally lends itself to a variety of measures for systemic risk. We consider two aspects of systemic risk that are particularly important in any crisis. Firstly, the magnitude of systemic losses, or *exposure*. Secondly, the pervasiveness of systemic failure, or *fragility*.

4.1 Systemic exposure

The total uncovered loss from a banking network, over horizon $[0,T]$ and evaluated at time-0, may be expressed as follows:

$$TL^{(Q)}_{[0,T]} = \sum_{i=1}^{M} \int_0^T e^{-\int_0^t r_s ds} \left(L^{(Q)}_{it} - \phi_i V^{(Q)}_{it} \right) dD^{(Q)}_{it} \tag{12}$$

Recall that ϕ_i is the fire-sale discount factor which reflects the bankruptcy costs for Bank i.

Note that the total uncovered loss is measured with respect to the banking configuration Q, as indicted by the superscript above. Clearly, a network where $Q = \mathbf{0}$ has no systemic exposure, since there are no interbank links. Rather, the losses associated with a network of $Q = \mathbf{0}$ are solely attributable to systematic and idiosyncratic risks.

Accordingly, the **systemic exposure** is measured by taking the incremental uncovered loss due to the banking network Q against the benchmark network $\mathbf{0}$ that has no interbank linkages:

$$\text{SystemicExp}^{(Q)}_{[0,T]}(\mathbf{A}) = E_0 \left[TL^{(Q)}_{[0,T]} - TL^{(\mathbf{0})}_{[0,T]} \middle| \mathbf{A} \right] \tag{13}$$

where the systemic exposure is defined in terms of a conditioning event of interest, $\mathbf{A}$, and a time period of interest, $[0,T]$. A stress event such as the stock market declining by at least 40% over next six months was, for example, used by Brownlees and Engle (2012) as the conditioning event. Naturally, few would worry about systemic upside risk corresponding to a booming market. The same conditioning event has, for example, been built into the SRISK, which is produced by a regularly updated operational system, pioneered by the Volatility Institute headed by Nobel laureate Robert Engle in Sterns School of Business, New York University, to measure systemic risks of financial institutions.

Next, in order to distinguish the impacts of systematic shocks from idiosyncratic shocks, we define $TL^{(\mathbf{0},*)}_{[0,T]}$, the total loss in the setup where bank assets and liabilities are driven purely by idiosyncratic risk but the total risk of an individual bank remains unchanged. The hypothetic system of interest is:

$$\frac{dV^*_{it}}{V^*_{it_-}} = \mu^*_i dt + \sigma^*_i dW_{it} + \frac{dL^*_{it}}{V^*_{it_-}} \tag{14}$$

$$\frac{dL^*_{it}}{L^*_{it_-}} = \varphi^*_i dt + \nu^*_i dB_{it} \tag{15}$$

where $\mu_i^*, \sigma_i^*, \varphi_i^*, \nu_i^*$ are such that the total volatilities of $\frac{dV_{it}^*}{V_{it\_}^*}$ and $\frac{dL_{it}^*}{L_{it\_}^*}$ are the same as those of the original dynamics where they are subject to systematic shocks characterized by common risk factors.

We are now ready to define the **systematic exposure** where banks have no interbank links:

$$\text{SystematicExp}_{[0,T]}^{(\mathbf{0})}(\mathbf{A}) = E_0 \left[TL_{[0,T]}^{(\mathbf{0})} - TL_{[0,T]}^{(\mathbf{0},*)} \middle| \mathbf{A} \right]. \tag{16}$$

Using the above hypothetical asset-liability system, we can also define the **totally-idiosyncratic exposure**, meaning that this exposure is measured under the assumption that total asset risk is converted into entirely idiosyncratic risk. In other words, the level of individual total asset risk remains unchanged but a bank's assets are no longer correlated with those of other banks.

$$\text{T-IdiosyncraticExp}_{[0,T]}^{(\mathbf{0})}(\mathbf{A}) = E_0 \left[TL_{[0,T]}^{(\mathbf{0},*)} \middle| \mathbf{A} \right]. \tag{17}$$

Since this quantity is measured using the null matrix of interbank links, it basically reflects the system's total exposure when there are no interbank exposures, bank assets are totally uncorrelated with each other, and total volatilities of individual banks' assets remain the same as the true system.

To assess the relative magnitude of the losses and facilitate the comparison across different banking systems, one may normalize the systemic and systematic risk exposures against GDP or aggregate banking capital.

By our definition above, systemic and systematic exposures need not be always positive, and this feature is not too difficult to understand. One would expect systemic exposure to increase with interbank linkages, and rightly so, since interbank exposures may cause some banks to default earlier than otherwise would be. But early default may not actually incur higher losses for two reasons. Firstly, an earlier default could face a higher recovery rate due to our use of a soft insolvency default trigger (i.e., a technically solvent bank can still default) and a fixed fire-sale discount factor (i.e., rising asset values make an early default experiencing a smaller dollar amount of the fire-sale discount). Secondly, with interbank linkages, a default within the banking system causes non-defaulting banks to mark down their assets and liabilities through the close-out netting arrangement, and in a way forces loss realizations that are still properly covered.

Systematic exposure would be negative if a risk factor moves in favor of some banks. If, for example, a bank has a negative correlation with the stock market index while other banks are positively correlated. When the conditioning event is a 40% or more stock market decline over six months, this bank will actually fare better under the market stress scenario, and hence help reduce the banking system's overall losses.

4.2 Systemic fragility

As an analogue to systemic exposure, **systemic fragility** measures the expected proportion of banks which will default, again benchmarking against a banking configuration matrix, **0**.

$$\text{SystemicFra}^{(Q)}_{[0,T]}(\mathbf{A}) = E_0 \left[\left. \frac{\sum_{i=1}^{M} \int_0^T dD_{is}^{(Q)} - \sum_{i=1}^{M} \int_0^T dD_{is}^{(\mathbf{0})}}{M} \right| \mathbf{A} \right]. \tag{18}$$

Likewise, we can define **systematic fragility** as

$$\text{SystematicFra}^{(\mathbf{0})}_{[0,T]}(\mathbf{A}) = E_0 \left[\left. \frac{\sum_{i=1}^{M} \int_0^T dD_{is}^{(\mathbf{0})} - \sum_{i=1}^{M} \int_0^T dD_{is}^{(\mathbf{0},*)}}{M} \right| \mathbf{A} \right] \tag{19}$$

where $D_{is}^{(\mathbf{0},*)}$ is the default indicator process generated by the hypothetical asset-liability processes in equations (14) and (15). Naturally, we also define the **totally-idiosyncratic fragility** as

$$\text{T-IdiosyncraticFra}^{(\mathbf{0})}_{[0,T]}(\mathbf{A}) = E_0 \left[\left. \frac{\sum_{i=1}^{M} \int_0^T dD_{is}^{(\mathbf{0},*)}}{M} \right| \mathbf{A} \right]. \tag{20}$$

Note that systemic fragility measures how pervasive bank failures are expected to be, but it does not actually reveal the severity of trouble for the banking system in terms of uncovered dollar losses. Systemic exposure, on the hand, provides an assessment on the expected aggregate losses to the banking system under stress.

For the same reason stated above, systematic fragility may also take negative values. However, systemic fragility is expected to be non-negative.

5 Dynamic scenario analysis and stress-testing

The two systemic risk measures – systemic exposure and fragility – discussed in the preceding section are based on some conditioning event, $\mathbf{A}$. They in effect measure the resilience of a banking system under a prescribed stress scenario. Thus, computing these measures is a kind of stress testing much like what financial regulators normally conduct. Stress scenarios used in macro stress testing are usually defined in terms of GDP, unemployment rate, etc., and are used to conduct "what-if" scenario analysis. In this paper, however, we will focus on stress scenarios directly defined in terms of the common risk factors that define the asset and liability dynamics. For example, we follow Brownlees and Engle (2012) to consider the stress event that the stock market declines by 40% or more over next six months. Since common risk factors are expected to be correlated, we must ensure our scenario analysis dynamically consistent among all common risk factors, which is accomplished by devising a novel bridge-sampling procedure.

Our dynamic model allows stochastic variables to evolve according to the prescribed system dynamics, taking into account the conditioning event. In this paper, we specify three market-wide risk factors – interest rate term spread, equity market index and exchange rate. We would like to condition a particular common risk factor to a prescribed shock over time, and evolve other risk factors in a dynamically consistent way. For example, consider an event where the stock market index falls at least 40% over a six-month period. Firstly, the fall is not static, which takes place over time in a random fashion. Secondly, the term spread and exchange rate, being correlated processes, must also move in a correlated manner. Over many simulations, a wide variety of scenarios are allowed to play out. This may capture important scenarios such as a confluence of events to which the system is particularly vulnerable to. Our implementation makes use of a Gaussian bridge sampler. The technical details for this bridge sampling implementation are provided in Appendix B.

We assume that the interest rate term spread is governed by an Ornstein-Uhlenbeck process, an assumption made popular by the term structure model of Vacicek (1977). In fact, the Vasicek (1977) model implies that the term spread will also follow an Ornstein-Uhlenbeck process with suitable changes to its parameters. The term spread dynamics is assumed to be

$$dR_t = \kappa(\bar{R} - R_t)dt + \eta_R dW_{Rt} \tag{21}$$

where κ is the mean reverting speed, $\bar{R}$ is the long run average term spread and η is the term spread volatility. We choose term spread instead of interest rate as one common risk factor because term spread is likely to have more influence on bank's assets and liabilities. Banking operations have much to do with maturity transformation from short-term deposits to longer-term loans, and term spread should therefore serve as a more direct driver in bank's performance. Our British banking sample seems to confirm this observation.

Other market risk factors may also affect bank's asset/liability dynamics. The other two common risk factors considered in this paper are stock market index (I_t) and exchange rate (e_t). We use the geometric Brownian motion to model both; that is,

$$d\ln I_t = \delta_I dt + \eta_I dW_{It} \tag{22}$$

$$d\ln e_t = \delta_e dt + \eta_e dW_{et} \tag{23}$$

where W_{It} and W_{et} are correlated with ρ_{Ie} being the correlation coefficient. The correlation coefficients between W_{It} and W_{Rt} is denoted by ρ_{IR}. Similarly, ρ_{eR} denotes the correlation coefficient between W_{et} and W_{Rt}. Although the three common risk factors are the same for the asset and liability dynamics, they can be effectively different by setting some factor loadings to zero.

6 An example of the UK banking network

We choose to demonstrate the model using the UK banking system for several reasons. Firstly, we are able to obtain its interbank exposures distribution. Secondly, the impact of the recent credit crisis on the UK financial system was relatively severe. The UK government implemented bailout packages in 2008 and 2009, and some UK banks had since been consolidated. Our systemic risk measures should deliver a clearer message using this banking system. Lastly, bilateral netting is permitted under English law, and an quantification of its benefit is in order.

6.1 Implementation details

We compute systemic (and systematic) exposure and fragility quarterly from Q1 of 2004 to Q4 of 2012. Our implementation uses daily data from a one-year moving window to estimate the models for the common risk factors and the UK bank asset-liability dynamics. Unless otherwise stated, we assess systematic and systemic risk over a time horizon of six months and treat one year as having 250 time points (mimicking 250 trading days). Expectations are computed with 500,000 simulated sample paths.

Mitchell and Pulvino (2011) found that certain assets traded at roughly 10% discount to their fundamental values during the crisis.[5] We thus adopt a fixed fire-sale discount factor of 0.9 for all banks.

6.1.1 Interbank exposures and banks' asset-liability dynamics

We consider a network of 15 UK banks, which includes all major ones. The distribution of interbank exposures was provided by K. Anand and P. Gai, and the same data were used in Anand, *et al* (2013). The summary statistics of one bank's fractional claim on another bank's liabilities are provided in Table 1. As the identities are not known to us, we randomly assigned each bank to a particular (unknown but unique) counterparty in the data. This assignment is done once so that it is time consistent for each quarter where the systemic risk is being computed.

Table 1: Summary statistics of interbank claims

	Mean	Median	StDev	Min	Max
To an individual bank	0.00376	0.0000463	0.00958	0	0.0579
Total claims of a bank	0.05258	0.0036960	0.07436	0	0.1788

In actual production, the network may be extended to a wider setting to include more firms. However, in the UK alone, there are roughly 300 listed financial firms and it is necessary to select a suitable subset that can adequately capture the risk. In this respect, Duffie (2011) described a "10-by-10-by-10" approach which provides some interesting thoughts on identifying important financial firms.

It is important to use the market value of assets, rather than book value, for assessing bank default. Our calibration of bank asset dynamics is also based on market value. Daily market values of assets for each bank are obtained from the Credit Research Initiative (CRI) database administered by the Risk Management Institute (RMI), National University of Singapore. The underlying method for estimating the market value of assets is based on the maximum likelihood estimation (MLE) method proposed in Duan (1994, 2000) with modifications to accommodate

[5] The 10% discount was measured with respect to convertible debentures. We are not aware of any other more appropriate fire sale discount factor for bank assets.

financial firms as described in Duan and Wang (2012). This MLE method has the advantage of factoring in a suitable fraction of other liabilities (total liabilities minus short-term and long-term debts) into the parameter estimation.[6] This is particularly important for financial firms such as banks because they typically have sizeable customer deposits that are classified as neither short-term nor long-term debt.

For the specific operational/credit risk, we use a common set of parameters for all banks, which is based on broad approximates using Basel's Results from the 2008 Loss Data Collection Exercise for Operational Risk. The frequency of operational event is 8.9 per year, with each event incurring an average loss of £32 and standard deviation of £250 per million of bank assets. Although this operational/credit risk has been incorporated into our analysis, its effect is expectedly negligible as compared to the impact of other factors.

Recall that we defined ξ_{it} such that $P(\xi_{it} = 1|\xi_{it-} = 0) = \frac{e^{\alpha_0+\alpha_1 h(V_{it},L_{it})}}{1+e^{\alpha_0+\alpha_1 h(V_{it},L_{it})}}$, which is a logistic function that relates a bank's likelihood of default to its level of assets and liabilities. In this example, a DTD measure is appropriate, given its statistically significant relationship with actual defaults. Using the default history and DTD of *all* financial firms in Europe, our calibration using monthly data (2004-2012) obtained from RMI yields $\alpha_0 = -7.323$ and $\alpha_1 = -1.355$, both are strongly significant. This resulting default triggering function is applied to all simulations. In order to keep track of DTDs during simulation, we initialize each bank with its actual DTD at the start of each period where systemic risk is being measured, and update DTD as market value of assets and liabilities change along each simulated sample path.

Note that there are two concepts of liabilities being used.[7] The first pertains to the total liabilities of the bank, as stated on its book. This is used for computing losses in the event of a default, since it forces a resolution of all its liabilities. The second concept relates to "adjusted liabilities", commonly known as "default point". This is used in DTD and assessing the likelihood of default. As discussed above, the "adjusted liabilities" account more appropriately the role of long-term and "other liabilities" in the default trigger. In short, our combined use is a simplified way for dealing with complex liabilities of firms.

For this simulation, we do not dynamically evolve liabilities in response to common risk factors over the course of six months, except when there is a default. When there is a default, adjusted liabilities are assumed to be affected by the same proportion as book liabilities.

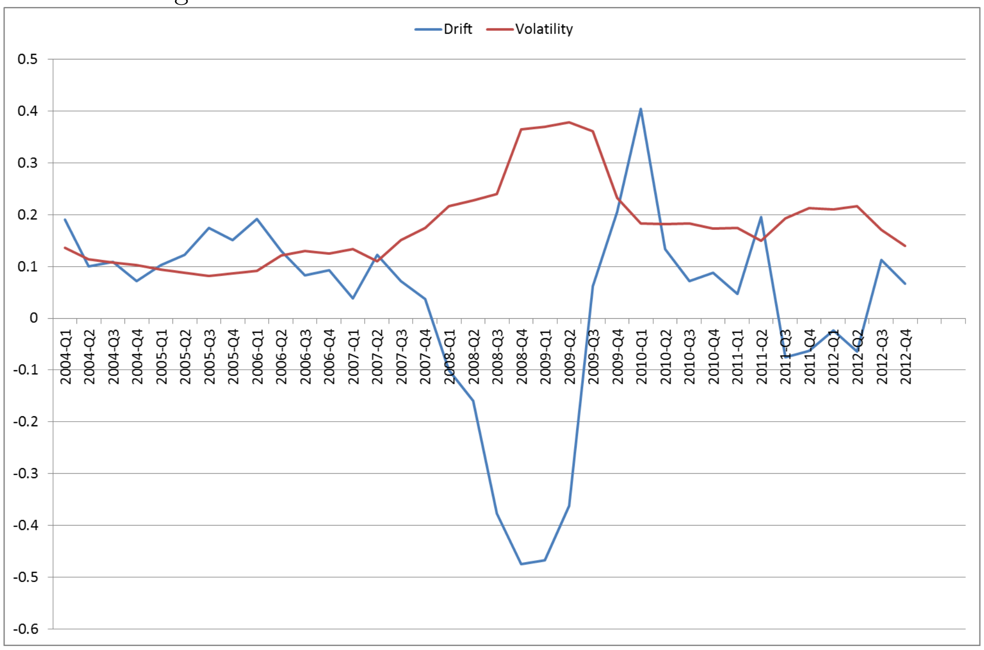
Where unavailable, "adjusted liabilities" had been supplemented by book values in the financial statements. Where it is also missing, we use the closest year to initiate the asset-liability dynamics[8].

[6] Alternative methods such as the well-known KMV methodology do not explicitly treat other liabilities which can be problematic in application to financial firms.

[7] In some earlier versions of this paper, we used adjusted liabilities for both default trigger and settlement, which resulted in lower systemic risk exposures.

[8] This only occurs occasionally for the small/unlisted banks.

Figure 1: Parameter estimates for the FTSE 100 index



6.1.2 Common risk factors

The dynamic common risk factors are the daily UK FTSE 100 index, trade-weighted British pound spot exchange index provided by Deutsche Bank, term spread between the 12-month and 1-month GBP LIBOR. The model for these three common risk factors have been described earlier. In addition, we add a common latent factor that does not have any time series dynamic. We add this latent factor to better reflect the degree of commonality (systematic risk) among British banks. This common latent factor is identified by applying a principal component analysis to the residuals of the bank asset returns after taking out the effect of the three dynamic common risk factors. For this analysis, we do not include common operational/credit events for the lack of comprehensive data on this.

In general, bank assets respond positively to increases in the UK FTSE index and negatively to a strengthening of the GBP. The response to interest rate term spread depends on the period in question. We provide detailed summary statistics of the estimates for the banks' factor loadings in Table 2.

The parameters of the common factor dynamics for each year in the 2004-2012 period are presented in Figures 1-3, and their numerical values are given in Table 4. The UK FTSE 100 index was generally drifting upwards from 2004 to 2007, but experienced downward drifts and high volatility over 2008 to 2009, during the credit crisis. A similiar trend can be observed for the GBP exchange rate. The GBP LIBOR interest rate term spread does not exhibit significant mean reversion in the full sample, much less for the shorter time window of one year. For the purpose of

Figure 2: Parameter estimates for the TWI GBP index

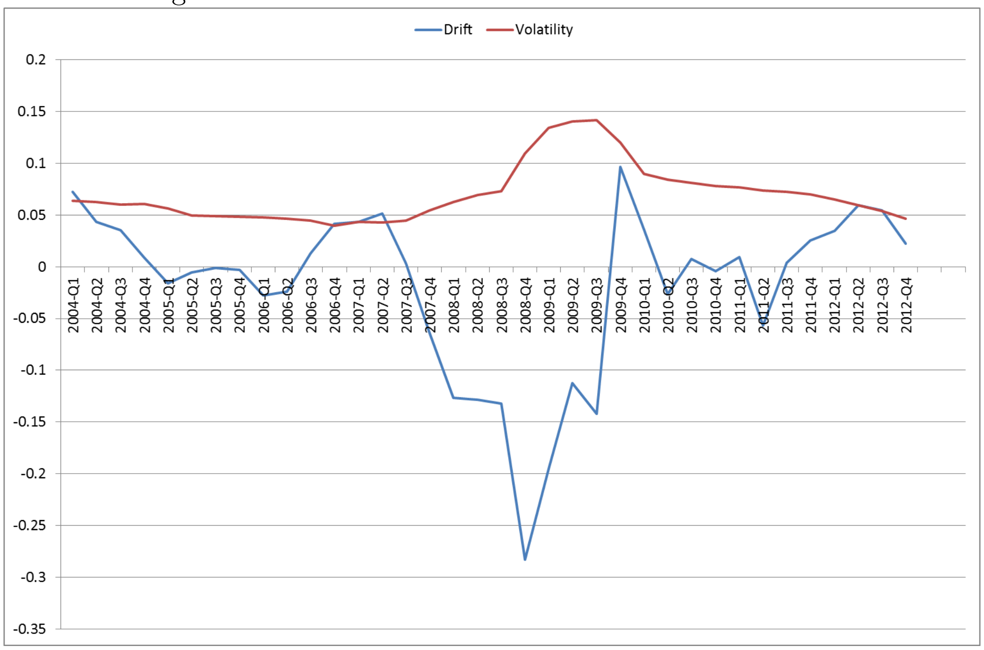
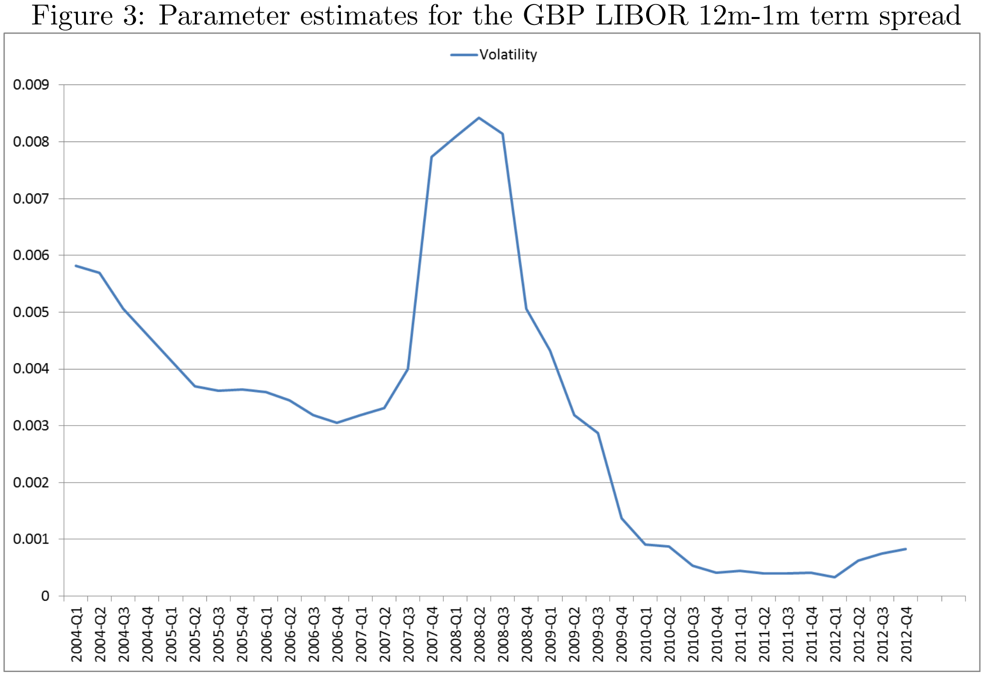


Figure 3: Parameter estimates for the GBP LIBOR 12m-1m term spread



our empirical analysis, it is treated as a unit root process with no mean reversion, i.e. $\kappa = 0$. If so desired, more refined modelling of the term spread is possible.

As the above three factors are not an exhaustive prescription of systematic risk, a common latent factor was included to capture the remaining effects. The R^2 of the bank assets against the three market risk factors alone yield an R^2 of 6% to 30%, while inclusion of the latent factor captured 24% to 70% of the variation in terms of R^2. Detailed summary statistics of the R^2 are provided in Table 5.

This latent factor is generated in a manner such that it is orthogonal to all dynamic common risk factors, and captures the maximum remaining common variation possible. Firstly, all UK banks are first subject to projection onto the three common dynamic risk factors. Then, the residuals of all banks are taken together to extract one principal component. We assumed that the latent factor followed geometric Brownian motion.

If, in any given year, the bank had insufficient data to compute its factor loadings, we used the average of all other banks with available data.

6.1.3 Conditioning event

The conditioning event in our implementation is that the UK stock market falls by at least 40% over a period of six months, similar to that prescribed in Brownlees and Engle (2012). Historically, a 40% drop in the FTSE 100 index did occur over the 2008-2009 period. It should be noted that our method is flexible enough to accommodate other conditioning events.

6.2 Time profile of total losses due to interbank linkages

Figure 4 shows the expected proportion of bank defaults (subject to the conditioning event) over the 2004-2012 period, broken down in systemic fragility and other components. Likewise, Figure 5 below shows our estimate of expected total uncovered losses broken down to systemic exposure and other components.[9]

Our measurement is responsive to the period of actual crisis in 2008-2009, where the overall risk spiked. It was at this point where systematic risk was high, accompanied by significant systemic risk. In periods of low systematic risk, systemic risk was low accordingly.[10] We will explore the driving relationship between systematic risk and systemic risk more in a later section.

The overall risk abated partially in late 2009 and more significantly thereafter in 2010. A possible factor for this could be consolidation of banks' capital structure through a series of mergers.

[9] These results are different from earlier versions of this paper. We had previously computed losses based on "adjusted liabilities" instead of total book liabilities. This issue was discussed in Section 6.1.1.

[10] Systematic fragility and exposure were slightly negative in 2005-Q4. One particular bank's (Northern Rock) assets were negatively correlated with the FTSE 100 Index, meaning that a 40% down in the FTSE 100 Index would drive its asset value up and hence avoid default.

Figure 4: Time profile of systemic fragility

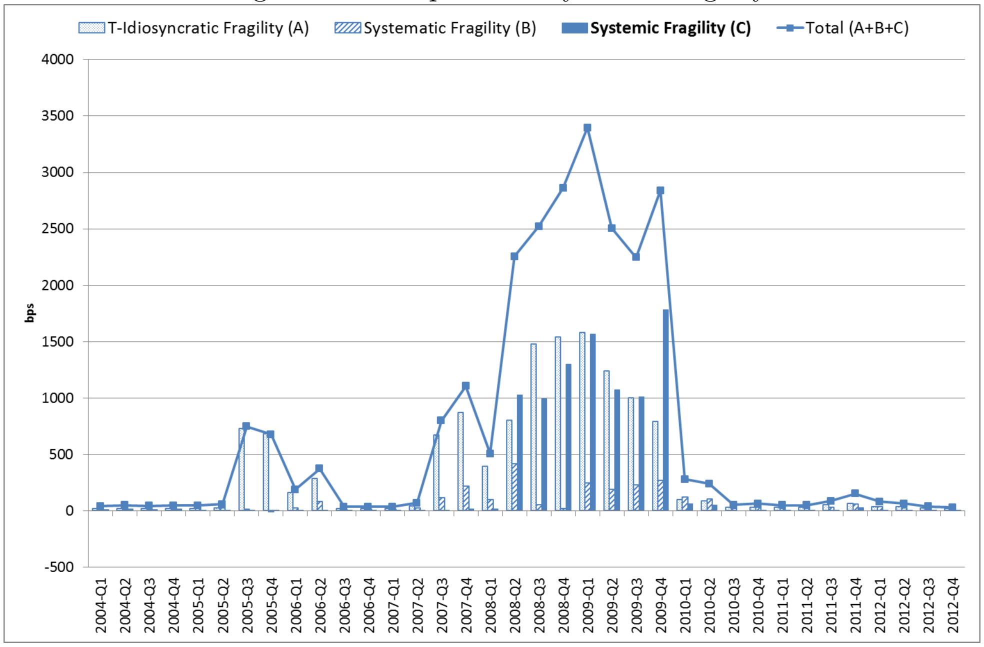
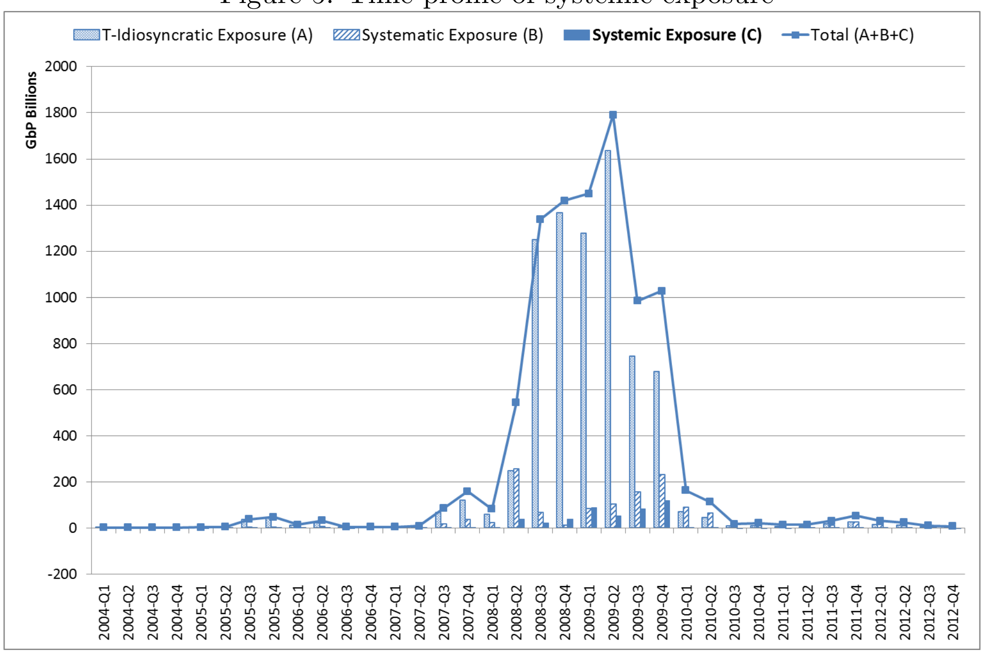


Figure 5: Time profile of systemic exposure



Banco Santander acquired both Bradford-Bingley and Alliance-Leicester, and HBOS-Lloyds and Britannia-Cooperative also merged. In addition, the UK government announced a bank rescue package.[11] Together, these actions could have improved the general capital standing of the banks.[12]

6.3 Dynamic common risk factors and systematic risk

Systematic risk (exposure or fragility) is directly and positively related with the market-wide common risk factors. This is intuitive: if the shocks are larger, banks' balance sheets suffer more from the impact, and their likelihood of default increases. To demonstrate this effect, we use the year ending 2009-Q4 as the basis for our comparison. We prescribe a range of conditional shocks on the equity index, of up to 40%. Figures 6 and 7 provide the result.

In the unconditional case, no shock is prescribed to the system, and the index is free to move up and down according to the system's dynamics. Hence, systematic risk (exposure or fragility) is almost zero. While banks may be correlated in their movements with the equity index, this does not translate to systematic risk per se, since the market may move up or down. In fact, if the market does sufficiently well, and bank assets are all positively correlated with the market, we would expect "negative systematic risk". Hence, a sufficiently large negative shock needs to be present in order to have a meaningful analysis of systematic risk.

When we restrict the event to a fall in index, that is a shock of at least 0%, the systematic effect begins to become apparent, and it grows as we strengthen the shock up to -40% of the equity index. Since bank assets are positively correlated with the equity index, they all fall together with it. A large fall decreases DTD significantly, leading to more defaults and higher systematic risk (exposure or fragility).

6.4 Systematic risk as a driver of systemic risk

Systematic risk should be thought of as a driver for systemic risk. Without common movements in banks' assets, it is difficult to contemplate multiple banks going into default at the same time. When systematic risk is zero, we expect systemic risk to also be close to zero. It is unlikely that idiosyncratic risk can cause a huge systemic effect.

Again using the year ending 2009-Q4 as the basis for our comparison, we analyze the sensitivity of systemic risk to the level of systematic vis-a-vis idiosyncratic risk. To conduct this analysis, we first determine the total variability in assets for each bank based on equation (1). The bank coefficients to the market risk factors and the Brownian motion are then scaled in a manner that attributes the desired proportion of risk to idiosyncratic and systematic components. We then subject the system to the same 40% down stress event and analyze the systemic effect.

[11] £500 billion bank rescue package, of which £50 billion comprised state investments in banks.

[12] Note that the effect of recapitalization and merger cannot be immediately reflected in our results, because the resulting balance sheet does not immediately become available. For quarterly (semiannual) release of financial statements, the delay can be up to three (six) months. For UK banks, most of the balance sheet data are based on semiannual statements. Quarterly financial statements are available for some banks in the later periods.

Figure 6: Stock market shock and impact on systematic fragility

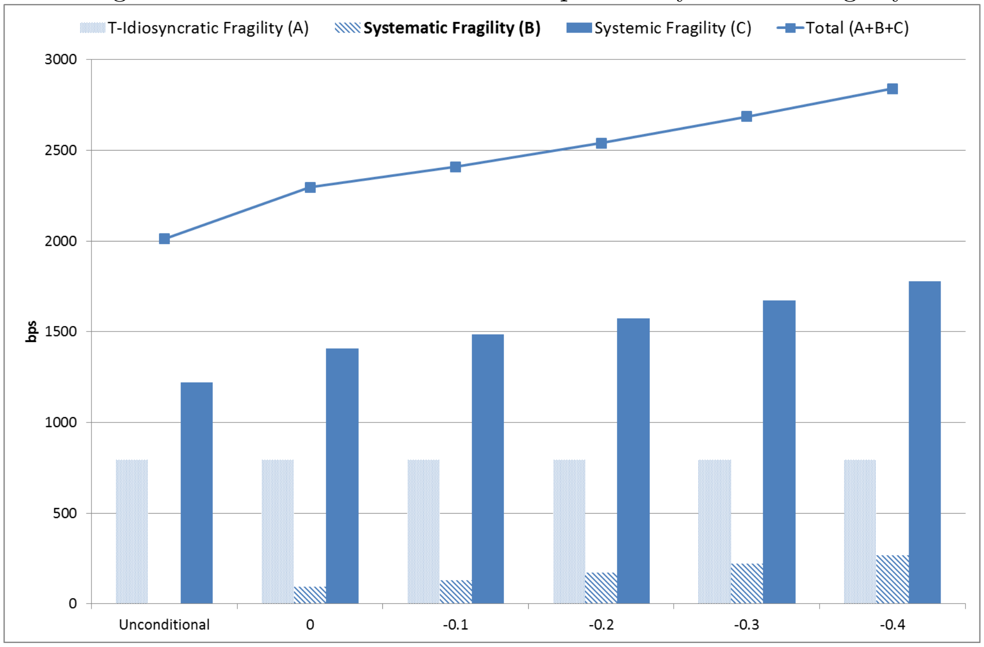


Figure 7: Stock market shock and impact on systematic exposure

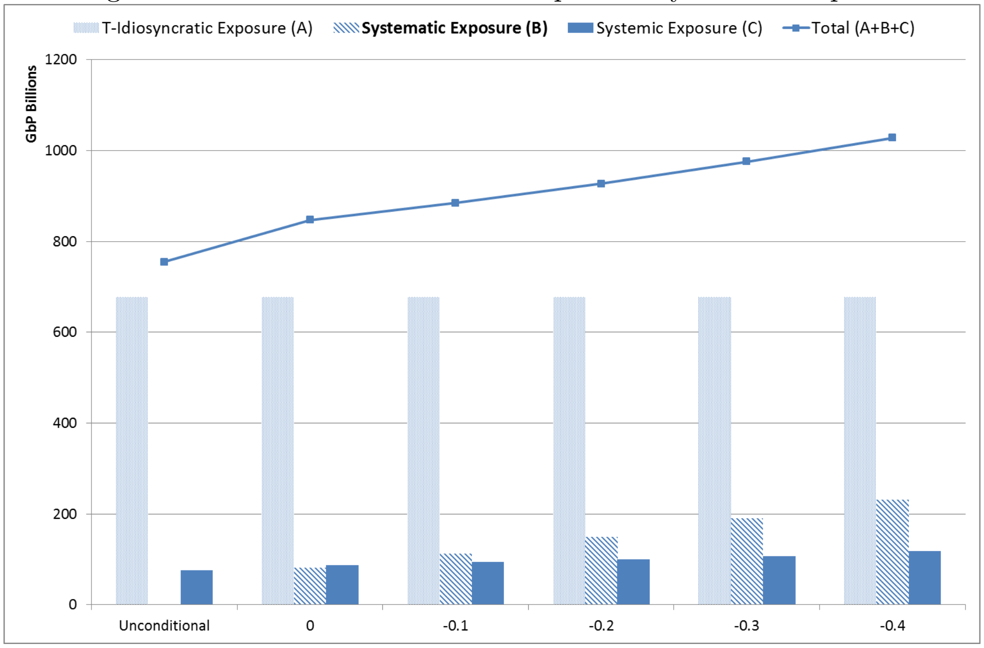


Figure 8: Systematic Risk as a Driver for Systemic Fragility

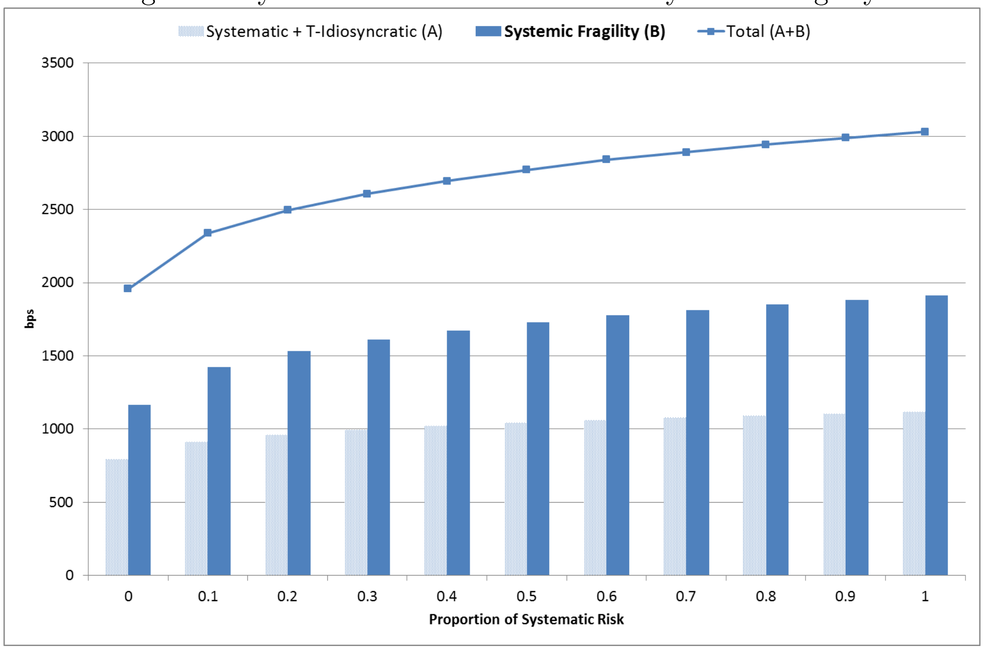


Figure 9: Systematic Risk as a Driver for Systemic Exposure

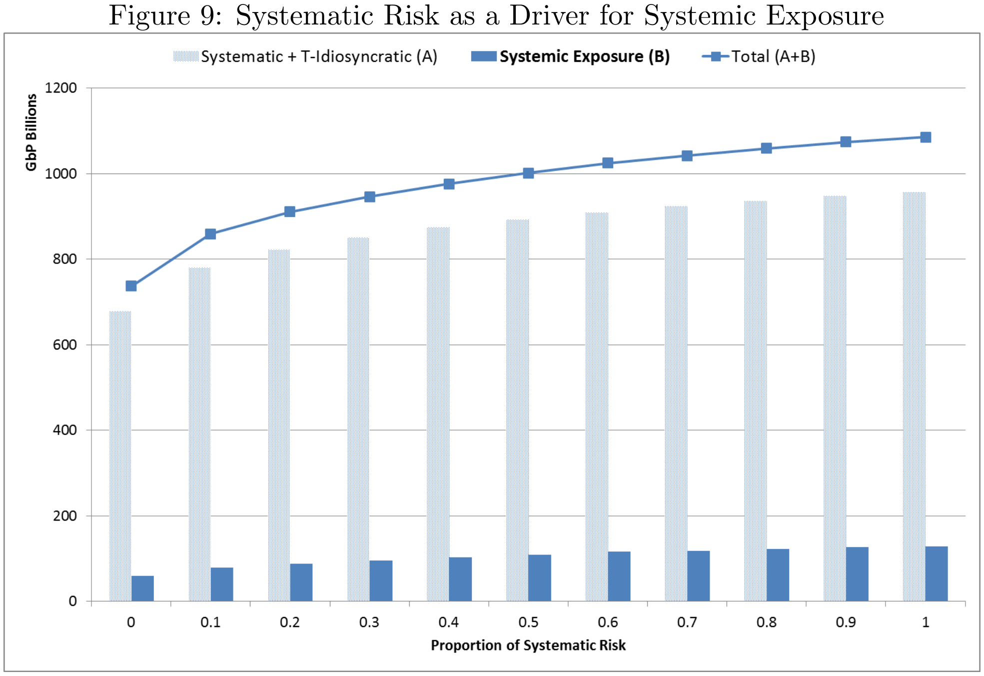


Figure 10: Effect of bilateral netting on systemic fragility

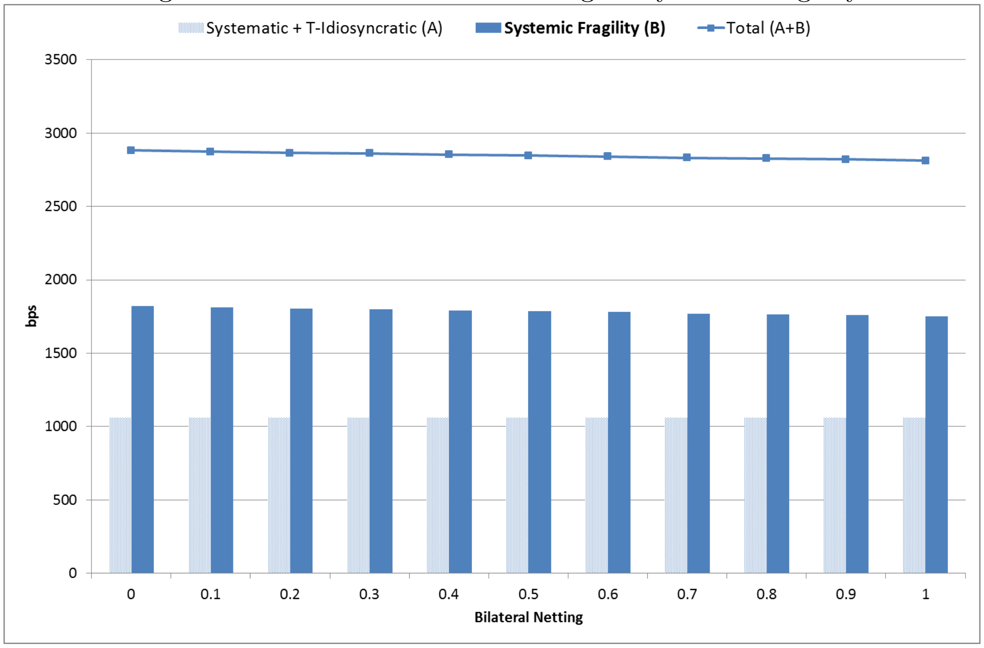


Figure 8 shows the systemic fragility in relation to the proportion of systematic risk present in the system (the total risk remains unchanged). At the point where bank assets respond only to idiosyncratic risk, systemic fragility is relatively low. As we introduce systematic effects into the system, banks become vulnerable simultaneously, and the risk of cascading defaults rises, as reflected in higher systemic fragility. This confirms our understanding that in principle, idiosyncratic shocks may trigger cascading defaults, but shocks to systematic risk factors, as opposed to banks' idiosyncratic elements, are more likely to drive cascading defaults and hence to cause higher systemic risk. The same conclusion holds true for systemic exposure as reflected in Figure 9.

6.5 Bilateral netting

In this section, we assess the effect of bilateral netting on systemic risk. Figures 10 and 11 show the systemic fragility and systemic exposure at different levels of netting. Based on the interbank exposure data available to us, interbank netting does not reduce systemic risk significantly. Both systemic fragility and systemic exposure are fairly invariant to the level of netting assumed. However and as expected, interbank losses shown in Figure 12 exhibited a decreasing relationship with the netting level.

We limit our conclusion of the weak effect of bilateral netting to the on-balance sheet interbank exposures between 2004-2007, which were made available to us. If off-balance sheet data were accessible, and prove to be large, the findings may be different.

Figure 11: Effect of bilateral netting on systemic exposure

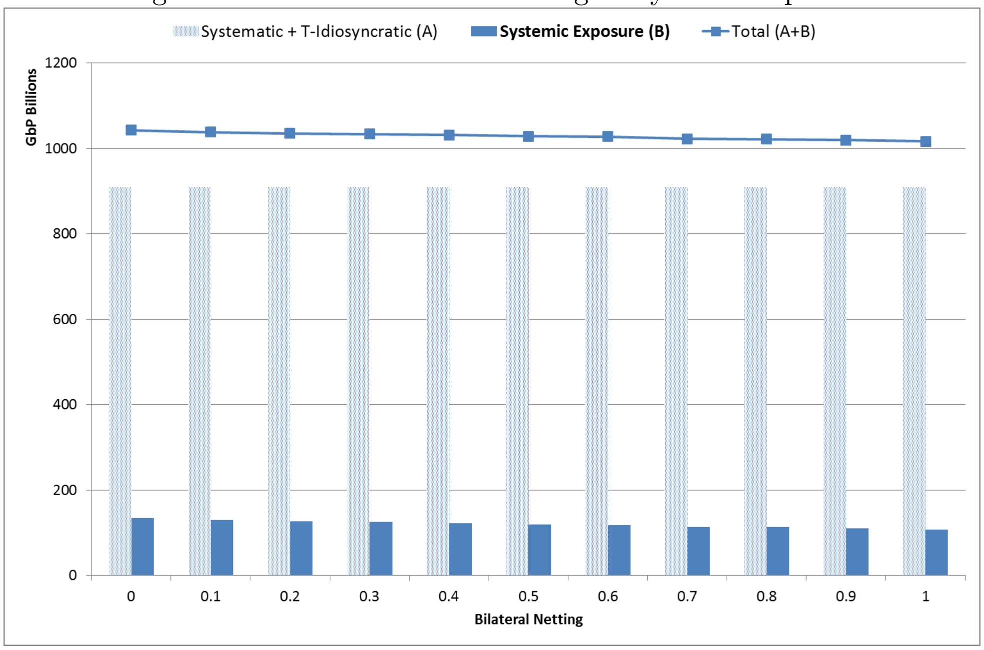
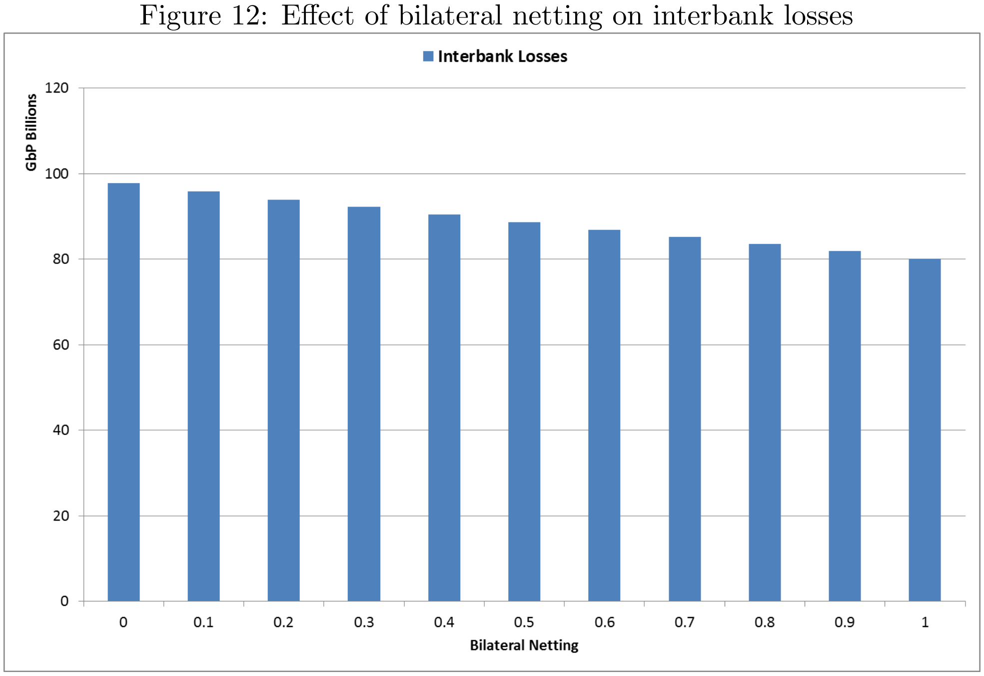


Figure 12: Effect of bilateral netting on interbank losses



7 Determining SIFIs

The 2008-09 global financial crisis has put a spotlight on financial institutions which are "too-big-to-fail" or "too-connected-to-fail", i.e., their failures might cause severe disruption to the overall financial system. New regulations proposed by authorities worldwide are in part aimed at controlling the magnitude of this problem. A key challenge has been to identify systemically important financial institutions (SIFIs). In this section, we present a methodology for ranking banks in terms of their marginal contributions to the overall systemic risk.

The FSB describes SIFIs as "financial institutions whose distress or disorderly failure, because of their size, complexity and systemic interconnectedness, would cause significant disruption to the wider financial system and economic activity." Our model is particularly suited for ranking banks by systemic importance, as it takes into account several important factors. Firstly, "systemic interconnectedness" is captured by our banking configuration matrix Q, which describes the extent of exposures between banks. A large bank with significant levels of interbank exposures would be more likely to cause a cascade of defaults and disrupt the financial system, thereby leading to a high systemic fragility and systemic exposure. Secondly, the size of a bank and its ability to absorb losses are captured by the market value of assets V_{it} and liabilities L_{it}. Indeed, the FSB has identified loss absorption capacity as a means to combat "too-big-to-fail" and banks' capital requirements have been increased under Basel III.

Importantly, our model is informative on the impact of a failure, and not only the risk of a failure. We note that the Basel Committee "is of the view that global systemic importance should be measured in terms of the impact that a failure of a bank can have on the global financial system and wider economy rather than the risk that a failure can occur."

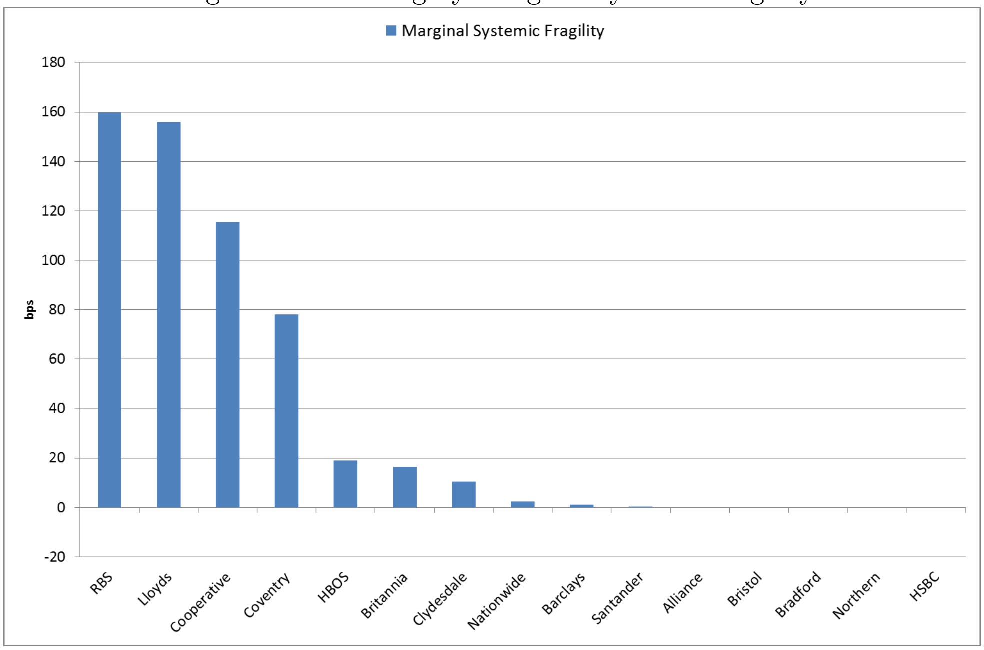
To aid in ranking systemically important banks, we introduce the concept of **marginal systemic fragility** and **marginal systemic exposure**. For Bank i, let Q_{-i} represents the banking configuration matrix Q with its i^{th} row and column replaced by a vector of zeros, i.e. Bank i along has no interbank exposures. The marginal systemic fragility and exposure for Bank i are defined as:

$$\text{MargSystemicFra}_{[0,T]}^{(Q,i)}(\mathbf{A}) = E_0\left[\left.\frac{\sum_{i=1}^{M}\int_0^T dD_{is}^{(Q)} - \sum_{i=1}^{M}\int_0^T dD_{is}^{(Q_{-i})}}{M}\right| \mathbf{A}\right]. \tag{24}$$

$$\text{MargSystemicExp}_{[0,T]}^{(Q,i)}(\mathbf{A}) = E_0\left[\left. TL_{[0,T]}^{(Q)} - TL_{[0,T]}^{(Q_{-i})}\right| \mathbf{A}\right] \tag{25}$$

The marginal systemic fragility represents the increase in expected bank default rate as a result of Bank i's interbank exposures, all others being held constant. The marginal systemic fragility of a particular bank should be assessed and ranked in relation to that of all other banks. If a bank introduces a significant level of systemic fragility to the network as compared to other banks, it should be deemed to be systemically important. The marginal systemic exposure provides a complementary measure in this regard, representing the total uncovered losses introduced by Bank i, instead of expected rate.

Figure 13: Ranking by marginal systemic fragility



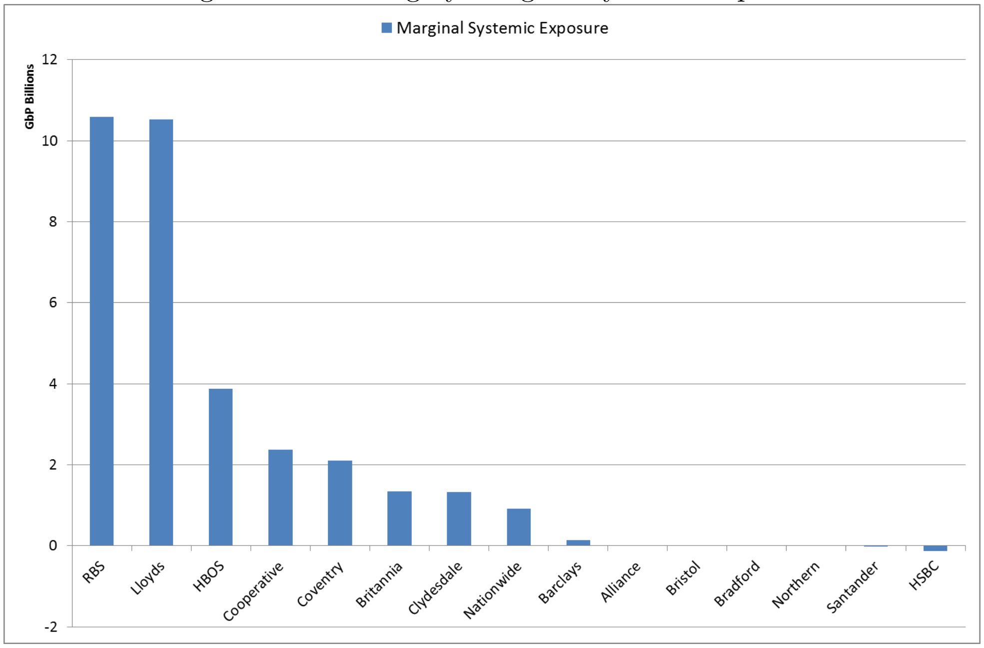
Figures (13) and (14) provide an example of rankings by systemic importance based on marginal systemic fragility and marginal systemic exposure, respectively. The figures present the results in Q4 of 2009 for the 11 banks (out of the original sample of 15 banks) still standing at the time. We emphasize that this is merely an example, the goal here is to demonstrate the methodology. As we do not know the identities of individual interbank exposures for our sample of 11 British banks then, we have simply applied the average interbank exposure for all banks in these graphs. In other words, the results in these figures reflect more of the characteristics of banks' balance sheets and the risk profiles of their assets. Use of actual interbank exposure data, which naturally reside with banking supervisors and central banks, will no doubt make the results more informative and relevant.

8 Conclusion

We introduce a structural model by which systemic, systematic and idiosyncratic risks may be separately identified and measured. Formally distinguishing the first two beyond just concepts and providing with explicit risk measurements are to our knowledge the first in the literature.

Two useful measures are proposed: systemic exposure and systemic fragility. These measures reflect the expected losses due to interbank linkages, and the latter measures the pervasiveness of bank defaults, conditional upon an appropriate stress event. Both of which are highly tied to the presence of systematic risk reflected in common dynamic risk factors. A novel bridge sampling

Figure 14: Ranking by marginal systemic exposure



technique is specifically designed for computing these two systemic measures by simulating the common risk factors under stress.

Through our explicit structural model of cascading defaults, we characterize systematic risk as the driver of systemic risk. In a banking system where there are no systematic effects, systemic risk is extremely low, because idiosyncratic risks are not able to drive multiple defaults. However, when systematic risks are introduced, banks suffer concurrently from a market impact and the risk of cascading defaults increases.

Under benign conditions, the systematic risk of the UK banking system empirically studied in this paper is close to zero, but increases in magnitude together with the stress in the UK stock market. Using actual data on a network of 15 UK banks over the period of 2004-2012, we find that our two measures of systemic risk are responsive to actual periods of the crisis around 2008.

Finally, we define marginal systemic fragility and marginal exposure, which measure the increase in potential disruption that a particular bank brings to the overall financial system. We expect both measures to be informative for identifying systemically important banks.

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Appendix A: Cascading defaults – a numerical example

We use a numerical example to demonstrate cascading defaults with and without bilateral netting. For this example, we use a 5-bank network configuration matrix as follows:

$$Q = \begin{bmatrix} 0 & 0.15 & 0.15 & 0.10 & 0.10 \\ 0.08 & 0 & 0.12 & 0.20 & 0.10 \\ 0.18 & 0.02 & 0 & 0.15 & 0.15 \\ 0.05 & 0.15 & 0.12 & 0 & 0.18 \\ 0.20 & 0.10 & 0.20 & 0 & 0 \end{bmatrix}. \tag{26}$$

According to the above network configuration matrix, 50% (sum of the first row) of Bank 1's total liabilities is owed to its interbank counterparties. Similar calculations apply to other banks. The above matrix is made to have the same row sum of 50% but to differ in their distributions over banks. Further assume

$$L_t = \begin{bmatrix} 200 & 0 & 0 & 0 & 0 \\ 0 & 200 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 300 & 0 \\ 0 & 0 & 0 & 0 & 100 \end{bmatrix}. \tag{27}$$

Thus,

$$\Pi_t = \begin{bmatrix} 0 & 16 & 18 & 15 & 20 \\ 30 & 0 & 2 & 45 & 10 \\ 30 & 24 & 0 & 36 & 20 \\ 20 & 40 & 15 & 0 & 0 \\ 20 & 20 & 15 & 54 & 0 \end{bmatrix}. \tag{28}$$

The above matrix implies that Bank 1 has a nominal claim against Bank 2 in the amount of \$16 while at the same has a nominal liability of \$30 to Bank 2.

The bankruptcy cost factor is assumed to be 0.8 for all banks. We assume that a bank default occurs with the following asset and liability values:

	Bank 1	Bank 2	Bank 3	Bank 4	Bank 5
Asset value:	225	195	120	305	115
Liability value:	200	200	100	300	100
Default indicator:	0	1	0	0	0

Note that the assumed liability values are the same as the ones used in obtaining equation (28).

To present a simpler illustration in this example, we will use a hard insolvency default trigger, i.e., a bank defaults when its asset value falls below the level of its nominal liabilities. Elsewhere in this paper, the soft insolvency default trigger defined by a logistic function described in section 3.2 is used.

Since Bank 2's asset value falls below its liabilities, it has to default. Bank 2's default hence causes all other banks to revise their asset and liability values.

A1. Cascading defaults without bilateral netting

First iteration:

	Bank 1	Bank 2	Bank 3	Bank 4	Bank 5
Asset value:	191.48	195	112.72	251.2	100.6
Liability value:	170	200	98	255	90
Default indicator:	0	1	0	1	0

Bank 1 revises its liability down by 30 because $\pi_{21} = 30$ as specified in equation (11). Note that ψ is set to zero because of no netting, and thus $\omega_{ij} = \pi_{ij}$. Its asset value goes down by 33.52 because $\pi_{21} + \pi_{12}\left(1 - \frac{\phi_2 V_2}{L_2}\right) = 30 + 16 \times \left(1 - \frac{0.8\times195}{200}\right) = 33.52$ as specified in equation (10). Similar calculations apply to Banks 3-5. As shown in the table, the asset and liability revisions due to Bank 2's default triggers Bank 4's default, which calls for further asset and liability revisions.

Second iteration:

	Bank 1	Bank 2	Bank 3	Bank 4	Bank 5
Asset value:	168.30	195	90.09	251.2	89.16
Liability value:	150	200	83	255	90
Default indicator:	0	1	0	1	1

Bank 4's default forces Bank 1's liability downward by 20 because $\pi_{41} = 20$. Bank 1's asset value goes down by 23.18 because $\pi_{41} + \pi_{14}\left(1 - \frac{\phi_4 V_4}{L_4}\right) = 20 + 15 \times \left(1 - \frac{0.8\times251.2}{255}\right) = 23.18$. Similar calculations apply to Banks 3 and 5. This time, the asset and liability revisions due to Bank 4's default triggers Bank 5's default, which calls for further asset and liability revisions.

Third iteration:

	Bank 1	Bank 2	Bank 3	Bank 4	Bank 5
Asset value:	144.15	195	70.94	251.2	89.16
Liability value:	130	200	68	255	90
Default indicator:	0	1	0	1	1

Bank 5's default forces Bank 1 to again revise its liability downward by 20 because $\pi_{51} = 20$. Bank 1's asset value needs to go down by 24.15 because $\pi_{51} + \pi_{15}\left(1 - \frac{\phi_5 V_5}{L_5}\right) = 20 + 20 \times \left(1 - \frac{0.8\times89.16}{90}\right) = 24.15$. Similar calculations apply to Bank 3. In this case, the asset and liability revisions due to Bank 5's default stop triggering any further default. Thus, the fixed point has been reached.

A2. Cascading defaults with full bilateral netting

Under full bilateral netting (i.e., $\psi = 1$), the fact that Bank 2 has defaulted brings us to that situation that Bank 2 has no obligation to Bank 1 but has claim against Bank 1 in the amount of \$14. Bank 2, however, owes Bank 3 a net amount of 22 for which Bank 3 can only hope for a partial recovery. Similar reasoning applies to other banks. Full netting forces Bank 2's assets and liability to go down prior to settling other debts. The amount equals 68 (i.e., $16 + 2 + 40 + 10 = 68$). The net exposures that other banks face are in column 2 of the following matrix:

$$\Omega_t = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 14 & 0 & 0 & 5 & 0 \\ 12 & 22 & 0 & 21 & 5 \\ 5 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 54 & 0 \end{bmatrix}. \tag{29}$$

First iteration:

	Bank 1	Bank 2	Bank 3	Bank 4	Bank 5
Asset value:	195	127	112.93	260	102.7
Liability value:	170	132	98	255	90
Default indicator:	0	1	0	0	0

Bank 1 revises its liability down by 30 because $\pi_{21} = 30$ as specified in equation (11). Its asset value goes down by 30 because $\pi_{21} + \omega_{12}\left(1 - \frac{\phi_2 \hat{V}_2}{\hat{L}_2}\right) = 30 + 0 \times \left(1 - \frac{0.8 \times 127}{132}\right) = 30$ as specified in equation (10). Similar calculations apply to Banks 3-5. Clearly, the asset and liability revisions due to Bank 2's default have not triggered further defaults. The fixed point is reached after one iteration. In contrast to the case without bilateral netting, cascading defaults do not take place in this particular case.

Appendix B: Event-based conditional sampling

The conditional sampling schemes for the three common dynamic risk factors – term spread, equity market index and exchange rate – are based on Gaussain bridge sampling and the specifics are described in B1 and B2. Our bridge sampling scheme is based on the Euler approximation of the three common risk factors' dynamics. Note that we are actually able to derive the exact discrete-time distribution for all three risk factors and thus perform the bridge sampling without the Euler approximation. Using the approximated version is to be consistent with our sampling procedure for the asset and volatility dynamics. For daily sampling (250 trading days a year) employed in this paper, sampling with or without the Euler approximation makes no material difference.

B1. Conditioning on an term spread event

Since the term spread process is Gaussian, we only need to know three quantities for constructing a bridge sampler.

$$\begin{aligned}
E_t(R_{t+k\Delta t}) &= \bar{R}\left[1-(1-\kappa\Delta t)^k\right]+(1-\kappa\Delta t)^k R_t \\
Var_t(R_{t+k\Delta t}) &= \frac{\eta^2\Delta t\left[1-(1-\kappa\Delta t)^{2k}\right]}{1-(1-\kappa\Delta t)^2} \\
Cov_t(R_{t+k\Delta t}, R_{t+(k+m)\Delta t}) &= \frac{\eta^2\Delta t(1-\kappa\Delta t)^m\left[1-(1-\kappa\Delta t)^{2k}\right]}{1-(1-\kappa\Delta t)^2}
\end{aligned}$$

The bridge sampler will be based on the following regression for $0 \le i \le k$:

$$R_{t+i\Delta t} = a_{k,i}(R_t) + b_{k,i}R_{t+k\Delta t} + \epsilon_{k,i}$$

where

$$\begin{aligned}
b_{k,i} &= \frac{(1-\kappa\Delta t)^{k-i}\left[1-(1-\kappa\Delta t)^{2i}\right]}{1-(1-\kappa\Delta t)^{2k}} \\
a_{k,i}(R_t) &= \bar{R}\left[1-(1-\kappa\Delta t)^i\right]+(1-\kappa\Delta t)^i R_t - b_{k,i}\left\{\bar{R}\left[1-(1-\kappa\Delta t)^k\right]+(1-\kappa\Delta t)^k R_t\right\}.
\end{aligned}$$

Moreover, $\epsilon_{k,i}$ is a normal random variable with mean 0 and variance equal to

$$Var(\epsilon_{k,i}) = \frac{\eta^2\Delta t\left[1-(1-\kappa\Delta t)^{2i}\right]\left[1-(1-\kappa\Delta t)^{2(k-i)}\right]}{\left[1-(1-\kappa\Delta t)^2\right]\left[1-(1-\kappa\Delta t)^{2k}\right]}.$$

As an example, we want sample the term spread paths over k periods that always originate from R_t and end with an $R_{t+k\Delta t} \ge R_t + 0.05$. We first sample $R_{t+k\Delta t}$ that is greater than or equal to $R_t + 0.05$ using a truncated normal random variable sampler. Note that the mean and variance of $R_{t+k\Delta t}$ conditional on the time-t information are given above. Corresponding to a simulated value of $R_{t+k\Delta t}$, we generate $R_{t+(k-i)\Delta t}$ using the regression equation for $i = k-1, k-2, \cdots, 1$. This path thus begins at R_t and ends at an $R_{t+k\Delta t} \ge R_t + 0.05$. Repeat the process to obtain as many paths as desired.

Based on the simulated term spread path, we need to sample the corresponding stock market index and exchange rate paths. Because their innovations are correlated with the term spread innovation, we need to derive the conditional processes and they are

$$\begin{aligned}
d\ln I_t &= \delta_I dt + \eta_I dW_{It} \\
&= \delta_I dt + \eta_I \rho_{IR} dW_{Rt} + \eta_I\sqrt{1-\rho_{IR}^2}\,dZ_{It} \\
d\ln e_t &= \delta_e dt + \eta_e dW_{et} \\
&= \delta_e dt + \eta_e \rho_{eR} dW_{Rt} + \eta_e\sqrt{1-\rho_{eR}^2}\,dZ_{et}
\end{aligned}$$

where Z_{It} and Z_{et} are two Wiener processes independent of W_{Rt}. The correlation coefficient between Z_{It} and Z_{et} is $\frac{\rho_{Ie}-\rho_{IR}\rho_{eR}}{\sqrt{(1-\rho_{IR}^2)(1-\rho_{eR}^2)}}$ if $\rho_{IR} \neq 1$ and $\rho_{eR} \neq 1$. Otherwise, the results are trivial. Note that dW_{Rt} in the above equations can be computed from the simulated term spread path. The remaining

random components in the asset and liability dynamics are independent of these three common dynamic risk factors. One can therefore simulate them easily and construct the corresponding asset and liability dynamics.

B2. Conditioning on an equity market or exchange rate event

The sampling scheme can be applied to either stock market index or exchange rate. We will describe the bridge sampling scheme using stock market index. Since $\ln I_t$ is Gaussian, we only need to know three quantities for constructing a bridge sampler.

$$
\begin{aligned}
E_t(\ln I_{t+k\Delta t}) &= \ln I_t + \delta_I k\Delta t \\
Var_t(\ln I_{t+k\Delta t}) &= \eta_I^2 k\Delta t \\
Cov_t(\ln I_{t+k\Delta t}, \ln I_{t+(k+m)\Delta t}) &= \eta_I^2 k\Delta t
\end{aligned}
$$

The bridge sampler will be based on the following regression for $0 \leq i \leq k$:

$$
\ln I_{t+i\Delta t} = c_{k,i} \ln I_t + d_{k,i} \ln I_{t+k\Delta t} + \varsigma_{k,i}
$$

where $d_{k,i} = \frac{i}{k}$ and $c_{k,i} = \frac{k-i}{k}$. Moreover, $\varsigma_{k,i}$ is a normal random variable with mean 0 and variance equal to $\frac{i(k-i)}{k}\eta_I^2\Delta t$.

Suppose that we want sample the stock market index paths over k periods that see an index level drop by more than 25%, i.e., the process originates from I_t and ends at $I_{t+k\Delta t} \leq 0.75 I_t$. We first sample $\ln I_{t+k\Delta t}$ that is less than or equal to $\ln I_t + \ln 0.75$ using a truncated normal random variable sampler. Corresponding to a simulated value of $\ln I_{t+k\Delta t}$, we generate $\ln I_{t+(k-i)\Delta t}$ using the regression equation for $i = k-1, k-2, \cdots, 1$. This path thus begins at I_t and ends at an $I_{t+k\Delta t} \leq 0.75 I_t$. Repeat the process to obtain as many paths as desired.

Based on the simulated stock market index path, we need to sample the corresponding term spread and exchange rate paths. We need to derive the conditional processes and they are

$$
\begin{aligned}
dR_t &= \kappa(\bar{R} - R_t)dt + \eta_R dW_{Rt} \\
&= \kappa(\bar{R} - R_t)dt + \eta_R \rho_{IR} dW_{It} + \eta_R\sqrt{1-\rho_{IR}^2}dY_{Rt} \\
d\ln e_t &= \delta_e dt + \eta_e dW_{et} \\
&= \delta_e dt + \eta_e \rho_{Ie} dW_{It} + \eta_e\sqrt{1-\rho_{Ie}^2}dY_{et}
\end{aligned}
$$

where Y_{Rt} and Y_{et} are two Wiener processes independent of W_{It}. The correlation coefficient between Y_{Rt} and Y_{et} is $\frac{\rho_{eR}-\rho_{IR}\rho_{Ie}}{\sqrt{(1-\rho_{IR}^2)(1-\rho_{Ie}^2)}}$ if $\rho_{IR} \neq 1$ and $\rho_{Ie} \neq 1$. Otherwise, the results are trivial. Note that dW_{It} in the above equations can be computed from the simulated stock index path.

The remaining random components in the asset and liability dynamics are independent of these three common dynamic risk factors. One can therefore simulate them easily and construct the corresponding asset and liability dynamics.

Table 2: Summary statistics of the banks' factor loadings

	Drift					IR Spread					UKX				
Quarter	Mean	Median	StDev	Min	Max	Mean	Median	StDev	Min	Max	Mean	Median	StDev	Min	Max
2004-Q1	-0.041	-0.054	0.049	-0.117	0.051	0.409	0.647	0.672	-1.203	1.002	0.126	0.114	0.058	0.047	0.228
2004-Q2	-0.028	-0.038	0.039	-0.089	0.057	0.325	0.501	0.563	-0.925	0.930	0.158	0.141	0.061	0.084	0.278
2004-Q3	-0.004	0.003	0.033	-0.075	0.049	-0.656	-0.650	0.520	-1.428	0.427	0.148	0.140	0.045	0.068	0.223
2004-Q4	0.006	0.008	0.030	-0.038	0.070	-0.637	-0.641	0.652	-1.492	0.620	0.151	0.158	0.053	0.052	0.219
2005-Q1	-0.018	-0.013	0.017	-0.045	0.002	-1.297	-1.446	0.948	-2.696	0.362	0.143	0.139	0.041	0.066	0.212
2005-Q2	-0.040	-0.043	0.028	-0.074	0.025	-1.763	-1.978	1.014	-3.406	-0.184	0.142	0.146	0.040	0.052	0.182
2005-Q3	-0.069	-0.036	0.117	-0.378	0.006	-0.004	-0.295	1.192	-1.259	2.984	0.094	0.133	0.149	-0.286	0.239
2005-Q4	-0.082	-0.049	0.114	-0.381	0.000	-0.031	-0.595	1.503	-1.005	3.885	0.083	0.105	0.100	-0.170	0.193
2006-Q1	-0.043	-0.017	0.097	-0.280	0.060	0.698	0.278	1.776	-0.827	5.302	0.097	0.099	0.053	-0.025	0.164
2006-Q2	-0.045	-0.006	0.132	-0.385	0.057	1.447	0.883	2.493	-0.605	7.950	0.109	0.101	0.031	0.079	0.175
2006-Q3	0.054	-0.012	0.190	-0.033	0.558	0.499	-0.170	1.671	-0.506	4.670	0.164	0.112	0.149	0.071	0.554
2006-Q4	0.054	-0.003	0.180	-0.028	0.531	0.791	0.233	1.376	-0.117	4.312	0.169	0.115	0.148	0.070	0.554
2007-Q1	0.041	-0.018	0.177	-0.039	0.511	0.898	0.190	2.058	-0.322	6.247	0.153	0.108	0.108	0.072	0.429
2007-Q2	0.024	-0.027	0.164	-0.042	0.460	0.561	-0.033	1.876	-0.322	5.544	0.171	0.109	0.133	0.079	0.508
2007-Q3	-0.073	-0.022	0.146	-0.454	0.016	-0.915	-0.499	1.104	-3.359	0.073	0.190	0.125	0.111	0.104	0.413
2007-Q4	-0.089	-0.030	0.180	-0.562	0.036	0.502	-0.133	1.874	-0.413	5.457	0.215	0.128	0.148	0.108	0.514
2008-Q1	-0.102	-0.030	0.195	-0.594	0.033	0.448	-0.287	2.086	-0.475	5.976	0.181	0.126	0.099	0.099	0.375
2008-Q2	-0.145	-0.045	0.245	-0.741	0.062	1.043	-0.070	2.762	-0.628	8.158	0.165	0.136	0.071	0.103	0.305
2008-Q3	-0.021	0.003	0.096	-0.193	0.106	0.717	-0.217	2.584	-0.743	7.522	0.147	0.152	0.096	-0.069	0.256
2008-Q4	0.113	0.052	0.363	-0.353	0.762	-3.175	-1.323	6.367	-18.725	3.177	0.100	0.108	0.284	-0.558	0.528
2009-Q1	-0.065	0.019	0.196	-0.420	0.110	2.399	-1.299	8.496	-2.289	22.934	0.232	0.118	0.238	0.089	0.785
2009-Q2	0.146	0.088	0.344	-0.295	0.809	8.054	-1.685	23.180	-6.033	64.132	0.254	0.116	0.288	0.089	0.926
2009-Q3	-0.162	-0.249	0.290	-0.626	0.189	-9.211	-1.950	19.279	-51.475	5.489	0.128	0.104	0.062	0.066	0.246
2009-Q4	-0.053	-0.067	0.152	-0.208	0.138	7.609	3.707	12.784	-3.836	27.351	0.194	0.118	0.187	0.092	0.574
2010-Q1	-0.018	-0.024	0.053	-0.092	0.056	17.112	-1.104	32.170	-12.097	60.760	0.259	0.125	0.247	0.061	0.650
2010-Q2	0.000	-0.001	0.045	-0.066	0.061	18.101	-0.379	33.172	-12.103	66.591	0.295	0.156	0.253	0.101	0.674
2010-Q3	0.166	0.065	0.275	-0.022	0.640	15.179	2.136	32.412	-4.480	72.853	0.314	0.187	0.298	0.087	0.824
2010-Q4	0.184	-0.008	0.272	-0.017	0.525	18.412	10.060	30.200	-8.693	67.678	0.365	0.194	0.340	0.066	0.859
2011-Q1	0.153	0.006	0.215	-0.010	0.395	3.099	-2.956	16.640	-11.536	30.926	0.305	0.167	0.279	0.061	0.710
2011-Q2	0.096	-0.011	0.149	-0.014	0.266	3.577	-9.210	24.025	-15.679	42.895	0.274	0.127	0.291	0.052	0.730
2011-Q3	-0.062	-0.054	0.073	-0.162	0.038	6.098	7.161	25.266	-17.616	45.010	0.134	0.101	0.068	0.071	0.211
2011-Q4	-0.058	-0.045	0.089	-0.191	0.055	-5.579	-7.177	9.553	-14.131	10.038	0.123	0.110	0.060	0.061	0.219
2012-Q1	-0.032	-0.027	0.065	-0.108	0.065	3.618	-3.860	18.230	-8.081	35.980	0.135	0.116	0.072	0.064	0.249
2012-Q2	0.000	-0.032	0.056	-0.052	0.073	-0.384	-0.444	4.372	-5.437	6.262	0.128	0.104	0.071	0.055	0.236
2012-Q3	0.057	0.063	0.045	0.002	0.116	1.785	2.921	3.128	-2.326	5.163	0.129	0.092	0.069	0.062	0.223
2012-Q4	0.079	0.044	0.091	0.004	0.229	3.125	3.850	3.924	-1.191	8.614	0.118	0.086	0.067	0.051	0.215

Table 3: Summary statistics of the banks' factor loadings (continued)

	FX					Latent factor					Volatility				
2004-Q1	0.028	0.042	0.039	-0.068	0.061	0.170	0.147	0.304	-0.248	0.906	0.038	0.037	0.009	0.027	0.053
2004-Q2	-0.009	-0.012	0.034	-0.070	0.044	0.102	0.058	0.337	-0.306	0.939	0.033	0.031	0.009	0.018	0.049
2004-Q3	-0.011	-0.006	0.044	-0.070	0.056	0.119	0.188	0.330	-0.576	0.658	0.030	0.030	0.007	0.020	0.042
2004-Q4	-0.044	-0.058	0.041	-0.102	0.047	0.101	0.184	0.337	-0.652	0.599	0.032	0.029	0.008	0.021	0.044
2005-Q1	-0.059	-0.063	0.027	-0.115	-0.016	0.282	0.327	0.188	-0.148	0.542	0.026	0.020	0.017	0.014	0.065
2005-Q2	-0.038	-0.035	0.038	-0.099	0.024	0.290	0.344	0.173	-0.127	0.486	0.026	0.018	0.017	0.014	0.065
2005-Q3	-0.179	-0.072	0.377	-1.168	0.087	0.125	0.025	0.328	-0.055	0.995	0.040	0.042	0.016	0.005	0.066
2005-Q4	-0.128	-0.024	0.403	-1.186	0.143	0.126	0.025	0.327	-0.053	0.995	0.037	0.040	0.013	0.004	0.048
2006-Q1	-0.126	0.000	0.401	-1.182	0.111	0.121	0.018	0.330	-0.055	0.996	0.032	0.034	0.013	0.003	0.047
2006-Q2	-0.189	-0.068	0.451	-1.376	0.066	0.120	0.016	0.330	-0.056	0.996	0.031	0.028	0.015	0.002	0.057
2006-Q3	-0.015	-0.020	0.069	-0.110	0.118	-0.119	-0.002	0.330	-0.998	0.013	0.027	0.024	0.014	0.002	0.053
2006-Q4	-0.016	-0.024	0.069	-0.101	0.125	-0.118	-0.001	0.331	-0.998	0.014	0.025	0.023	0.013	0.002	0.050
2007-Q1	-0.076	-0.058	0.133	-0.405	0.081	-0.118	-0.001	0.331	-0.998	0.014	0.023	0.021	0.012	0.002	0.041
2007-Q2	-0.069	-0.031	0.170	-0.494	0.124	-0.118	-0.001	0.331	-0.998	0.014	0.020	0.018	0.011	0.002	0.035
2007-Q3	0.134	0.007	0.311	-0.116	0.893	-0.191	-0.043	0.289	-0.885	-0.024	0.035	0.024	0.020	0.019	0.078
2007-Q4	0.059	-0.006	0.174	-0.097	0.472	-0.148	-0.020	0.317	-0.977	0.007	0.041	0.026	0.030	0.021	0.105
2008-Q1	0.136	0.009	0.350	-0.109	1.040	-0.140	-0.014	0.321	-0.987	0.000	0.047	0.028	0.039	0.022	0.119
2008-Q2	0.139	0.002	0.386	-0.088	1.155	-0.278	-0.071	0.344	-0.845	-0.028	0.091	0.051	0.120	0.029	0.406
2008-Q3	-0.073	0.022	0.290	-0.780	0.175	-0.209	-0.008	0.410	-0.913	0.186	0.104	0.048	0.143	0.029	0.480
2008-Q4	0.240	0.050	0.659	-0.130	1.985	0.513	0.003	1.311	-0.796	2.926	0.175	0.069	0.188	0.044	0.534
2009-Q1	-0.001	0.006	0.054	-0.076	0.089	0.456	-0.029	1.549	-0.978	3.958	0.143	0.117	0.130	0.020	0.419
2009-Q2	-0.089	0.011	0.276	-0.767	0.053	0.498	-0.028	1.641	-0.978	4.222	0.155	0.129	0.133	0.022	0.416
2009-Q3	0.018	0.034	0.061	-0.084	0.084	0.269	0.020	0.399	-0.126	0.711	0.120	0.123	0.068	0.045	0.194
2009-Q4	-0.039	-0.047	0.125	-0.235	0.130	-0.315	-0.278	0.381	-0.835	0.054	0.141	0.149	0.094	0.039	0.276
2010-Q1	-0.163	-0.001	0.335	-0.707	0.117	0.255	-0.004	0.411	-0.099	0.767	0.100	0.068	0.079	0.027	0.200
2010-Q2	-0.076	0.010	0.210	-0.409	0.126	-0.259	0.001	0.408	-0.779	0.090	0.096	0.067	0.070	0.028	0.187
2010-Q3	-0.051	0.002	0.178	-0.346	0.111	-0.279	-0.036	0.391	-0.860	0.011	0.107	0.050	0.098	0.030	0.258
2010-Q4	0.004	-0.005	0.067	-0.066	0.113	-0.267	-0.043	0.401	-0.921	0.012	0.085	0.053	0.072	0.031	0.208
2011-Q1	0.049	0.014	0.102	-0.039	0.224	-0.271	-0.046	0.398	-0.920	0.007	0.085	0.048	0.071	0.035	0.206
2011-Q2	-0.014	-0.049	0.151	-0.165	0.240	-0.267	-0.043	0.401	-0.928	0.008	0.079	0.045	0.068	0.034	0.197
2011-Q3	-0.085	-0.055	0.094	-0.250	-0.012	-0.219	-0.034	0.436	-0.998	-0.004	0.043	0.042	0.027	0.003	0.077
2011-Q4	-0.046	-0.064	0.035	-0.069	0.014	-0.384	-0.463	0.257	-0.574	0.057	0.037	0.032	0.019	0.017	0.066
2012-Q1	0.005	0.005	0.033	-0.033	0.039	-0.386	-0.477	0.252	-0.624	0.010	0.042	0.036	0.012	0.033	0.062
2012-Q2	-0.052	-0.035	0.078	-0.147	0.032	-0.355	-0.346	0.304	-0.808	-0.010	0.045	0.044	0.005	0.040	0.052
2012-Q3	-0.071	-0.040	0.107	-0.185	0.028	-0.342	-0.315	0.322	-0.841	-0.001	0.038	0.036	0.012	0.028	0.056
2012-Q4	-0.050	-0.002	0.114	-0.183	0.083	-0.335	-0.242	0.332	-0.896	-0.075	0.029	0.021	0.015	0.018	0.055

Table 4: Parameter estimates for the common risk factors

Quarter	UKX Drift	UKX Volatility	FX Drift	FX Volatility	IR Spread Volatility
2004-Q1	0.19	0.14	0.072	0.064	0.0058
2004-Q2	0.10	0.11	0.043	0.063	0.0057
2004-Q3	0.11	0.11	0.036	0.060	0.0051
2004-Q4	0.07	0.10	0.009	0.061	0.0046
2005-Q1	0.10	0.09	-0.016	0.056	0.0042
2005-Q2	0.12	0.09	-0.006	0.050	0.0037
2005-Q3	0.17	0.08	-0.001	0.049	0.0036
2005-Q4	0.15	0.09	-0.003	0.048	0.0036
2006-Q1	0.19	0.09	-0.028	0.048	0.0036
2006-Q2	0.13	0.12	-0.024	0.047	0.0035
2006-Q3	0.08	0.13	0.013	0.044	0.0032
2006-Q4	0.09	0.13	0.041	0.040	0.0031
2007-Q1	0.04	0.13	0.044	0.044	0.0032
2007-Q2	0.12	0.11	0.052	0.043	0.0033
2007-Q3	0.07	0.15	0.003	0.045	0.0040
2007-Q4	0.04	0.17	-0.064	0.055	0.0077
2008-Q1	-0.10	0.22	-0.127	0.063	0.0081
2008-Q2	-0.16	0.23	-0.128	0.070	0.0084
2008-Q3	-0.38	0.24	-0.132	0.073	0.0081
2008-Q4	-0.48	0.36	-0.283	0.110	0.0051
2009-Q1	-0.47	0.37	-0.195	0.134	0.0043
2009-Q2	-0.36	0.38	-0.113	0.140	0.0032
2009-Q3	0.06	0.36	-0.142	0.142	0.0029
2009-Q4	0.21	0.23	0.096	0.120	0.0014
2010-Q1	0.41	0.18	0.036	0.090	0.0009
2010-Q2	0.13	0.18	-0.027	0.084	0.0009
2010-Q3	0.07	0.18	0.008	0.081	0.0005
2010-Q4	0.09	0.17	-0.004	0.078	0.0004
2011-Q1	0.05	0.18	0.010	0.077	0.0004
2011-Q2	0.20	0.15	-0.057	0.074	0.0004
2011-Q3	-0.08	0.19	0.004	0.072	0.0004
2011-Q4	-0.06	0.21	0.025	0.070	0.0004
2012-Q1	-0.02	0.21	0.035	0.065	0.0003
2012-Q2	-0.06	0.22	0.060	0.059	0.0006
2012-Q3	0.11	0.17	0.055	0.054	0.0008
2012-Q4	0.07	0.14	0.023	0.047	0.0008

Table 5: R^2 of the banks' asset value processes with a latent factor

	R^2 Inc Latent Factor					R^2 Exc Latent Factor				
Quarter	Mean	Median	StDev	Min	Max	Mean	Median	StDev	Min	Max
2004-Q1	0.361	0.337	0.237	0.071	0.940	0.124	0.129	0.071	0.032	0.271
2004-Q2	0.342	0.255	0.254	0.095	0.974	0.183	0.180	0.090	0.068	0.352
2004-Q3	0.479	0.463	0.183	0.116	0.803	0.179	0.198	0.100	0.039	0.351
2004-Q4	0.483	0.486	0.169	0.118	0.730	0.193	0.238	0.113	0.008	0.351
2005-Q1	0.680	0.808	0.304	0.061	0.896	0.141	0.142	0.072	0.011	0.257
2005-Q2	0.703	0.847	0.307	0.055	0.914	0.113	0.115	0.051	0.011	0.178
2005-Q3	0.249	0.165	0.290	0.050	1.000	0.078	0.078	0.039	0.022	0.129
2005-Q4	0.242	0.145	0.290	0.071	1.000	0.059	0.059	0.026	0.024	0.105
2006-Q1	0.259	0.162	0.287	0.080	1.000	0.078	0.091	0.039	0.020	0.122
2006-Q2	0.326	0.275	0.266	0.090	1.000	0.146	0.177	0.070	0.028	0.239
2006-Q3	0.353	0.274	0.265	0.108	1.000	0.192	0.225	0.088	0.027	0.295
2006-Q4	0.384	0.298	0.254	0.126	1.000	0.219	0.248	0.096	0.025	0.335
2007-Q1	0.420	0.344	0.233	0.246	1.000	0.211	0.221	0.085	0.020	0.303
2007-Q2	0.431	0.376	0.226	0.252	1.000	0.199	0.213	0.077	0.019	0.292
2007-Q3	0.517	0.481	0.195	0.269	0.944	0.264	0.264	0.068	0.141	0.364
2007-Q4	0.480	0.448	0.210	0.244	0.993	0.301	0.328	0.092	0.154	0.448
2008-Q1	0.501	0.477	0.218	0.250	0.996	0.206	0.184	0.083	0.083	0.358
2008-Q2	0.443	0.414	0.242	0.177	0.915	0.191	0.154	0.078	0.090	0.344
2008-Q3	0.537	0.465	0.297	0.115	0.996	0.168	0.193	0.123	0.016	0.359
2008-Q4	0.494	0.423	0.221	0.154	0.890	0.160	0.137	0.119	0.014	0.357
2009-Q1	0.524	0.439	0.263	0.189	0.996	0.153	0.126	0.157	0.017	0.460
2009-Q2	0.554	0.466	0.282	0.187	0.995	0.156	0.126	0.152	0.021	0.482
2009-Q3	0.537	0.467	0.272	0.152	0.848	0.236	0.088	0.265	0.021	0.739
2009-Q4	0.448	0.402	0.260	0.119	0.864	0.174	0.171	0.135	0.012	0.352
2010-Q1	0.522	0.390	0.263	0.297	0.855	0.139	0.082	0.113	0.051	0.305
2010-Q2	0.527	0.358	0.261	0.327	0.867	0.175	0.104	0.136	0.053	0.336
2010-Q3	0.467	0.457	0.264	0.246	0.892	0.202	0.227	0.121	0.058	0.366
2010-Q4	0.435	0.407	0.326	0.151	0.960	0.175	0.147	0.077	0.119	0.308
2011-Q1	0.414	0.376	0.340	0.107	0.959	0.141	0.105	0.069	0.090	0.258
2011-Q2	0.377	0.317	0.367	0.057	0.967	0.090	0.073	0.047	0.057	0.172
2011-Q3	0.351	0.227	0.365	0.131	1.000	0.144	0.151	0.064	0.049	0.227
2011-Q4	0.617	0.750	0.326	0.099	0.929	0.172	0.172	0.069	0.083	0.274
2012-Q1	0.560	0.657	0.274	0.091	0.780	0.167	0.152	0.064	0.090	0.262
2012-Q2	0.475	0.501	0.292	0.076	0.839	0.144	0.151	0.053	0.075	0.202
2012-Q3	0.486	0.410	0.326	0.064	0.929	0.126	0.135	0.051	0.063	0.186
2012-Q4	0.498	0.400	0.289	0.272	0.969	0.126	0.141	0.058	0.041	0.196