

# An Improvement to The Relative Efficiency With Price Uncertainty: An Application to The Bank Branches

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## Abstract

This article describes the method of measuring relative efficiency, when the input and output prices are unknown. In a situation, where only the bound of input prices for the cost efficiency and the bound of output prices for the revenue efficiency are known, measurement of relative efficiency consists of two cases: optimistic and pessimistic perspective. The main object of this article is to study the pessimistic relative efficiency that eventually, with the computation of assessment of optimistic, it gives an interval efficiency for each DMU. Finally we apply the method in the analysis of bank branches activity.

*Keywords:* DEA, Cost efficiency, Revenue efficiency, Efficiency interval.

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## 1. Introduction

The present article is focused on the measurement of relative efficiency in the bank branches. We will particularly consider the computation of pessimistic cost efficiency and the pessimistic revenue efficiency. The cost efficiency estimates the capability of the current output production in the minimum cost. The revenue efficiency estimates the maximal benefit obtained from the outputs with the current input. The efficiency measurement of DMUs - in an envelopment analysis- dates back to the work of Debreu(1951) and Koopmans(1957).They provided the first measure of efficiency which is called 'coefficient of resource of utilization'. Farrell (1957) extended their work and proposed the measurement of cost efficiency by taking into account the economic context. In a new evaluation of efficiency, Cooper (1996) explained that the cost efficiency measurement in the Farrell's model, has a limited value in actual applications. The exact knowledge of prices is difficult and the range of prices will be changed in a short period of time.

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The extension of the model of Farrell's cost efficiency with the fixed prices to a model of price uncertainty was first done by Thompson(1996) and Schaffnit(1997). They described the optimistic cost efficiency in situation of input price uncertainty for DMUs and when only the minimal and maximal bound of price are estimable. Camanho and Dayson (2005) showed a model for the computation of cost efficiency measurement with the pessimistic input-price uncertainty. Their model has some problems which will be discussed in the next chapter. In the present paper we will offer a new model for the computation of pessimistic cost efficiency measurement that has no the problem of previous models and according to the existence of an integer constraint in the model of its structure, it has the form of an MIP.

## 2. Models of efficiency measurement

### 2.1 Cost efficiency measurement

To determine the cost efficiency measurement, Farrell designed the model of minimal cost as:

$$\begin{aligned}
 & \text{Min} \quad \sum_{i=1}^m p_{ijo} x_i^o \\
 & \text{s.t :} \\
 & \quad \sum_{j=1}^n x_{ij} \lambda_j = x_i^o \quad i = 1, \dots, m \\
 & \quad \sum_{j=1}^n y_{rj} \lambda_j \geq y_{rjo} \quad r = 1, \dots, s \\
 & \quad \lambda_j \geq 0, \quad j = 1, \dots, n \\
 & \quad x_i^o \geq 0, \quad i = 1, \dots, m
 \end{aligned} \tag{1}$$

in which  $p_{ijo}$  is the price of input  $i$  for the  $DMU_{jo}$  under assessment.  $x_i^o$  is a variable that at the optimal solution gives the amount of input  $i$  to be employed by  $DMU_{jo}$  in order to produce the current output at minimal cost. As it mentioned earlier, input prices are fixed but they can be different between  $DMUs$ . So the cost efficiency is -with the fixed and known prices for inputs- as follows:

$$ce_{jo} = \frac{\sum_{i=1}^m p_{ijo} x_i^o}{\sum_{i=1}^m p_{ijo} x_{ijo}}$$

in this manner, the revenue efficiency model with the fixed and known prices for outputs, is as follows:

$$\begin{aligned}
 & \text{Max} \quad \sum_{i=1}^m q_{r_{jo}} y_r^o \\
 & \text{s.t.} : \\
 & \quad \sum_{j=1}^n x_{ij} \lambda_j \leq x_{i_{jo}} \quad i = 1, \dots, m \\
 & \quad \sum_{j=1}^n y_{rj} \lambda_j \geq y_r^o \quad r = 1, \dots, s \\
 & \quad \lambda_j \geq 0 \quad j = 1, \dots, n \\
 & \quad y_r^o \geq 0 \quad r = 1, \dots, s
 \end{aligned} \tag{2}$$

in which  $q_{r_{jo}}$  is the price of output  $r$  for the  $DMU_{jo}$  under assessment.  $y_r^o$  is a variable that at the optimal solution gives the amount of output  $r$  to be employed by  $DMU_{jo}$  in order to produce maximal benefit obtained from outputs with the current inputs. As a result, revenue efficiency is as follows:

$$re_{jo} = \frac{\sum_{i=1}^m q_{r_{jo}} y_r^o}{\sum_{i=1}^m q_{r_{jo}} y_{r_{jo}}}$$

We can rewrite the measure of cost efficiency in a multiple case as the relative value of the input weights must be equal to the relative value of input prices observed at each  $DMU$ , so:

$$\frac{v_{i^a}}{v_{i^b}} = \frac{P_{i^a_{jo}}}{P_{i^b_{jo}}}, \quad i^a, i^b = 1, \dots, m$$

In this formula  $v_{i^a}$  and  $v_{i^b}$  are the input weights used for the cost efficiency assessment in a multiple case and  $P_{i^a_{jo}}$  and  $P_{i^b_{jo}}$  are the fixed and known input prices for  $DMU_{jo}$ . So the cost efficiency model in the multiple cases and with known prices is as follows:

$$\begin{aligned}
 & \text{Max} \quad k = \sum_{r=1}^s u_r y_{rjo} \\
 & \text{s.t.} : \\
 & \quad \sum_{i=1}^m v_i x_{ij} = 1 \\
 & \quad \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j = 1, \dots, n \\
 & \quad v_{i^a} - \frac{P_{i^a_{jo}}}{P_{i^b_{jo}}} v_{i^b} = 0 \\
 & \quad i^a < i^b, \quad i^a, i^b = 1, \dots, m \\
 & \quad u_r \geq \varepsilon, \quad r = 1, \dots, s.
 \end{aligned} \tag{3}$$

But as it mentioned before, Cooper (1996) described that the exact knowledge of prices is difficult and the range of prices will be changed in a short period of time. As a result, the model shown above has no any kind of actual application.

## 2.2 The measure of cost efficiency with price uncertainty

Thompson and Schaffint(1997) explained the model of optimistic cost efficiency measurement with price uncertainty. Their model in the multiple cases is as follows:

$$\begin{aligned}
 & \text{Max} \quad \sum_{r=1}^s u_r y_{rj_0} \\
 & \text{s.t :} \\
 & \quad \sum_{i=1}^m v_i x_{ij_0} = 1 \\
 & \quad \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n \\
 & \quad \frac{P_{i^a j_0}^{\min}}{P_{i^b j_0}^{\max}} \leq \frac{v_{i^a}}{v_{i^b}} \leq \frac{P_{i^a j_0}^{\max}}{P_{i^b j_0}^{\min}} \\
 & \quad i^a < i^b, \quad i^a, i^b = 1, \dots, m \\
 & \quad u_r \geq \varepsilon, \quad r = 1, \dots, s
 \end{aligned} \tag{4}$$

in which  $P_{i^a j_0}^{\max}, P_{i^a j_0}^{\min}$  are the upper and lower bounds of price of input  $i^a$  for  $DMU_{j_0}$  respectively, that means:

$$P_{i^a j_0}^{\min} \leq v_{i^a} \leq P_{i^a j_0}^{\max}$$

The model above is in the optimistic case. The easiest idea for obtaining the pessimistic case is the changing of objective function from maximization to minimization. The solution of the model became approximately zero and it is only the constraint  $u_r \geq \varepsilon, r = 1, \dots, s$  which prevents of becoming zero. Camanho and Dyson (2005) offered the model of cost efficiency price uncertainty in the pessimistic perspective. At first, they described a replaced formula for the multiple model of CCR and then introduced their model of cost efficiency with price uncertainty in a pessimistic perspective as below:

$$\begin{aligned}
\text{Min } \psi_{jojp} &= \sum_{r=1}^s u_r y_{rjo} \\
\text{s.t. :} & \\
& \sum_{i=1}^m v_i x_{ij} = 1 \\
& \sum_{r=1}^s u_r y_{rjp} - \sum_{i=1}^m v_i x_{ijp} = 0 \\
& \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n \\
& \frac{P_{i^a jo}^{\min}}{P_{i^b jo}^{\max}} \leq \frac{v_{i^a}}{v_{i^b}} \leq \frac{P_{i^a jo}^{\max}}{P_{i^b jo}^{\min}} \\
& i^a < i^b, \quad i^a, i^b = 1, \dots, m \\
& u_r \geq \varepsilon, \quad r = 1, \dots, s
\end{aligned} \tag{5}$$

in which the index  $jp$  represents the peer  $DMU_{jp}$  under assessment of  $DMU_{jo}$ . This model may have no feasible solution that means  $DMU_{jp}$  is not suitable as a peer for  $DMU_{jo}$ . In other words, this model will be only feasible if  $DMU_{jp}$  is located on the frontier of  $PPS$ . Meanwhile, in order to obtain an efficiency score for the  $DMUs$  under analysis via the model obtained above it requires solving  $n^2$  linear programming models. Plus to this, the model mentioned above, in application with multiple output dimensions leads to very low pessimistic estimates, without a clear managerial interpretation and it can give a suitable solution when it used for the single output.

### 3. An improvement

In this section we introduce a new model for the relative efficiency measurement in the pessimistic perspective. This model is an MIP and has no deficiencies of the previous models.

#### 3.1 The model of cost efficiency with price uncertainty (pessimistic perspective)

The main idea is as follows: We consider the multiple model of  $CCR$  in a situation of an optimistic perspective:

$$\begin{aligned}
\text{Max } & \frac{\left[ \frac{\sum_{r=1}^s \bar{u}_r y_{rjo}}{\sum_{i=1}^m \bar{v}_i x_{ij} } \right]}{\text{Max}_{1 \leq j \leq n} \left[ \frac{\sum_{r=1}^s \bar{u}_r y_{rj}}{\sum_{i=1}^m \bar{v}_i x_{ij}} \right]} \\
\text{s.t. } & \text{for all } i, r \quad \bar{u}_r, \bar{v}_i \geq 0
\end{aligned}$$

Now, we use the following change of the variables:

$$t = \frac{1}{\text{Max}_{1 \leq j \leq n} \left[ \frac{\sum_{r=1}^s \bar{u}_r y_{rj}}{\sum_{i=1}^m \bar{v}_i x_{ij}} \right]}$$

$$\begin{cases} t\bar{u}_r = u_r \\ t\bar{v}_i = v_i \end{cases}$$

As a result it gives:

$$\text{Max} \quad \frac{\sum_{r=1}^s u_r y_{rj_0}}{\sum_{i=1}^m v_i x_{i j_0}}$$

s.t :

$$\text{Max} \left\{ \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \right\} = 1 \quad 1 \leq j \leq n$$

$$\bar{u}_r, v_i \geq 0 \quad \text{for all } i, j$$

It is clear that this is the multiple model of CCR. So in a pessimistic perspective we have:

$$\text{Min} \quad \frac{\sum_{r=1}^s u_r y_{rj_0}}{\sum_{i=1}^m v_i x_{i j_0}}$$

s.t :

$$\text{Max}_{1 \leq j \leq n} \left\{ \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \right\} = 1$$

$$\text{for all } i, r, \quad \bar{u}_r, v_i \geq 0$$

If we use the change of variables again and introduce slack variables, we have:

$$\text{Min} \quad \sum_{r=1}^s u_r y_{rj_0}$$

s.t :

$$\sum_{i=1}^m v_i x_{i j_0} = 1$$

$$\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + s_j = 0, \quad j = 1, \dots, n \quad (6)$$

$$u_r \geq 0 \quad r = 1, \dots, s$$

$$v_i \geq 0$$

$$s_j \geq 0 \quad j = 1, \dots, n.$$

It is clear that model (6) is equivalent to (5) if there is an  $s_t$ ,  $t \in \{1, \dots, n\}$  and  $s_t = 0$ .

The only question that remains is how to appoint  $s_t$  equal to zero. This matter will be done by definition of a binary variable and adding some constraints in order making it linear.

$$\begin{aligned}
 & \text{Min} \quad \sum_{r=1}^s u_r y_{rjo} \\
 & \text{s.t.} : \\
 & \quad \sum_{i=1}^m v_i x_{ijo} = 1 \\
 & \quad \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + s_j = 0 \quad j = 1, \dots, n \\
 & \quad s_j \leq (1 - \rho_j) T \quad j = 1, \dots, n \\
 & \quad \sum_{j=1}^n \rho_j \geq 1 \\
 & \quad u_r \geq 0, \quad v_i \geq 0, \quad s_j \geq 0, \quad \rho_j \in \{0, 1\}, \text{ for all } i, j, r.
 \end{aligned} \tag{7}$$

In (7)  $T$  is a large positive and  $(u^*, v^*, \rho^*, s^*)$  is an optimum solution. If  $\rho_k = 1$  then  $s_k = 0$  and this leads to the favorite result.

Now, we are able to compute the cost efficiency with price uncertainty, using the model mentioned above and as a result we have:

$$\begin{aligned}
 & \text{Min} \quad \sum_{r=1}^s u_r y_{rjo} \\
 & \text{s.t.} : \\
 & \quad \sum_{i=1}^m v_i x_{ijo} = 1 \\
 & \quad \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + s_j = 0 \quad j = 1, \dots, n \\
 & \quad s_j \leq (1 - \rho_j) T \quad j = 1, \dots, n \\
 & \quad \sum_{j=1}^n \rho_j \geq 1 \\
 & \quad \frac{P_{i^a jo}^{\min}}{P_{i^b jo}^{\max}} \leq \frac{v_{i^a}}{v_{i^b}} \leq \frac{P_{i^a jo}^{\max}}{P_{i^b jo}^{\min}} \\
 & \quad i^a < i^b, \quad i^a, i^b = 1, \dots, m \\
 & \quad u_r \geq 0, \quad v_i \geq 0, \quad s_j \geq 0, \quad \rho_j \in \{0, 1\}, \text{ for all } i, j, r.
 \end{aligned}$$

If  $\rho_j = 1$  then  $s_j = 0$  and consequently we have an efficient *DMU*. The fourth constraint guarantees that at least one efficient *DMU* and the model -as we will show in the section 4-gives the acceptable results with having multiple outputs.

### 3.2 The model of revenue efficiency with uncertainty prices

In this section we extend the model discussed in the previous section. Now consider the problem below with an output nature:

$$\begin{aligned} \text{Min} \quad & \frac{\left[ \sum_{r=1}^s \bar{v}_i x_{i jo} \right]}{\left[ \sum_{i=1}^m \bar{u}_r y_{r jo} \right]} \\ & \min \left[ \frac{\sum_{r=1}^s \bar{v}_i x_{ij}}{\sum_{i=1}^m \bar{u}_r y_{rj}} \right] \\ \text{s.t. :} \quad & \bar{u}_r, \bar{v}_i \geq 0 \quad \text{for all } i, j. \end{aligned}$$

For making linear the model above we use the changing of variable

$$\frac{1}{t} = \min \left[ \frac{\sum_{i=1}^m \bar{v}_i x_{ij}}{\sum_{r=1}^s \bar{u}_r y_{rj}} \right] \quad \text{and} \quad \begin{cases} t \bar{u}_r = u_r \\ t \bar{v}_i = v_i \end{cases}$$

Then we have

$$\begin{aligned} \text{Min} \quad & \left[ \frac{\sum_{i=1}^m v_i x_{i jo}}{\sum_{r=1}^s u_r y_{r jo}} \right] \\ \text{s.t. :} \quad & \min \left[ \frac{\sum_{i=1}^m v_i x_{ij}}{\sum_{r=1}^s u_r y_{rj}} \right] = 1 \\ & \bar{u}_r, \bar{v}_i \geq 0 \quad \text{for all } i, j. \end{aligned}$$

Adding the slack variable we have:

$$\begin{aligned} \text{Min} \quad & \sum_{i=1}^m v_i x_{i jo} \\ \text{s.t. :} \quad & \sum_{r=1}^s u_r y_{r jo} = 1 \\ & - \sum_{r=1}^s u_r y_{r jo} - s_j = 0 \sum_{i=1}^m v_i x_{i jo}, \quad j = 1, \dots, n \\ & u_r, v_i \geq 0, \quad \text{for all } i, j. \end{aligned}$$



If in the model shown above there is an  $s_j$  which is equal to zero then we have an efficient *DMU*. Just like the previous section, now we gain the form below, using the binary variable and maximization of the objective function to the model of revenue efficiency:

$$\begin{aligned}
 & \text{Max} \quad \sum_{i=1}^m v_i x_{i j_0} \\
 & \text{s.t.} : \\
 & \quad \sum_{r=1}^s u_r y_{r j_0} = 1 \\
 & \quad - \sum_{r=1}^s u_r y_{r j_0} - s_j = 0 \quad \sum_{i=1}^m v_i x_{i j_0} \quad j = 1, \dots, n \\
 & \quad \quad \quad s_j \leq (1 - \rho_j) T \quad j = 1, \dots, n \\
 & \quad \quad \quad \sum_{j=1}^n \rho_j \geq 1 \\
 & \quad \quad \quad s_j \geq 0 \quad j = 1, \dots, n \\
 & \quad \quad \quad v_i \geq 0 \quad i = 1, \dots, m \\
 & \quad \quad \quad u_r \geq 0 \quad r = 1, \dots, s \\
 & \quad \quad \quad \rho_j \in \{0, 1\} \quad j = 1, \dots, n.
 \end{aligned}$$

Now adding weight restrictions of the prices of output, we obtain the model of revenue efficiency with price uncertainty in a pessimistic perspective. For adding the restrictions of the prices of output, we need to know the minimal and maximal prices for each output. If we show the minimal and maximal prices of each output  $r$ , as  $\mu^{\min}$  and  $\mu^{\max}$  then it leads to the model shown below which will be used for revenue efficiency measurement with price uncertainty in situation of a pessimistic perspective:

$$\begin{aligned}
 & \text{Max} \quad \sum_{i=1}^m v_i x_{i j_0} \\
 & \text{s.t.} : \\
 & \quad \sum_{r=1}^s u_r y_{r j_0} = 1 \\
 & \quad - \sum_{r=1}^s u_r y_{r j_0} - s_j = 0 \quad \sum_{i=1}^m v_i x_{i j_0} \quad j = 1, \dots, n \\
 & \quad \quad \quad s_j \leq (1 - \rho_j) T \quad j = 1, \dots, n \\
 & \quad \quad \quad \sum_{j=1}^n \rho_j \geq 1 \\
 & \quad \quad \quad s_j \geq 0 \quad j = 1, \dots, n \\
 & \quad \quad \quad v_i \geq \varepsilon \quad i = 1, \dots, m
 \end{aligned}$$

$$\frac{\mu_{r^a,jo}^{\min}}{\mu_{r^b,jo}^{\max}} \leq \frac{u_{i^a}}{u_{i^b}} \leq \frac{\mu_{r^a,jo}^{\max}}{\mu_{r^b,jo}^{\min}}$$

$$r^a < r^b, \quad r^a, r^b = 1, \dots, S$$

$$\rho_j \in \{0, 1\} \quad j = 1, \dots, n.$$

#### 4. An Empirical study

In order to provide a numerical illustration of the proposed approach, an example is given. Consider the data set in table 1 consisting of 25 bank branches each consuming three inputs to produce four outputs.

Inputs include number of staff, number of computer terminals and spaces, and output include deposits, loans, charge and benefits. The normalized data are listed in table 1. Running the model proposed in this paper yield to the results that are listed in table 2. The second column of the table gives the revenue efficiency interval. The cost efficiency interval is given in last column.

Table 1  
The data for the example

$DMU_j$	I1	I2	I3	O1	O2	O3	O4
1	0.4642	0.1179	0.1501	0.4642	0.1179	0.1501	0.3899
2	0.2522	0.1069	0.0995	0.2522	0.1069	0.0995	0.1823
3	0.1173	0.1508	0.1728	0.1763	0.1508	0.1728	0.1741
4	0.1624	0.0969	0.1250	0.1624	0.0969	0.1250	0.1097
5	0.1804	0.1289	0.1367	0.1804	0.1289	0.1367	0.0964
6	0.2090	0.1335	0.1470	0.2090	0.1335	0.1470	0.1395
7	0.1516	0.1325	0.1358	0.1516	0.1325	0.1358	0.0828
8	0.2293	0.0795	0.0712	0.2293	0.0795	0.0712	0.2354
9	0.3284	0.0951	0.0691	0.3284	0.0951	0.0691	0.2952
10	0.1259	0.1042	0.0670	0.1259	0.1042	0.0670	0.0429
11	0.3745	0.2322	0.2214	0.3745	0.2322	0.2214	0.1906
12	0.3521	0.0960	0.0743	0.3521	0.0960	0.0743	0.2463
13	0.2514	0.2048	0.2169	0.2514	0.2048	0.2169	0.1977
14	0.3810	0.0951	0.1175	0.3810	0.0951	0.1175	0.2331
15	0.6625	0.3665	0.3992	0.6625	0.3665	0.3992	1.0000
16	0.8265	1.0000	0.8969	0.8265	1.0000	0.8969	0.5690
17	1.0000	0.9863	1.0000	1.0000	0.9863	1.0000	0.3570
18	0.6009	0.4059	0.4902	0.6009	0.4059	0.4902	0.5915
19	0.6617	0.7322	0.6873	0.6617	0.7322	0.6873	0.9370
20	0.5464	0.6271	0.4119	0.5464	0.6271	0.4119	0.2558
21	0.7287	0.6280	0.5972	0.7287	0.6280	0.5972	0.5177
22	0.4038	0.1389	0.1789	0.4038	0.1389	0.1789	0.4870
23	0.6186	0.4516	0.3959	0.6186	0.4516	0.3959	0.3662
24	0.7309	0.4598	0.3239	0.7309	0.4598	0.3239	0.8213
25	0.3250	0.3135	0.2257	0.3250	0.3135	0.2257	0.2235

Table 2  
revenue and cost efficiency interval

$DMU_j$	Revenue efficiency interval	Cost efficiency interval
1	[ 0.0451 , 1.0000 ]	[ 0.3303 , 1.0000 ]
2	[ 0.4455 , 0.8489 ]	[ 0.5072 , 0.9999 ]
3	[ 0.2910 , 0.9431 ]	[ 0.2926 , 0.9999 ]
4	[ 0.2892 , 0.8412 ]	[ 0.2775 , 1.0000 ]
5	[ 0.2665 , 0.7823 ]	[ 0.2189 , 0.9998 ]
6	[ 0.3095 , 0.8056 ]	[ 0.2931 , 0.9999 ]
7	[ 0.2432 , 0.8117 ]	[ 0.1919 , 0.9999 ]
8	[ 0.4659 , 0.9915 ]	[ 0.5140 , 1.0000 ]
9	[ 0.3872 , 1.0000 ]	[ 0.3682 , 1.0000 ]
10	[ 0.2126 , 0.7928 ]	[ 0.1749 , 1.0000 ]
11	[ 0.3046 , 0.7593 ]	[ 0.2569 , 0.9997 ]
12	[ 0.3378 , 0.9473 ]	[ 0.3703 , 1.0000 ]
13	[ 0.2878 , 0.8636 ]	[ 0.2867 , 0.9999 ]
14	[ 0.3468 , 0.9175 ]	[ 0.3284 , 1.0000 ]
15	[ 0.0265 , 1.0000 ]	[ 0.4501 , 1.0000 ]
16	[ 0.0264 , 0.8910 ]	[ 0.1959 , 1.0000 ]
17	[ 0.1973 , 0.7905 ]	[ 0.1155 , 0.9991 ]
18	[ 0.3317 , 0.9208 ]	[ 0.3476 , 1.0000 ]
19	[ 0.3407 , 1.0000 ]	[ 0.2771 , 1.0000 ]
20	[ 0.2061 , 0.8637 ]	[ 0.1631 , 1.0000 ]
21	[ 0.2875 , 0.8370 ]	[ 0.2683 , 0.9995 ]
22	[ 0.5532 , 1.0000 ]	[ 0.4042 , 1.0000 ]
23	[ 0.3163 , 0.8030 ]	[ 0.2718 , 0.9996 ]
24	[ 0.5002 , 1.0000 ]	[ 0.5127 , 1.0000 ]
25	[ 0.2836 , 0.8807 ]	[ 0.2687 , 0.9999 ]

## 1. Conclusion

In this paper we have presented a method to describe the relative efficiency measurement, when the input and output prices are unknown. The main object in this paper is to study the pessimistic relative efficiency that eventually, with the computation of assessment of optimistic, it gives interval efficiency for each DMU. We have applied the method in the analysis of bank branches activity.

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