Non-Normal Real Estate Return Distributions by Property Type in the U.K.

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Abstract: Investment risk models with infinite variance provide a better description of distributions of individual property returns in the IPD database over the period 1981 to 2003 than Normally distributed risk models, which mirrors results in the U.S. and Australia using identical methodology. Real estate investment risk is heteroscedastic, but the Characteristic Exponent of the investment risk function is constant across time yet may vary by property type. Asset diversification is far less effective at reducing the impact of non-systematic investment risk on real estate portfolios than in the case of assets with Normally distributed investment risk. Multi-risk factor portfolio allocation models based on measures of investment codependence from finite-variance statistics are ineffectual in the real estate context.

Key words: Asset-specific risk, return distributions, non-Normality, diversification, institutional investing

As institutional investors expand their options for investment opportunities to a global arena, it is helpful to have an understanding of the behavioral characteristics of assets that might be purchased individually or in portfolios. If there are characteristic performance differences among assets in different countries, these differences might lead to differences in portfolio strategies for the global investor. However, if there are similarities among investment characteristics, then investors could realize efficiencies by extending effective strategies in the home country to foreign soil.

The data and analysis of this paper extend the research presented by Young and Graff (1995). In that empirical study of disaggregated NCREIF data in the U.S., Young and Graff found that cross-sectional annual returns were not Normally distributed during any year between 1978 and 1992. Additionally, the authors found that both the skewness and the magnitude of real estate risk changed over time. In a working paper, Young (2005) extends the time period to 2003 and finds nearly identical results. Graff, Harrington, and Young (1997) examine the shape of Australian institutional real estate returns with similar results, thereby leading to the suspicion that the findings are universal. This work carries the work one step further by applying the same methodology to U.K. data supplied by IPD.

All previous U.K. studies of property return distributions mentioned in the literature review below have utilized IPD data in one way or another. However, this study uses all available individual asset returns for each of the years 1981 to 2003—amounting to 269,853 return observations in total.

With these individual institutional-grade property performance data from IPD, it is possible to test empirically the presumptions that property return distributions have finite variance and are

Gaussian Normal. The purpose of this study is to test whether property return distributions have finite variance, and to examine the implications of the test results for real estate portfolio construction, investment, and management.

Stable Distributions

Normal distributions are stable and are the only stable distributions with finite variance. Other examples of stable distributions are the well-known Cauchy distributions. Although most stable distributions and their probability densities cannot be described in closed mathematical form, their characteristic functions—and the logarithms of the characteristic functions—can be written in closed form.[1] The log characteristic functions of stable distributions have the following form for cases where $\alpha \neq 1$:

$$\psi(t) = i\delta t - |\gamma t|^{\alpha}\left[1 - i\beta \operatorname{sgn}(t)\tan(\pi\alpha/2)\right] \quad (1)$$

The four parameters α, β, γ, and δ in Equation (1) completely characterize the distribution.

The Characteristic Exponent α lies in the half-open interval (0,2] and measures the rate at which the tails of the density function decline to zero. The larger the value of the Characteristic Exponent α, the faster the tails shrink toward zero. When α=2.0, the distribution is Normal.

While the means (first moments) of stable distributions with Characteristic Exponents α>1.0 do exist, variances (second moments) do not exist—i.e., are infinite—for those distributions with Characteristic Exponents α<2.0.

The Skewness Parameter β lies in the closed interval [-1,1], and is a measure of the asymmetry of the distribution. The closer the Characteristic Exponent α is to the upper limit of the permissible range—i.e., the value 2.0—the less significance the skewness has in terms of shifting the shape of the distribution away from the corresponding symmetric distribution. At the limit α=2.0, the Normal distribution, the Skewness Parameter β becomes irrelevant and all stable distributions are symmetric.

The Scale Parameter γ lies in the open interval $(0, \infty)$, and is a measure of the spread of the distribution. If α=2.0, the Scale Parameter γ is directly proportional to the standard deviation: $\gamma=\sigma/\sqrt{2}$. However, the Scale Parameter γ is finite for all stable distributions, despite the fact that the standard deviation is infinite for all α<2.0. Thus, the Scale Parameter γ can be regarded as a generalization of the standard deviation.

The Location Parameter δ may be any real number, and is a rough measure of the midpoint of the distribution. A change in δ simply shifts the graph of the distribution left or right, hence the term "location."

Previous Studies

There is a significant and growing body of literature that suggests that returns for asset classes are not Normally distributed. For instance, Young and Graff (1995) examined the annual returns distributions for U.S. institutional private real estate over the period 1980-1992 Using the NCREIF database. They decomposed individual property data (grouped by type) into two components: the mean return for a property type in any one year and a residual return for the individual property in that year. Then, using the methodology suggested by McCulloch (1986),

[1] The Normal and Cauchy distributions are the only stable distributions for which probability densities can be expressed in closed form in terms of elementary mathematical functions.

they found that the characteristic parameter alpha for the whole sample, at 1.48 was significantly below the value of 2.0 that characterizes the Normal distribution, a result that held for the great majority of years and property types. The beta parameters, the measure of skewness, were typically negative: for the whole sample and significantly different from zero at the 99% confidence level.

These findings broadly confirm those of Miles and McCue (1984) and Hartzell, Hekman, and Miles (1986) who find evidence of non-Normality in terms of skewness and kurtosis, and Myer and Webb (1994) who provide evidence of non-Normal kurtosis and autocorrelation in private real estate returns. In similar vein, Byrne and Lee (1997) using the Jarque Bera test examined quarterly returns for sector/region disaggregations of the NCREIF index between 1983 and 1994 and found that for ten of the sixteen sub-sectors Normality was rejected. Consistent with earlier findings, they found positive kurtosis and, typically, negative skewness.

In the U.K. research by Booth, Matysiak, and Ormerod (2002) for the Investment Property Forum tested the Normality of the 37 Investment Property Databank (IPD) sectors for which monthly data existed over the period from December 1986 to December 2000, with the exception of data for shopping centres and retail warehouses where data was from December 1994 and from December 1991 respectively. using standard statistical tests, Normality was rejected in 35 out of the 37 markets segments. The authors also examined monthly returns in the 37 sectors *relative* to the IPD Monthly Index, i.e., the monthly returns of each segment minus the returns of the market index. This is important for those fund managers who are active managers as the risk they face is tracking error risk, their performance relative to the market benchmark rather than total return. In 34 out of 37 cases the hypothesis of Normality was rejected, due to ‘fat tails.’ In other words, the risk of a large movement is greater than would be the case with a Normal distribution.

In a similar vein, Lee (2002) examined the distributional properties of the IPD monthly total returns data over the same period. The data divided into a number of property types and geographical regions making a total of 31 real estate market segments. The returns of the market index (IPDMI) were represented by the value-weighted performance of all the properties within the database. Lee (2002) found that that all market segments displayed significant positive skewness, except for offices in the City of London, which showed significantly negative skewness. In contrast, returns of the market index were fairly symmetric. The market and all the markets segments showed significant positive excess kurtosis (i.e., greater than 3). Thus, all the time-series data are leptokurtic and so display greater peakedness than expected from Normally distributed data. As a consequence, all the market segments exhibited significant departures from Normality at the 1% significance level, as shown by the Jarque-Bera (JB) statistic, while the returns of the market index were significant at the 10% level. Lee (2002) also found that the correlograms for the squared returns of the 31 market segments and the IPD index showed that beyond the first two lags the autocorrelation coefficients become increasing significant as the lag length increased, which implies that non-linear dependency is prevalent in all market segments and the market index. Strong autocorrelation in the squared returns is also a symptom of changing unconditional or conditional variance, which implies that the variance of all market segments and the market index may be time dependent.

Lizieri and Ward (2000) used the BestFit program to find the distribution that best characterized direct real estate data in the U.K.. The direct real estate data comprised monthly total returns for the period from December 1986 to December 1998 as reported by the IPD with series for all property and sub-indices for specified regional and property types. Lizieri and Ward

(2000) found that irrespective of region or property type, in general, the most appropriate distribution appeared to be the Logistic distribution, but that even here it is rejected in most cases. The main reason for the inappropriate fit was the leptokurtic nature of the real estate returns.

Interestingly, de-smoothing the data to account for alleged aberrant behavior attributed to the appraisal-based nature of capital returns did not result in returns distributions that are easier to model or that conform to Normality. Which the authors found resulted from the high proportion of returns that are close to zero, indicative of a thinly-traded market and slow arrival of information, resulting in static individual valuations. In other words, Lizieri and Ward (2000) found that de-smoothing monthly data does not correct for the thinness of the trading in the property market.

Lizieri and Ward (2000) then examined quarterly data arguing that if the atypical behavior of property returns can be explained by the thinness of the market and the lack of liquidity and trading, they expected to see the distributions to conform more closely to a Normal distribution over longer trading intervals. The analysis of quarterly data was consistent with this conjecture. Returns were easier to model and the Normal distribution was favored on a number of tests both for the aggregate index and at sub-sector level.

Brown (1991) and Brown and Matysiak (2000) investigated the returns distributional properties of individual data in the U.K.. Brown (1991) used individual data for 135 properties which comprised 39 Offices, 46 Retail and 50 Industrials over the period January 1979 to December 1982. On average the authors found that the data were more positively skewed and leptokurtic (peaked) relative to the Normal distribution, particularly in the Retail sector. In other words, the results would seem to rule out the proposition that the returns are drawn from a Normal distribution. However, when the data was combined into portfolios, the data began to approach the Normal distribution. In addition, when quarterly and half yearly data were used Brown (1991) finds that as the holding period gets longer the distribution of individual returns tends towards the Normal distribution, as suggested by Fama (1965). Results corroborated by Brown and Matysiak (2000) who used monthly returns of 100 individual properties; 40 Retail properties, 30 Office properties and 30 Industrial properties over the period December 1987 to November 1997.

Maurer, Reiner, and Sebastian (2004) have also compared the distributional properties of U.S., U.K., and German direct real estate returns. Using quarterly returns over the period 1987-2002 and three statistical test (Jarque/Bera, Anderson/Darling, and Shapiro/Wilks), Maurer et al (2004) find that in the case of the U.K. real estate market Normality could not be rejected as there was no significant skewness nor excess kurtosis. In contrast, both the U.S. and German real estate markets exhibited significant departures from Normality, but for different reasons. In the case of the German data Normality was rejected due to significant *positive* skewness, while the U.S. data could not be classified as Normal due to significant *negative* skewness and leptokurtosis (peakedness). In other words, real estate data in the U.K. conforms to the Normal distribution if quarterly nominal data are used confirming the findings of Lizieri and Ward (2000); Brown (1995) and Brown and Matysiak (2000), but it is questionable if the assumption of Normality can be used for German and U.S. data. However, once the data were de-smoothed, the results changed with Normality being accepted for Germany but rejected for the U.K. and the U.S. data. Nonetheless, when annual data were used the assumption of Normality could not be rejected for any country, either using the appraisal-based or de-smoothed data.

Finally, in Australia, Graff, Harrington, and Young (1997) examined the distributional characteristics of annual direct real estate data from the Property Council of Australia's Performance Index, over the period 1984-1996. Using the same methodology as Young and Graff (1995), the authors found that the mean alpha parameter, at 1.59 was again significantly below the value of 2.0 characteristic of a Normal distribution. However, the betas did not give any clear indication of skewness.

In summary, a number of studies have examined the distributional properties of direct real estate market data at the individual, sub-market, or index level in a number of countries, with generally broadly similar results. At the individual level, the data exhibit non-Normality, mainly due to excess kurtosis (peakedness) and significant skewness, although the skew can be positive or negative depending on the country. Normality is also rejected for sub-market and indices, again for the same reasons, especially if high frequency (monthly) data are used. However, when longer holding period (quarterly and annual) data are used, Normality is less likely to be rejected and, when it is rejected, this is usually due to the existence of excess of kurtosis rather than skewness. This is true for the raw or appraisal-based real estate data and for de-smoothed returns. In other words, lack of Normality in real estate data is mainly due to the thinness and the lack of liquidity of the market which cannot be corrected by any de-smoothing process.

Data Description

Property performance data in the U.K. are compiled by IPD using cash flow records supplied by major institutional investors, such as insurance companies, pension funds, and quoted property companies, that participate in its benchmarking services. All previous U.K. studies of property return distributions mentioned in the literature review have utilized IPD data in one way or another. However, this study uses all available individual asset returns for each of the years 1981 to 2003—amounting to 269,853 return observations in total. These are unleveraged returns on directly-held real estate assets. The main exclusions from the data were returns of assets in the year in which they were traded and returns from the 'Other' property category, which represents minor sectors of U.K. institutional investment, such as agricultural land or leisure, and which stood at only 3% of the value of the IPD annual index at the end of 2004 (IPD, 2005).

Reported returns are based on income and asset value changes (i.e., capital gains) as determined by appraisal. All appraisals in the IPD databank are conducted by external, third-party valuers. For some properties, monthly and quarterly frequency returns were available, but this study analyzes the full dataset of monthly, quarterly, and annually valued properties by examining annual frequency returns.

Before beginning the data analysis, each discrete annual sample return r_t in the data set was converted to its continuously compounded equivalent, $\ln(1 + r_t)$. These returns were then examined across all properties and within the Office, Retail, and Industrial property types.

Real Estate Return Model

A comparison of the data in the property type sub-indices reveals significant differences among the annual returns. Our real estate market model assumes that expected variations in annual property returns due to differences in property type account for all of the differences in returns on properties in the IPD database.

We assume that the observed annual total return on each commercial property p during the calendar year t is of the following form:

$$R_t(p) = \mu_t(h(p)) + \varepsilon_t(p) \quad (2)$$

where $h()$ is the property type (Office, Retail, or Industrial), $\mu_t()$ is the expected total return during year t as a function of property type, and $\varepsilon_t(p)$ is a stable (possibly, infinite-variance) random variable. In addition, we assume that, for each $t \geq 1981$, the $\varepsilon_t()$ are independent identically distributed random variables with Characteristic Exponent $\alpha_t > 1.0$ and zero mean, and that $\varepsilon_{t_1}(p_i)$ and $\varepsilon_{t_2}(p_j)$ are independent for all $t_1 \neq t_2$ and all i and j.[2]

Under these assumptions, the random variable $\varepsilon_t(p)$ corresponds to the asset-specific investment risk of property p during period t, while the systematic and market sector real estate risk is described by the function $\mu_t(h(\;))$.

Conventional approaches toward empirical real estate research have assumed the Normal probability distribution of asset-specific risk as an act of faith, and then apply statistical techniques to obtain descriptions of systematic and market-sector risk. By contrast, the tests of this paper examine asset-specific investment risk under the assumptions of our model, with the objectives of (1) confirming or rejecting real-world applicability of the model, and (2) obtaining additional statistical information about the likely shape of real estate investment risk. In particular, the focus of this investigation is the test of a model for the distributional form of $\varepsilon_t(p)$, the asset-specific risk.

Tests and Results

Exhibits 1a to 4a show the distributions of continuously compounded annual total returns for the years 1981-2003: (1) in the aggregate, and (2) by each of three property types. Superimposed upon the sample histograms are Normal densities with the corresponding means and standard deviations.[3] In each case, the sample density function is more peaked near the mean than the corresponding Normal density, has weaker shoulders and fatter tails (i.e., is leptokurtotic), and is negatively skewed. These distinctions can be seen more clearly in the graphs of the differences between each sample density and the corresponding Normal density, Exhibits 1b to 4b.

Before fitting stable distributions to the sample data, we corrected for possible extraneous data dispersion due to changing expected return by reducing each annual return by the corresponding sample mean for that calendar year and property type (cf. Equation (2)). The means are shown in Exhibit 5 for purposes of completeness, but will not be needed in the subsequent discussion.

McCulloch's (1986) quantile methodology was used to fit a stable distribution to each set of residuals arranged by calendar year and property type. To test whether the parameters varied during the sample period, stable parameters were estimated for sets composed of the residuals aggregated across calendar years and property types. These results are tabulated in Exhibit 5 and are displayed graphically together with one and two standard deviation error bands in Exhibits 6 to 9 for the parameters α, β, and γ (δ is irrelevant because the Location Parameter is an estimator for the mean and we adjusted for the effect of varying means).

2 The assumption that $\alpha_t > 1.0$ guarantees that the mean of $\varepsilon_t(p)$ exists.

3 There are 50 "bins" in the histogram that span the range from minus to plus five standard deviations. Because some samples extend beyond this range, all the samples beyond plus or minus five standard deviations are included in the two extreme bins.

In the case of Characteristic Exponents α_t estimated by calendar year and property type, 100% (69 of 69) were distinct statistically from 2.0—the Characteristic Exponent of the Normal distribution—with 95% confidence and 99% (68 of 69) were distinct from 2.0 with 99% confidence. In the case of residuals aggregated across property type (the first panel of Exhibit 5), all twenty-three sample Characteristic Exponents α_t were distinct from 2.0 with 99% confidence.

In the case of the Skewness Parameter β_t for all residuals aggregated across property type, 83% (19 of 23) were statistically significant (i.e., non-zero) with 99% confidence.

Exhibit 6 displays the sample Characteristic Exponents α_t of both the aggregated and individual property type residuals. It appears that α_t could be time-invariant. However, Exhibit 9 that shows graphical representations of these data, suggests that α_t likely varies across property type. From Exhibit 5, for the entire 1981 to 2003 sample period, estimates of Characteristic Exponents together with their standard errors are 1.448 ±0.004 for all three property types combined, 1.431 ±0.007 for Office properties, 1.471 ±0.006 for Retail properties, and 1.425 ±0.009 for Industrial properties.

By contrast, Exhibit 7 shows clearly that β_t is not time-invariant. Indeed, β_t for all properties displayed a roughly cyclic pattern throughout the test), and seem to track one another especially the Office and Industrial results.

Exhibit 8 shows clearly that the Scale Parameter γ is not time-invariant either in the aggregate or by property type. The general time-series patterns, however, are quite similar with roughly the same peaks and valleys. Since γ is the stable infinite-variance measure of risk, this means that asset-specific risk is heteroscedastic.

The three graphs of Exhibit 9 show the Characteristic Exponent, the Skewness, and the Scale Parameter for each property type and the aggregate over the full 1981 to 2003 time period along with the one- and two-sigma error bands. In terms of Skewness and Scale Parameter, all three property type results differ statistically from one another.

Because all twenty-three sample estimators for α_t are asymptotically Normal, the proposition that the true values are all equal (i.e., that α_t is time-invariant) can be tested by using the fact that, when it is true,

$$\sum w_i \left(x_i - \bar{x}\right)^2$$

is distributed as χ^2 on twenty-two degrees of freedom, where each weight w_i is given by the reciprocal of the asymptotic variance of x_i, and $\bar{x}$ is the weighted average of x_i (weighted by the w_i).

The last column of Exhibit 10 shows the year-by-year χ^2 components for the sample Characteristic Exponents with the total for the twenty-three-year period at the bottom of the column. The total is 521.99, which is substantially larger than the 0.05 significance level of 33.92 for twenty-two degrees of freedom. Although there are some exceptionally high χ^2 results in the year-by-year components, none of these years is notable in terms of market events or circumstances that could lead to the speculation that valuers had difficulty exceptional uncertainty as a cause of these nominal outliers.

The χ^2 test can also be used to test whether, for each year during the sample period, the individual property type α estimates are consistent with the hypothesis that the true values of α for the various property types are identical. More precisely, for each year in the sample period, let P_t be the hypothesis that the true values of α for the three property types in year *t* are identical (note that this does not assume that the true value for α is time-invariant). By computing the weighted average of sample property type α's for each year, the analog of the χ^2 test described above can be applied to test hypothesis P_t. This time the critical χ^2 value is 7.81, i.e., the 0.05 significance level of the χ^2 function for two degrees of freedom.

The resulting twenty-three χ^2 values are shown in the next-to-last column of Exhibit 10 (the corresponding χ^2 for the data aggregated across the sample period is shown at the bottom of the column). In only 48% of the cases (11 of 23), the observed sample value is below the 0.05 significance level.

The analogous χ^2 test for β_t can be used to test the proposition that β_t was time-invariant during the test period. The last column of Exhibit 11 shows the year-by-year χ^2 components of the Skewness Parameter with the total for the twenty-three-year period at the bottom of the column. The total is 2058.97, which is enormously larger than the 0.05 significance level of 33.92 for twenty-two degrees of freedom. Thus, there is no reasonable possibility that β_t was time-invariant during the sample period. Likewise, the individual property type β estimates for each year of the sample period show little similarity. In only 39% of the cases (9 of 23), the observed sample value is below the 0.05 significance level.

The above analysis implies that (1) real estate investment risk during the sample period was heteroscedastic; (2) during virtually all sample sub-periods and across property type, stable infinite-variance skewed asset-specific risk functions with a Characteristic Exponent α of approximately 1.448 modeled the observed distributions of return residuals better than Normally distributed risk candidates; and (3) property type differences in the Characteristic Exponent across property types are likely, certainly Retail properties showed notable differences from Office and Industrial over the full 1981 to 2003 sample period.

Implications for Portfolio Management

In the era of Modern Portfolio Theory, the central task of portfolio management is considered to be the optimization of the portfolio return/risk trade-off, subject to investment policy constraints on construction of portfolios. This involves asset selection and allocation to achieve two independent objectives: (1) minimization of the combined effect of asset-specific risk, and (2) optimization of the trade-off between portfolio return and systematic/sector risk.

The approach to this problem most often taken in portfolio research is: (a) specify the largest tolerable combined asset-specific risk; (b) calculate the minimum number of assets necessary to ensure that the combined effect of asset-specific risk is below the critical threshold; and (c) solve the trade-off problem under the additional constraint that investment funds be diversified among at least the number of assets determined in (b) for each permissible sector

To see what is involved in satisfying the additional constraint imposed by (b) above, it is helpful to make the following simplifying assumptions: all asset-specific risk functions are stable with the same Characteristic Exponent α and have the same Skewness Parameter β, all individual assets have the same level of asset-specific risk (proxied by the Scale Parameter γ of the distribution for the common asset-specific risk function), and the same percentage of the total portfolio value is invested in each component asset in the optimal portfolio. Then, letting *p* represent the portfolio, *f* the common asset-specific risk function, and using the relation between Scale Parameters of sums of stable random variables described in Equation (2):[4]

$$\gamma_p^{\alpha} = \gamma_{(1/n)f_1}^{\alpha} + \ldots + \gamma_{(1/n)f_n}^{\alpha}$$

4 Cf. Fama and Miller (1972), pp. 268-270, and Fama (1965b).

$$= n\gamma^{\alpha}_{(1/n)f} = n(1/n)\gamma^{\alpha}_{f}$$
$$= n(1/n)^{\alpha}\gamma^{\alpha}_{f} = n^{(1-\alpha)}\gamma^{\alpha}_{f}$$

This implies that:

$$\gamma_p = n^{(1/\alpha)-1}\gamma_f \quad (3)$$

Exhibit 13 shows the impact of varying α upon reduction in asset-specific risk for various numbers of properties in a portfolio. For any given $\alpha>1.0$, the reduction in asset-specific risk increases with increasing n. As α diminishes to 1.0 from its upper limit of 2.0, the reduction in asset-specific risk likewise diminishes for any given $n> 1$.

The sample value $\alpha=1.448$ from the preceding section implies the following practical estimate for the effect of portfolio diversification on asset-specific risk reduction:

$$c_p \approx n^{-0.309}c_f \quad (3')$$

A typical closed-end real estate fund or client separate account has 10 to 20 properties, and large open-end real estate funds might have about 100 properties. Under the above assumptions, the magnitude of combined asset-specific risk for such a closed-end fund or client separate account is between 40% and 49% of the magnitude of asset-specific risk for a single property portfolio. However, if the asset-specific risk were Normally distributed, the combined asset-specific risk would be between 22% and 32% of the magnitude of asset-specific risk for a single property portfolio.

Similarly, the magnitude of combined asset-specific risk for open-end fund of 100 properties is 24% of the magnitude of asset-specific risk for a single property portfolio. However, if the asset-specific risk were Normally distributed, the combined asset-specific risk would be just 10% of the magnitude of asset-specific risk for a single property portfolio.

Alternatively, if the question of risk reduction is rephrased to ask the number of assets n_k needed in a portfolio to achieve a reduction of asset-specific risk by a specified factor of k, then the answer is as follows: n_k is the smallest integer at least as large as k raised to the power 1/0.309. In mathematical notation,

$$n_k = k^{\alpha/(\alpha-1)} + 1 \approx k^{3.23} + 1 \quad (4)$$

This implies that the number of properties in a portfolio needed to achieve a four-fold reduction in the magnitude of combined asset-specific risk is 88—compared with only 16 properties if asset-specific risk were Normally distributed. Similarly, the number of properties in a portfolio needed to achieve a ten-fold reduction in combined asset-specific risk is 1,698—compared with 100 properties if asset-specific risk were Normally distributed. In other words, if purchases are restricted to institutional-grade properties, equally weighted investments in one-fifth of the properties currently in the IPD data base would be needed to achieve a ten-fold reduction in the magnitude of combined asset-specific risk.

The effect of varying α upon the portfolio size needed to achieve risk reduction by various specified factors k is shown in Exhibit 14.

Conclusions

The empirical results in this study support the existing real estate literature in emphasizing that it is unsafe to assume Normality of property returns. For the U.K. annual IPD data, Normality was emphatically rejected

When sub-sector returns were analyzed, Normality was rejected in almost all cases. Individual (continuously compounded) annual property returns in the IPD database are not

Normally distributed for calendar years during the period 1981-2003, with only two exceptions each for Office and Retail properties.

For each calendar year t in that interval, there is a stable infinite-variance distribution with Characteristic Exponent α_t such that the return on each property for year t can be represented as the average (mean) return for that year on properties of the same commercial type plus a random sample from the stable distribution for that year, and furthermore that these samples are independent for distinct properties or calendar years. These stable distributions can be considered to represent real estate asset-specific risk.

The data analysis strongly implies that both the skewness and magnitude of real estate asset-specific risk change over time, i.e., real estate risk is heteroscedastic with respect to both the amount of risk and the shape of the risk distribution.

However, the analysis also supports the conclusion that there is a single value for the Characteristic Exponent of asset-specific risk across both calendar year and property type. A statistical estimate of this common value for the Characteristic Exponent a together with a 95% confidence interval around this value is 1.448 ±0.004, based on a sample distribution of 269,853 annual property returns over the twenty-three-year sample period. This interval is so far from 2.0—the value for a Normal distribution—that it has profound implications for real estate portfolio management.

Because real estate investment risk has infinite variance, there is no way to measure codependence among property risk functions with the statistical tools currently available. In particular, sample correlations used in multi-factor MPT real estate risk models are fictitious products of flawed data analysis methodology, and do not measure true risk codependence.

The fact that the distribution of property returns appears to behave in a different way from those of equities and bonds, has implications for asset allocation models based on the standard deviation (or variance) as the measure of risk. The inclusion of real estate returns, especially when measured over small intervals, alongside other asset classes in optimizing procedures may produce misleading results. Consequently, Byrne and Lee (1997) and Coleman and Mansour (2004) recommend alternatives to mean-variance analysis. Byrne and Lee (1997) advocate Mean Absolute Deviation (MAD) optimization, which is less sensitive to departures from Normality yet produces portfolio compositions very similar to mean-variance analysis, Byrne and Lee (2005). Coleman and Mansour (2005) suggest the use of more flexible statistical distributions to account for the skewness and leptokurtic nature of real estate returns in the optimization process.

A final observation concerns the accuracy of appraisal-based returns data relative to transaction-based data. The fact that thousands of appraisals by real estate professionals across the country over a twenty-three-year period form sample distributions with nearly indistinguishable Characteristic Exponents across calendar years by property types suggests strongly that the real estate community has a common perception of asset value and the sources of that value that have remained constant across changing market regimes of liquidity, credit access, and supply and demand of product.

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Exhibit 1a
Distribution of Log Annual Total Return Residuals
IPD, All Properties, 1981 to 2003

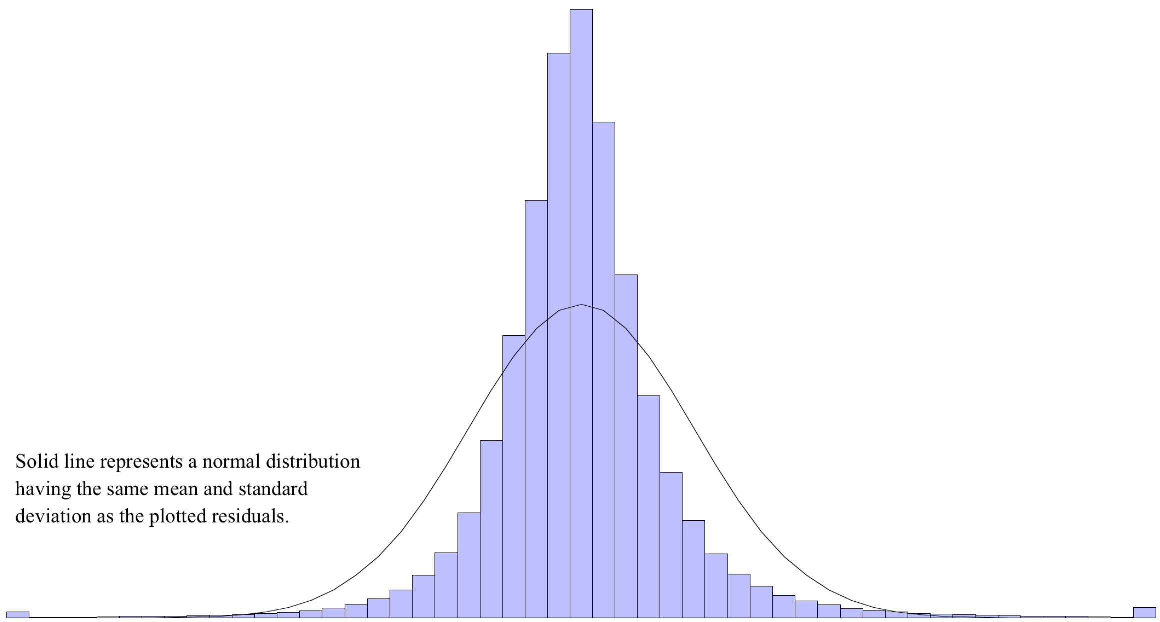

Exhibit 1b
Difference in Frequency, Log Annual Total Return Residuals to Normal Distribution
IPD, All Properties, 1981 to 2003

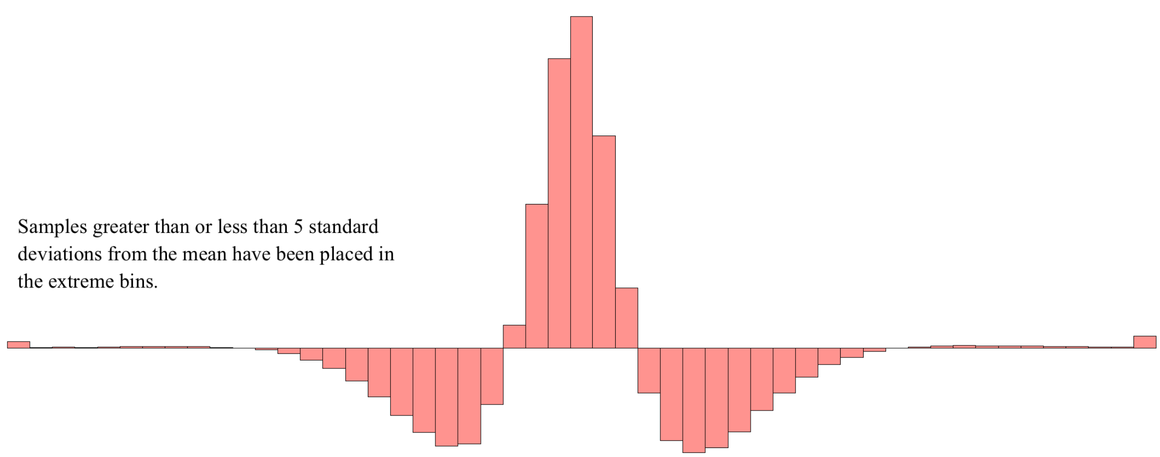

Exhibit 2a
Distribution of Log Annual Total Return Residuals
IPD, Office Properties, 1981 to 2003

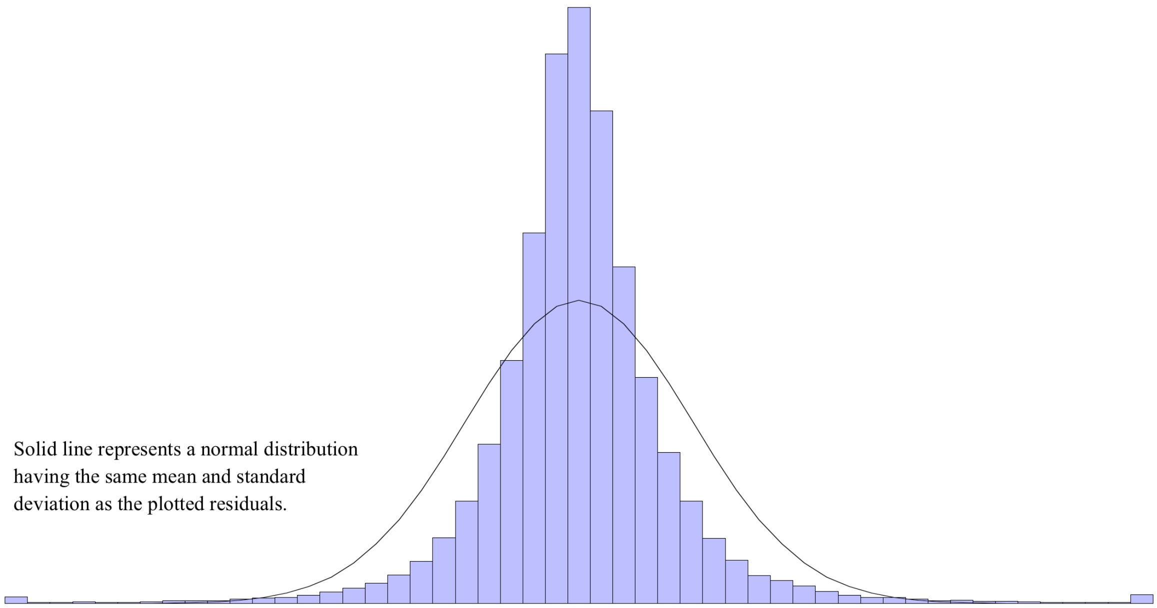

Exhibit 2b
Difference in Frequency, Log Annual Total Return Residuals to Normal Distribution
IPD, Office Properties, 1981 to 2003

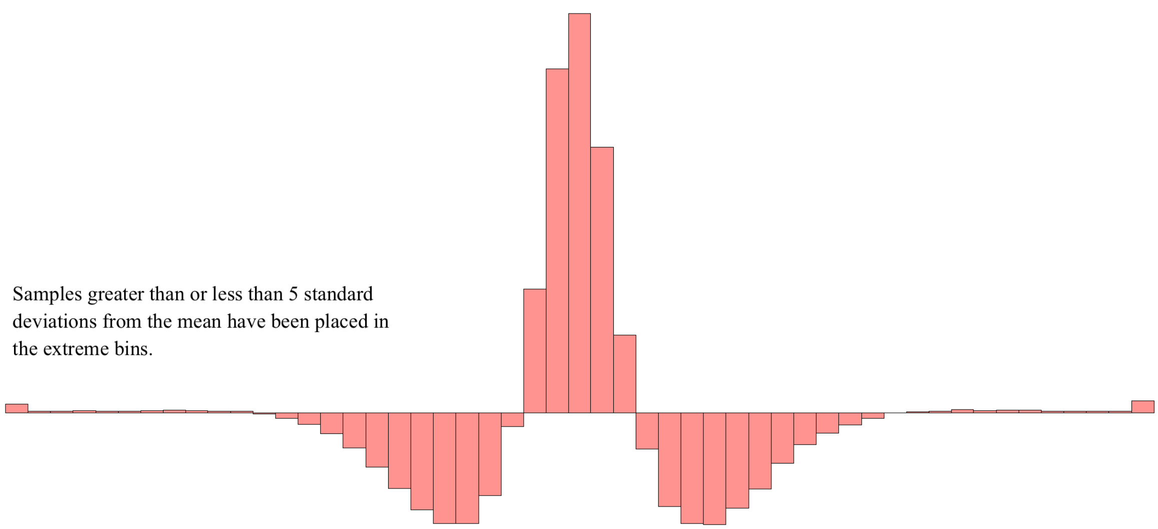

Exhibit 3a
Distribution of Log Annual Total Return Residuals
IPD, Retail Properties, 1981 to 2003

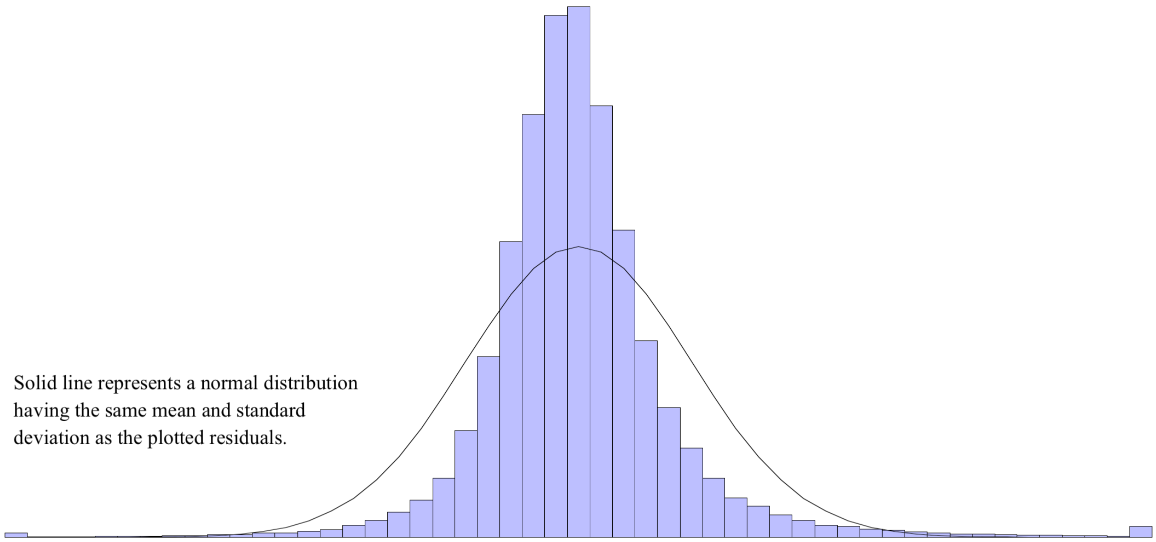

Exhibit 3b
Difference in Frequency, Log Annual Total Return Residuals to Normal Distribution
IPD, Retail Properties, 1981 to 2003

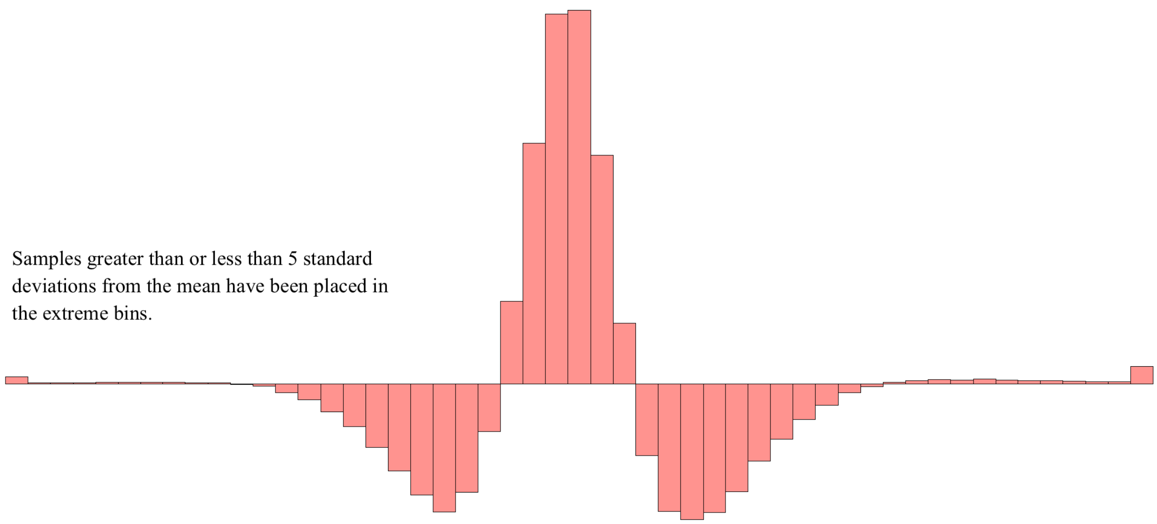

Exhibit 4a
Distribution of Log Annual Total Return Residuals
IPD, Industrial Properties, 1981 to 2003

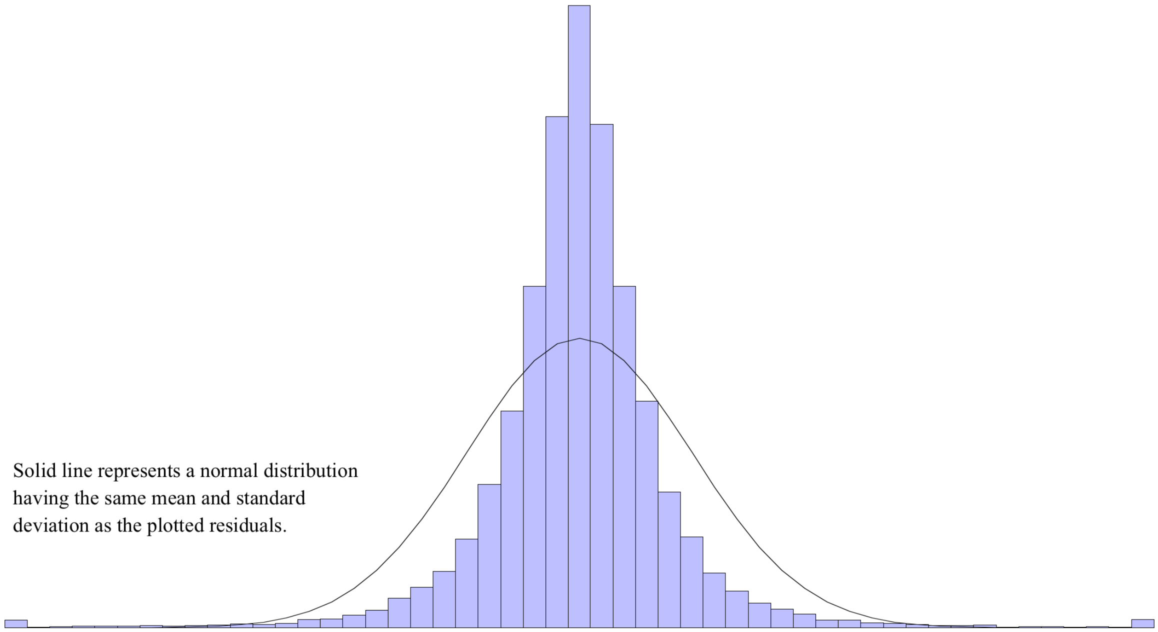

Exhibit 4b
Difference in Frequency, Log Annual Total Return Residuals to Normal Distribution
IPD, Industrial Properties, 1981 to 2003

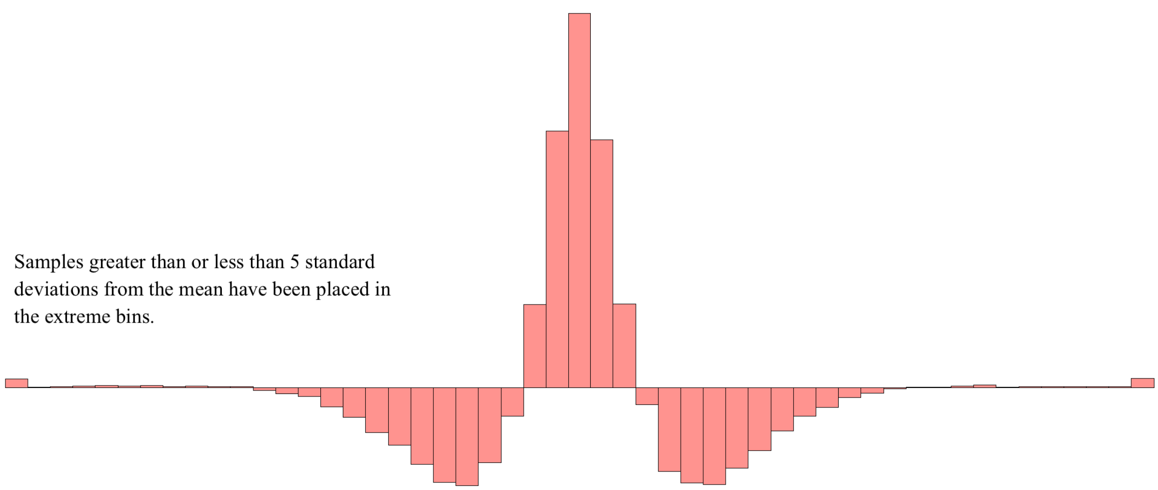

Exhibit 5
Stable Distribution Parameters for IPD Property Database
Log Annual Total Return Residuals & Mean Returns & Number of Properties

All Properties Combined:

Year or Period	α	β	γ	Mean Return	Number of Properties
2003	1.423 **	-0.026	0.046	0.107	9,133
2002	1.417 **	-0.089 **	0.047	0.096	9,787
2001	1.433 **	0.004	0.047	0.068	10,413
2000	1.446 **	0.123 **	0.052	0.083	11,361
1999	1.440 **	0.300 **	0.049	0.129	11,796
1998	1.414 **	0.148 **	0.049	0.102	12,600
1997	1.409 **	0.261 **	0.056	0.132	12,642
1996	1.322 **	-0.007	0.043	0.080	12,983
1995	1.469 **	-0.282 **	0.056	0.020	13,356
1994	1.459 **	-0.177 **	0.064	0.094	12,789
1993	1.517 **	-0.265 **	0.087	0.165	12,363
1992	1.527 **	-0.596 **	0.079	-0.013	12,428
1991	1.500 **	-0.248 **	0.081	0.015	11,892
1990	1.632 **	-0.011	0.088	-0.067	11,309
1989	1.597 **	0.602 **	0.088	0.157	11,126
1988	1.743 **	1.000 **	0.099	0.274	11,493
1987	1.615 **	1.000 **	0.086	0.207	12,123
1986	1.434 **	0.344 **	0.068	0.123	12,341
1985	1.461 **	0.344 **	0.066	0.107	12,042
1984	1.349 **	0.399 **	0.063	0.127	11,853
1983	1.338 **	0.305 **	0.060	0.116	11,539
1982	1.324 **	0.428 **	0.064	0.114	11,475
1981	1.283 **	0.537 **	0.069	0.173	11,009
1981-03	1.448 **	0.136 **	0.066	0.104	269,853

Exhibit 5 (continued)
Stable Distribution Parameters for IPD Property Database
Log Annual Total Return Residuals & Mean Returns & Number of Properties

Office Properties:

Year or Period	α	β	γ	Mean Return	Number of Properties
2003	1.441 **	-0.371 **	0.055	0.041	2,551
2002	1.504 **	-0.321 **	0.059	0.045	2,849
2001	1.390 **	0.098 *	0.047	0.081	2,987
2000	1.370 **	0.379 **	0.052	0.130	3,075
1999	1.422 **	0.333 **	0.052	0.140	3,216
1998	1.394 **	0.175 **	0.054	0.113	3,479
1997	1.309 **	0.242 **	0.056	0.131	3,564
1996	1.271 **	-0.262 **	0.045	0.063	3,783
1995	1.389 **	-0.319 **	0.061	0.016	3,963
1994	1.406 **	-0.270 **	0.068	0.083	3,917
1993	1.541 **	-0.739 **	0.110	0.129	3,803
1992	1.551 **	-1.000 **	0.101	-0.094	3,830
1991	1.644 **	-0.781 **	0.111	-0.072	3,752
1990	1.645 **	-0.121 *	0.096	-0.080	3,576
1989	1.692 **	0.829 **	0.099	0.198	3,456
1988	1.750 **	1.000 **	0.106	0.293	3,531
1987	1.706 **	1.000 **	0.104	0.221	3,765
1986	1.476 **	0.172 **	0.076	0.096	3,892
1985	1.357 **	0.167 **	0.060	0.088	3,823
1984	1.273 **	0.105 **	0.055	0.094	3,725
1983	1.275 **	0.114 **	0.050	0.087	3,616
1982	1.318 **	0.337 **	0.058	0.102	3,595
1981	1.366 **	0.495 **	0.066	0.158	3,373
1981-03	1.431 **	0.053 **	0.072	0.089	81,121

Exhibit 5 (continued)
Stable Distribution Parameters for IPD Property Database
Log Annual Total Return Residuals & Mean Returns & Number of Properties

Retail Properties:

Year or Period	α	β	γ	Mean Return	Number of Properties
2003	1.510 **	0.158 **	0.045	0.145	3,966
2002	1.526 **	0.113 **	0.047	0.130	4,327
2001	1.513 **	-0.063 *	0.051	0.053	4,896
2000	1.535 **	-0.072 *	0.055	0.046	5,842
1999	1.434 **	0.350 **	0.045	0.117	6,268
1998	1.399 **	0.129 **	0.046	0.089	6,758
1997	1.453 **	0.295 **	0.058	0.130	6,826
1996	1.306 **	0.081 **	0.042	0.083	6,936
1995	1.520 **	-0.286 **	0.054	0.019	7,072
1994	1.509 **	-0.091 **	0.062	0.097	6,691
1993	1.570 **	0.015	0.079	0.181	6,493
1992	1.523 **	-0.434 **	0.069	0.025	6,502
1991	1.503 **	-0.136 **	0.072	0.045	6,236
1990	1.605 **	0.017	0.086	-0.076	5,967
1989	1.529 **	0.490 **	0.083	0.106	5,950
1988	1.786 **	1.000 **	0.098	0.241	6,246
1987	1.546 **	1.000 **	0.077	0.193	6,422
1986	1.357 **	0.561 **	0.062	0.147	6,326
1985	1.408 **	0.575 **	0.064	0.141	6,086
1984	1.345 **	0.733 **	0.066	0.166	5,962
1983	1.338 **	0.482 **	0.066	0.152	5,798
1982	1.305 **	0.626 **	0.068	0.143	5,760
1981	1.265 **	0.680 **	0.075	0.200	5,663
1981-03	1.471 **	0.257 **	0.066	0.111	138,993

Exhibit 5 (continued)
Stable Distribution Parameters for IPD Property Database
Log Annual Total Return Residuals & Mean Returns & Number of Properties

Industrial Properties:

Year or Period	α		β		γ	Mean Return	Number of Properties
2003	1.324	**	0.094	*	0.038	0.112	2,616
2002	1.215	**	-0.074	*	0.032	0.094	2,661
2001	1.311	**	0.024		0.036	0.083	2,530
2000	1.369	**	0.090	*	0.043	0.114	2,444
1999	1.488	**	0.198	**	0.054	0.147	2,312
1998	1.472	**	0.119	*	0.048	0.120	2,363
1997	1.508	*	0.258	**	0.050	0.143	2,252
1996	1.280	**	0.080	*	0.036	0.098	2,264
1995	1.549	**	-0.335	**	0.057	0.028	2,321
1994	1.458	**	-0.114	*	0.063	0.105	2,181
1993	1.581	**	-0.458	**	0.080	0.181	2,067
1992	1.591	**	-0.951	**	0.070	0.018	2,096
1991	1.527	**	-0.161	*	0.063	0.090	1,904
1990	1.610	**	0.003		0.077	-0.013	1,766
1989	1.698	**	0.914	**	0.087	0.248	1,720
1988	1.772	**	1.000	**	0.098	0.354	1,716
1987	1.854	**	1.000	**	0.087	0.228	1,936
1986	1.448	**	-0.080	*	0.058	0.102	2,123
1985	1.679	**	-0.227	*	0.072	0.045	2,133
1984	1.426	**	-0.060		0.056	0.074	2,166
1983	1.428	**	-0.041		0.052	0.064	2,125
1982	1.344	**	-0.132	*	0.051	0.055	2,120
1981	1.234	**	0.220	**	0.050	0.123	1,973
1981-03	1.425	**	-0.025	*	0.056	0.110	49,739

Statistically significant confidence of non-Normality $\alpha \neq 2.0$) or skewness ($\beta \neq 0$):
** = 99% confidence
* = 95% confidence

α is the Characteristic Exponent, and only equals 2.0 for the Normal distribution
β is the Skewness Parameter in the range -1.0 to +1.0
γ is the (positive) Scale Parameter which measures the spread of the distribution about δ

Note: The means are shown in Exhibit 5 for purposes of completeness, but will not be needed for discussion or analysis in the body of this article.

Exhibit 6

Characteristic Exponent "Alpha" of Distributions of Log Annual Total Return Residuals

IPD 1981 to 2003

(bands indicate plus and minus one and two standard deviations)

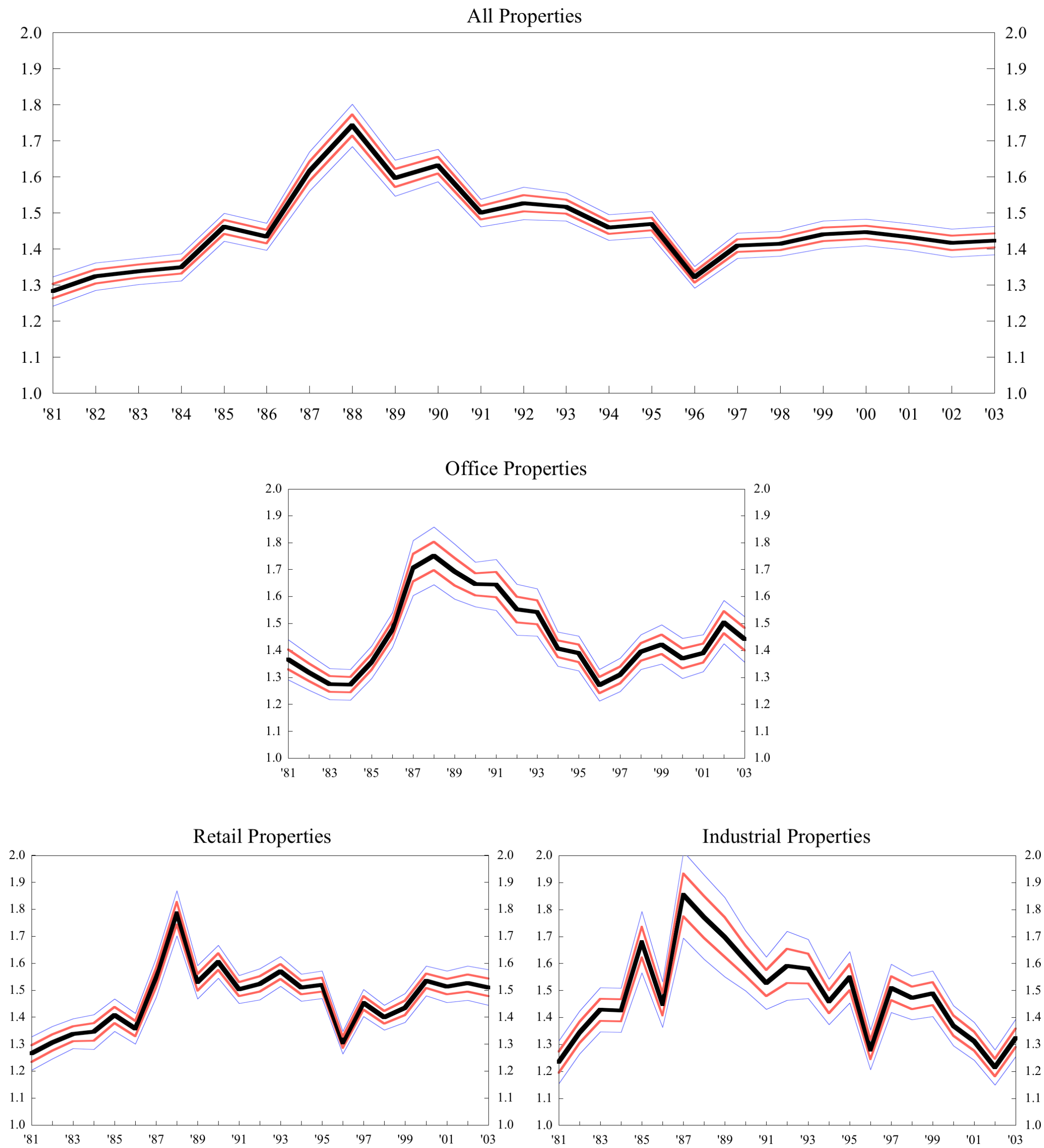

Exhibit 7

Skewness Parameter "Beta" of Distributions of Log Annual Total Return Residuals

IPD 1981 to 2003

(bands indicate plus and minus one and two standard deviations)

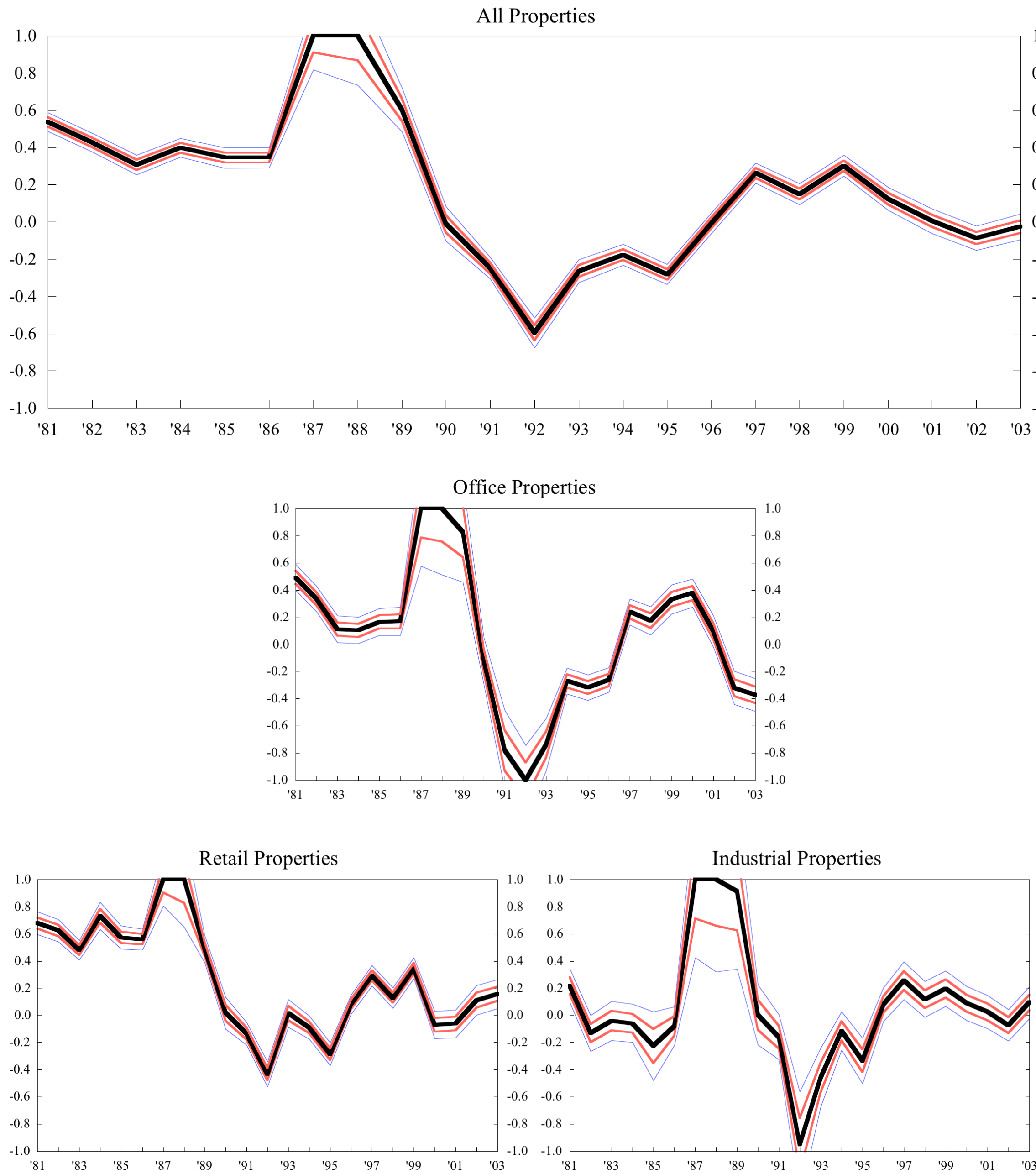

Exhibit 8

Scale Parameter "Gamma" of Distributions of Log Annual Total Return Residuals

IPD 1981 to 2003

(bands indicate plus and minus one and two standard deviations)

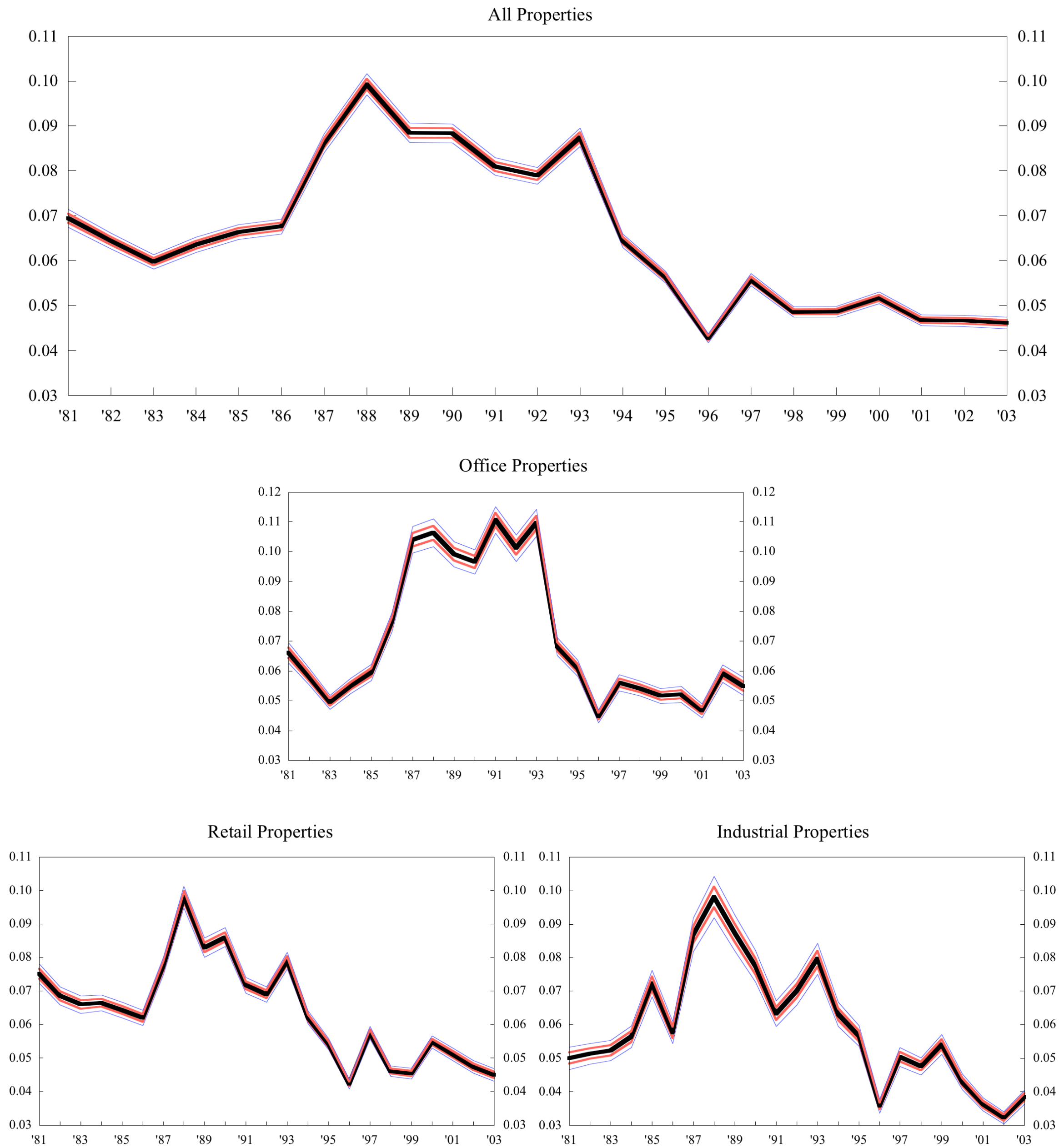

Exhibit 9

Three Parameters of Distributions of Log Annual Total Return Residuals by Property Type

IPD 1981 to 2003

(bands indicate plus and minus one and two standard deviations)

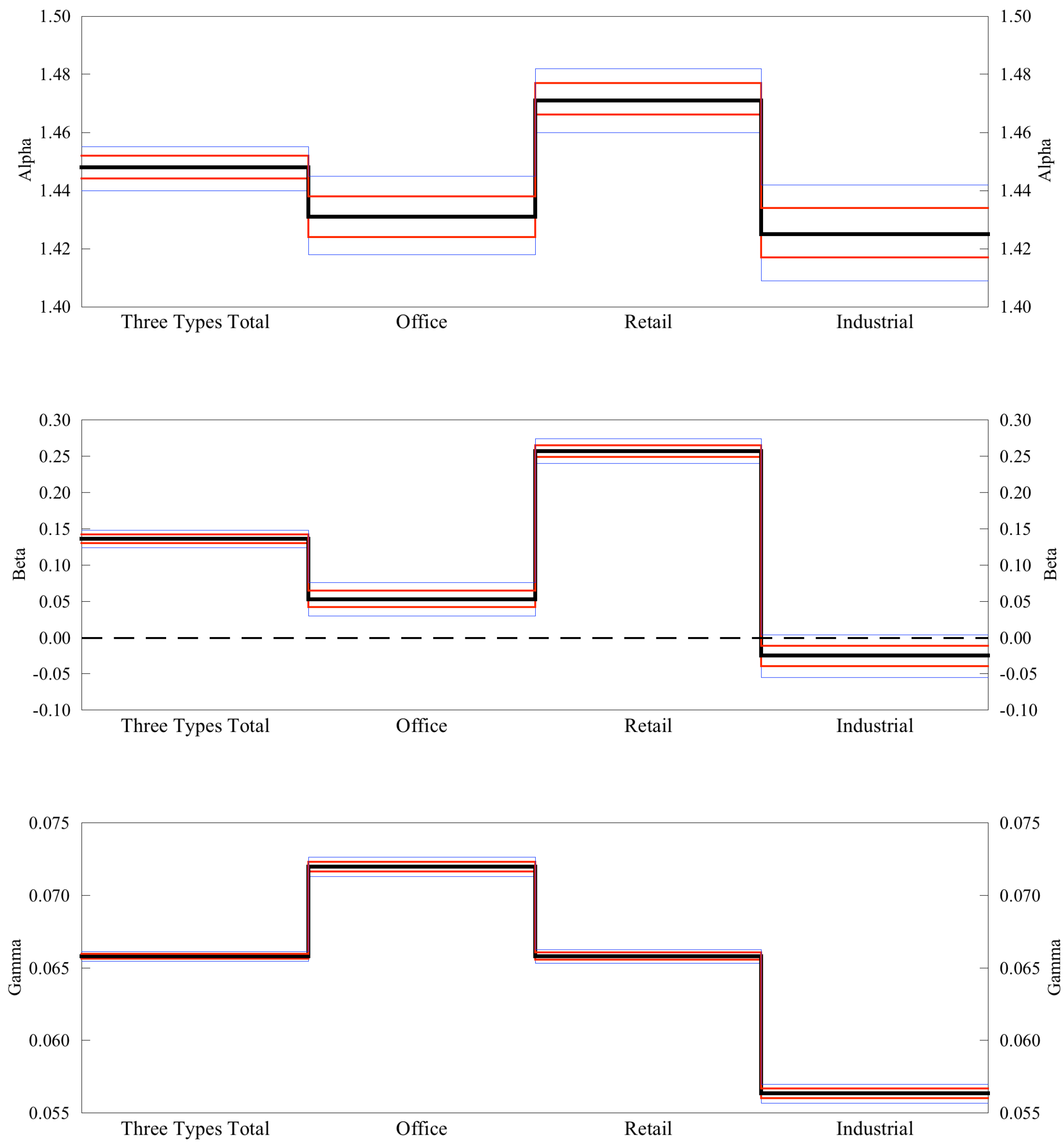

Exhibit 10
Characteristic Exponent α for IPD Property Database
Log Annual Total Return Residual Distributions
All Properties, Properties by Type, and Chi-square Goodness of Fit Results

Year or Period	All Properties	Office	Retail	Industrial	Annual Property-Type χ^2	Annual Components of Sample Period χ^2
2003	1.423	1.441	1.510	1.324	15.17	0.55
2002	1.417	1.504	1.526	1.215	54.83	1.23
2001	1.433	1.390	1.513	1.311	20.82	0.06
2000	1.446	1.370	1.535	1.369	18.79	0.19
1999	1.440	1.422	1.434	1.488	1.58 *	0.02
1998	1.414	1.394	1.399	1.472	2.78 *	1.88
1997	1.409	1.309	1.453	1.508	18.40	2.66
1996	1.322	1.271	1.306	1.280	1.04 *	57.55
1995	1.469	1.389	1.520	1.549	12.53	2.95
1994	1.459	1.406	1.509	1.458	6.65 *	1.47
1993	1.517	1.541	1.570	1.581	0.41 *	16.88
1992	1.527	1.551	1.523	1.591	1.05 *	15.63
1991	1.500	1.644	1.503	1.527	6.88 *	10.61
1990	1.632	1.645	1.605	1.610	0.61 *	72.88
1989	1.597	1.692	1.529	1.698	10.04	39.62
1988	1.743	1.750	1.786	1.772	0.28 *	107.18
1987	1.615	1.706	1.546	1.854	15.47	42.23
1986	1.434	1.476	1.357	1.448	8.37	0.04
1985	1.461	1.357	1.408	1.679	25.38	1.39
1984	1.349	1.273	1.345	1.426	9.80	22.14
1983	1.338	1.275	1.338	1.428	9.45	30.47
1982	1.324	1.318	1.305	1.344	0.63 *	35.80
1981	1.283	1.366	1.265	1.234	6.79 *	58.55
1981-03	1.448	1.431	1.471	1.425	31.80	
1981-03 χ^2		16.92	10.55	4.33		521.99

* Statistically significant confidence of 95% that the Characteristic Exponent α is identical for the calendar year across all three property types.

Exhibit 11
Skewness Parameter β for IPD Property Database
Log Annual Total Return Residual Distributions
All Properties, Properties by Type, and Chi-square Goodness of Fit Results

Year or Period	All Properties	Office	Retail	Industrial	Annual Property-Type χ^2	Annual Components of Sample Period χ^2
2003	-0.026	-0.371	0.111	0.094	48.40	21.02
2002	-0.089	-0.321	0.090	-0.074	27.67	46.33
2001	0.004	0.098	-0.437	0.024	4.41 *	15.39
2000	0.123	0.379	-0.098	0.090	40.22	0.09
1999	0.300	0.333	0.064	0.198	4.00 *	35.46
1998	0.148	0.175	0.180	0.119	0.63 *	0.32
1997	0.261	0.242	0.028	0.258	0.81 *	23.05
1996	-0.007	-0.262	0.348	0.080	38.59	25.34
1995	-0.282	-0.319	-0.720	-0.335	0.42 *	238.38
1994	-0.177	-0.270	-0.402	-0.114	8.39	121.68
1993	-0.265	-0.739	0.618	-0.458	53.70	167.00
1992	-0.596	-1.000	-1.000	-0.951	22.38	321.94
1991	-0.248	-0.781	-1.000	-0.161	17.75	173.57
1990	-0.828	-0.121	0.613	0.003	1.75 *	9.59
1989	0.602	0.829	0.227	0.914	5.05*	63.03
1988	1.000	1.000	0.041	1.000	0.00 *	42.92
1987	1.000	1.000	0.185	1.000	0.00 *	90.66
1986	0.344	0.172	0.225	-0.080	78.67	60.22
1985	0.344	0.167	0.012	-0.227	62.53	56.39
1984	0.399	0.105	1.000	-0.060	117.73	107.64
1983	0.305	0.114	0.077	-0.041	63.46	42.23
1982	0.428	0.337	0.112	-0.132	96.59	137.16
1981	0.537	0.495	0.768	0.220	37.83	259.16
1981-03	0.136	0.053	0.257	-0.025	265.23	
1981-03 χ^2		48.36	145.28	71.59		2058.97

* Statistically significant confidence of 95% that the Skewness Parameter β is identical for the calendar year across all three property types.

Exhibit 12
Risk Reduction for Various α and Number of Assets

	Number of Assets						
α	1	2	4	8	10	20	100
2.00	1	0.707	0.500	0.354	0.316	0.224	0.100
1.90	1	0.720	0.519	0.373	0.336	0.242	0.113
1.80	1	0.735	0.540	0.397	0.359	0.264	0.129
1.70	1	0.752	0.565	0.425	0.387	0.291	0.150
1.60	1	0.771	0.595	0.459	0.422	0.325	0.178
1.50	1	0.794	0.630	0.500	0.464	0.368	0.215
1.40	1	0.820	0.673	0.552	0.518	0.425	0.268
1.30	1	0.852	0.726	0.619	0.588	0.501	0.346
1.20	1	0.891	0.794	0.707	0.681	0.607	0.464
1.10	1	0.939	0.882	0.828	0.811	0.762	0.658
1.00	1	1.000	1.000	1.000	1.000	1.000	1.000
0.90	1	1.080	1.167	1.260	1.292	1.395	1.668

Exhibit 13
Number of Assets Needed for Risk Reduction by the Factor *k*

	Factor *k*						
α	1	2	4	8	10	20	100
2.00	1	4	16	64	100	400	10,000
1.90	1	5	19	81	130	558	16,682
1.80	1	5	23	108	178	846	31,623
1.70	1	6	29	156	269	1,445	71,969
1.60	1	7	41	256	465	2,948	215,444
1.50	1	8	64	512	1,000	8,000	1,000,000
1.40	1	12	128	1,448	3,163	35,778	10,000,000
1.30	1	21	407	8,192	21,545	434,307	4.6×10^8
1.20	1	64	4,096	262,144	1,000,000	6.4×10^7	1.0×10^{12}