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Unsteady MHD free convective flow past a vertical porous plate immersed in a porous medium with Hall current, thermal diffusion and heat source

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Abstract

An attempt has been made to study the unsteady MHD free convective flow past a vertical porous plate immersed in a porous medium with Hall current, thermal diffusion and heat source. Analytical solution has been found depending on the physical parameters including the Hartmann number M, the Prandtl number Pr, the Grashof number for heat transfer Gr, the Grashof number for mass transfer Gc, the Schmidt number Sc, the Hall parameter m, the Soret number S_0, heat source S, frequency parameter Ω. The influence of these parameters on velocity, temperature, species concentration, and shearing stress at the plate are demonstrated graphically and the results obtained are discussed. It is found that the concentration at the plate-surface increases under Soret effect. Further, it is observed that the Soret effect causes the main-flow shear stress to rise and the cross-flow shear stress to fall. It is also found that a decrease in the Soret effect leads to an increase in both the main flow and cross-flow velocities.

2000 Mathematics subject classification: 76 W 05
Keywords: Free convection, MHD, thermal diffusion, Hall effect

1. Introduction

In recent years, the analysis of hydromagnetic convection flow involving heat and mass transfer in porous medium has attracted the attention of many scholars because of its possible applications in diverse fields of science and technology such as – soil-sciences, astrophysics, geophysics, nuclear power reactors etc. In geophysics, it finds its applications in the design of MHD generators and accelerators, underground water energy storage system etc. It is worth-mentioning that MHD is now undergoing a stage of great enlargement and differentiation of subject matter. These new problems draw the attention of the researchers due to their varied significance, in liquid metals, electrolytes and ionized gases etc. The MHD in the present form is due to contributions of several notable authors like Shercliff (1965), Ferraro and Plumpton (1966) and Crammer and Pai (1973).

The heat and mass transfer effects on a flow along a vertical plate in the presence of a magnetic field was investigated by Elbashbeshy (1997). The influence of combined natural convection from a vertical wavy surface due to thermal and mass diffusion was studied by Hossain and Ross (1999). Chen (2004) investigated the effects of heat and mass transfer in MHD free convection from a vertical surface. In addition, the applications of the effect of Hall current on the fluid flow with variable concentration have been seen in MHD power generators, astrophysical and meteorological studies as well as in plasma physics. The Hall effect is due merely to the sideways magnetic force on the drifting free charges. The electric field has to have a component transverse to the direction of the current density to balance this force. In many works on plasma physics, the Hall effect is disregarded. But if the strength of magnetic field is high and the number density of electrons is small, the Hall effect can not be ignored as it has a significant effect on the flow pattern of an ionized gas. Hall effect results in a development of an additional potential difference between opposite surfaces of a conductor for which a current is induced perpendicular to both the electric and magnetic field. This current is termed as Hall current. Model studies on the effect of Hall current on MHD convection flows have been carried out by many authors due to application of such studies in the problems of MHD generators and Hall accelerators. Some of them are Aboeldhab (2001), Dutta *et al.* (1976), Acharya *et al.* (2001) and Biswal *et al.* (1994). In the above studies the effect of heat source/sink effect was not considered. The study of heat transfer problems in presence of heat source/sink is quite important in the field of industrial technology. Ostrach (1952, 1954, 1958), Raptis (1982) carried out a number of analytical studies for different types of heat transfer problems in presence of heat generators. The problem concerning MHD free convection and mass transfer flow with heat source and thermal diffusion was studied by Singh (2001). Recently, Sharma *et al.* (2007) have investigated the Hall effect on MHD mixed convective flow of a viscous incompressible fluid past a vertical porous plate immersed in a porous medium with heat source/sink. Recently, Nikodijevic *et al.* (2009) have studied the generalized similarity method in unsteady two-dimensional MHD boundary layer on the body with time varying temperature.

In the above mentioned works, the thermal diffusion (Soret) effect was not taken into account in the species continuity equation. The flux of mass caused due to temperature gradient is known as the Soret effect or thermal diffusion. The experimental investigation of the thermal diffusion effect on mass transfer related problems was first performed by Charles Soret in 1879. There after its effect is termed as Soret effect in the honor of his name. In general the Soret effect is of smaller order of magnitude than the effect described in Fick's law and is often ignored in mass transfer process. Though this effect is quit small, but the devices may be arranged to produce very sharp temperature gradient so that the separation of components in mixtures are affected. Eckert and Drake (1972) have emphasized that the Soret effect assumes significance in cases concerning isotope separation and in mixtures between gases with very light molecular weight (H_2 , H_e) and the medium molecular weight (N_2 , air).

Based on Eckert and Drake's work (1972) many other investigators have carried out model studies on the Soret effect in different heat and mass transfer problems. Some of them are Dursunkaya and Worek (1992), Kafoussias and Williams (1995), Sattar and Alam (1994), Alam *et al.* (2005a, 2005b, 2006a, 2006b, 2006c and 2007), Raju *et al.* (2008).

In view of the significance of the Soret effect as well as Hall effect, we have proposed in the present paper to investigate the unsteady MHD free convective flow past a vertical porous plate in porous medium with Hall current, thermal diffusion and heat source. Here our main objectives are to study the effect of Soret number and Hall parameter on the flow and transport characteristics. Our work is an extension to the work done by Sharma *et al.* (2007) to consider the effect of thermal diffusion on the flow and heat and mass transfer.

2. Basic equations

The equations governing the motion of an incompressible viscous electrically conducting fluid in presence of a magnetic field are-
the equation of continuity:

$$\vec{\nabla}.\vec{v} = 0 \tag{1}$$

the momentum equation:

$$\rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}.\vec{\nabla})\vec{v}\right] = -\vec{\nabla}p + \vec{J}\times\vec{B} + \rho\vec{g} + \mu\nabla^2\vec{v} - \frac{\mu}{k}\vec{v} \tag{2}$$

the energy equation:

$$\rho C_p\left[\frac{\partial T}{\partial t} + (\vec{v}.\vec{\nabla})T\right] = \kappa\nabla^2 T + \phi + \frac{\overline{J}^2}{\sigma} + \rho C_p S(T - T_\infty) \tag{3}$$

the species continuity equation:

$$\frac{\partial C}{\partial t} + (\vec{v}.\vec{\nabla})C = D\nabla^2 C + D_T\nabla^2 T \tag{4}$$

the Kirchhoff's first law:

$$\vec{\nabla}.\vec{J} = 0 \tag{5}$$

the general Ohm's law, taking Hall effect into account

$$\vec{J} + \frac{\omega_e\tau_e}{B_0}(\vec{J}\times\vec{B}) = \sigma(\vec{E} + \vec{v}\times\vec{B} + \frac{1}{e\eta_e}\vec{\nabla}p_e) \tag{6}$$

the Gauss's law of magnetism:

$$\vec{\nabla}.\vec{B} = 0 \tag{7}$$

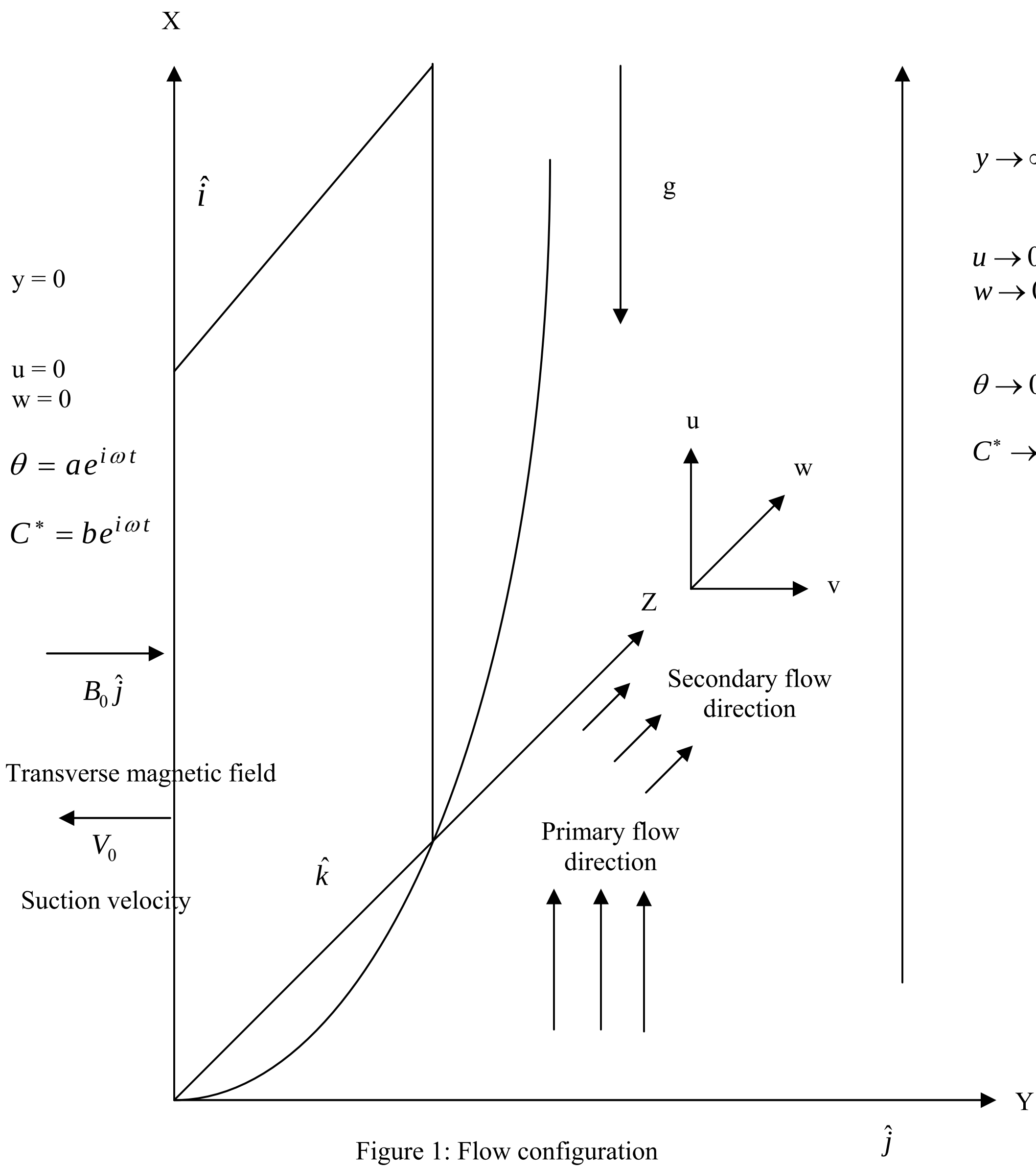

Figure 1: Flow configuration

We now consider an unsteady flow of an electrically conducting fluid past an infinite vertical porous flat plate coinciding with the X-axis y = 0, taking into account the thermal diffusion, Hall current and heat source in presence of a uniform transverse magnetic field. Our investigation is restricted to the following assumptions:

(i) All the fluid properties except the density in the buoyancy force term are constant.
(ii) The plate is electrically non-conducting.
(iii) The magnetic Reynolds number is so small that the induced magnetic field may be neglected.
(iv) p_e is constant.
(v) $\vec{E} = 0$.

We introduce a coordinate system (x, y, z) with X-axis vertically upwards, Y-axis normal to the plate directed into the fluid region and Z-axis along the width of the plate. Let $\vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$ be the velocity, $\vec{J} = J_x\hat{i} + J_y\hat{j} + J_z\hat{k}$ be the current density at the point $P(x, y, z, t)$ and $\vec{B} = \vec{B}_0\hat{J}$ be the applied magnetic field, $\hat{i}, \hat{j}, \hat{k}$ being unit vectors along X- axis, Y-axis and Z-axis respectively. Since the plate is of infinite length in X and Z- direction, therefore all the quantities except possibly the pressure are independent of x and z.

Now,
The equation (1) gives

$$\frac{\partial v}{\partial y} = 0 \tag{8}$$

which is trivially satisfied by

$$v = -V_0 \tag{9}$$

where V_0 is a constant and $V_0 \rangle 0$

Therefore the velocity vector $\vec{v}$ is given by

$$\vec{v} = u\hat{i} - V_0\hat{j} + w\hat{k} \tag{10}$$

Again the equation (7) is satisfied by

$$\vec{B} = \vec{B}_0\,\hat{j} \tag{11}$$

Also the equation (5) reduces to

$$\frac{\partial J_y}{\partial y} = 0$$

which shows that $J_y =$ constant (12)

Since the plate is non-conducting, $J_y = 0$ at the plate and hence $J_y = 0$ at all points in the fluid.

Thus the current density is given by

$$\vec{J} = J_x\hat{i} + J_z\hat{k} \tag{13}$$

Under the assumption (iv) and (v), the equation (6) takes the form

$$\vec{J} + \frac{m}{B_0}(\vec{J} \times \vec{B}) = \sigma(\vec{v} \times \vec{B}) \tag{14}$$

Where $m = \omega_e \tau_e$ is the Hall parameter.

The equations (10), (11), (13), and (14) yield,

$$\left.\begin{aligned} J_x &= \frac{\sigma B_0}{1+m^2}(mu - w) \\ J_z &= \frac{\sigma B_0}{1+m^2}(u + mw) \end{aligned}\right\} \tag{15}$$

With the foregoing assumptions and the usual boundary layer and Boussinesq approximation, the equations (2), (3) and (4) reduce to the following:

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2(u + mw)}{\rho(1+m^2)} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\upsilon u}{k} \tag{16}$$

$$\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial y} = \upsilon\frac{\partial^2 w}{\partial y^2} + \frac{\sigma B_0^2(mu - w)}{\rho(1+m^2)} - \frac{\upsilon w}{k} \tag{17}$$

$$\frac{\partial(T - T_\infty)}{\partial t} + v\frac{\partial(T - T_\infty)}{\partial y} = \frac{\kappa}{\rho C_p}\frac{\partial^2(T - T_\infty)}{\partial y^2} + S(T - T_\infty) \tag{18}$$

$$\frac{\partial(C - C_\infty)}{\partial t} + v\frac{\partial(C - C_\infty)}{\partial y} = D\frac{\partial^2(C - C_\infty)}{\partial y^2} + D_T\frac{\partial^2(T - T_\infty)}{\partial y^2} \tag{19}$$

In equation (18) the viscous dissipation and Ohmic dissipation are ignored and in equation (19), the term due to chemical reaction is supposed to be absent.

Now using

$$v = -V_0, T(y,t) - T_\infty = \theta(y,t) \text{ and } C(y,t) - C_\infty = C^*(y,t)$$

Subject to the boundary conditions

$$t \le 0: u(y,t) = w(y,t) = 0, \theta = 0, C^* = 0 \quad \text{for all } y$$

$$\left.\begin{aligned} &y = 0: u(0,t) = w(0,t) = 0, \theta(0,t) = ae^{i\omega t}, C^*(0,t) = be^{i\omega t} \\ &y \to \infty: u(\infty,t) = w(\infty,t) = 0, \theta(\infty,t) = 0, C^*(\infty,t) = 0 \end{aligned}\right\} (t > 0) \tag{20}$$

Let us introduce the following dimensionless quantities:

$$\eta=\frac{V_0 y}{\upsilon},\ t'=\frac{V_0^2 t}{4\upsilon},\ u'=\frac{u}{V_0},\ w'=\frac{w}{V_0},\ \theta'=\frac{\theta}{a},\ C'=\frac{C^*}{b},\ Gr=\frac{4g\beta\upsilon a}{V_0^3},\ Gc=\frac{4g\beta^*\upsilon b}{V_0^3},\ M=\frac{4B_0^2\sigma\upsilon}{\rho V_0^3},\ Pr=\frac{\upsilon\rho C_p}{\kappa},$$

$$Sc=\frac{\upsilon}{D},\ k'=\frac{V_0^2 k}{4\upsilon^2},\ S'=\frac{4S\upsilon}{V_0^2},\ S_0=\frac{D_T a}{\upsilon b}. \tag{21}$$

All the physical variables are defined in the Nomenclature.

Equations (16), (17), (18) and (19) transform to the following non-dimensional forms, respectively (dropping the dashes)

$$\frac{\partial u}{\partial t}-4\frac{\partial u}{\partial \eta}=4\frac{\partial^2 u}{\partial \eta^2}-\frac{M}{1+m^2}(u+mw)+Gr\theta+GcC-\frac{u}{k} \tag{22}$$

$$\frac{\partial w}{\partial t}-4\frac{\partial w}{\partial \eta}=4\frac{\partial^2 w}{\partial \eta^2}+\frac{M}{1+m^2}(mu-w)-\frac{w}{k} \tag{23}$$

$$\frac{\partial \theta}{\partial t}-4\frac{\partial \theta}{\partial \eta}=\frac{4}{Pr}\frac{\partial^2 \theta}{\partial \eta^2}+S\theta \tag{24}$$

$$\frac{\partial C}{\partial t}-4\frac{\partial C}{\partial \eta}=\frac{4}{Sc}\frac{\partial^2 C}{\partial \eta^2}+4S_0\frac{\partial^2 \theta}{\partial \eta^2} \tag{25}$$

The corresponding boundary conditions (20) in non-dimensional forms are (dropping the dashes):

$$t\le 0: u(\eta,t)=w(\eta,t)=0, \theta=0, C=0 \text{ for all } \eta=0$$

$$\left.\begin{array}{l}\eta=0: u(0,t)=w(0,t)=0, \theta(0,t)=e^{i\omega t}, C(0,t)=e^{i\omega t}\\ \eta\to\infty: u(\infty,t)=w(\infty,t)=0, \theta(\infty,t)=0, C(\infty,t)=0\end{array}\right\}(t>0) \tag{26}$$

3. Method of solutions

The equations (22) and (23) can be combined using the complex variable

$$\psi=u+iw \tag{27}$$

This gives the combined equation as

$$\frac{\partial^2\psi}{\partial\eta^2}+\frac{\partial\psi}{\partial\eta}-\frac{1}{4}\frac{\partial\psi}{\partial t}-\frac{1}{4}\left[\frac{M}{1+m^2}(1-im)+\frac{1}{k}\right]\psi=-\frac{1}{4}Gr\theta-\frac{1}{4}GcC \tag{28}$$

Now introducing the non-dimensional parameter $\Omega=\frac{4\upsilon\omega}{V_0^2}$ and using equation (27), the boundary condition in (26) are transformed to:

$$\left.\begin{array}{l}\psi(0,t)=\psi(\infty,t)=0, C(0,t)=e^{i\Omega t}\\ \theta(0,t)=e^{i\Omega t}, \theta(\infty,t)=0, C(\infty,t)=0\end{array}\right\} \tag{29}$$

Substituting $\theta(\eta,t)=e^{i\Omega t}f(\eta)$ in equation (24), we have

$$f''(\eta)+Pr f'(\eta)-\left(\frac{i\Omega Pr}{4}-\frac{S Pr}{4}\right)f(\eta)=0 \tag{30}$$

The equation (30) can be solved under the boundary conditions,

$$f(0)=1,\ f(\infty)=0 \tag{31}$$

Hence the solution is

$$f(\eta)=e^{-A_1\eta},\ \text{where}\ A_1=\frac{Pr+\sqrt{Pr^2+Pr(i\Omega-S)}}{2}$$

$$\Rightarrow \theta(\eta,t)=e^{i\Omega t-A_1\eta} \tag{32}$$

Separating equation (32) in to real and imaginary parts and taking the real part only we get

$$\theta_r(\eta,t)=e^{-x_2\eta}\cos(\Omega t-y_2\eta) \tag{33}$$

Where $x_2 = \dfrac{Pr + x_1}{2}, \quad x_1 = \left\{ \dfrac{(Pr^2 - S\,Pr) + Pr\sqrt{(Pr - S)^2 + \Omega^2}}{2} \right\}^{\frac{1}{2}}$

$$y_2 = \frac{y_1}{2}, \quad y_1 = \left\{ \frac{Pr\sqrt{(Pr - S)^2 + \Omega^2} - (Pr^2 - S\,Pr)}{2} \right\}^{\frac{1}{2}} \tag{34}$$

Again substituting $C(\eta, t) = e^{i\Omega t} g(\eta)$ in equation (25), we have

$$g^{//}(\eta) + Sc\,g^{/}(\eta) - \frac{i\Omega S_c}{4} g(\eta) = -S_0 Sc f^{//}(\eta) \tag{35}$$

The equation (35) can be solved under the boundary conditions,

$$g(0) = 1, g(\infty) = 0 \tag{36}$$

Consequently the solution is

$$g(\eta) = A_4 e^{-A_2 \eta} + A_3 e^{-A_1 \eta}$$

$$\Rightarrow C(\eta, t) = e^{i\Omega t}(A_4 e^{-A_2 \eta} + A_3 e^{-A_1 \eta}) \tag{37}$$

Separating equation (37) in to real and imaginary parts and considering the real part only we obtain,

$$C_r(\eta, t) = e^{-x_4 \eta}\{L_3 \cos(\Omega t - y_4 \eta) - M_2 \sin(\Omega t - y_4 \eta)\}$$

$$+ e^{-x_2 \eta}\{L_2 \cos(\Omega t - y_2 \eta) - M_2 \sin(\Omega t - y_2 \eta)\} \tag{38}$$

where

$$x_4 = \frac{Sc + x_3}{2}, x_3 = \left\{ \frac{Sc^2 + Sc\sqrt{Sc^2 + \Omega^2}}{2} \right\}^{\frac{1}{2}}, y_4 = \frac{y_3}{2}, y_3 = \left\{ \frac{Sc\sqrt{Sc^2 + \Omega^2} - Sc^2}{2} \right\}^{\frac{1}{2}} \tag{39}$$

Also substituting $\psi = e^{i\Omega t} F(\eta)$ in equation (28) we arrive at,

$$F^{//}(\eta) + F^{/}(\eta) - \frac{1}{4}\left[i\Omega + \frac{M}{(1 + m^2)}(1 - im) + \frac{1}{k} \right] F(\eta) = -\frac{1}{4} Gr f(\eta) - \frac{1}{4} Gc\, g(\eta) \tag{40}$$

The equation (40) can be solved under the boundary conditions,

$$F(0) = 0 \text{ and } F(\infty) = 0 \tag{41}$$

Therefore the solution is

$$F(\eta) = A_{11} e^{-A_6 \eta} + A_{10} e^{-A_1 \eta} - A_8 e^{-A_2 \eta}$$

$$\Rightarrow \psi(\eta, t) = e^{i\Omega t}(A_{11} e^{-A_6 \eta} + A_{10} e^{-A_1 \eta} - A_8 e^{-A_2 \eta}) \tag{42}$$

Separating equation (42) in to real and imaginary parts, and then using (27), we have

$$u = e^{-x_6 \eta}\{L_{11} \cos(\Omega t - y_6 \eta) - M_{11} \sin(\Omega t - y_6 \eta)\} + e^{-x_2 \eta}\{L_{10} \cos(\Omega t - y_2 \eta) - M_{10} \sin(\Omega t - y_2 \eta)\}$$

$$- e^{-x_4 \eta}\{L_8 \cos(\Omega t - y_4 \eta) - M_8 \sin(\Omega t - y_4 \eta)\} \tag{43}$$

$$w = e^{-x_6 \eta}\{L_{11} \sin(\Omega t - y_6 \eta) + M_{11} \cos(\Omega t - y_6 \eta)\} + e^{-x_2 \eta}\{L_{10} \sin(\Omega t - y_2 \eta) + M_{10} \cos(\Omega t - y_2 \eta)\}$$

$$- e^{-x_4 \eta}\{L_8 \sin(\Omega t - y_4 \eta) + M_8 \cos(\Omega t - y_4 \eta)\} \tag{44}$$

Where $x_6 = \dfrac{1 + x_5}{2}, \ x_5 = \left\{ \dfrac{(1 + L_4) + \sqrt{(1 + L_4)^2 + M_4^2}}{2} \right\}^{\frac{1}{2}},$

$$y_6 = \frac{y_5}{2}, \qquad y_5 = \left\{ \frac{\sqrt{(1 + L_4)^2 + M_4^2} - (1 + L_4)}{2} \right\}^{\frac{1}{2}}$$

The shearing stress at the wall along x-axis is given by

$$\tau_1 = \left(\frac{\partial u}{\partial \eta} \right)_{\eta = 0} = L_{11} y_6 + M_{11} x_6 + L_{10} y_2 + M_{10} x_2 - L_8 y_4 - M_8 x_4 \tag{45}$$

and the shearing stress at the wall along z-axis is given by

$$\tau_2 = \left(\frac{\partial w}{\partial \eta}\right)_{\eta=0} = M_{11}y_6 - L_{11}x_6 + M_{10}y_2 - L_{10}x_2 - M_8 y_4 + L_8 x_4 \tag{46}$$

The constants involved in the above discussion have been obtained but not presented here for the sake of brevity.

4. Discussion of the results

In order to attain a physical insight into the problem, we have carried out numerical calculations for the velocity field, temperature field, concentration field and shearing stresses at the plate due to primary and secondary velocity fields for different values of $S_0, m, \Pr, Sc$ keeping the values of S, Ω, Gr, Gc, $\Omega t, M$ and k fixed. The value of $\Pr$ is taken to be 0.71 which corresponds to air at 25^0 C temperature and one atmosphere pressure respectively. The value of Grashof number for heat transfer is assumed to be $Gr = 10$ and Grashof number for mass transfer is taken to be $Gc = -5$, corresponding to the heated plate $(Gc\langle 0)$. The values of the Schmidt number are chosen to be Sc =0.78 (for Ammonia) and Sc =0.30 (for Helium) at 25^0 C temperature and one atmosphere pressure. The numerical results obtained are discussed in figures 2 to 11.

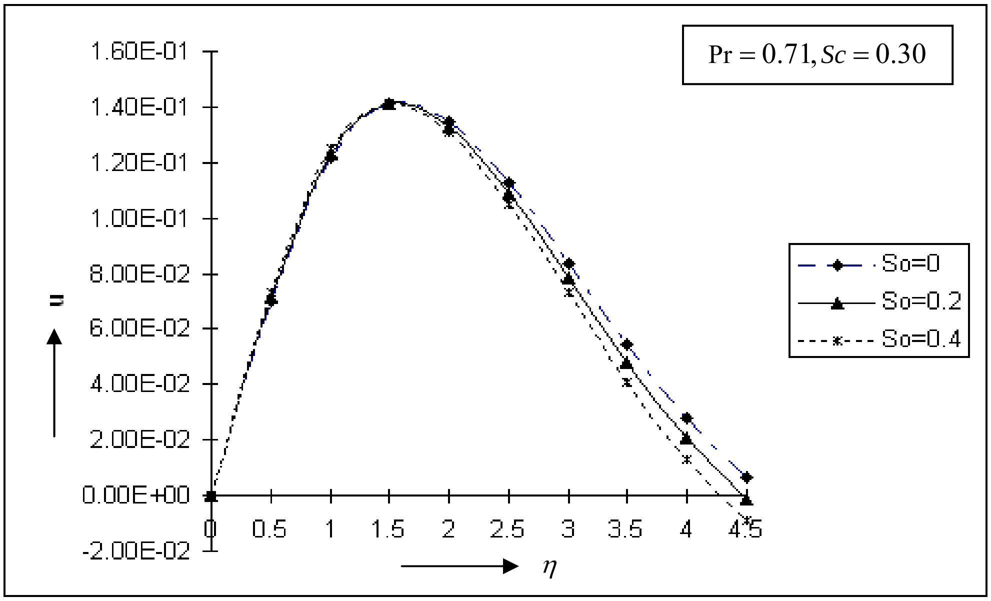

Figure 2: Variation of velocity component u against η

when $S = 1, \Omega = 1, Gr = 10,\ Gc = -5, \Omega t = \pi/2, m = 0.5, M = 5, k = 1$.

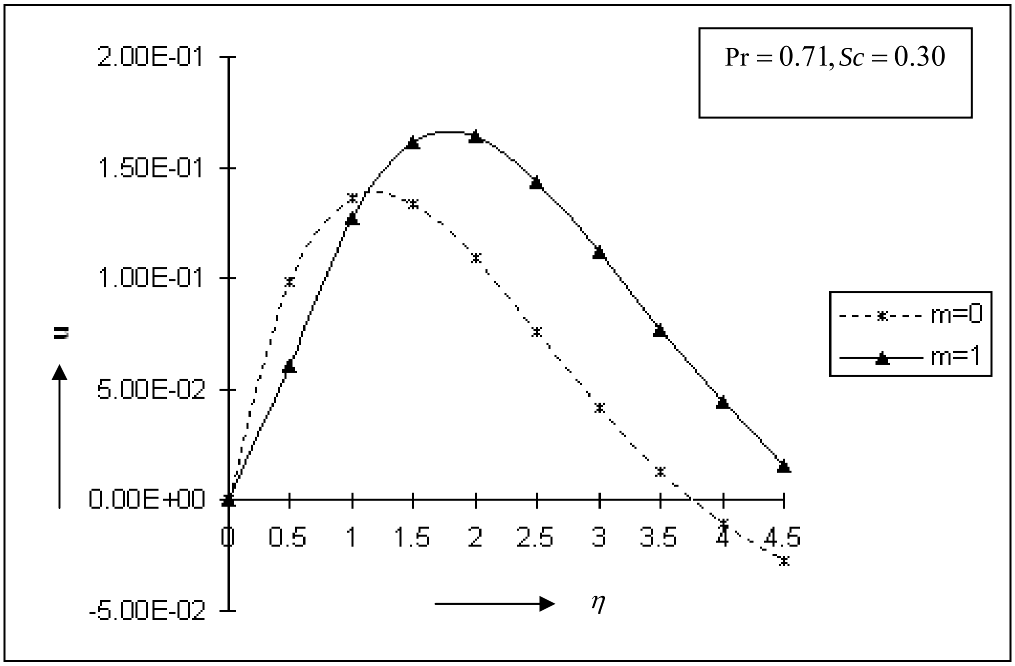

Figure 3: Variation of velocity component u against η

when $S=1, \Omega=1, Gr=10,\ Gc=-5, \Omega t=\pi/2, M=5, S_0=0.2, k=1$

Figures 2 and 3 given above depict the variation of the velocity component u against η under the influence of Soret number S_0 and Hall parameter m respectively. It is seen from figure 2 that the velocity component u increases as S_0 decreases. It may be noted from (21) that as S_0 decreases, D_T decreases. This leads to a fall in chemical thermal diffusivity and a subsequent increase in the species concentration gradient. Consequently, the buoyancy forces due to concentration differences increase, thereby causing the main flow component u to increase. We also notice from this figure that this increase is significant for large values of η only. From figure 3 we observe that u rises as m (Hall parameter) decreases for small values of η whereas it illustrates a reversal of the aforementioned trend as $\eta \to \infty$.

The following figures 4 and 5 exhibit the change of behavior of the velocity component w against η under the effect of Soret number and Hall parameter respectively. From figure 4 it is clear that w increases in sign as S_0 decreases. Again, figure 5 depicts that w rises as Hall parameter rises for small values of η whereas this behavior takes a reverse trend as $\eta \to \infty$. The Soret effect on ‘u’ and ‘w’ is negligible near the plate-surface, but is noteworthy at large distances from the plate-surface.

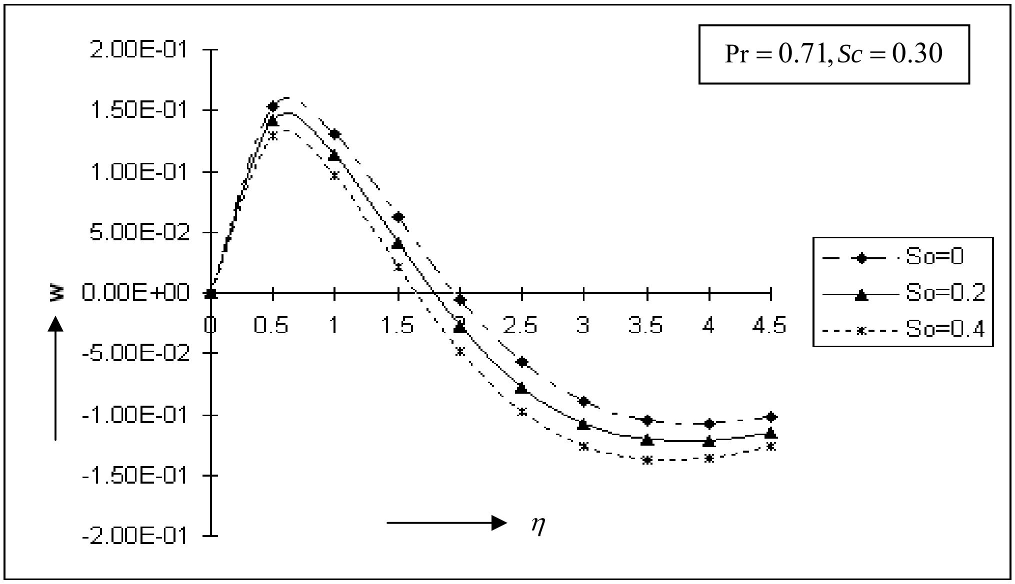

Figure 4: Variation of velocity component w against η

when $S=1, \Omega=1, Gr=10,\ Gc=-5, \Omega t=\pi/2, m=0.5, M=5, k=1$

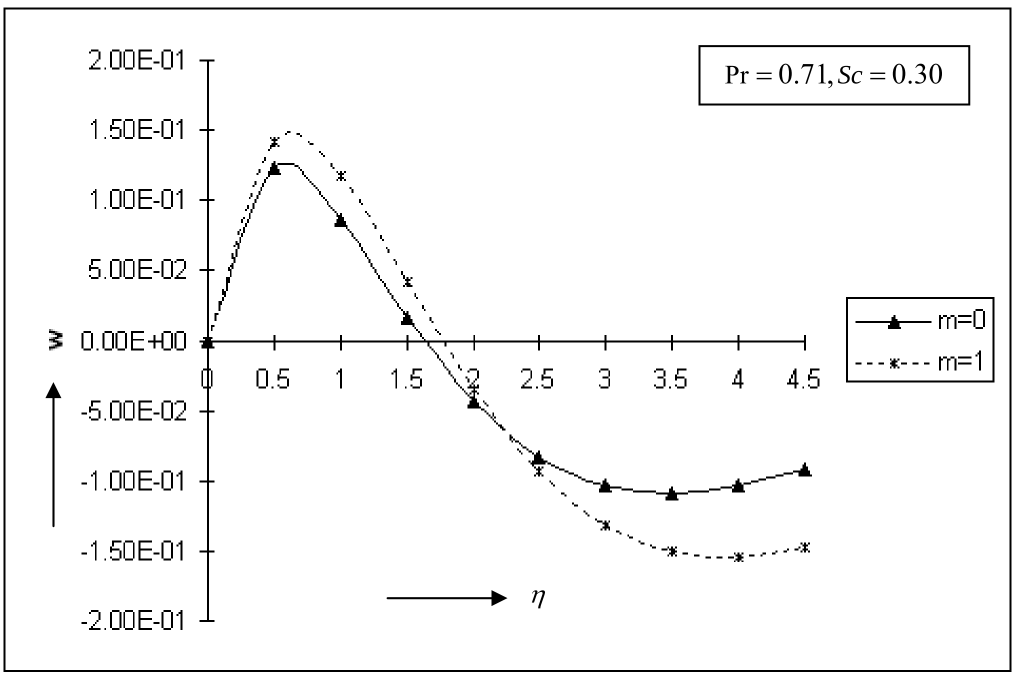

Figure 5: Variation of velocity component w against η

when $S=1, \Omega=1, Gr=10,\ Gc=-5, \Omega t=\pi/2, M=5, S_o=0.2, k=1$

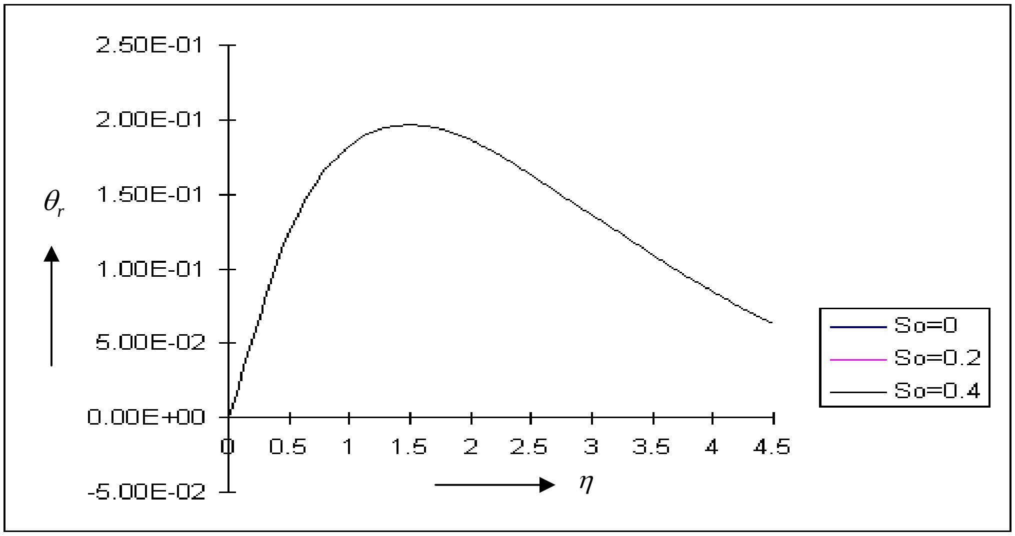

Figure 6: Variation of temperature θ_r against η

when $S = 1, \Omega = 1, Gr = 10,\ Gc = -5, \Omega t = \pi/2, m = 0.5, M = 5, k = 1$.

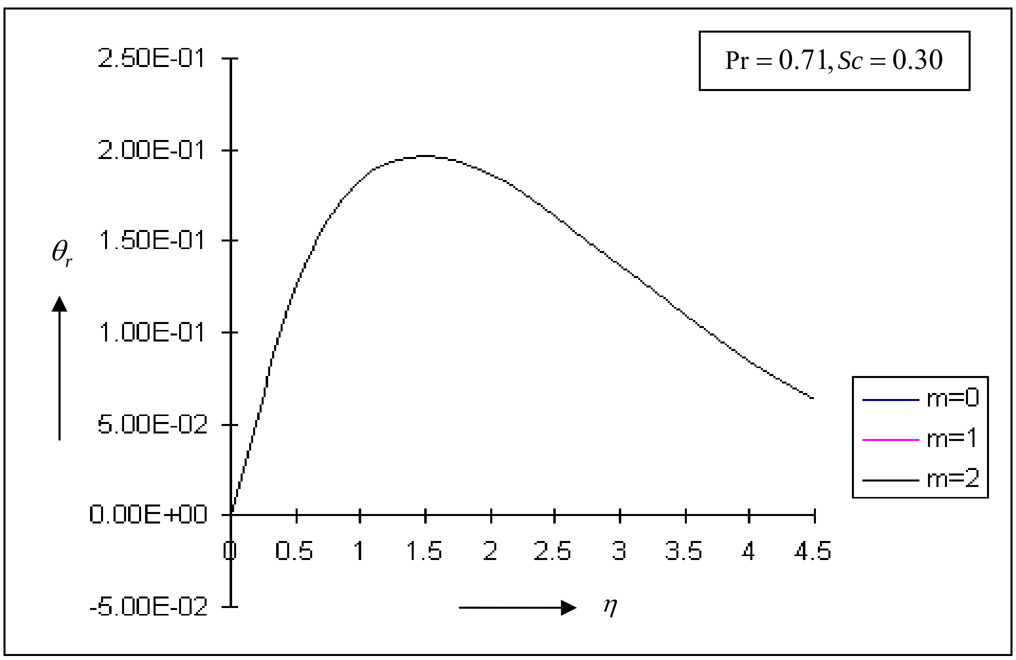

Figure 7: Variation of temperature θ_r against η

when $S = 1, \Omega = 1, Gr = 10,\ Gc = -5, \Omega t = \pi/2, M = 5, S_0 = 0.2, k = 1$.

In figures 6 and 7 the effects of Soret number S_0 and Hall parameter m on the temperature field θ_r against η are shown, respectively. We see from figures 6 and 7 that θ_r increases sharply for small values of increasing η, irrespective of the choice of values of S_0 and m. The same figures also indicate a steady fall in θ_r for comparatively larger values of η i.e. as $\eta \to \infty$, regardless of the choice of values of S_0 and m. This is attributable to the fact that a heated plate is considered and hence, near the hot plate, the fluid temperature rises sharply and then again declines with increasing distance from the plate-surface.

The variation of the concentration C_r at the plate under the influence of Soret number S_0 and Hall m parameter are presented in the following figures 8 and 9 respectively. From figure 8, it is inferred that C_r increases as S_0 increases whereas from figures 8 and 9 we notice that C_r increases sharply for small values of η and then again declines steadily as $\eta \to \infty$, for any value of S_0 and m. Further, the Soret effect on C_r is negligible near the plate-surface and is marked at large distances from the plate-surface. A rise in S_0 causes a greater chemical thermal diffusivity at the plate's surface. Hence, C_r rises as S_0 increases.

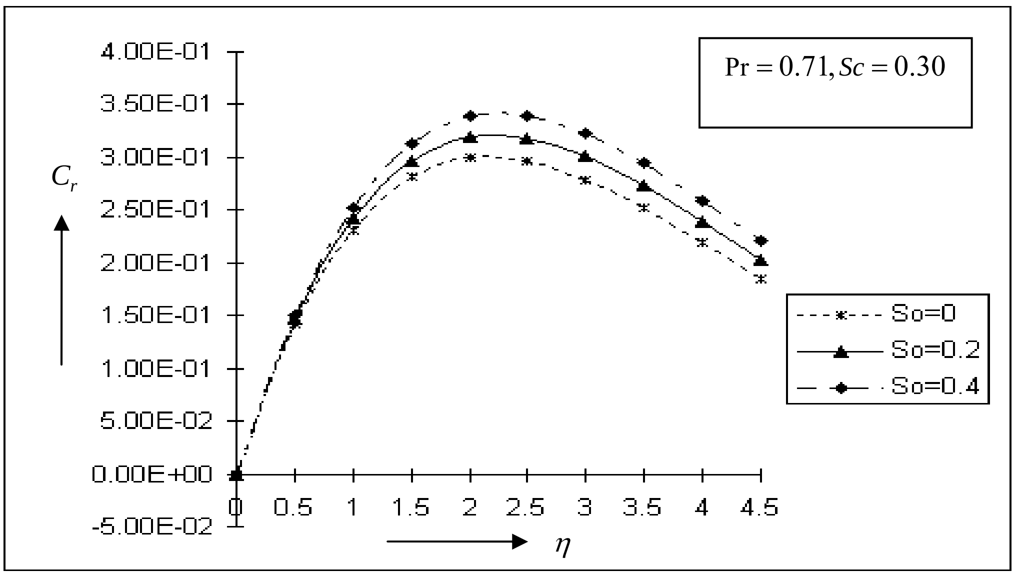

Figure 8: Variation of concentration C_r against η

when $S = 1, \Omega = 1, Gr = 10, Gc = -5, \Omega t = \pi/2, m = 0.5, M = 5, k = 1$.

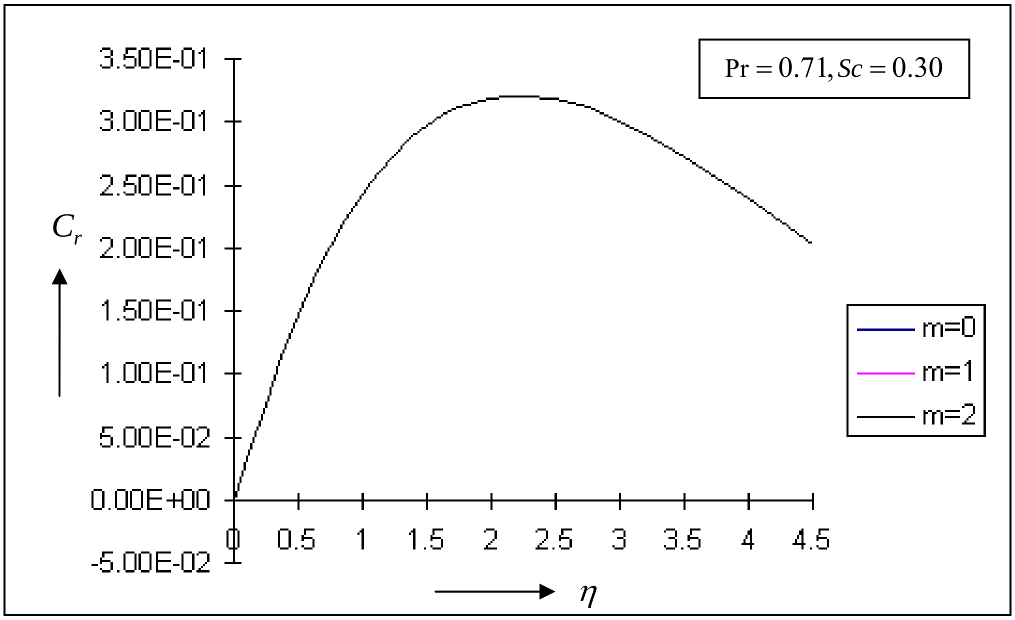

Figure 9: Variation of concentration C_r against η

when $S = 1, \Omega = 1, Gr = 10, Gc = -5, \Omega t = \pi/2, M = 5, S_0 = 0.2, k = 1$.

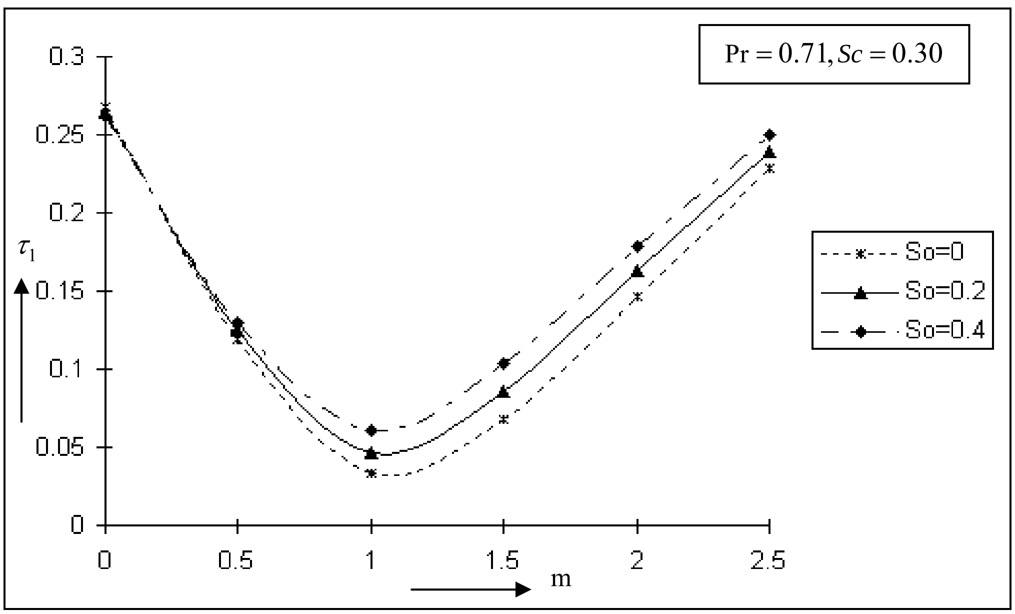

Figure 10: Variation of shearing stress τ_1 against m

when $S = 1, \Omega = 1, Gr = 10, Gc = -5, M = 5, k = 1$.

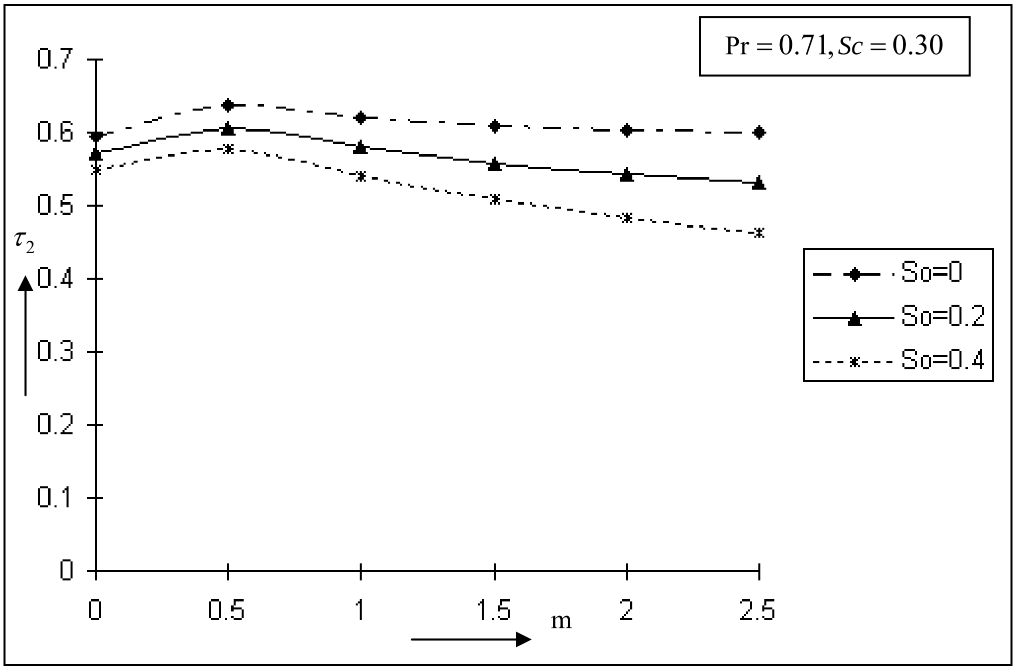

Figure 11: Variation of shearing stress τ_2 against m

when $S = 1, \Omega = 1, Gr = 10, Gc = -5, M = 5, k = 1$.

The above figures 10 and 11 respectively demonstrate the variations of the shearing stresses τ_1 and τ_2 against the Hall parameter m, under the Soret effect. It is seen that τ_1 increases for increasing values of S_0, in figure 10. A rise in S_0 causes an increase in chemical thermal diffusivity at the plate's surface. The increased molecular activity at the plate surface causes τ_1 to increase. Again, figure 11 exhibits a rise in τ_2 for decreasing values of S_0. A fall in S_0 causes a decrease in chemical thermal diffusivity at the plate's surface, thereby causing the cross-flow shear stress τ_2 to rise. The same figures also indicate that the Soret effect on τ_1 and τ_2 assumes significance for larger values of m. Further, we note from these figures that for small values of m, τ_1 falls sharply and τ_2 rises. However, for larger values of m, we note that τ_1 increases sharply and τ_2 falls steadily.

The graphs for $S_0 = 0$ (i.e. in absence of Soret effect) are almost identical with the graphs in the problem (sans Soret effect) studied by Sharma *et al*. (2007). This clearly supports validity of our results, when compared to those obtained by Sharma *et al*. (2007).

5. Conclusions

a) A decrease in the Soret effect leads to an increase in both the main flow and cross-flow velocities. Also, the Soret effect on the main flow and cross-flow fields is noteworthy at large distances from the plate surface.
b) In the fluid region close to the plate-surface, the hall effect causes the main flow-field to decrease and the cross-flow field to increase. However, at large distances from the plate surface, the hall effect causes the main-flow field to rise and the cross-flow field to fall.
c) The buoyancy effects are significant near the plate-surface, as observed from the figures 2 to 5. This is characterized by the steep rise in the velocity fields namely 'u' and 'w', near the plate-surface.
d) The Soret effect causes the main-flow shear stress to rise and the cross-flow shear stress to fall. The concentration at the plate-surface increases under Soret effect.

e) The similarity in the temperature and concentration profiles indicates that heat and mass transfer phenomenon are analogous processes.

Nomenclature

$\vec{B}$ **[-]**	magnetic induction vector
B_0 [Tesla]	intensity of the applied magnetic field
C [-]	dimensionless species concentration of the fluid
C_p [J/kg K]	specific heat at constant pressure
C_∞ [kmol/m^3]	species concentration far away from the plate
C^* [kmol/m^3]	species concentration of the fluid at the plate
D [m^2 s^{-1}]	coefficient of chemical molecular mass diffusivity
D_T [M^{1}L^{-1}T^{-1}K^{-1}]	coefficient of chemical thermal diffusivity
$\vec{E}$ [-]	electric field
e [Coulomb]	electron charge
Gr [-]	Grashof number for heat transfer
Gc [-]	Grashof number for mass transfer
g [m s^{-2}]	acceleration due to gravity
$\vec{J}$ [-]	electric current density
κ [W/mK]	thermal conductivity
k []	permeability of the porous medium
M [-]	magnetic field parameter (Hartmann number)
m [-]	Hall parameter
Pr [-]	Prandtl number
p_e [-]	electron pressure
S [-]	source parameter
Sc [-]	Schmidt number
S_0 [-]	Soret number
T [K]	temperature of the fluid in the boundary layer
T_∞ [K]	fluid temperature far away from the plate
t [T]	time
u [-]	x-component of the velocity vector
$\vec{V}$ [-]	velocity vector
V_0 [m s^{-1}]	reference velocity
w [m s^{-1}]	z-component of $\vec{V}$

Greek symbols

β [K^{-1}]	coefficient of volume expansion for heat transfer
β^* [m^3/k mol]	coefficient of volume expansion for mass transfer
ρ [Kg/m^{-3}]	fluid density in the boundary layer
υ [m^2 s^{-1}]	kinematic viscosity
σ [Ω^{-1} m^{-1}]	electrical conductivity
θ [-]	dimensionless temperature
ϕ [W m^{-3}]	frictional heat
Ω [-]	non-dimensional frequency parameter

ω_e [-] electron frequency

ω_i [-] ion frequency

τ_e [T] electron collision time

τ_i [T] ion collision time

η_e [Coulomb] electron charge

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