On the Off-Axis Strength Test for Anisotropic Materials[1]

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ABSTRACT

Difficulty in the experimental determination of the interaction component of strength tensor of boron-epoxy composites by off axis tests is discussed. Although off axis data agree well with the prediction of the strength tensor theory, the uniaxial data cannot be used to back calculate the interaction term. The data also showed that responses due to normal and shear stresses can be uncoupled in the linear as well as nonlinear regions.

INTRODUCTION

DETERMINATION OF THE ultimate strength of fiber-reinforced composite materials subjected to biaxial states of stress has recently been the subject of considerable interest [1–4]. Recently, Tsai and Wu [1] introduced a failure criterion for anisotropic materials which offers a significant improvement in

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operational simplicity over many current failure criteria. The basic assumption of this criterion is that there exists a failure surface in the stress space of the following form:

$$F_i \sigma_i + F_{ij} \sigma_{ij} = 1 \quad (i, j = 1\text{–}6) \tag{1}$$

F_i and F_{ij} are strength tensors of second and fourth rank, respectively. When consideration is limited to specially orthotropic materials, subjected to planar states of stress, Equation (1) reduces to the following:

$$F_1\sigma_1 + F_2\sigma_2 + F_6\sigma_6 + F_{11}\sigma_1{}^2 + F_{22}\sigma_2{}^2 + F_{66}\sigma_6{}^2 + 2F_{12}\sigma_1\sigma_2 = 1 \tag{2}$$

Both F_6 and F_{66} may be determined by pure shear tests, while F_1, F_2, F_{11}, and F_{22} may be determined by uniaxial tension and compression tests. F_{12} alone must be determined in a biaxial test. Previous investigations have shown that the off-axis tensile coupon, possesses the potential for relatively inexpensive determination of selected biaxial strength properties of composite materials [5–7]. However, recent studies have indicated that the off-axis tensile coupon is not effective in determination of F_{12} for graphite-epoxy. It is, therefore, the objective of this paper to examine the off-axis tensile test and determine its utility as a biaxial characterization specimen for boron-epoxy (AVCO 5505).

SPECIMEN DESIGN AND TEST

There are two goals in the design and test of the off-axis tensile specimen. First, the existence of a statically determinant, uniaxial state of stress within the test section must be assured. Recent analytic studies have revealed that the first goal may be accomplished by establishing the specimen geometry such that the length-to-width ratio is a practical maximum [8]. In the current study the maximum specimen length was limited to 14 inches by the physical dimensions of the Instron test machine. Hence, the specimen length-to-width ratio was controlled by varying the specimen width. The proper specimen width was chosen by examining the stress field in the specimen of maximum anisotropic behavior, $\theta = 15^\circ$, and reducing the difference between the predicted state of stress [8] and the desired state of stress to less than 5 percent. A specimen width of 0.5 inch was chosen for the boron-epoxy characterization study.

The second goal of the design and test of the off-axis tensile specimen is to assure that the elastic responses in the in-plane shear and transverse tension modes are consistent and that failure occurs within the specimen test section. In an effort to establish the consistency of the elastic response of each off-axis specimen, electrical resistance strain rosettes were attached to the specimens and monitored during test. The strain rosette provided both the normal and shear strain components in a coordinate system parallel and perpendicular to the axis of the specimen. By transforming these strain components and the uniaxial state of stress to the fiber coordinate system, it was possible to examine separately the shear and normal elastic responses. The consistency of the shear response of each specimen is illustrated in Figure 1 where a composite of the responses of the 15°, 30°, 45°, and

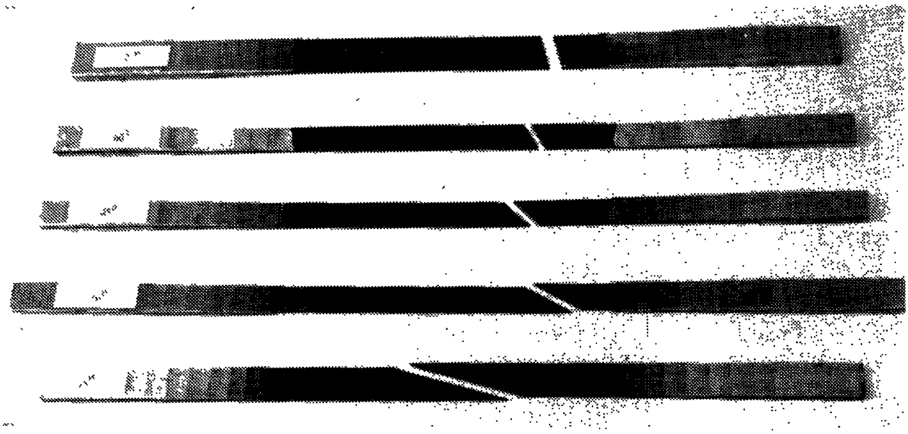

Figure 1. Off-axis tensile coupon specimens.

60° off-axis coupons is superimposed upon the established shear response for boron-epoxy [9]. After correcting the transverse normal strain for the effects of Poisson induced strain, the consistency of the transverse response could be

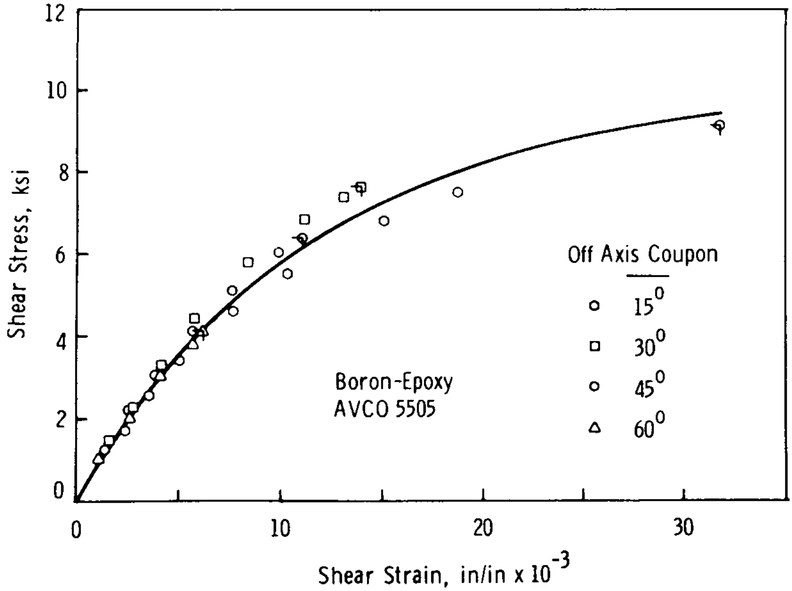

Figure 2. Composite shear stress-strain response.

examined. These results, shown in Figure 2, reveal excellent agreement between the composite and established transverse tension responses. It is significant to note that the normal and shear responses decouple not only within the linear-elastic region, as expected, but also within the non-linear region of response. Rosen recently substantiated this result while examining the non-linear response of the ± 45° boron-epoxy coupon [10]. In order to insure test section failures, each specimen was reinforced in the region of load introduction with highly tapered glass-epoxy end tabs. The fiber orientation of the end tabs was selected to be equal to that of each specimen in an effort to match the anisotropic behavior of coupon and end tab. Failed specimens shown in Figure 3 reveal that this technique was successful.

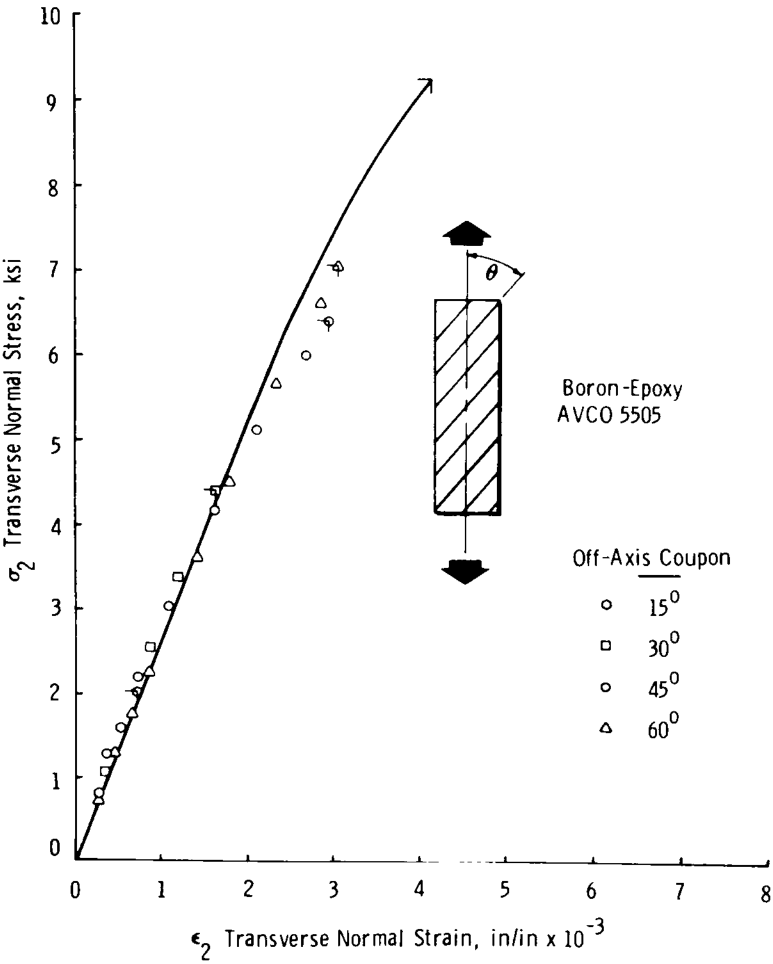

Figure 3. Composite transverse normal stress-strain response.

FAILURE CRITERIA

The utility of the off-axis specimen for biaxial characterization of a composite material such as boron-epoxy is determined by the ability and sensitivity of the specimen to establish the failure envelope for the material. For the purposes of this study, we employed the off-axis tensile coupon to determine the strength tensors of the Tsai-Wu strength criterion [1]. Material strengths for the boron-epoxy material system (AVCO 5505) utilized in determination of F_i and F_{ij} are as follows:

$$X = 1.88 \times 10^5 \text{ psi} \qquad Y = 0.9 \times 10^4 \text{ psi} \qquad S = 1.0 \times 10^4 \text{ psi}$$

$$X' = 3.61 \times 10^5 \text{ psi} \qquad Y' = 4.5 \times 10^4 \text{ psi} \qquad S' = 1.0 \times 10^4 \text{ psi}$$

Where X and X' are the tensile and compressive strength in the fiber direction, respectively; Y and Y' are tensile and compressive strength normal to the fiber direction, respectively; and S and S' are positive and negative shear strengths, respectively. These material strengths allow determination of all the F_i and F_{ij} except F_{12}.

$$F_1 = 2.55 \times 10^{-6} \text{in}^2/\text{lb} \qquad F_{11} = 1.47 \times 10^{-11} \text{in}^4/\text{lb}^2$$

$$F_2 = 8.89 \times 10^{-5} \text{in}^2/\text{lb} \qquad F_{22} = 2.47 \times 10^{-9} \text{ in}^4/\text{lb}^2$$

$$F_6 = 0.00 \qquad F_{66} = 1.00 \times 10^{-8} \text{ in}^4/\text{lb}^2$$

As mentioned earlier, it is necessary to determine the stress coordinates of a single biaxial failure point in order to determine F_{12}. This can be achieved by utilization of the state of stress in the fiber coordinate system of the off-axis tensile coupon as given by the following:

$$\sigma_1 = \frac{S}{2} (1 + \cos 2\theta)$$
$$\sigma_2 = \frac{S}{2} (1 - \cos 2\theta) \tag{3}$$
$$\sigma_6 = \frac{S}{2} \sin 2\theta$$

where S is the tensile strength and θ is the fiber orientation of the specimen. Combining Equations (2) and (3) yields an equation for determination of F_{12}.

$$2F_{12} = [4 - 2F_1 S(1 + \cos 2\theta) - 2F_2 S(1 - \cos 2\theta) - 2F_6 S \sin 2\theta - F_{11}S^2 (1 + \cos 2\theta)^2 - F_{22}S^2 (1 - \cos 2\theta)^2 - F_{66}S^2 \sin^2 2\theta] / S^2(1 - \cos^2 2\theta) \tag{4}$$

Off-axis specimens of four fiber orientations were employed in this study and their experimentally determined strengths are given in Table 1. Employing the average strength of the coupons, F_{12} was determined for each of the four fiber orientations. The values determined for F_{12} are tabulated in Table 2.

Table 1.

θ, Degrees	S, psi	S avg., psi	θ, degrees	S, psi	S avg., psi
15	29,952		30	17,684	
15	36,242	33,546	30	18,226	17,866
15	34,445		30	17,730	
45	12,114		30	17,860	
45	12,589	12,312	60	8,966	9,275
45	12,232		60	9,585	

Table 2

θ, Degrees	$F_{12} \times 10^{-10}\, in^4/lb^2$
15	− 0.58
30	− 6.82
45	− 4.76
60	+29.50

In order that the failure surface intercept all stress axes and form a closed geometric surface, F_{12} must satisfy the following stability criterion

$$F_{12}^2 \leqslant F_{11}F_{22} \tag{5}$$

Hence, F_{12} is constrained to a value within the interval $-1.91 \times 10^{-10} \leqslant F_{12} \leqslant +1.91 \times 10^{-10}$. The data presented in Table 2 reveals, however, that only the value of F_{12} determined by test of the 15° off-axis coupon falls within this interval and satisfies the stability criterion. Since, the consistency of the elastic responses of the specimens of each fiber orientation has been demonstrated, variations in laminate quality and experimental test procedures may be eliminated as a source of the large variation observed in the experimental results for F_{12}. Hence, the sensitivity of the off-axis tensile test must be considered as a potential source of the large data variation. Tsai and Wu observed this phenomenon for a graphite-epoxy material system [1]. In order to ascertain the sensitivity of the off-axis tensile specimen for determination of F_{12}, it is necessary to examine the functional dependence of F_{12}

upon S, the tensile strength of the specimen as given in Equation (4). Employing properties given earlier for boron-epoxy (AVCO 5505), Equation (4) is illustrated in Figure 4. These results show that a ±5 percent variation in the strength of the off-axis tensile specimen, S results in a variation in F_{12} of greater than ±1000 percent. Clearly, the off-axis tensile test exhibits very little sensitivity for

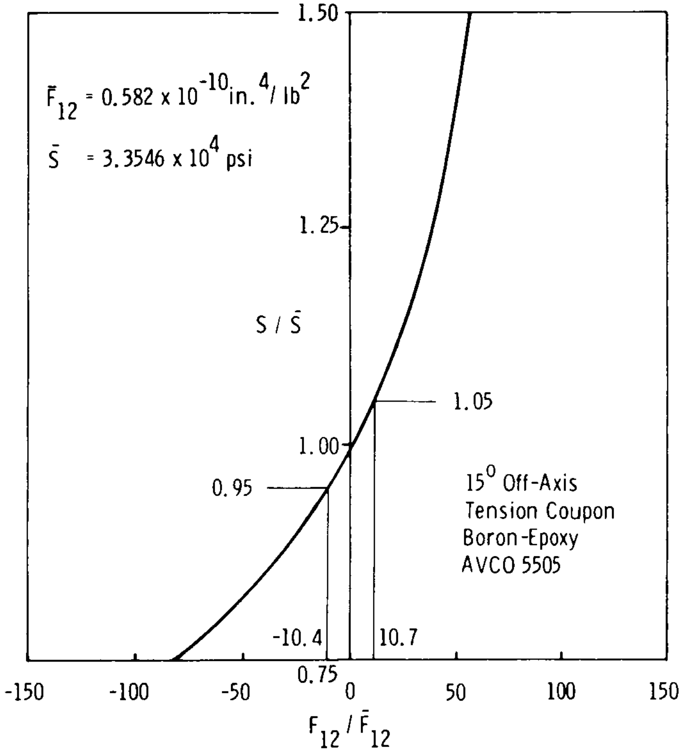

Figure 4. Off-axis tensile strength versus F_{12}.

determination of F_{12} for boron-epoxy and, therefore, may be considered as the source of the variation in the observed experimental data. The authors of reference [1] determined the off-axis compression test to be superior to the tensile test for determination of F_{12} for graphite-epoxy. Figure 5 reveals that the off-axis compression test exhibits excellent sensitivity in this determination where a ±5 percent variation in S results in only a ±3 percent variation in F_{12}. However, the off-axis compression test possesses certain practical disadvantages. First, the coupon specimen is unacceptable as an off-axis compression test since the specimen geometry which insures a statically determinant, uniaxial state of stress within the test section, yields a geometrically unstable test. Therefore, the off-axis compression test can be accomplished only when the cylindrical test specimen

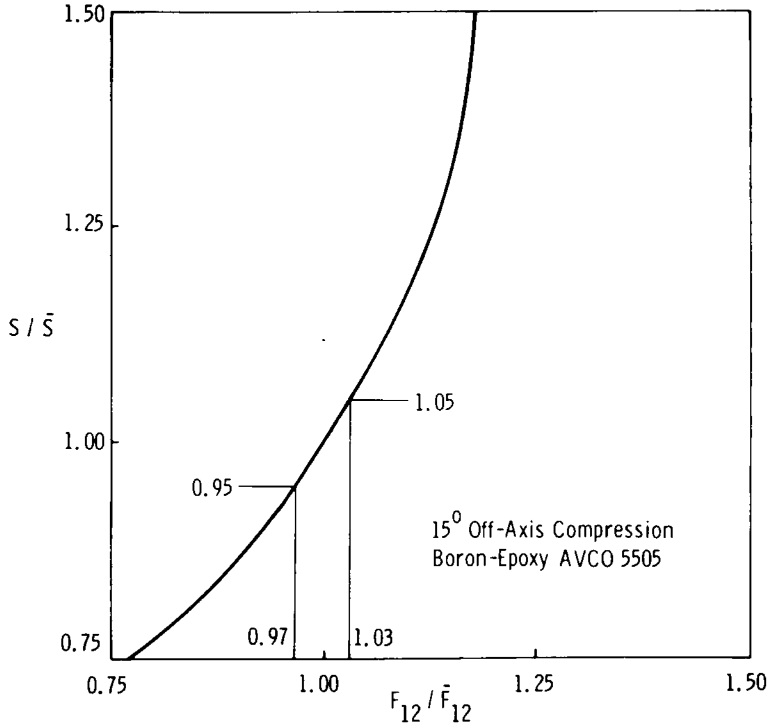

Figure 5. *Off-axis compression strength versus F_{12}.*

geometry is employed. When the off-axis cylindrical specimen is subjected to axial loads, shear deformation, resulting from the anisotropic behavior of the off-axis specimen, produces a rotation of one end of the specimen with respect to the other end. Another disadvantage of the off-axis compression test, therefore, is that the required experimental apparatus must allow free-rotation of one end of the specimen while applying an axial compressive load.

If the value determined by the 15° off-axis specimen is taken as an accurate estimate of F_{12}, we may compare the off-axis tensile strength of the boron-epoxy material system with predictions of the Tsai-Wu strength criterion [1]. Figure 6 reveals excellent agreement between this strength criterion and the experimental data. It is significant to note that both the strength criterion and the experimental data reveal that the failures of the 15°, 30°, 45°, and 60° specimens exhibit an interaction between the inplane shear and transverse tension stress components which results in strengths which are diminished from that predicted by maximum stress theory. The excellent agreement between the experimental data and predictions of the Tsai-Wu strength criterion is not sufficient to guarantee this strength criterion to be an accurate representation of failure throughout the stress space. This fact is demonstrated by the inability of the off-axis tensile test to accurately

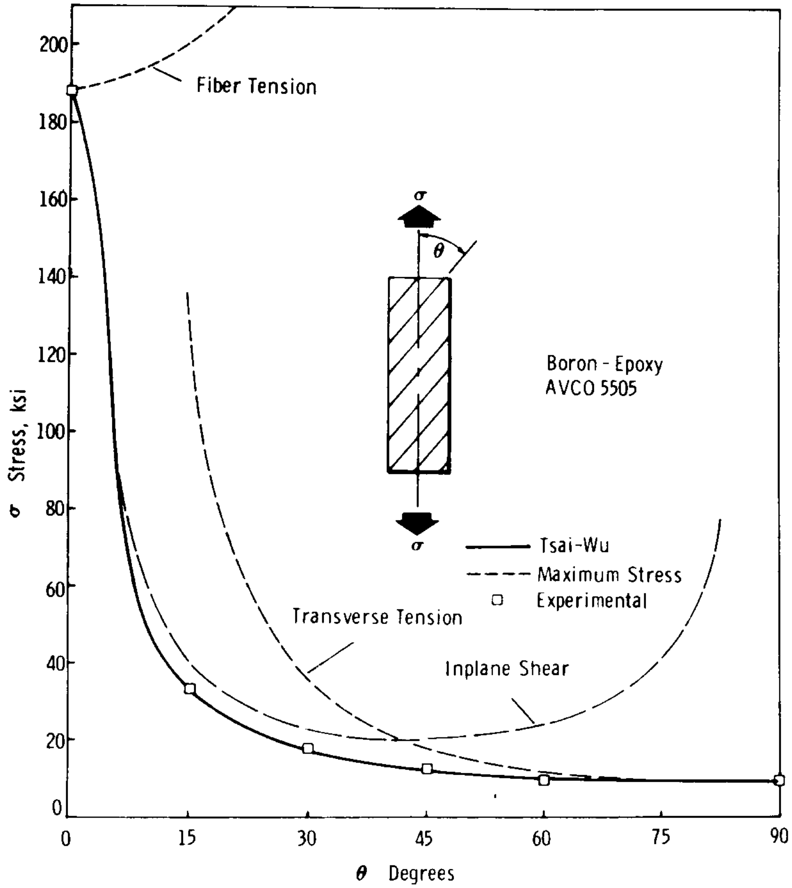

Figure 6. Off-axis tensile strength.

determine F_{12}. The good agreement between experiment and prediction is due, in a large part, to the fact that variations in F_{12} result in only second order changes in off-axis tensile strength predictions. In fact, the maximum distortional energy strength criterion yields off-axis tensile strength predictions which do not differ significantly from the Tsai-Wu criterion. Tsai-Wu criteria corresponding to two values of F_{12} (-0.58 and -4.76×10^{-10}) are shown in Figure 7. In addition, results for the maximum distortional energy criterion are compared to the Tsai-Wu criterion for prediction of off-axis tensile strength.

CONCLUSIONS

The results of this study have revealed that the off-axis tensile test is not adequate to discern differences in failure criteria for the boron-epoxy material system.

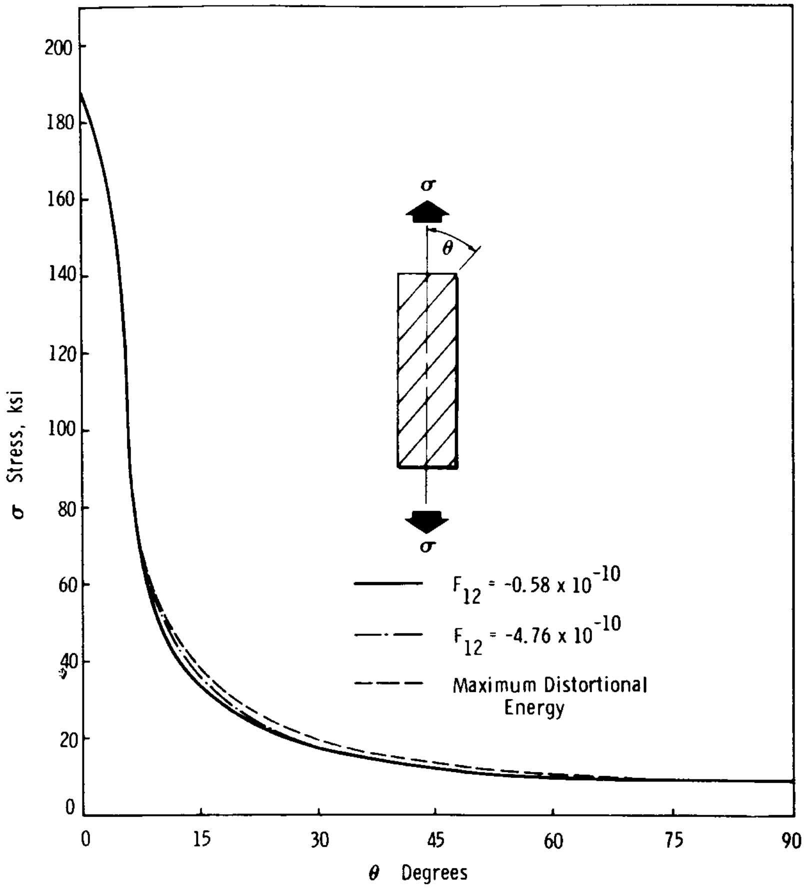

Figure 7. Influence of F_{12} on the off-axis tensile strength.

In addition, it is shown to possess very little sensitivity for determination of the Tsai-Wu criterion interaction term, F_{12} and should be employed only as an aid in verification of tubular characterization specimen results.

A second conclusion which may be drawn from this study is that the normal and shear responses for boron-epoxy may be decoupled within the non-linear response region. This result should simplify the non-linear analysis of boron-epoxy structural laminates.

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