Heterogeneous inference with maps

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Abstract:

Since Tolman's paper in 1948, psychologists and neuroscientists have argued that cartographic representations play an important role in cognition. These empirical findings align with some theoretical works developed by philosophers who promote a pluralist view of representational vehicles, stating that cognitive processes involve representations with different formats. However, the inferential relations between maps and representations with different formats have not been sufficiently explored. Thus, this paper is focused on the inferential relations between maps cartographic and linguistic representations. To that effect, I appeal to heterogeneous inference with ordinary maps and sentences. In doing so, I aim to build a model to bridge the gap between cartographic and linguistic thought.

1. Introduction

Since Tolman's paper in 1948, psychologists and neuroscientists have argued that cartographic representations play a crucial role in animal and human spatial cognition (Gallistel, 1990; O'Keefe & Nadel, 1978; Tolman, 1948). These empirical findings align with some theoretical works developed by philosophers who have posited cognitive maps to explain cognitive capacities such as our navigational skills (Heck 2007, Peacocke 1992b, Braddon-Mitchell y Jackson 1996). Generally speaking, this hypothesis promotes a pluralist view of representational vehicles since it implies that cognitive processes involve representations with different formats: that is, representations with cartographic and linguistic formats (Camp, 2007; Casati & Varzi, 1999; Rescorla, 2009b).[1]

In broad terms, cartographic systems are defined by exploiting a spatial isomorphism (such as metrical, physical, topological, and so on.). Also, they are typically composed of both symbolic and iconic elements. By exploiting a spatial isomorphism, cartographic systems differ from linguistic systems in exciting ways. Nevertheless, since they also include symbolic elements and exploit multiple abstraction levels, they also differ from purely iconic representations. Thus, cartographic systems can be thought of as *sui generis* representational systems that differ from –and relate to– iconic and linguistic representations in significant manners.[2]

In the last fifteen years, some philosophers have been interested in distinguishing maps from sentences and other kinds of representational structures.[3] In doing so, they have argued that whereas sentences have a propositional structure, maps have a non-propositional, geometrical structure. This distinction derives from the thesis that languages have a predicative structure, whereas there seems to be nothing in maps that plays the role of predicates (Camp 2018, Rescorla 2009c).[4] As a result, there is a gap between cartographic and linguistic representations, making it difficult to explain how they might relate to each other semantically and logically.

Now, even if predication and propositional structure were taken to be exclusive to linguistic systems, philosophers tend to agree that maps are structured in ways that can sustain inferential processes (see Aguilera, 2016; Camp, 2007; Casati and Varzi, 1999; Rescorla, 2009). However, most proposals have only focused on the capacity of maps for making inferences within cartographic systems. For example, in *Thinking with Maps*, Camp (2007) has argued that maps can be extended to represent functional relations such as negation, disjunction, conditional, and quantification. Consequently, these maps would have the kind of logical structure required for deductive inferences. Rescorla, in turn, has argued that maps lack the resources for predication (2009c) and quantification (2009b). According to him, sentences have a logical form, whereas maps have a geometrical form. However, Rescorla claims that maps enable inferential processes through their geometrical structure (2009b).[5] In a similar vein, Heck (2007) denies that maps can represent logical connectives and quantifiers. Hence, they claim that the structure of maps differs from the structure of sentences (i. e. they say that while sentences have a propositional structure, the structure of maps is not propositional). However, unlike Camp and Rescorla, he takes a further step and claims that maps can interact with sentences in rational ways. However, he does not tell us how those interactions are.

Despite this general commitment to a pluralistic framework, the inferential relations between maps and representations with different formats, such as sentences, have not been sufficiently explored.[6] As I hypothesize, part of this derives from the assumption that linguistic representations have –but cartographic representations lack– a predicative structure. Since logical inferences depend on how representational contents are structured or vehiculized, it is difficult to see how cartographic representations might logically interact with sentences.

In contrast to this generalized idea, this paper aims to bridge the gap between cartographic and linguistic representations by appealing to heterogeneous inference. This step is necessary on pain of leaving our cognitive maps isolated from the rest of our cognitive organization. In this direction, I will argue that cartographic representations can logically interact with linguistic representations through heterogeneous inferences. Heterogeneous inferences are robust inferences that combine representations that belong to different representational systems. There are remarkable studies on heterogeneous inferences in the literature (see Barwise & Ethemendy's Hyperproof in 1996). However, most of these studies focused on the logical interaction between linguistic representations and diagrams (see Shin, 1994; and Allwein & Barwise, 1996) and images (see Barceló, 2012). Based on these studies, I want to advance the case for heterogeneous inferences involving cartographic representations. In particular, I want to argue that maps have the kind of structure required to participate as premises in heterogeneous inferences. To do so, I will argue that maps have a predicative structure. While the main participants of this debate understand the notion of predicate exclusively in terms of the Predicate Calculus (Camp, 2018; Casati & Varzi, 1999; Rescorla, 2009c), I endorse a primitive notion of predication. Thus, I will argue that cartographic representations can participate in heterogeneous inferences with sentences by having a predicative structure that partially overlaps with the sentences' predicative structure.

As I will argue, predication is a function that can be exploited independently from linguistic vehicles. This notion of predication can be traced back to Sellars's work (1981). Following Sellars, I will distinguish this primitive form of predication from the notion of a propositional function. Then, I will focus on the format and functions of some examples of external, concrete maps, and their relations to sentences to shed light on how different sorts of mental representations might interact. Precisely, based on the case of subway maps, I will argue that cartographic representations have different resources for predication and that these resources are central to participate in heterogeneous inferences with sentences.

The paper's structure runs as follows: in the second section, I will briefly analyze some recent views according to which cartographic representations do not make room for predication. As we will see, these views rest on an intellectualized notion of predicates, derived from the Predicate Calculus analysis of predication. In the third section, starting from Sellars's view on predication, I will argue that maps have resources for primitive predication. However, to give a complete account of maps, Sellars' notion of predication needs further development. To do that, I introduce a third category of predication based on Burge's (2010) notion of attributives. Thus, I will split the notion of predication into three different kinds that might be named: Sellarsian and Burgean predication, which can be found in cartographic systems, and Fregean Predicates, exclusive to linguistic systems. Finally, in the fourth section, I will argue that maps can participate in heterogeneous inferences. I will conclude the paper by pondering some benefits that derive from making heterogeneous inferences with maps. Based on the previous analysis, I will show that maps can interact with sentences in rational ways. This is a philosophical exploration meant to take out maps and language from their cognitive isolation.

2. Some thoughts about predication

With some differences, Camp (2018) and Rescorla (2009) militate against the predicative view of maps. Their position derives, in part, from a logical view of predication that can be traced back to Frege's and Tarski's works. According to this view, predicates are propositional functions (Gamut 1991): functional-argumentative structures that map objects to truth-values (Rescorla, 2009, p. 179). Like functional expressions, predicates contain "argument-places" –i. e., open positions– that can be filled with either denoting terms or bounded variables (Rescorla, 2009, p. 177). From a syntactic point of view, predicates are thus characterized as expressions of form F(x), which have the function of returning a truth value when saturated with one of the many values that may take the variable x. So, when the predicate F(x) receives a proper value as an input, say *a*, it becomes a meaningful proposition which returns a truth-value as an output (see 1 below):

(1) F(a) → TRUE/FALSE

Now, what is interesting and compelling about this notion of predication is that it is also possible that propositional functions become meaningful propositions without being saturated with proper values. This may happen when the variables inside the propositional function are bounded to quantifiers (see 2 and 3 below).

(2) $\forall x\ F(x)$

(3) $\exists x\ F(x)$

Under this interpretation, the syntactic distinction between propositional functions and their values correlates with a distinction at the level of vehicles between, on the one hand, predicate symbols – such as ‘P(x)’, ‘Q(x)’, ‘R(x)’ – which represent unsaturated propositional functions and, on the other hand, constant symbols – such as ‘a’, ‘b’, ‘c’ – which stand for the values that saturate the propositional functions. When quantifiers bound variable symbols, such as ‘x’, ‘y’, and ‘z’, they range over all the possible values that the propositional functions may take as input and become meaningful representations. Due to this distinction at the level of vehicles, predicates can be explicitly detached from their values. From a cognitive point of view, this means that the comprehension of predicates involves an abstract comprehension of the instantiation relation: what it means for any entity to have the property or relation denoted by P.

Now, based on the logical conception of predicates described above, Camp (2018) and Rescorla (2009) argue that maps do not feature predication. Let us present their arguments. Firstly, Rescorla argues that map regions or grids cannot play the role of *predicates*, that is, argument-places or propositional functions. He states that if grids were open propositional functions, placing a marker inside a grid in a map (see 4) would mean that the corresponding object or property occupies the corresponding grid in physical space. However, the argument goes, whereas deleting a singular term from sentences yields an unsaturated representation, which cannot be evaluable as true or false; what results from deleting a marker from a map is still truth-evaluable.

> If we delete “Jack” from “Jack is hungry” what remains is no longer evaluable as true or false. It does not represent the world as being a certain way. If we delete marker *A* from map M_1, what remains is evaluable as true or false. It still represents the world as being a certain way. (Rescorla 2009c, 180)

(4)

	1	2	3	4	5
x		A			C
y				C	C
z		B	B	B	

Besides, following Casati and Varzi (1999), Rescorla claims that maps elicit the "absence intuition." According to his argument, if map-markers were predicates, the absence of a marker in a map region represents that the corresponding property is absent in the represented region. Nevertheless, nothing like the absence intuition arises from ordinary predication (Rescorla, 2009c, 182). In a similar vein, Camp (2018) argues that such grids cannot play the semantic role of *names*. According to Camp, it would be difficult to determine how a collection of unsaturated map-predicates would syntactically differ from a map with a world location assigned. Furthermore, she says that "having assigned world-locations is not essential to a map being either syntactically well-formed or semantically interpreted" (p. 10). Besides, Camp suggests that the metaphysical distinction between objects and properties, which is closely mirrored by the distinction between 'saturated' subjects and 'unsaturated' predicates in language, is not crucial in maps (p. 11). So, she concludes that grids do not function either like names or predicates. Instead, she claims that the propositional analogs to grids might be quantifiers.

I think, however, that both Rescorla's and Camp's views are misleading. To see this, let us consider firstly Camp's verdict about map grids as analogs to quantifiers. Quantifiers are the mark of generality. They are logical devices that help us represent the relationship of instantiation between objects and predicates in an abstract way. As such, they range over the extensions of a discursive domain without referring to any particular item. That is why they allow us to express general thought, be it existential or universal. To analyze if Camp's analogy of maps with quantifiers holds, we can compare maps with other non-linguistic representational systems explicitly designed to hold quantifiers. Venn diagrams, for instance, are designed to hold quantifiers. Venn diagrams represent logical relations between sets by exploiting an isomorphism between spatial regions and sets (Barwise and Etchemendy 1996, 10), and spatial points and elements of such sets. Thus, any point in a region represents membership. Because of using this particular kind of isomorphism, Venn Diagrams can represent the logical relationships of instantiation. Indeed, like sentences, Venn diagrams can take constants and variables as arguments. So, even though they lack particular symbols for quantification, Venn diagrams represent general thoughts (both universal or existential) (Shin 1994).

In contrast, maps exploit a different kind of isomorphism: a physical spatial isomorphism. According to this, maps regions represent physical space and the spatial relations between objects, such as localization and distance. Consequently, an element in a map region represents the localization of an object in a physical space. However, this is very different from representing objects in sets like Venn diagrams. Thus, whereas Venn diagrams can hold quantification in virtue of representing logical relations, it is unlikely that maps can hold quantification by representing spatial relations.

Secondly, let us consider the argument from the absence-intuition. According to this argument, the absence of a marker in a map region represents that the corresponding property is absent in the represented region. Rescorla says that the absence intuition holds because maps replicate the geometric features of the represented regions. So, representing a property's spatial distribution implies representing where it is located and where it is *not* located. Nevertheless, nothing like the absence intuition arises from ordinary predication (Rescorla, 2009c, p. 182). Moreover, the argument goes, if map-markers were predicates, unmarked maps regions would lack truth-conditions. Syntactically speaking, deleting a marker in a map would produce an incomplete expression –just as deleting a singular expression from a propositional function. Nevertheless, nothing like this happens with maps. Maps' semantics –Rescorla argues– is quite different from the semantics of languages. Thus, if maps sustain the absence intuition, they never contain unsaturated locations, and deleting a marker from a map represents the absence of the denoted property (Rescorla, 2009c; Camp, 2018, p. 13).

Rescorla's considerations purport to show that there are significant differences between maps and sentences. However, I do not believe that the absence intuition is as strong as it is suggested. In this direction, Camp (2018) suggests that although the absence intuition holds for navigational maps for general purposes, this intuition becomes weaker when it comes to informal maps with more restricted purposes or less epistemic authority. In the same line, Bronner has argued that we can explain the absence intuition by distinguishing what a map *conveys* from what it literally *represents* (Bronner 2015). Thus, he places the query in a pragmatic dimension. Besides, the absence intuition may be analyzed as material rules of inference, which entitle to infer the absence of an entity or property from the absence of the element that represents it. A material rule of inference, like this, falls outside the map, yet it could be useful for interpreting and using the map for specific purposes. So, the argument based on the absence intuition gets blurred.

In the next section, we will see that most of these arguments against the predicative nature of maps fade out if we abandon the logical notion of predication, which identifies predicates with propositional functions. In my view, cartographic representations lack a quantificational structure. Contrary to Camp and Rescorla, I argue that cartographic representation makes room for predication, but only in singular or referential contexts. As we will see, this proposal will provide a useful distinction within the broad notion of predication and, more importantly, it will be useful to provide an account of the logical interactions between maps and sentences.

3. Maps and Predication

In the previous section, we examined some arguments against the idea that maps can hold predication (Camp 2018, Rescorla 2009c). We have seen that this non-predicative view of maps relies on a logical tradition started by Frege. According to this, predicates are propositional functions. Henceforward, I will refer to this notion of predication as "Fregean predicates". This view of predication has many virtues; mainly, it accurately analyzes the inner structure of sentences. Also, it provides comprehension of general thoughts that represent relations between predicates. However, there are other views of predication, less demanding but not less promising. Some examples may be found in Sellars (1981), Burge (2010), and others.

The goal of this section is to challenge the non-predicative view of maps. In particular, I want to argue that, once one departs from the notion of predication as it is derived from the Predicate Calculus, maps can be viewed as hosting a primitive predicative structure. As a starting point, I will take Sellars's view of predication, conceiving predication as conceptually prior to propositional functions. Based on this view, I will argue that there are good reasons for attributing a primitive form of predication to maps. Sellars's ideas provide a notion of predication for basic representational systems. So, they are extremely useful to analyze some of the forms in which maps predicate. However, maps are hybrid representational systems that are enriched by symbolic elements. On the one hand, maps exploit spatial relations between those symbols to represent spatial relations between the elements they represent. On the other hand, maps exploit iconic and symbolic elements that exhibit different degrees of semantic arbitrariness – like other conventional symbols. So, I will push Sellars' ideas further and split the concept of predication into different notions. To do that, I will take Burge's notion of attributive. As we will see later, these moves will be particularly useful for highlighting how maps and languages predicate and, fundamentally, how maps and language interact.

According to Sellars, predication is conceptually prior to the notion of predicate as the Predicate Calculus conceives it. In basic cases, predication requires only a *referential element,* along with a *characterizing* or *attributive* function. The referential element plays the role of representing particular entities. In contrast, the characterizing function plays the role of attributing properties to individual entities by modifying the mode in which the referential element represents such individuals, without representing properties or relations –as it has been generally thought (Sellars 1981). In contrast to the traditional idea that predicates stand for properties, for Sellars, predication does not play *such* a representational role. In his view, predication makes sense in referential structures by characterizing particulars, and this is clear in that the predicative function does not require a predicate term either (Sellars 1981, 334).

Sellars's proposal is involved in an in-depth metaphysical and semantic discussion about predication that I do not want to get into here. In particular, he endorses a variety of nominalism together with a non-relational view of predication that denies the traditional conception of predication as a relation to abstract entities (Sellars 1981, 332; deVries 2005). To our purposes, Sellars's view of predication is a powerful theoretical tool for distinguishing how predication operates in different representational systems. In particular, I believe that Sellars's proposal is promising for understanding how maps predicate. Thus, whereas I am committed to some aspects of Sellars' semantics, I will keep neutral about his metaphysics.

According to Sellars, the distinction between predicating and referring is exclusively functional. In this aspect, his proposal takes us apart from the view of Camp and Rescorla since it provides a cue to soften the constrictions for predication at the level of vehicles, blurring the distinction between predicate and referential elements. To illustrate this, let us focus on Sellars's distinction between two kinds of representational systems:

Basic representational systems (BRS):

On the one hand, there are basic representational systems where manipulating predicates does not require manipulating predicate-symbols. In this primitive form, predication is not conceived as predicate terms: namely, semantically isolated open symbols that can receive referential representations as inputs. On the contrary, predication is nothing but an abstract modifier of referential representations.

Linguistic systems (LS):

On the other hand, in linguistic systems, both the referential and attributive functions are conveyed by distinctive symbols: a subject-symbol, ‘s’, and a predicate-symbol, *P*.

In other words, whereas in the linguistic case, the syntactic distinction between predicating and referring correlates with a distinction between referential symbols and predicate symbols (analyzed, respectively, as arguments and propositional functions), BRS do not exploit a distinctive symbol or vehicle for predication besides the referential structure. Let us take as an example a slightly changed hypothetical representational system (Sellars 1981, 334) composed of symbols representing particular entities, and where the spatial relations between the symbols represent the spatial relations of the entities denoted by them (see 5 below).

(5) A

B

This example represents that the entity denoted by A is above the entity denoted by B. Linguistically, the same configuration of entities may be represented as follows (6):

(6) A *is above* B

According to Sellars, both representations have a predicative structure: Whereas in “A is above B”, a specific element plays the predicative role of *being above*, in (5), it is the spatial arrangement of A and B what plays the predicative role. Sellars exploits this example in the context of his defense of nominalism. According to Sellars, this example shows that predicate terms are dispensable, and it is part of an argument against a relational conception of predication. According to it, predication can be analyzed in terms of arrangements of particulars instead of relations to abstract entities (deVries 2005).[7] In our discussion, this example shows that predication can run without predicate terms, given that BRS do not employ a specific representational *vehicle* to play the predicative role of being above. Instead, that role is played by the spatial relation holding between A and B.

It is worth noticing that, in Sellars' view of predication, the distinction between predicative function and referential elements does not depend on having an unsaturated or saturated structure, as the Frege-Tarski tradition sustains. For this logical tradition, predicates have a propositional structure that allows them to be open and then saturated with referential elements. For Sellars' view, instead, the distinction between predicating and referring depends on the function that a symbol – or an arrangement of it – can play in cognition. However, at an abstract functional level, cartographic and linguistic predication behaves in partially overlapped but different ways. In other words, as in 5 and 6 above, whereas the *represented* fact is the same, the *representing* facts are different.[8] Firstly, like 5 and 6 shows, both kinds of predication *attribute* the same property or relation. Hence, their *contents* overlap. Secondly, each predicative structure is different: on the one hand, we have a propositional function that can be bound to quantifiers to express general thoughts and complex propositions; on the other, we have a spatial arrangement of elements representing a spatial relation between particular items. Nevertheless, at an abstract level, both cases count as predicative structures. Along this line, Sellars establishes a distinction between propositional and logical form: BRS and LS share a propositional form: they involve referring and characterizing functions that play inferential roles. However, only LS's sentences have a logical form since they include logical connectives and quantifiers (Sellars 1981, 341).[9]

Sellars's view of predication is a powerful theoretical tool for distinguishing how predication operates in different representational systems. In particular, I believe that Sellars's proposal is promising for understanding how maps predicate spatial relations like position, connection, and so on. Like BRS, maps are characterized by exploiting an isomorphic relation. This means that the spatial relations of the elements in a map represent spatial relations between the object represented by those elements. However, maps are hybrid representational systems. Like linguistic systems, they compose of symbols: more or less semantically arbitrary elements, such as labels, markers, draws, and so on, with different syntactic functions. So, we need to develop further Sellars's ideas to include other (non-spatial) forms of predication.

Along this line, we can take Burge's (2010) concept of attribution to understand how cartographic elements predicate. According to Burge, attributives involve general and referential elements that pick-up particulars and attribute properties to them (2010): "Attributives are characterizers. When they are applied, attributives function to characterize entities picked out by the referential applications" (Burge 2018, 90). Like Sellars' notion of predication, Burge's attributives range over referential context by characterizing particulars entities. Besides, I will assume a formal view of predicates and singular expressions. According to it, if two tokens of a symbol type explicitly refer to the same entity, they operate as a noun or singular term. Respectively, if different tokens of a symbol type do not co-refer (hence refer to different particulars), the symbol has a predicative function.[10]

Thus, based on this extension of Sellars' theory, we can distinguish two different ways in which maps predicate: On the one hand, there are predicative structures driven by the map's spatial relations (such as connectivity, and localization) between the specific elements of the map. I will call this kind of cartographic predication "*Sellarsian* predication", in allusion to Sellars' BRS. On the other, there are cases of predication driven by semantically arbitrary elements. In allusion to Burge (2010), I will call this kind of predication "*Burgean* predication".[11] Since they are driven for semantically arbitrary symbols, Burgean predication differs from Sellarsian predication. However, both forms of predication share the function of characterizing particulars by attributing properties or relations to entities picked-up by referential elements (Burge 2018, Sellars 1981). To continue with our analysis, let us focus on the London Tube Map (see figure 1).

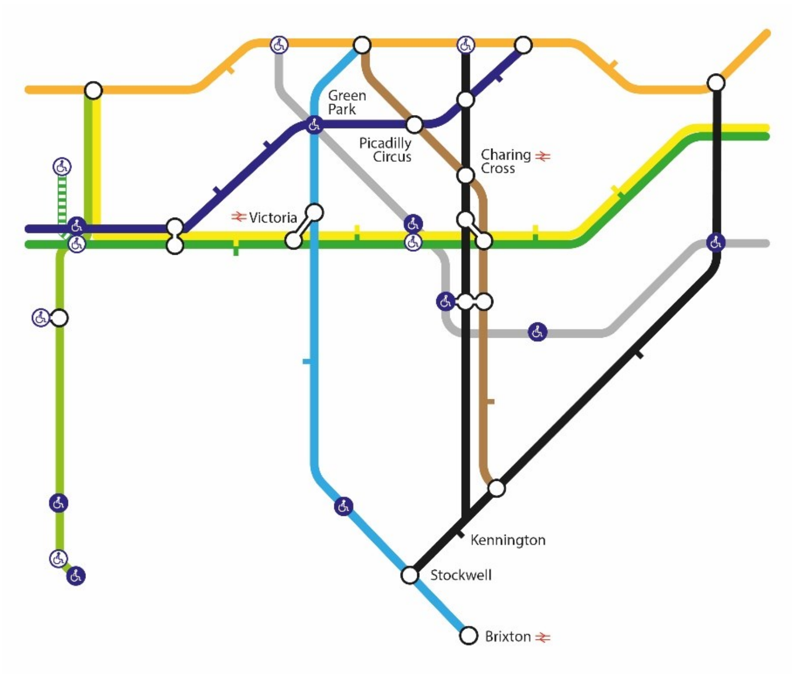


Figure 1: a piece of map inspired by London's Tube map

As we will see with this example, maps or cartographic representations are in a middle way between Sellars's BRS and LS. Cartographic structures are composed of semantically arbitrary elements that can be distinguished according to their functional role. Thus, the Tube Map we are focused on employs a color code to individuate routes, that is, individual train services (Lloyd, Rodgers and Roberts 2018, 2), and labels to individuate stations (see 7 below). These elements play the role of singular expressions: Insofar as tokens of the same label (such as "Victoria", "Oxford Circus", "Bond Street", etc.) have the same referent, they functionally behave in the same way that singular expressions do in languages. Similarly, the same color distributed along different (but continuous) segments of a path denotes a particular route. Therefore, we are entitled to treat labels and colors as nouns or referential structures.

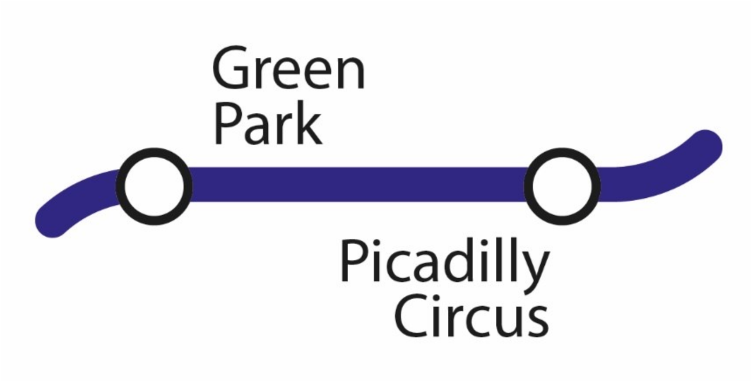
In contrast, circles and lines play predicative roles by attributing properties to particular entities (see 8). Different tokens of circle denote different interchange stations, which in turn are individuated by labels (such as "Victoria", "Oxford Circus", "Bond Street", etc.). Similarly, different tokens of lines attribute the property of being a subway line to different entities, which are individuated by the color code (such as the green subway line, the blue subway line, etc.) (Lloyd, Rodgers and Roberts 2018). Unlike Sellars's BRS, this form of cartographic predication is driven by semantically arbitrary vehicles. So, they are cases of Burgean predication. Despite their arbitrariness, these predicative elements differ from their linguistic counterparts: Unlike propositional functions, they cannot hold variables and quantifiers. Since Burgean predicative elements play the function of attributing properties to entities pick up in referential applications, they are always applied to particular entities, as in this case, to particular stations. In this respect, like Sellars's predication, they do not represent properties as abstract entities but modify how particular entities are represented.

(7) Tube map's referential elements:

(8) Tube Map's Burgean predicative elements:

On the other hand, the Tube map is an example of a topological structure; consequently, Burgean predicative elements are bounded to spatial coordinates or relations. Topological representations are cartographic representations that abstract from physical and metrical properties and relations to represent relations of connectivity, continuity, and convergence between objects. Since this map exploits a topological isomorphism – metrically distorted – the topological relations between the map elements represent topological relations between the entities represented by the map. For example, the spatial relations holding between the blue line, Green Park station, and Piccadilly Circus station (see figure 1) represent that Green Park and Piccadilly Circus stations are connected by the blue line. Besides, the Tube Map divides the topological space into different zones. Stations are distributed on concentric circles that demarcate the zone where each station belongs. Like the BRS, the Tube map does not employ a distinctive predicative symbol to represent the relation of *being connected* that holds between the blue line, Green Park and Piccadilly Circus. Likewise, it does not require an extra element to represent that a station is placed in a particular zone.[12]Since these relations are represented by the spatial relationship between the elements of the map (See 9 below), I will call this kind of cartographic predicates "*Sellarsian* predication", in allusion to Sellars' BRS.

(9) Tube map's Sellarsian predication (*being connected)*:



Besides, Burgean and Sellarsian predication can be distinguished from what I have called "*Fregean* predicates", or predicate terms, exclusive to linguistic systems. In section 2, we saw that Fregean predicates are propositional functions that can be applied outside referential structures since they can be detached from their attributive function. Thus, these predicates represent properties and relations as detached from particular entities. This is the case of predicates that are part of general thoughts and range over a domain of object without references to particular entities.[13]

To sum up, I have argued that maps represent spatial relations by exploiting a spatial isomorphism. They attribute such *relations* by localizing representational elements on the map that stand for particular subway lines and stations, but maps do not employ a predicative element besides those markers. That is why maps hold Sellarsian predication. Furthermore, maps can represent general properties by exploiting explicit predicative-symbols. Like linguistic predicates, these predicative elements – Burgean predication – exploit symbolic elements. However, Burgean predicative elements cannot be identified with propositional functions for two reasons: firstly, because unlike unsaturated argument-places, they are attributives that modify referential elements; secondly, because when they are employed in a map, they are subject to Sellarsian predication.

Consequently, cartographic predication is governed by combinatorial rules which map onto spatial relations.

At last, in this section, I have split the notion of predication into different kinds. As I have argued, these three notions of predication are different since they involve different kinds of vehicles and structures; however, they might have overlapping representational contents. Someone could ask why should we split the notion of predication. We will face this question in the following section. There, we will see that in virtue of having a different but overlapping predicative structure, maps can logically interact with sentences.

4. Heterogeneous inferences with maps

According to the traditional view of inference, only sentences can play a role in formal inferences since apparently the kind of syntactic structure (i. e. logical form) required to stand in logical relations to one another is exclusive to linguistic representations (Bermúdez, 2010; Brandom, 2010; Davidson, 1983, Fodor, 2008; Peacocke, 1992; McDowell, 1994). This view of reasoning has been subject to much criticism (Aguilera, 2016; Allwein & & Barwise, 1996; Hurley & Nudds, 2006; Vigo & Allen, 2009). In this section, I want to challenge the traditional view of reasoning by appealing to heterogeneous inferences, that is, logical inferences that combine representations that belong to different representational systems. I will argue that maps can stand in logical relation with sentences through heterogeneous inferences (see also Aguilera & Castellano, forthcoming). Contrary to the traditional view, we saw that Camp's (2007) and Rescorla's (2009b) have argued that maps can stand in logical relations. However, they have focused on the capacity of maps to make inferences within cartographic systems. Since formal inferences are dependent on the structure of representational content, and they argue that cartographic representations lack the predicative structure, it gets difficult to see how cartographic representations can logically interact with sentences.

In contrast to such a non-predicative view of maps, in the previous section, I have argued that maps are hybrid representations with a predicative structure. Thus, if maps and sentences have resources for predication, the semantic gap between linguistic and cartographic structures narrows down. Based on the predicative view of maps developed above, in this section, I will argue that cartographic representations logically interact with linguistic representations by appealing to heterogeneous inferences. I believe that the notion of heterogeneous inferences is worthwhile to explain how maps interact with other representational formats. I borrow the notion of heterogeneous inference from Barwise and Etchemendy (1996), who have mainly focused on the role of diagrams in mathematical proofs. These authors put forward an informational approach to inference. According to it, inferring is the task of extracting implicit information from some explicitly presented information. In particular, making an inference involves getting information from a representational content with the independence of how that content is structured. In this aspect, I slightly separate from Barwise and Etchemendy's informational approach to endorse a functional approach, centered on the structure of maps and sentences.

Thus, along with a functional approach, I will argue that maps can participate in formal inferences with sentences by having a predicative structure. Although there are significant differences between linguistic and cartographic systems and their cognitive functions, we will see that predication is the joint that links linguistic and cartographic representations. Along this line, I want to analyze heterogeneous inferences involving linguistic and cartographic premises. In the context of recent debates on representational vehicles (Beck, 2013; Camp, 2007; Rescorla, 2009; Heck, 2007), and based on the empirical hypothesis that our cognition manipulates mental representations with different formats, I aim to provide an account of the rational interactions between representations with cartographic and linguistic formats. In particular, I believe that heterogeneous inferences may help to understand the logical relationships between mental representations with different formats, i.e., cartographic and linguistic. I will base my analysis on a case of inference with the map presented above. My idea is that by reflecting on the format of *ordinary* representations, we can sketch an understanding of the relations between *mental* representations. Although this proposal is mainly exploratory, it provides a powerful model for explaining the logical interactions between cartographic and linguistic representations. Hereunder, we will focus on a case of reasoning with maps.

Imagine that Sally is walking through Buckingham Palace Road, London, and needs a journey to Brixton. She looks at the tube map and finds that the nearest connection is by Victoria Station to the south. Then Sally walks to the south band of Victoria station. There, she revises the map and realizes that whereas Victoria is in zone 1, Brixton is in zone 2. Sally has also heard that single journeys from zones 1 to 2 cost £5.90 by cash. Based on this information, Sally concludes that her journey will be £5.90 instead of £4.40, the regular fee. We can suppose that she makes the following inference (see HI1):

HI1

1. The fee for journeys between stations localized in zone 1 and 2 is £5.90
2. 

∴ The fee for a journey from Brixton to Victoria is £ 5.90

Firstly, I want to argue that this is a case of heterogeneous reasoning, which combines representations belonging to heterogeneous systems – linguistic and cartographic – that interact in rational ways to sustain an inference. On the one hand, the linguistic premise conveys general information about the Tube system fees. It states that the fee for journeys between stations localized in zone 1 and stations localized in zone 2 is £5.90. This is a general statement. For playing this role, the linguistic premise puts together different predicates such as journey, localization, and fee. On the other hand, the cartographic premise conveys specific information concerning the connections between subway lines and stations and the zones where they are located. In particular, it represents –among many other things– that Victoria is an interchange station, that Brixton is an interchange station too, that the blue subway line connects both stations, and that Victoria is placed in zone 1 whereas Brixton in zone 2.

Secondly, I want to argue that semantic and syntactic relations are held between the linguistic and the cartographic premises in the reasoning. The linguistic and the cartographic premises exploit a set of overlapping predicative structures, such as 'journey', 'interchange station', 'being located'. By sharing these predicative structures, the reasoning can be drawn. Nevertheless, whereas the linguistic premise employs Fregean predication, the cartographic premise instead exploits Sellarsian and Burgean predication. So, linguistic and cartographic predicative structures are different, albeit partially overlapping: Although they differ on the *representing* facts, that is, in the representational resources devoted to predication, the facts *represented*, namely, the contents, are the same. Let us think, as an example, on the predicate *being located*. In the linguistic case, the predicate *can be analyzed* in terms of propositional functions, specifically, as a binary predicate, L, with two argument places, L (x, y), devoted to stations and zones. In the cartographic case, instead, the location of an item is represented by the location of a marker – a circle token – in a region of the map – zone 2 –, for example. In contrast to the linguistic case, the map does not need an element – such a predicate term – to represent the spatial relation of being located, but only locating a marker in a specific zone on the map.

It is worth noticing that, firstly, the cartographic premise of this inference exploits both Burgean and Sellarsian predication: The inferential role of the map not only depends on the functional role of its elements such as labels, circles, lines, etc. but also on the way they are spatially related. On the one hand, the inference relies on contents conveyed by the map's specific elements, such as that Victoria and Brixton are interchange stations.

On the other hand, it also depends on specific contents conveyed by the spatial relations of those elements, such as that Victoria station *is located in* zone 1, that Brixton, in zone 2, and that the blue light subway line connects both stations. Moreover, in the particular inferential transition we have focused on, while the map premise exploits both kinds of cartographic predication, the occurrence of Sellarsian predication plays a critical role. Secondly, it is worth noticing that the linguistic premise plays a central cognitive role in pointing out which piece of information from the map will be extracted to be part of the inferential processes. Notably, it points out the *referential* and *predicative* elements of the cartographic premise that need to be exploited to drive the inference.

Thus, this inference is sensitive to the semantic and syntactic relations between a linguistic premise, on the one hand, and a cartographic premise, on the other. Moreover, I have argued that there are also significant differences in the kind of predication employed on the premises. Consequently, it cannot be understood in terms of the classical view of inference. Someone might try to account for Sally's inferential process in purely linguistic terms (see I. 1):

I. 1

1. Victoria connects to Brixton
2. Tickets for single journeys from zone 1 to zone 2 cost £ 5.90
3. Victoria is in zone 1
4. Brixton is in zone 2

∴ A ticket for a journey from Victoria to Brixton costs £ 5.90

However, there is no *a priori* reason to suppose that Sally's inferential process exploited linguistic rather than cartographic representations. In our view, such an account would overlook that maps are representational devices with a predicative structure and, as such, that the tube map can play an inferential role in Sally's reasoning. Although Sally knows that tickets for single journeys from zone 2 to zone 1 cost £ 5.90, she got the information that Victoria is in zone 1 and Brixton is in zone 2 entirely from the tube map. Moreover, that information is part of the content expressed by the map. Someone might insist that to make an inference, she needs to translate the map's content into a sentence. However, it would be redundant for Sally to represent that Victoria is in zone 1 linguistically, and Brixton is in zone 2 since this information is already represented and available on the map. Furthermore, Sally would require a higher cognitive effort to translate the map's content into linguistic premises rather than just using the map as a premise in the context of her reasoning.[14]

It could be objected that the map-premise is unnecessary, that it does not play an inferential role. However, if we remove the map, the conclusion cannot be obtained only from the linguistic premise since the zoning information is exclusively coded on the map, but not on the sentence. Of course, someone can say that the map is nothing but a sentence expressed in a fancy way. According to this objection, the map has an inferential role since it is linguistic, after all. In this sense, maps would be like those books for kids that include some icons within the sentences to facilitate children reading abilities: "The 🐢 was ☺ for leaving the cave." So, the idea is that like in those books where icons and symbols replace words, symbolic elements in maps behave just like words.

Nevertheless, as I have already argued, besides having a semantic value, each map element is part of a web of spatial relations. And sentences are not like this! More specifically, the Tube map represents Victoria station as placed in zone 1 and as an interchange station connected to Brixton and other stations by the blue line, and so on. Hence, it would be difficult to say which sentence – among different options – best expresses the content of the map that plays an inferential role in the reasoning (Barceló 2012, Stainton 2006). Besides, what would be the point of choosing one sentence – from different candidates – like the one that expresses the content of the map. For instance, we can paraphrase the map premise into the following sentences:

a. Victoria is placed in zone 1
b. Brixton is placed in zone 2
c. Victoria is an interchange station
d. Victoria is connected to Brixton by the blue line
e. Victoria is connected to different stations by the blue line
f. Victoria is connected to different stations by the green line
g. Victoria is connected to different stations by the yellow line, and so forth.

However, we cannot pick up just one sentence without losing the content of the map. Besides, it is not clear that a conjunction of all those sentences expresses the map's content. The map's content is determined not only by the semantic value of its elements but also by how they are related. Thus, the maps represent relations between its elements that i) could be overlooked if the map-premise was translated into sentences, and ii) their translation would demand a high degree of computational effort, both in processing and in the use of representational recourses. Something similar can be said about how the content of the map is structured. If the map premise were nothing but a sentence, and each sentence expresses one proposition, since we cannot choose one sentence from many different options, we cannot choose what proposition captures the map's content either. Likewise, it is not clear if a conjunctive proposition could do that job either.

As we have argued above, by attributing relations without predicate symbols, compared to sentences, Sellarsian predication reduces the need for representational elements, alleviating the representational burden of maps. Thus, there are particularities on *how* the information is presented and organized in the map: Firstly, the map is economical, it represents spatial relations between its elements efficiently, and uses a minimum of representational resources. Secondly, the map is holistic: It represents information in a relational organic way (Camp 2018, Heck 2007, Braddon-Mitchell y Jackson 1996). For instance, the Tube map represents at once the connections between stations and subway lines. These particularities might not impinge on the map's content but on how that content is represented. Furthermore, *how* that content is represented might be crucial in terms of the reasoning and inferential processes. Suppose that we separate a privileged set of sentences, sufficient to infer every single connection between the stations and destinations as they are represented in a map. However, whereas those connections could be inferred from the selected set of sentences, they are already represented on the map, ready to be used on further inferences.

Besides, if premises are individuated in terms of their consequences or inferential role, it is not clear that the map premise is equivalent to its translation to a sentence or conjunction of sentences. On the one hand, in our example, from the map premise is possible to infer further information besides the fee for a journey from Victoria to Brixton. So, for instance, from 1 and 2 on (H. I. 1), we can infer that a journey from Victoria to Stockwell also costs £ 5.90; we can see that there is a connection to Kennington by the black line, that Kennington is in zone 2, and then infer that a journey cost £ 5.90; that since Charing Cross is in zone 1, the fee is cheaper, and so forth. On the other hand, by combining a linguistic premise with a map, it is possible to extract information that would be inaccessible solely from the map. In our example above, the map represents – among other things – a connection between Victoria and Brixton by the light blue line, and that Victoria and Brixton are in zone 1 and 2, respectively.

Nevertheless, nothing on the map states what the fee for a journey is. That information can only be inferred from the combination of the map with the sentence. Moreover, the linguistic premise determines among different pieces of information from the map which one – the zoning –will play an inferential role to arrive at a conclusion – the fee. Besides, it is central to remark that whereas the fees are linguistically represented, the zoning information instead is represented on the map. It would be possible to represent the zoning information in a list; however, it would lose the connections between the stations. Of course, one can enumerate the connections, but at the price of providing an extremely long list, that, after all, would miss the zoning information. Thus, heterogeneous inferences increase the representational and inferential values of maps and sentences.

5. Final words

This paper endorses the hypothesis that our cognition manipulates representations with different formats that coexist and interact with each other. It is mainly focused on rational processes between different kinds of representations and, more specifically, on inferences between representations with cartographic and linguistic formats.

In doing so, I have developed a predicative view of cartographic representations. To do that, I have argued that the notion of predication can be detached from the Predicate Calculus. As a starting point, I have based on Sellars's view of predication, and I have distinguished Sellarsian and Burgean predication, from Fregean predicates. Whereas Fregean predicates are paradigmatic of linguistic systems, Sellarsian and Burgean predication can be found in cartographic systems. Despite their differences in their structure and function, these kinds of predication have partial but significant overlapping.

According to my argument, by having a predicative structure, maps can logically interact with sentences. To do that, I have explored cases of inferences between ordinary (non-mental) maps and sentences, and I have argued that maps can participate in heterogeneous inferences with sentences. The analysis provided shows that heterogeneous inferences enhance not only the cognitive power of maps but also of sentences in terms of the consequences that can be obtained and the economy of cognitive resources. Although this proposal is mainly exploratory, it looks forward to bridging the gap between cartographic and linguistic thoughts.

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[1] Although neo-fregeans who do not endorse the LOTH establish a differentiation at the level of content between cognitive maps and propositional thought, they need to explain the rational relations between them.

[2] See, however, (Johnson 2015).

[3] Camp, 2007; Casati & Varzi, 1999; Heck, 2007; Rescorla, 2009a; 2009b; 2009c.

[4] Casati & Varzi (1999) argue for a predicative propositional semantics for maps. However, they differentiate maps from sentences by pointing that maps elicit the absence intuition. I will say more about this later.

[5] In particular, Rescorla (2009a) argues for the case of inductive inferences with maps.

[6] See, however, Aguilera & Castellano (forthcoming).

[7] As I said before, I do not want to get into an in-depth metaphysical debate here. Doing so would require changing the goal of this paper: namely, to figure out how different representational formats interacts.

[8] I want to thank one of the anonymous referees for suggesting this distinction.

[9] The notion of propositional structure or propositional form has been widely scrutinized in contemporary debates (Grzankowski and Montague 2018). In particular, in some theories, the notion of propositional and logical form collapse (Heck 2007, King 1996). For the sake of argument, I will not question Sellars's notion here.

[10] It is crucial to distinguish the case of predicates – where different tokens of a symbol type do not co-refer – from homonymous – where different individuals have the same name.

[11] Strictly speaking, Burge (2010) distinguish attributive representations from pure predicates. According to Burge, in predication, the attribution is inhibited; that is, it is not veridical of any particular entity. However, since they play the attributive function we are looking for, i. e. they are bounded to referential context, I will refer to this attributive as "Burgean predicates".

[12] Burge says something similar about pictures: "It is a mistake to identify a specific part of the picture that serves as a representational constituent that represents any relation that a picture depicts. If one object is depicted as to the left of another object, with some distance between them, there is no answer as to what part of the picture specifically represents the relation to the left of. The spatial relation is depicted, but no part of the picture corresponds specifically and proprietarily to the space between the entities" (Burge 2018, 95).

[13] In other words, the reference of quantificational structures is indeterminate and includes all the objects in a domain (deVries, 2005, p. 69).

[14] Here, I am analyzing a case of reasoning using a paper or e-map. As one anonymous referee stated, it is an empirical question of whether subjects do translate cartographic representations to linguistic representations. As such, it requires further empirical research. However, *ex hypothesi*, we manipulate cartographic and linguistic *mental* representations. Thus, the question is whether it is necessary to translate the content of cartographic representations into linguistic representations. Moreover, I justify a negative answer by providing a model based on the notion of heterogeneous inference.

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