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Performance Analysis and Optimization of a *N*-Class Bipolar Network

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ABSTRACT A wireless network with unsaturated traffic and N classes of users sharing a channel under random access is analyzed here. Necessary and sufficient conditions for the network stability are derived, along with simple closed formulas for the stationary packet transmission success probabilities and mean packet delays for all classes under stability conditions. We also show, through simple and elegant expressions, that the channel sharing mechanism in the investigated scenario can be seen as a process of partitioning a well-defined quantity into portions, each portion assigned to each user class, the size of which determined by system parameters and performance metrics of that user class. Using the derived expressions, optimization problems are then formulated and solved to minimize the mean packet delay and to maximize the channel throughput per unit of area. These results indicate that the proposed analysis is capable of assessing the trade-off involved in radio-resource management when different classes of users are considered.

INDEX TERMS Design optimization, queueing analysis, stochastic processes, wireless networks.

I. INTRODUCTION

Efficient use of radio resources has always been an important aspect in the deployment of large-scale wireless communications systems [1]. This situation is now more exacerbated due to the growth of the number of applications that require higher data rates like video streaming services [2]. On top of it, the number of terminals is exponentially increasing; in a not very far future, there will be more machines communicating to each other than humans [3]. In a new world of the Internet of Things (IoT), wireless communication systems will need to serve both human- and machine-type communications while being capable of delivering data rates of up to tens of Gb/s, latency in the order of milliseconds, and reduced energy consumption to 10% of the current values. All these are set as targets for the fifth-generation cellular system (5G System) [4].

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In one scenario envisioned for the 5G systems, a number of subnetworks will co-exist in the same geographic area, sharing radio resources. Each of these subnetworks will be dedicated to serve a particular type of application and/or scenario, with its own requirements, such as coverage, transmission rates and maximum acceptable latency [5]. It seems to be a consensus in the academic and industrial communities that the goals imposed on 5G systems will only be achieved through the use of heterogeneous networks [6]. Although extensively studied by the scientific community, there is still a lack of understanding of the theoretical limits of interference-limited wireless networks where the relative positions of terminals and their activities are unknown.

We are interested here in studying the performance of a network composed by N classes of users that share the same radio resources, namely radio spectrum and transmit power. Packets arrive at the transmitters randomly, which are modeled by queues where packets waiting for transmission are stored. Each of these N classes has its own characteristics (e.g., terminal density, transmit power, and traffic intensity) and quality of service requirements (communication link quality and maximum tolerable mean delay). IoT scenarios covered by 5G systems can be viewed in this way since thousands of wireless terminals connected to sensors access the wireless network to transmit their messages [7]. Each sensor is associated with a specific application that imposes different requirements: automatic control systems may required millisecond-delay and 99.999% reliability [8], while telemetry applications may accept minutes of delay and weaker reliability constraints [9]. The wireless connections may involve an access point (cellular mode) or direct device-to-device links (D2D mode) [10].

In the scenario investigated here, links of all user classes share the same channel, causing mutual interference, which in turn makes the queues of the terminals coupled, since the transmission success probability of a transmitter (i.e., the service rate of the associated queue) depends on the state of the queues of other transmitters (whether their queues are either empty or non-empty). The analysis of networks with coupled queues is still a fertile research field, especially when the *capture model* is adopted [11]. This model assumes that a packet is successfully received if the corresponding signal to interference plus noise ratio (SINR) at the receiver is above a certain threshold (in contrast to the *collision model* [11], according to which a packet transmission is successful only if there are no concurrent transmissions). The concept of stochastic dominance was employed in several works found in the literature (see, for instance, [12], [13]) to determine the conditions for queue stability under coupled queues scenarios. Stochastic dominance is based on the comparison of the original system network with a simpler and sub-optimal system that, for example, transmits dummy packets for a given set of users. If it is proven that the sub-optimal network is stable, then the original (dominated) network is stable as well [14].

Stamatiou and Haenggi [15] investigated the scenario described above, by combining the use of the stochastic dominance technique with stochastic geometry models and queueing theory results. They studied the stability and the delay of random networks, where terminals are located according to a Poisson point processes (PPP), and obtained necessary and sufficient conditions for stability in a network with one or two classes of users. Afterwards, many other works explored this combination between queueing theory and stochastic geometry to study random networks (see, for instance, [16]–[18] and references therein).

The present work extends the results presented by Stamatiou and Haenggi [15], generalizing the formulation that describes the behavior of users in a random network with N classes of users. We assume that each transmitter (generated by a "mother" PPP) chooses the closest terminal within the pool of potential receivers generated by its "son" PPP to transmit its packet, as illustrated in Fig. 1. We establish the necessary and sufficient conditions relating user densities,

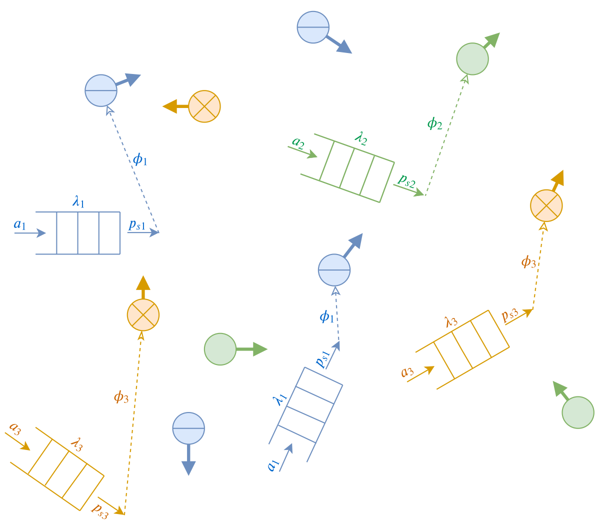


FIGURE 1. Example of a bipolar high-mobility random network with $N = 3$ user classes (one for each color). The queues represent the transmitters, and the potential receivers are represented by circular shapes of the corresponding color. Each transmitter communicates with the closest potential receiver, as it is shown by the dashed lines. Unconnected circles represent inactive receivers. The quantities λ, a, p_s and ϕ are related to density of users, rate of arrival of packets, rate of service of packets and link quality, respectively.

transmit power levels and traffic intensities that ensure stability for the terminal queues of *all* classes. For the case of stable networks, we show that the portion of the radio resource allocated to each class of users is well-defined by a simple expression relating its average delay, intensity of traffic, density of terminals, and the minimum acceptable link signal-to-interference ratio of that class. Our results evince the interplay among stability, traffic intensity, density of users and outage probability in a scenario where terminals share radio resources, through a simple formulation that allows for insights into the existing trade-offs among key network parameters for N classes of users (Proposition 2 and Corollary 1).

The scenario considered in this paper was studied by other authors, as discussed in the following paragraphs. Liu *et al.* [18] derived analytic expressions for the mean delay and the throughput of the channel, but their solutions are not in closed form[1] [19]. Also, they considered static PPPs and, in this case, it is not possible to find the conditions for network stability, since for *almost*[2] all PPPs there is a subset of unstable users, *i.e.*, users with a transmission distance greater than a critical value, such that their queues are unstable.

Zhong *et al.* [20] also studied the problem of stability in Poisson networks under random access, but for a static network. The authors introduce the concept of ϵ-stability, according to which a network is said ϵ-stable when the

[1] In this present paper, a closed form expression is defined as a finite combination of elementary functions, which are limited to sum, multiplication, exponentiation and their inverses in the field of complex numbers $\mathbb{C}$.

[2] *Almost* in the sense of having probability measure one.

portion of unstable queues in the network is less than ϵ. The necessary condition and the sufficiency condition for ϵ-stability are then determined. However, the authors do not analyze the queueing delay.

Another work that explores this combination of stochastic geometry and queueing theory was presented by Gharbieh *et al.* [21], where the authors compare the performance (in terms of transmission success probability, buffer queue length, access delay time, and scalability) of scheduled protocol with random access protocol for the uplink in the context of IoT. The conclusion is that the best performing protocol depends on the operational scenario regarding terminal densities and traffic rates.

Gharbieh *et al.* [22] investigated the delay and stability in cellular wireless networks where base stations serve IoT devices, which are modeled as queues with geometric arrival. The performance of power-ramping and backoff transmission strategies was studied, by modeling the time evolution of queues using discrete time Markov chains. Due to the interaction among queues, caused by mutual interference, the proposed Markov chains were solved using iterative algorithms.

In another related work, Zhong *et al.* [23] studied the delay in wireless networks, considering however the downlink of a heterogeneous cellular network, with K tiers of base stations. Downlink traffic arrives at the base stations in bursts, to be delivered to users, which are spatially distributed according to a Poisson point process. Each user has a delay requirement in terms of maximum acceptable mean packet delay. Different scheduling mechanisms are investigated, including random access, FIFO and robin-round scheduling.

Yang and Quek also investigated the downlink performance of a wireless network composed of user terminals served by small access points (SAP) [24]. The locations of SAPs and user terminals are modeled by means of Poisson point processes, and SAPs are equipped with a set of queues, each one dedicated to a user terminal. The SAPs share the same downlink channel, causing mutual interference among transmissions, which in turn results in a interacting queues scenario. The main focus of the paper is on the effects of spatial geometry of interfering terminals and the traffic intensity on the SAP coverage. Numerical results show that higher traffic intensity requires lower SINR threshold if the coverage is to remain unchanged.

Chen *et al.* [25] studied delay and throughput in a cognitive radio network, in which secondary users share the channel with a single primary user. Secondary users are allowed to access the channel with a probability that depends on the length of the queue of the primary user. Using results from queueing theory and stochastic geometry, the authors developed an analytic framework to investigate the relationship between the delay of the primary users and the throughput of the secondary network. Optimization problems are then proposed and solved to maximize the secondary network throughput under delay constraints for the primary users.

Following a different approach from the ones aforementioned, Kountouris *et al.* investigated the delay in wireless networks using tools from stochastic network calculus [26]. For a static Poisson network, they derived bounds on the delay violation probability and on the effective capacity distribution, using network calculus.

A. MAIN CONTRIBUTIONS

The main contributions of our work when compared to the results presented in the aforementioned papers, particularly the one presented by Stamatiou and Haenggi [15], are twofold. Firstly, we have extended the analysis of stability and delay in random-access wireless network to the case of a network with an arbitrary number N of user classes. Secondly, we have expanded this analysis to show that the channel sharing mechanism in the investigated scenario can be seen as a process of partitioning a fixed and well-defined quantity into portions, each portion allotted to each user class, the size of which varying in accordance with the user class parameters.

More specifically, the contributions of the paper are:

- We propose a tractable scenario to study the performance and stability of a bipolar network with an arbitrary number N of classes of users sharing the same channel;
- a simple and elegant expression relating mean delays, arrival rates, user densities, mean link distances and bit rates of all N classes is derived for the case of stable network. This expression clearly shows that each class of user takes a well-defined portion of the available finite resource in the RF channel (Proposition 2);
- a closed form [19] solution to the fixed-point system of equations that determine the stationary transmission success probabilities for N user classes is found;
- an intuitive equation is presented relating link quality, packet arrival rate, density of users and stationary mean delay (Proposition 1);
- we prove the necessary and sufficient conditions that determine whether a given network is stable (Theorems 1 and 2);
- we establish a simple necessary condition for stability that does not depend on the transmit powers (Corollary 1);
- the optimum transmit powers per user class that achieve the optimum stationary mean delays for each user class (Proposition 3) are derived;
- the optimum packet arrival rates per user class that achieve the maximum channel throughput per unit of area (Proposition 4) are derived;
- we conclude that depending on the channel and user classes, the best strategy to maximize channel throughput is to share the channel, instead of using one single class per channel;

The paper is organized as follows: Section II describes the model used throughout the paper and provides some important results from the literature to be used in the following sections; Section III presents the main results of the paper, *i.e.*, necessary and sufficient conditions for stability when we

TABLE 1. Notations and symbols used in the paper.

Symbol	Definition/explanation
$\mathbb{Z}, \mathbb{Z}_+$	set of integers and non-negative integers
$\mathbb{R}, \mathbb{R}_+$	set of real numbers and non-negative reals
$\alpha \in (2, \infty)$	path loss exponent
$\delta \in (0, 1)$	$\triangleq 2/\alpha$
$N \in \mathbb{Z}_+\backslash\{0\}$	number of user classes
$\mathcal{N}$	$\triangleq \{1, 2, \ldots, N\}$
$n \in \mathcal{N}$	refers to the n-th user class
$p_n \in (0, 1)$	medium access probability
$a_n \in (0, 1)$	packet arrival rate per time slot
$\boldsymbol{a} \in (0, 1)^N$	$= (a_1, a_2, \ldots, a_N)$
$p_{s,n} \in (0, 1)$	packet success probability
$\theta_n \in \mathbb{R}_+$	SIR threshold for successful communication
$\overline{R}_n \in \mathbb{R}_+$	average transmission distance
$D_n \in (1, \infty)$	average packet transmission delay
$\boldsymbol{D} \in (1, \infty)^N$	$= (D_1, D_2, \ldots, D_N)$
$P_n \in \mathbb{R}_+$	transmission power
$\boldsymbol{P} \in \mathbb{R}_+^N$	$= (P_1, P_2, \ldots, P_N)$
$\Phi_n \subset \mathbb{R}^2$	Poisson point process for the transmitters
$\lambda_n \in \mathbb{R}_+$	density of Φ_n
$\phi_n \in \mathbb{R}_+$	$\triangleq 4\,\overline{R}_n^2\,\theta_n^\delta\,\pi\delta/\sin(\pi\delta)$
$\|\|\cdot\|\|$	euclidean norm
$\mathbb{1}_{\{\cdot\}}$	indicator function

have N interacting user classes, and shows a simple expression for the stationary mean delay and the packet success probability; Section IV applies the obtained results in two general scenarios: one scenario optimizes the transmission power of different user classes with different delay requirements sharing the same channel and the other optimizes the throughput per unit of area; Section V concludes the paper.

The notations used in the paper are summarized in Table 1.

II. PRELIMINARIES

A. SYSTEM MODEL

We consider a network composed by N classes of users that share the same RF channel under Aloha protocol. Time is slotted and for each time slot $t \in \mathbb{N}$ and each user class $n \in \mathcal{N} \triangleq \{1, 2, \ldots, N\}$, we have a homogeneous Poisson point process (PPP) denoted by $\Phi_n(t) \subset \mathbb{R}^2$ of density λ_n, which represents the position of the sources. These PPPs are independent from each other and from the past. Each transmitter (TX) of user class n transmits with power P_n. The position of the transmitters are given by $\{X_{in}(t)\}_i$, $i \in \mathbb{N}$, i.e., $\Phi_n(t) = \{X_{in}(t)\}_i$. More precisely, for each time slot the position $X_{in}(t)$ of the i-th TX is reallocated following the high-mobility random walk model [27]. The i-th TX of user class n communicates with a receiver (RX) located at $Y_{in}(t)$. Thus, the distance between the i-th transmitter of class n and its destination is given by $R_{in}(t) = ||X_{in}(t) - Y_{in}(t)||$. In this work, we assume that each transmitter is associated to a "son" PPP that models the locations of its potential receivers. The receiver associated to the i-th transmitter of class n is chosen as the closest point in the respective son PPP to the point $X_{in}(t)$. As a consequence, $\{R_{in}(t)\}_t$ are iid Rayleigh random variables[3] [28, Eq. (2.35)]. Rayleigh distributed TX-RX separation distance has been used in several other works investigating similar scenarios (e.g., [29]–[31]). We denote the mean transmission distance $\mathbb{E}[R_{in}(t)]$ by $\overline{R}_n$. The occupation of the buffer at each TX is represented by its queue length $\{Q_{in}(t)\}$ of infinite capacity. The packet arrival probability at each queue is denoted by a_n and the medium access probability by p_n. Within each slot, the first event to take place for each TX with a non-empty queue is the medium access decision with probability p_n. If it is granted access and the signal to interference ratio (SIR)[4] is greater than a threshold $\theta_n > 0$, a packet is successfully transmitted and leaves the queue. Then, we have the arrival of the next packet with probability a_n. The last event to take place is the displacement of the transmitters and destinations.

The queue lengths of the i-th TX, user class n are Markov Chains represented by

$$Q_{in}(t+1) = (Q_{in}(t) - B_{in}(t))_+ + A_{in}(t), \quad t \in \mathbb{N},$$

where $(\cdot)_+ \triangleq \max\{\cdot, 0\}$, $\{A_{in}(t)\}$ are iid Bernoulli random variables of parameter a_n and represents the arrival process,

$$B_{in}(t) = e_{in}(t)\,\mathbb{1}_{\mathrm{SIR}_{in} > \theta_n}$$

represents the departure process, where $\{e_{in}(t)\}$ are iid Bernoulli random variables of parameter p_n, the constant θ_n represents the SIR threshold for successful communication, and the SIR of user i and user class n is given by

$$\mathrm{SIR}_{in} = \frac{P_n\, h_{in,in}\, ||X_{in} - Y_{in}||^{-\alpha}}{\sum_{(j,k)\neq(i,n)} P_k\, h_{jk,in}\, e_{jk}\, \mathbb{1}_{Q_{jk}>0}\, ||X_{jk} - Y_{in}||^{-\alpha}}, \qquad (1)$$

where the dependence on t has being omitted, $\{h_{jk,in}(t)\}_t$ are iid exponential distributed random variables of parameter one and represent the Rayleigh fading coefficient from the j-th TX of class k to the i-th RX of class n. Coefficient $\alpha > 2$ is the path loss exponent.

Remark 1: The numerator of (1) represents the power of the signal received by a given RX that was transmitted by its corresponding TX. The denominator of (1) represents the sum of the interfering signal powers from all other TXs transmitting at that time slot, i.e., TX with non-empty queues.

B. DEFINITIONS AND LITERATURE RESULTS

This subsection is devoted to present the usual definitions found in the literature and some known results that are useful to prove the main results of the paper. Let us start by defining the concept of stability in a queueing system. Let us use the definition proposed by Szpankowski [14], presented next.

Definition 1: A queue $Q(t)$ is stable if for $x \in \mathbb{Z}_+$

$$\lim_{t\to\infty} \mathbb{P}(Q(t) < x) = F(x) \quad \text{and} \quad \lim_{x\to\infty} F(x) = 1,$$

where $F : \mathbb{R}_+ \longrightarrow [0, 1]$ is the limiting distribution function.

The system network is stable when the queue of the typical user from n-th class is stable for all $n \in \mathcal{N}$. The stability

[3] The iid random variables for the TX-RX separation distance are of grave importance for the theoretical model. Otherwise, there is at least one unstable queue and, consequently, the queueing network is unstable.

[4] We assume thermal noise is negligible; refer to [32] for further details.

region $\mathcal{R}$ is given by the set of arrival rates $\boldsymbol{a} \in [0, 1)^N$ that makes the system network stable.

Throughout the paper, stationary success transmission probability $p_{s,n}$ refers to the limiting probability of a successful transmission from a typical user of class n, *i.e.*,

$$p_{s,n} \triangleq \lim_{t\to\infty} \mathbb{P}(\mathrm{SIR}_{i,n}(t) > \theta_n).$$

This probability does not depend on the user i, by symmetry. The stationary mean delay D_n to transmit packets of class n is defined as the limiting (as t tends to infinity) expected time a packet spends in the buffer and the server.

The following results from the literature are used in many proofs throughout the paper. In a wireless network, let us assume that *(i)* the separation distance between a given pair TX - RX is equal to r, *(ii)* the positions of the interferers (users who will transmit packets in a given time slot) follow a PPP of density λ_{eff}, and *(iii)* every transmitter has the same transmit power. Then, the probability of a successful transmission between TX and RX is given by [33, Sec. III.A]

$$\begin{aligned}\mathbb{P}(\mathrm{SIR} > \theta) &= \mathbb{E}[\mathrm{e}^{-\theta r^\alpha I}] \\ &= \exp\left(-\pi\, \Gamma(1+\delta)\Gamma(1-\delta)\, \theta^\delta\, r^2\, \lambda_{\mathrm{eff}}\right), \end{aligned} \quad (2)$$

where $\delta \triangleq 2/\alpha$ and $I \triangleq \sum_{X\in\Phi} ||X||^{-\alpha}$ is the interference received by RX normalized by the transmit power and Φ is a PPP of density λ_{eff}, which is the effective density of active sources. This result can be generalized for the case of N classes of interferes, each class $n \in \mathcal{N}$ with an effective density $\lambda^{(n)}_{\mathrm{eff}}$ and transmit power P_n. The proof with two user classes was presented by Yin *et al.* [34, Proposition 3] and it is easily extended to N user classes as follows. Let the analyzed transmitter be from class $k \in \mathcal{N}$, then

$$\begin{aligned}\mathbb{P}(\mathrm{SIR}_k > \theta) &= \mathbb{E}\left[\exp\left(-\frac{\theta\, r^\alpha}{P_k} \sum_{n\in\mathcal{N}} P_n\, I_n\right)\right] \\ &= \prod_{n\in\mathcal{N}} \mathbb{E}\left[\exp\left(-\theta\, r^\alpha \frac{P_n}{P_k} I_n\right)\right] \\ &= \prod_{n\in\mathcal{N}} \exp\left(-\pi r^2\, \theta^\delta\, \frac{\pi\,\delta}{\sin(\pi\,\delta)} \frac{P_n^\delta}{P_k^\delta} \lambda^{(n)}_{\mathrm{eff}}\right) \\ &= \exp\left(-\pi r^2\, \theta^\delta\, \frac{\pi\,\delta}{\sin(\pi\,\delta)} \sum_{n\in\mathcal{N}} \frac{P_n^\delta}{P_k^\delta} \lambda^{(n)}_{\mathrm{eff}}\right), \end{aligned} \quad (3)$$

where we used Euler's reflexion formula $\Gamma(1+\delta)\Gamma(1-\delta) = (\pi\,\delta)/\sin(\pi\,\delta)$ and I_n is the interference from the n-th class normalized by the transmit power. It is assumed that $\{I_n\}_{n\in\mathcal{N}}$ is iid.

III. MULTIPLE-CLASS NETWORK

As described in Section II, we consider a network with N classes of users. The following proposition presents the stationary success probability and mean delay when transmitting a packet in a stable network. The results that guarantee stability are presented later in the paper, in Theorem 1.

Proposition 1: If the network is stable, then the stationary success probability and mean delay for a typical user of class $n \in \mathcal{N}$ are given by

$$p_{s,n} = \left(1 + \frac{\phi_n}{P_n^\delta} \frac{\sum_j P_j^\delta\, a_j \lambda_j}{1 - \sum_j \phi_j\, a_j \lambda_j}\right)^{-1}, \quad (4)$$

$$D_n = \frac{1 - a_n}{p_n\, p_{s,n} - a_n}, \quad (5)$$

where the sums are taken over the set of user classes $\mathcal{N}$, $\delta \triangleq 2/\alpha$, and

$$\phi_n \triangleq \frac{4\,\overline{R}_n^2\, \theta_n^\delta\, \pi\,\delta}{\sin(\pi\,\delta)}. \quad (6)$$

Proof: The buffer (plus server) is a discrete time Geo/Geo/1 queue [15] and the equation for the delay D_n is found in the literature [35, Chapter 4.6]. As $t \to \infty$, the effective PPP density of active sources $\lambda^{(n)}_{\mathrm{eff}}$ for each user class $n \in \mathcal{N}$ converges (by hypothesis) to $\lambda_n\, p_n\, \rho_n$, where $\rho_n = a_n/(p_n\, p_{s,n})$ is the load of the queue (or the probability of having a non-empty queue), which is the ratio between the arrival rate and the service rate of packets. Thus, $\lambda^{(n)}_{\mathrm{eff}} = \lambda_n\, a_n/p_{s,n}$.

Then, to calculate the transmission success probability $p_{s,n}$, we use (3), which assumes that the link distance between TX and RX is constant. Thus, by deconditioning the transmission success probability on $R_{i\,n}$, we take into account that $R_{i\,n}$ is Rayleigh distributed, that is

$$\begin{aligned}p_{s,n} &= \lim_{t\to\infty} \mathbb{P}(\mathrm{SIR}_{i\,n}(t) > \theta_n) \\ &= \int_0^\infty \lim_{t\to\infty} \mathbb{P}(\mathrm{SIR}_{i\,n}(t) > \theta_n \mid R_{i\,n}(t) = r) f_{R_n}(r)\,\mathrm{d}r \\ &= \int_0^\infty \frac{\pi\, r}{2\overline{R}_n^2} \exp\left\{-\frac{\pi\, r^2}{4\overline{R}_n^2}\left(1 + \phi_n \sum_{k\in\mathcal{N}} \frac{P_k^\delta}{P_n^\delta} \lambda^{(k)}_{\mathrm{eff}}\right)\right\} \mathrm{d}r \\ &= \left(1 + \frac{\phi_n}{P_n^\delta} \sum_{k\in\mathcal{N}} P_k^\delta \frac{a_k \lambda_k}{p_{s,k}}\right)^{-1}. \end{aligned} \quad (8)$$

This expression can be rearranged as

$$\frac{P_n^\delta}{\phi_n}\left(\frac{1 - p_{s,n}}{p_{s,n}}\right) = \sum_{k\in\mathcal{N}} P_k^\delta \frac{a_k \lambda_k}{p_{s,k}}. \quad (9)$$

Note that the right-hand side of (9) does not depend on n. Then, for all $j \in \mathcal{N}$, we can write

$$\frac{P_j^\delta}{\phi_j}\left(\frac{1 - p_{s,j}}{p_{s,j}}\right) = \frac{P_n^\delta}{\phi_n}\left(\frac{1 - p_{s,n}}{p_{s,n}}\right). \quad (10)$$

For each j, we can solve the above equation for $p_{s,j}$ and plug it into the sum of (8). Then, we can solve it for $p_{s,n}$, which ends the proof. □

The following theorem shows the conditions for which the network is stable, *i.e.*, it presents the region formed by all arrival rates $\boldsymbol{a}$ that make the system stable.

Theorem 1: A necessary and sufficient condition for the network stability is that $\boldsymbol{a} \in \bigcup_{\nu\in\mathcal{V}} \mathcal{C}_\nu$, where $\mathcal{V}$ is the space of all bijective functions from $\mathcal{N}$ to $\mathcal{N}$ (or permutations) and

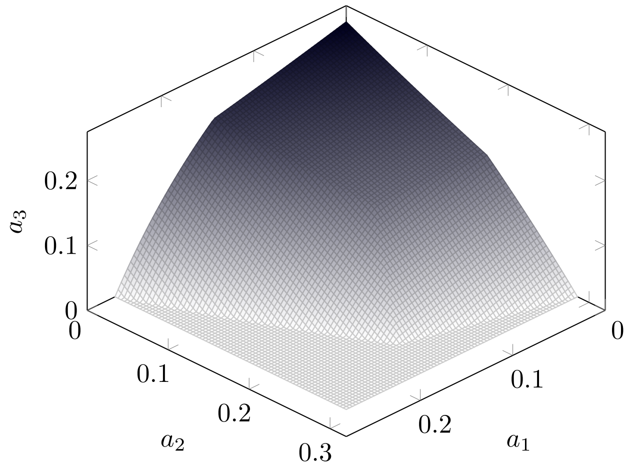


FIGURE 2. Stability region $\mathcal{R}$ according to theorem 2 for $p_1 = 1/3$, $p_2 = 2/3$, $p_3 = 1$, $\phi_1\lambda_1 = 1$, $\phi_2\lambda_2 = 2$, $\phi_3\lambda_3 = 3$, $\phi_1/P_1 = 1/3$, $\phi_2/P_2 = 1/2$, $\phi_3/P_3 = 1$.

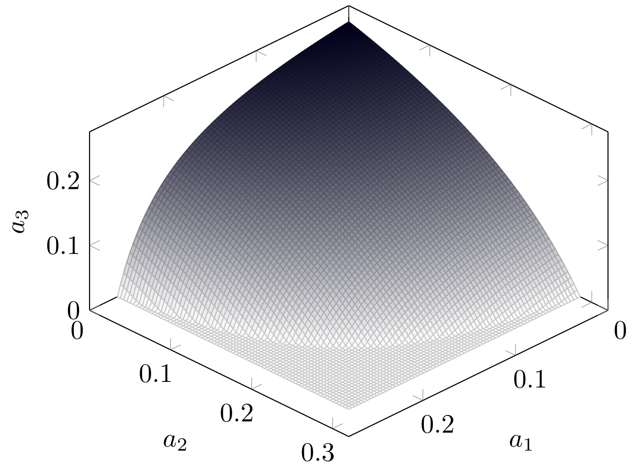


FIGURE 3. Maximum stability region $\mathcal{S}_0$ according to corollary 1 for $p_1 = 1/3$, $p_2 = 2/3$, $p_3 = 1$, $\phi_1\lambda_1 = 1$, $\phi_2\lambda_2 = 2$, $\phi_3\lambda_3 = 3$.

$\mathcal{C}_\nu$ is defined in (7), as shown at the bottom of this page, with the convention $\sum_{k=1}^{0} \cdot = 0$.

Proof: See Appendix B. □

Theorem 1 requires verifying $N \times N!$ inequalities, whereas the following theorem is equivalent and it involves only N inequalities. Thus, Theorem 2 presents a simpler form of verifying the conditions for stability.

Theorem 2: The system network is stable iff $\boldsymbol{a} \in \mathcal{R}$, where

$$\mathcal{R} \triangleq \Bigg\{\boldsymbol{a} \in [0,1)^N \;\Big|\; a_n < p_n, \frac{\phi_n}{P_n^\delta}\frac{a_n}{p_n - a_n} < \frac{1 - \sum_k \phi_k a_k \lambda_k}{\sum_k P_k^\delta a_k \lambda_k} \quad \forall n \in \mathcal{N}\Bigg\}.$$

Proof: See Appendix C. We show that $\mathcal{R} = \bigcup_{\nu \in \mathcal{V}} \mathcal{C}_\nu$. □

Remark 2: The convoluted conditions of Theorem 1 are extensively reduced in Theorem 2, for which we can see that as the density of users λ_n increases or the quantity ϕ_n (which is inversely related to link quality, see (6)) increases for some $n \in \mathcal{N}$, then the stability region $\mathcal{R}$ decreases for all user classes. On the other hand, if the transmission power P_n increases for some $n \in \mathcal{N}$, then the stability region $\mathcal{R}$ increases for the n-th class and decreases for all other classes. Surprisingly, when the access probability p_n varies, the only affected class regarding stability region is the n-th class (as long $p_n > a_n$).

Figure 2 shows an example of stability region for $N = 3$ classes, where we have used only three non-linear inequalities instead of 18. The following corollary establishes a simple result on stability, which is used in Section IV to propose and solve optimization problems regarding delay and throughput.

Corollary 1: There exists a vector of transmit powers $\boldsymbol{P} \in \mathbb{R}_+^N$ such that the network is stable iff $\boldsymbol{a} \in \mathcal{S}_0$, where

$$\mathcal{S}_0 = \left\{\boldsymbol{a} \in [0,1)^N \;\Big|\; 0 \leq \sum_{n \in \mathcal{N}} \frac{\phi_n \lambda_n}{\frac{1}{a_n} - \frac{1}{p_n}} < 1\right\}.$$

Proof: First, let us show that $\mathcal{R} \subset \mathcal{S}_0$ for all $\boldsymbol{P} \in \mathbb{R}_+^N$. If $\boldsymbol{a} \in \mathcal{R}$, then for all $n \in \mathcal{N}$

$$\frac{\phi_n}{P_n^\delta}\frac{a_n}{p_n - a_n} < \frac{1 - \sum_k \phi_k a_k \lambda_k}{\sum_k P_k^\delta a_k \lambda_k}.$$

Multiplying both sides of the above equation by $P_n a_n \lambda_n$ and summing over all $n \in \mathcal{N}$ result in

$$\sum_{n \in \mathcal{N}} \phi_n \lambda_n \frac{p_n a_n}{p_n - a_n} < 1$$

after some manipulations. Thus, $a \in \mathcal{S}_0$.

Now, let us show that $\mathcal{S}_0 \subset \mathcal{R}$ for some $\boldsymbol{P} \in \mathbb{R}_+^N$. In particular, let us choose $P_n = \phi_n a_n/(p_n - a_n)$, $n \in \mathcal{N}$. Then, the inequalities that describe the region $\mathcal{R}$ can be rewritten as one unique inequality

$$1 < \frac{1 - \sum_k \phi_k a_k \lambda_k}{\sum_k \phi_k a_k^2 \lambda_k/(p_k - a_k)}$$

that does not depend on n anymore. It is easy to show that this inequality is the same as the one that defines the region $\mathcal{S}_0$. Thus, if $\boldsymbol{a} \in \mathcal{S}_0$, then $\boldsymbol{a} \in \mathcal{R}$ for that choice of $\boldsymbol{P}$ (or any scalar multiple). This ends the proof. □

Figure 3 shows the region of arrival rates, according to Corollary 1, for which it is possible to find transmit powers that make the network stable. On the other hand, out of this

$$\mathcal{C}_\nu \triangleq \left\{\boldsymbol{a} \in [0,1)^N \;\Big|\; 0 \leq \frac{\phi_{\nu(n)}}{P_{\nu(n)}^\delta}\frac{a_{\nu(n)}}{p_{\nu(n)} - a_{\nu(n)}} < \frac{1 - \sum_{k=1}^{n-1} \phi_{\nu(k)} a_{\nu(k)} \lambda_{\nu(k)}}{\sum_{k=1}^{n-1} P_{\nu(k)}^\delta a_{\nu(k)} \lambda_{\nu(k)} + \sum_{k=n}^{N} P_{\nu(k)}^\delta p_{\nu(k)} \lambda_{\nu(k)}} \quad \forall n \in \mathcal{N}\right\} \tag{7}$$

region, the system is always unstable. It is worth mentioning that ϕ_n, defined in (6), is related to the quality of the link between receiver and transmitter for class $n \in \mathcal{N}$; the larger is the value of ϕ_n, the poorer is the quality of the link. Also, the stability region $\mathcal{R}$ showed in Fig. 2 is contained in $\mathcal{S}_0$. This is expected, since we used the same parameters for both sets and Corollary 1 considers the best case scenario, where we can choose the transmit powers $\boldsymbol{P}$ for each $\boldsymbol{a}$.

From now on, we assume that whenever there is a packet in the buffer, the corresponding TX attempts to transmit, *i.e.*, the medium access probability $p_n = 1$ for all $n \in \mathcal{N}$. We discuss in Appendix A the validity of this assumption and the high-mobility assumption. The motivation is that, when the access probability of all classes is equal to one, we maximize the stability region $\mathcal{R}$. This is easy to see with the inequalities of Theorem 2, where the right-hand side does not depend on p_n and the left-hand side decreases monotonically with p_n. Thus, the stability region is maximized when $p_n = 1$ for all $n \in \mathcal{N}$. The same occurs in Corollary 1. This result is surprising and might be explained from the fact that we have independence between adjacent time slots and, therefore, for each time slot there is a new scenario (a new effective PPP). Then, it makes sense to always try re-transmission. This approach also minimizes the mean delay according to (5), since the success probability $p_{s,n}$ in (4) does not depend on the access probability in a stable network.

Using Proposition 1, Proposition 2 is introduced, which presents an equation that relates all performance parameters independently of the transmission powers. Also, the conditions for stability are extensively simplified, see Corollary 1.

Proposition 2: If the network is stable and $p_n = 1$ for all $n \in \mathcal{N}$, then the following identities hold (at stationary state):

$$\sum_{n\in\mathcal{N}} \phi_n\,\lambda_n \frac{D_n}{D_n-1}\,\frac{a_n}{1-a_n} = 1, \tag{11}$$

and

$$\frac{\phi_j}{P_j^\delta}\left(\frac{D_j}{D_j-1}\,\frac{1}{1-a_j}-1\right) = \frac{\phi_k}{P_k^\delta}\left(\frac{D_k}{D_k-1}\,\frac{1}{1-a_k}-1\right)\ \forall j,k\in\mathcal{N}.$$

Proof: We start with the terms of the sum,

$$\begin{aligned}\phi_n\,\lambda_n \frac{D_n}{D_n-1}\,\frac{a_n}{1-a_n} &\overset{\text{(i)}}{=} \phi_n\lambda_n\frac{a_n}{1-p_{s,n}}\\ &= P_n^\delta\frac{\lambda_n\,a_n}{p_{s,n}}\left(\frac{\phi_n}{P_n^\delta}\,\frac{p_{s,n}}{1-p_{s,n}}\right)\\ &\overset{\text{(ii)}}{=} \frac{P_n^\delta\frac{\lambda_n a_n}{p_{s,n}}}{\sum_j P_j^\delta\frac{\lambda_j a_j}{p_{s,j}}},\end{aligned}$$

where (i) comes from (5) with $p_n = 1$ and (ii) comes from (9). Summing over $\mathcal{N}$ ends the proof of the first identity. For the second relation of Proposition 2, we use (5) once again to find

$$\frac{\phi_n}{P_n^\delta}\left(\frac{D_n}{D_n-1}\,\frac{1}{1-a_n}-1\right) = \frac{\phi_n}{P_n^\delta}\,\frac{p_{s,n}}{1-p_{s,n}}.$$

Comparing this expression with (10) ends the proof. □

Proposition 2 is an elegant form to see that a channel is a limited resource regarding traffic intensity and delay. Let us rewrite the identity (11) in terms of physical parameters,

$$\sum_{n=1}^{N} 4\,\lambda_n\,\overline{R}_n^2\,\theta_n^{2/\alpha}\,\frac{D_n}{D_n-1}\,\frac{a_n}{1-a_n} = \frac{\sin(2\pi/\alpha)}{2\pi/\alpha}. \tag{12}$$

Note that $\frac{a_n}{1-a_n}$ and $\frac{\sin(2\pi/\alpha)}{2\pi/\alpha}$ are monotonic increasing functions and $\frac{D_n}{D_n-1}$ is a monotonic decreasing function. The right hand-side of (12) can be seen as the amount of resource available to all users of the channel. Larger α results in higher $\frac{\sin(2\pi/\alpha)}{2\pi/\alpha}$, meaning that a larger amount of resource is available to users. This can be explained recalling that larger path loss exponent leads to stronger isolation among links sharing the channel and, consequently, more users can be accommodated in the network. Therefore, the larger the path loss exponent α, the larger (smaller) the terms λ_n, $\overline{R}_n$, θ_n, a_n (D_n) can be. The identity (12) also tells us that the n-th class of user takes a well-defined portion of the amount of resource available in the network, which is given by the n-th term in the summation. This means that the values of λ_n, $\overline{R}_n$, θ_n, a_n and D_n for a given class n can be adjusted, while keeping the quantity $\lambda_n\,\overline{R}_n^2\,\theta_n^{2/\alpha}\,\frac{D_n}{D_n-1}\,\frac{a_n}{1-a_n}$ unchanged. For instance, we can make a direct exchange between decreasing the delay D_n and decreasing the arrival rate of packets a_n (by controlling the ratio of transmit power levels), such that the term $\frac{D_n}{D_n-1}\,\frac{a_n}{1-a_n}$ remains constant; or else, increase the arrival rate of packets and decrease the density of users, such that the term $\lambda_n\,\frac{a_n}{1-a_n}$ remains constant. Therefore, Proposition 2 reveals, through a simple expression, the interplay among traffic intensity, mean delay, density of users, link distance, and outage probability, when the network is stable.

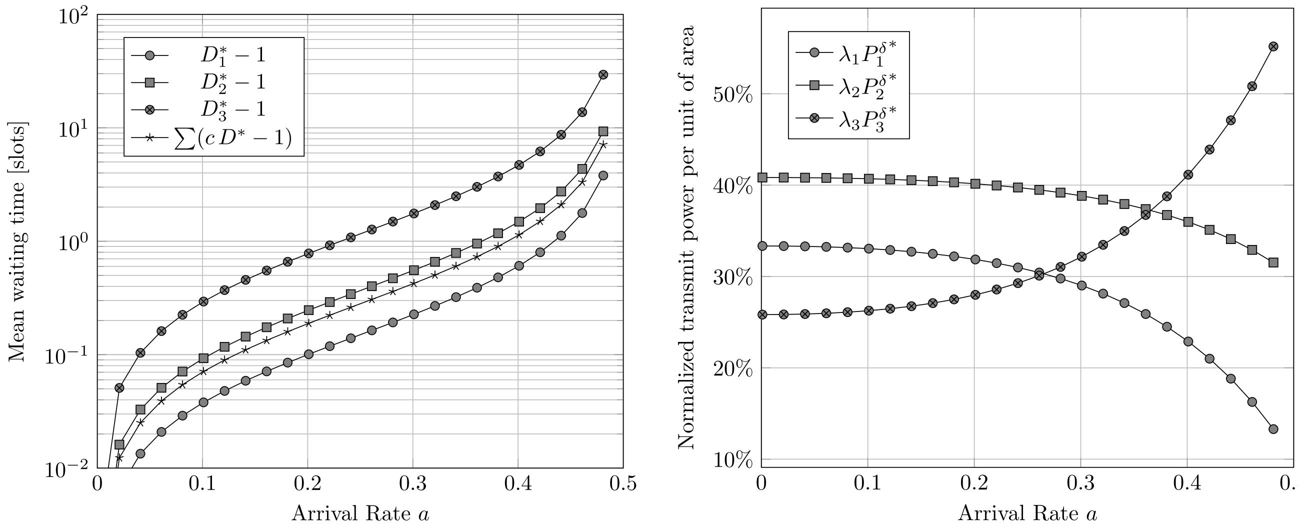
Remark 3: Corollary 1 and Proposition 2 are the simplest and the most meaningful results of the present paper, as they translate the behavior of the network in simple equations, which do not directly depend on the transmission powers.

IV. INTERPRETATION AND APPLICATION

In this section, we solve two optimization problems using the proposed formulation, applied to scenarios of different classes of terminals sharing a radio channel.

A. DELAY OPTIMIZATION

Let us consider the scenario with N classes sharing a channel. Each class may represent a particular user application, with each application having a different delay requirement in the network. Let us suppose we are interested in adjusting the transmit power of each user class, such that the weighted average delay among all classes is minimized. This problem is addressed as follows. For fixed arrival rates of vector $\boldsymbol{a}$ that satisfies Corollary 1, *i.e.*, for $\boldsymbol{a} \in \mathcal{S}_0$, let us minimize the delays $\boldsymbol{D}$ by changing the ratio between the transmit powers $\boldsymbol{P}$. Each user class requires a different response time, then we weight the optimization problem with the vector $(c_1, c_2, \ldots, c_N) \in \mathbb{R}_+^N$. The larger the coefficient of a class, the smaller the resulting mean delay to deliver packets for that



(a) Optimum delays (b) Optimum transmit powers

FIGURE 4. These figures represent the optimization of a 3-class network with the following parameters: $a_1 = a_2 = a_3 = a$, $\phi_1\,\lambda_1 = 0.1$, $\phi_2\,\lambda_2 = 0.3$, $\phi_3\,\lambda_3 = 0.6$ and $c_1 = \frac{10}{16}$, $c_2 = \frac{5}{16}$, $c_3 = \frac{1}{16}$.

class. Then, we have

$$\min_{\boldsymbol{P}\in\mathbb{R}_+^N} \sum_{n\in\mathcal{N}} c_n D_n = \min_{\boldsymbol{P}\in\mathbb{R}_+^N} \sum_{n\in\mathcal{N}} \frac{c_n\,(1-a_n)}{\left(1+\frac{\phi_n}{P_n^\delta}\,\frac{\sum_j P_j^\delta\, a_j\lambda_j}{1-\sum_j \phi_j\, a_j\lambda_j}\right)^{-1} - a_n}, \tag{13}$$

where D_n is given by Proposition 1. Note that as thermal noise is not considered in our model, we have a degree of freedom for the optimum solution $\boldsymbol{P}^*$, which agrees with the formulation in (13).

Proposition 3: The minimum of the optimization problem (13) *is attained by*

$$P_n^{*\delta} = \frac{\beta}{\lambda_n a_n}\left(\frac{a_n\,\mathcal{A}_n}{1-\sum_k \mathcal{A}_k} + \frac{\sqrt{c_n\,\mathcal{A}_n}}{\sum_k \sqrt{c_k\,\mathcal{A}_k}}\right), \quad n\in\mathcal{N}, \tag{14}$$

where β is any positive real constant, $\mathcal{A}_n \triangleq \phi_n\,\lambda_n\,\frac{a_n}{1-a_n}$ and the sums are over $\mathcal{N}$.

Proof: Since we have one degree of freedom for the solution $\boldsymbol{P}^*$, let us set $\sum_j P_j^\delta\, a_j\lambda_j = 1 - \sum_j \phi_j\, a_j\lambda_j$ to extensively simplify the algebraic manipulations. Then, we use the Karush-Kuhn-Tucker conditions [36, Section 3.3.1] in the *Lagrangian* function

$$\mathcal{L}(\boldsymbol{P},\mu) = \sum_{n\in\mathcal{N}} \frac{c_n(1-a_n)}{\left(1+\frac{\phi_n}{P_n^\delta}\right)^{-1} - a_n} + \mu\left[\sum_{j\in\mathcal{N}} P_j^\delta\, a_j\lambda_j - \left(1 - \sum_{j\in\mathcal{N}} \phi_j\, a_j\lambda_j\right)\right],$$

where $\mu \in \mathbb{R}$ is the *Lagrange* multiplier. The objective function is strictly convex (the Hessian is a diagonal matrix with positive eigenvalues) and the feasible region is a hyperplane, therefore the solution is the global optimum. Now, we return to the original problem that does not have the artificial constraint. Thus, we multiply the solution by an arbitrary constant $\beta > 0$ to obtain the general solution. □

It is interesting to note that we must have $\sum_k \mathcal{A}_k < 1$, by Corollary 1. Therefore, $P_n^{*\delta}$ is always a positive quantity. Also, if $c_n = \mathcal{A}_n$ for all $n \in \mathcal{N}$, then the optimum delays are all equal and given by $D_1 = D_2 = \cdots = D_N = \left(1 - \sum_k \mathcal{A}_k\right)^{-1}$. Thus, we can always choose transmit powers, such that we have the same mean delay for all classes!

As an example, let us consider a 3-class network, where Class 1 has a more restrictive delay requirement than Class 2, which is more restrictive than Class 3. We consider that all classes have the same arrival rate of packets, *i.e.*, $a_1 = a_2 = a_3 = a$. Figure 4(a) shows the expected waiting time of a packet before a successful transmission, which is $D_n^* - 1$, $(n = 1, 2, 3)$ since a transmission takes exactly one time slot. As expected, the optimization resulted in monotonic increasing functions and $D_1^* < D_2^* < D_3^*$ for all a.

Figure 4(b) shows the normalized[5] transmit powers per unit of area $\lambda_n P_n^{*\delta}$ as a function of a. In this case, we do not have a clear hierarchy among the transmit powers, as it depends on the traffic intensity. For $n \in \mathcal{N}$, if the network is close to saturation, *i.e.*, $\sum_k \mathcal{A}_k$ tends to 1, then the normalized $\lambda_n P_n^{*\delta}$ approaches $\mathcal{A}_n / \sum_k \mathcal{A}_k$ and, at first order, it does not depend on the coefficients $c_1, c_2, \ldots, c_N$. On the other hand, if the network is at low traffic, *i.e.*, $\sum_k \mathcal{A}_k$ tends to 0, then the normalized $a_n\lambda_n P_n^{*\delta}$ approaches $\sqrt{c_n\mathcal{A}_n}/\sum_k \sqrt{c_k\mathcal{A}_k}$.

B. THROUGHPUT OPTIMIZATION

Now, let us maximize the total throughput of the channel per unit of area with the constraint that the system is stable. Since each TX performs re-transmissions until the packet is

[5]Whenever we refer to normalized $f_n P_n^{*\delta}$, it means that we choose β in Proposition 3 such that $\sum_k f_k P_k^{*\delta} = 1$ and f_n is any function that depends on n, for example $f_n = \lambda_n$ or $f_n = a_n\lambda_n$.

correctly received by the intended RX, then all packets are successfully transmitted (eventually) in a stable system. Thus, the throughput per TX is given by the packet arrival rate a_n. The density of users per unit of area is given by λ_n, then the throughput of the n-th user class per unit of area is simply $\lambda_n a_n$ and the throughput per unit of area of the entire system is the sum of the throughput for all classes $n \in \mathcal{N}$. Using Corollary 1, we can formulate the optimization problem as $\max_{\boldsymbol{a} \in \mathcal{S}_0} \sum_n \lambda_n a_n$. However, the strict inequality in the region $\mathcal{S}_0$ of Corollary 1 results in an optimization problem that is not well-posed. In this case, if the optimum solution lies in the boundary of the feasible region, then the solution does not exist. To circumvent this problem, we propose a new region $\mathcal{S}_\epsilon \subset \mathcal{S}_0$ by adding an arbitrarily small parameter $\epsilon \in (0, 1)$ in the inequality, *i.e.*, the new region is given by

$$\mathcal{S}_\epsilon = \left\{ \boldsymbol{a} \in [0, 1)^N \mid \sum_{n \in \mathcal{N}} \phi_n \lambda_n \frac{a_n}{1 - a_n} \leq 1 - \epsilon \right\}.$$

As the parameter ϵ increases, the system becomes less sensible to perturbations[6]. Now, the optimization problem is posed as

$$\max_{\boldsymbol{a} \in \mathcal{S}_\epsilon} \sum_{n \in \mathcal{N}} \lambda_n a_n. \quad (15)$$

The following proposition presents the solution to the optimization, *i.e.*, the optimum arrival rates $\boldsymbol{a}^*$ that maximize the throughput per unit of area and maintain the system stable.

Proposition 4: If

$$\sum_{k \in \mathcal{N}} \lambda_k \phi_k \left(\sqrt{\frac{\max_n \phi_n}{\phi_k}} - 1 \right) < 1 - \epsilon, \quad (16)$$

then the solution of (15) *is attained by*

$$a_n^* = 1 - \frac{\sum_k \lambda_k \sqrt{\phi_n \phi_k}}{1 - \epsilon + \sum_k \lambda_k \phi_k}, \quad n \in \mathcal{N}, \quad (17)$$

where the sums are over $\mathcal{N}$. If the inequality (16) *is not satisfied, then the m-th class is excluded, where $m = \arg\max_n \phi_n$, and the inequality is checked again.*

Proof: It is a direct application of the Karush-Kuhn-Tucker conditions [36, Section 3.3.1] in the *Lagrangian* function

$$\mathcal{L}(\boldsymbol{a}, \mu) = \sum_{n \in \mathcal{N}} \lambda_n a_n + \mu \left[\sum_{k \in \mathcal{N}} \phi_k \lambda_k \frac{a_k}{1 - a_k} - (1 - \epsilon) \right],$$

where $\mu \in \mathbb{R}$ is the Lagrange multiplier associated with the constraint of stability. Equation (16) guarantees that the solution $\boldsymbol{a}^* \in [0, 1)^N$.

The objective function is convex (affine function) and the region $\mathcal{S}_\epsilon$ is strictly convex (the Hessian of the function that defines the region is a diagonal matrix with negative eigenvalues), therefore the presented solution is the global optimum and it is unique. □

[6]When the system parameters suffer a sufficiently small change, the system remains stable

TABLE 2. Network parameters for fig. 5.

Parameters	Values
$(\lambda_1, \lambda_2, \lambda_3)$	$= (1, 2, 3)$
(ϕ_1, ϕ_2, ϕ_3)	$= (0.3, 0.5, 0.4)$
(c_1, c_2, c_3)	$= (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

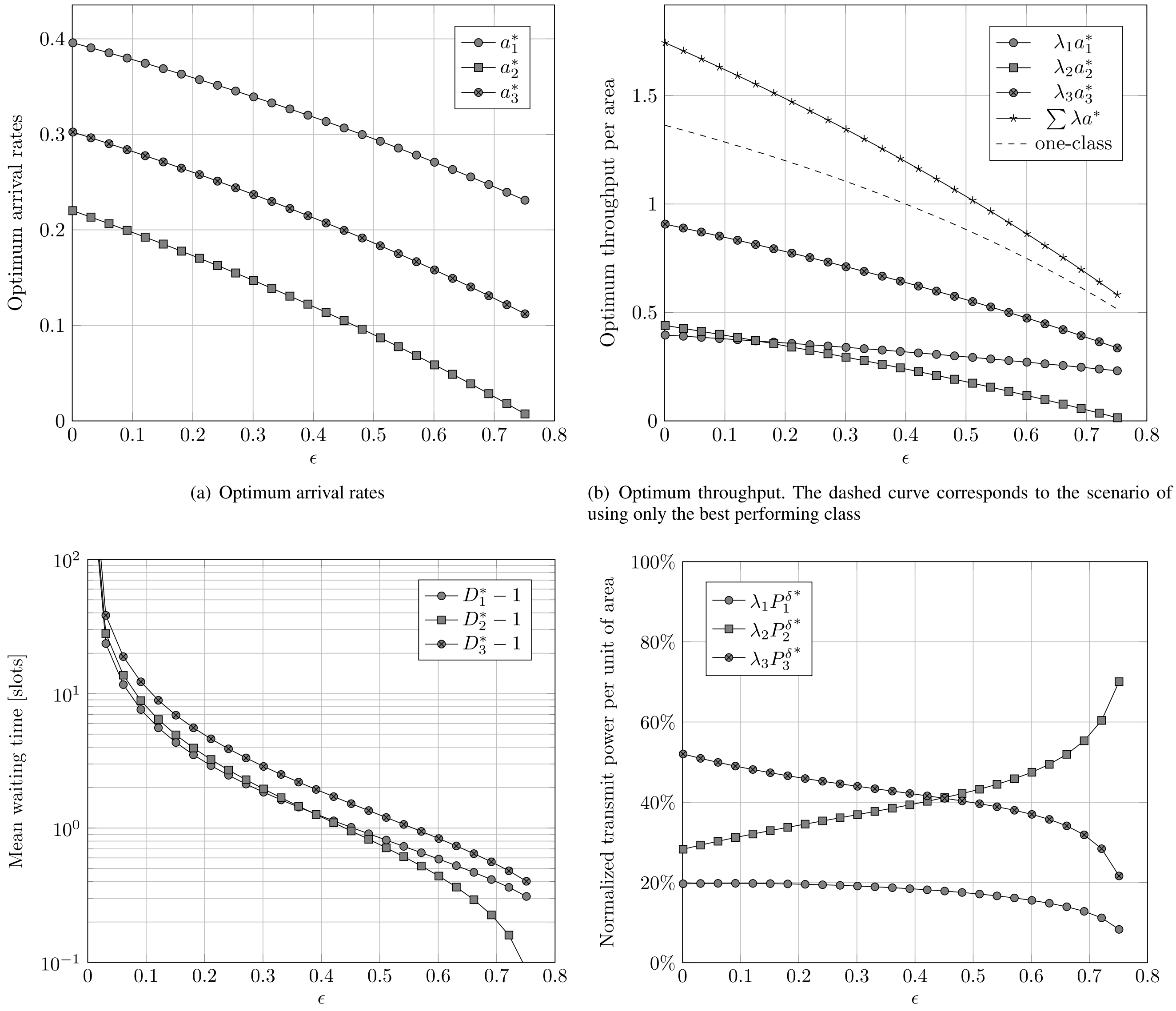
In the optimization (15), we still have freedom to choose the transmit powers $\boldsymbol{P}$, as long the network remains stable. The best way of choosing $\boldsymbol{P}$ is by minimizing the delays, which we have already done in Subsection IV-A, Proposition 3, where the arrival rates $\boldsymbol{a} \in \mathcal{S}_\epsilon \subset \mathcal{S}_0$ are fixed. When the optimization is performed in this sequence (maximization of throughput, then minimization of delay), we have the optimum throughput (per unit of area) and the optimum delays for the optimum configuration of arrival rates. Later in this section, we illustrate this procedure with a numerical example.

In order to solve the optimization problem (15) we did not have to handle with the transmit powers $\boldsymbol{P}$ directly, which would make the solution and the problem formulation more cumbersome. This shows the usefulness of Corollary 1.

Let us illustrate the throughput optimization problem with a system for which the parameters are shown in Table 2. Figure 5(a) shows the optimum arrival rates $\boldsymbol{a}^*$ that maximizes the throughput per unit of area.

It is quite interesting that the optimum solution is not necessarily solely activating the class with the best link quality (i.e., the class with the smallest ϕ, which is Class 1 in this example). In Fig. 5(b) it is shown the optimum throughput for each class and the total throughput of the system. For comparison, we plotted a dashed curve representing the total throughput if we only use the best performing user class, regarding throughput. The dashed curve is below the optimum total throughput for all ϵ. Therefore, the best solution is always a combination of all user classes, as long as (16) is satisfied. On the other hand, if this equation is not satisfied, it means that there is at least one user class with a bad link quality, such that it is better (regarding throughput efficiency) to reallocate this user class to another channel.

Now that we have, for each ϵ, the arrival rate configuration $\boldsymbol{a}^*(\epsilon)$ which gives the maximum throughput, we can use Proposition 3 to find the best configuration of transmit powers $\boldsymbol{P}^*(\epsilon)$ that minimizes the sum of the mean delays for each optimum configuration of arrival rates $\boldsymbol{a}^*(\epsilon)$. Figure 5(c) shows the result of this optimization, which is a direct application of (14). It is worth noting that as we increase ϵ the system is farther from instability, which corresponds to having a smaller delay to transmit packets, as we can see in Fig. 5(c), and a smaller throughput, as shown in Fig. 5(b). Figure 5(d) shows the optimum distribution of power per unit of area required by each user class. Notice that the first user class, which has the best link quality, uses the smallest power per unit of area. However, this behavior is more intricate; it



(a) Optimum arrival rates

(b) Optimum throughput. The dashed curve corresponds to the scenario of using only the best performing class

(c) Optimum delays

(d) Optimum transmit powers

FIGURE 5. These figures represent the optimization of the throughput and mean delay of a 3-class network with the parameters given in table 2.

also depends on the density of users of the corresponding class. Notice, for example, the inversion between user class 2 and 3 as we increase ϵ in Fig. 5(d).

Another interesting and direct result from Corollary 1 is to provide an upper bound for the total throughput per unit of area in a stable system, which is given by $1/(\min_n \phi_n)$. This is not a tight bound, however it is interesting on its own, due to its simplicity and the fact that it does not depend on the density of users. The proof follows

$$\begin{aligned}\sum_{k\in\mathcal{N}} a_k\lambda_k &\leq \frac{1}{\min_n \phi_n}\sum_{k\in\mathcal{N}} \phi_k\, a_k\, \lambda_k \\ &< \frac{1}{\min_n \phi_n}\sum_{k\in\mathcal{N}} \phi_k\, \lambda_k\, \frac{a_k}{1-a_k} \\ &< (\min_n \phi_n)^{-1}, \qquad (18)\end{aligned}$$

where the last inequality comes from Corollary 1.

V. CONCLUSION

In this paper, it is proposed a modified model to study the stability and delay of slotted Aloha in high-mobility Poisson bipolar networks. The model presented in the paper considers a scenario where transmitters with buffer communicate with the closest receiver belonging to a son Poisson point process. We derived necessary and sufficient conditions for stability in a network with N user classes; we also provided simple closed-form expressions for the packet success probability and mean delay. As shown by the results in the paper, the advantage of using this model as a base to model other network effects is its analytic tractability. As an example, we were able to derive simple conditions to verify the stability of a interference-limited network with undetermined transmit powers (see Corollary 1). We also solved (analytically and in closed form) two optimization problems regarding the

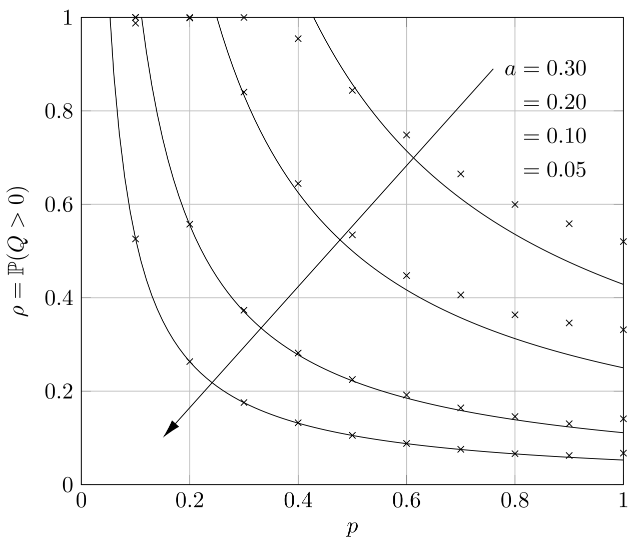


FIGURE 6. Queue load ρ as a function of the access probability p. Simulation results with a static network are presented in marks and the theoretical results with the high-mobility assumption are presented in curves.

minimization of the delays in a network (see Proposition 3) and maximization of the total throughput per unit of area (see Proposition 4). An interesting insight from the optimization problem is that the best solution to maximize the throughput of a channel is not necessarily using solely the user class with the best link quality, *i.e.*, a mix with other user classes may result in a better use of the channel. All in all, this paper provides a simple way to evaluate the existing trade-offs involved in the design of wireless networks when different classes of nodes co-exist. In the future, we plan to further extent this analysis to more practical machine-type communications (5G) scenarios, including long range technologies and transmission of critical messages.

APPENDIX A
HIGH-MOBILITY ASSUMPTION

In this appendix, we address the high-mobility assumption, which may not be realistic in real wireless networks, since the mobility of transmitters does not change drastically between adjacent time slots. Therefore, the independence assumption would not hold. Nevertheless, in a stable wireless network which has a small packet arrival rate a per user or a small access probability p, the correlation might be sufficiently small such that the independence (high-mobility) assumption is reasonable. In [29] the authors show that if the access probability p is sufficiently small, then the independence assumption provides a good approximation.

When the packet arrival rate a or the access probability p are small, the typical user sees a significantly different PPP of transmitters, which justifies the independence (high-mobility) assumption. We verified this claim through simulations and the result is shown in Fig. 6, where it is used one user class with $\lambda c \overline{R}^2 = \pi/4$, $\alpha = 3$ and $\theta = 1$.

The mean load of the queues ρ, which is equivalent to the percentage of queues with packets to transmit, are plotted as a function of the access probability p for several values of arrival rate a. As expected, for small values of a or p, the theoretical model presents good estimations of the average queue load. It is important to emphasize that we did not plot the mean delay D, because in a static PPP there might exist a set of unstable users, whose queues and delays tend to infinity. This would raise the average delay to infinity too. Then, we chose to plot the mean load ρ, which is equal to 1 for unstable users and does not tend to infinity as the mean delay D.

To establish Proposition 2, we suppose that the access probability p is equal to one for all users. In the context of high-mobility this approach makes sense, since the typical user sees a different interference scenario for each time slot. Thus, it makes sense to attempt a re-transmission every time slot until the packet is successfully transmitted. This also minimizes the mean delay D, which is in accordance with (5), as $p_{s,n}$ does not depend on p_n in a stable network.

There is another scenario, which does not require high-mobility to achieve spatial independence between adjacent time slots. This scenario is a network that uses the frequency-hopping scheme over a set of channels [37]. For each time slot there is a different PPP pattern, since the transmitting nodes select with equal probability one channel to transmit. Thus, the spatial correlation between time slots decreases with the number of channels available for selection.

APPENDIX B
PROOF OF THEOREM 1

Proof: Using the concept of stochastic dominance [38, Section 2.1.2], it is possible to derive necessary and sufficient conditions for stability. In the dominant network, all the user classes in the set $\mathcal{D} \subset \mathcal{N}$ transmit dummy packets. If the dominant network is stable, then the original network is stable. On the other hand, if the queues of the user classes in $\mathcal{D}$ are not empty in the original network, then this system behaves exactly as the dominant network (both systems are *indistinguishable* [14, Section 3.2]). Therefore, if the dominant network is unstable, then the original network will be unstable as well. In order to have necessary and sufficient conditions, we must perform this verification for all $\mathcal{D} \subset \mathcal{N}$.

Let us start with $\mathcal{D} = \mathcal{N}$, *i.e.*, all users transmit dummy packets. For each step of the verification, we remove the stable user class from the set $\mathcal{D}$. This procedure repeats until the set $\mathcal{D}$ becomes empty. In order to attain stability of the dominant network we must have an arrival rate smaller than the service rate [39]. Thus, a sufficient condition for the first user class stability is, for any queue i of this class (by symmetry),

$$a_1 < p_1 \, \mathbb{P}(\widetilde{\mathrm{SIR}}_{i,1} > \theta_1) = p_1 \left(1 + \frac{\phi_1}{P_1^\delta} \sum_{k=1}^{N} P_k^\delta p_k \lambda_k\right)^{-1},$$

where $\widetilde{\mathrm{SIR}}$ represents the signal-interference ratio in the dominant network and the second equality comes from the same procedure to obtain (8) with $\lambda_{\text{eff}}^{(n)} = \lambda_n$ (all users are active, since every TX transmits dummy packets).

This guarantees stability for the first user class. Let us remove it from the set $\mathcal{D}$. Then, we calculate the stationary success probability of the first user class $\widetilde{p}_{s,1}^{(1)}$ for this dominant network. At steady state, we have

$$\widetilde{p}_{s,1}^{(1)} = \left(1 + \frac{\phi_1}{P_1^\delta}\left(P_1^\delta p_1 \lambda_1 \frac{a_1}{p_1 \widetilde{p}_{s,1}^{(1)}} + \sum_{k=2}^{N} P_k^\delta p_k \lambda_k\right)\right)^{-1},$$

which can be solved for $\widetilde{p}_{s,1}^{(1)}$,

$$\widetilde{p}_{s,1}^{(1)} = \frac{1 - \phi_1 \lambda_1 a_1}{1 + \frac{\phi_1}{P_1^\delta}\sum_{k=2}^{N} P_k^\delta p_k \lambda_k}.$$

The next step is to verify the conditions of stability for the second user class, when the first user class is at steady state. After that, we remove the second user class from the set $\mathcal{D}$ and calculate the stationary success probability of the two stable user classes in the dominant network. We repeat these steps until we remove all user classes, *i.e*, $\mathcal{D} = \{\}$. We show this by induction; we suppose stability of the user classes $1, 2, \ldots, j-1$. Let $\mathcal{D} = \{j, j+1, \ldots N\}$; the j-th user class is stable, given that all the user classes in $\mathcal{N} \setminus \mathcal{D}$ are stable, when

$$\begin{aligned} a_j &< p_j \mathbb{P}(\widetilde{\mathrm{SIR}}_{i,j} > \theta_j) \\ &= p_j \left(1 + \frac{\phi_j}{P_j^\delta}\left(\sum_{k=1}^{j-1} P_k^\delta \lambda_k \frac{a_k}{\widetilde{p}_{s,k}^{(j)}} + \sum_{k=j}^{N} P_k^\delta p_k \lambda_k\right)\right)^{-1}, \end{aligned} \tag{19}$$

where $\widetilde{p}_{s,k}^{(j)}$ is the k-th user class success probability ($1 \leq k < j$) at steady state in the dominant network at the j-th step. To calculate this probability, we must solve the following system of equations. For $k \in \{1, 2, \ldots, j-1\}$

$$\widetilde{p}_{s,k}^{(j)} = \left(1 + \frac{\phi_k}{P_k^\delta}\left(\sum_{\ell=1}^{j-1} P_\ell^\delta \lambda_\ell \frac{a_\ell}{\widetilde{p}_{s,\ell}^{(j)}} + \sum_{\ell=j}^{N} P_\ell^\delta p_\ell \lambda_\ell\right)\right)^{-1}.$$

Using an analogous approach as the one presented in the proof of Proposition 1, we have that for $k \in \{1, 2, \ldots, j-1\}$,

$$\widetilde{p}_{s,k}^{(j)} = \left(1 + \frac{\phi_k}{P_k^\delta} \frac{\sum_{\ell=1}^{j-1} P_\ell^\delta a_\ell \lambda_\ell + \sum_{\ell=j}^{N} P_\ell^\delta p_\ell \lambda_\ell}{1 - \sum_{\ell=1}^{j-1} \phi_\ell a_\ell \lambda_\ell}\right)^{-1}.$$

Comparing the last two equations, it is easy to see that

$$\sum_{\ell=1}^{j-1} P_\ell^\delta \lambda_\ell \frac{a_\ell}{\widetilde{p}_{s,\ell}^{(j)}} + \sum_{\ell=j}^{N} P_\ell^\delta p_\ell \lambda_\ell = \frac{\sum_{\ell=1}^{j-1} P_\ell^\delta a_\ell \lambda_\ell + \sum_{\ell=j}^{N} P_\ell^\delta p_\ell \lambda_\ell}{1 - \sum_{\ell=1}^{j-1} \phi_\ell a_\ell \lambda_\ell}.$$

Finally, we can use this result to rewrite (19) as, for all $j \in \mathcal{N}$,

$$0 \leq \frac{\phi_j}{P_j^\delta} \frac{a_j}{p_j - a_j} < \frac{1 - \sum_{k=1}^{j-1} \phi_k a_k \lambda_k}{\sum_{k=1}^{j-1} P_k^\delta a_k \lambda_k + \sum_{k=j}^{N} P_k^\delta p_k \lambda_k}.$$

This concludes the proof, since the extension for the other partitions of $\mathcal{N}$ is analogous. □

APPENDIX C
PROOF OF THEOREM 2

Proof: The proof consists of showing that the set $\mathcal{R}$ is equal to the set defined in Theorem 1. First, let us prove that $\bigcup_{\nu \in \mathcal{V}} \mathcal{C}_\nu \subset \mathcal{R}$. For that we suppose $\boldsymbol{a} \in \mathcal{C}_\nu$ and we show $\boldsymbol{a} \in \mathcal{R}$ for all $\nu \in \mathcal{V}$ by induction. For simplicity of exposition let us take $\mathcal{C}_\nu$ with $\nu : n \longmapsto n$, $n \in \mathcal{N}$. We assume the inequality

$$\frac{\phi_{N-j}}{P_{N-j}^\delta} \frac{a_{N-j}}{p_{N-j} - a_{N-j}} < \frac{1 - \sum_{k=1}^{N} \phi_k a_k \lambda_k}{\sum_{k=1}^{N} P_k^\delta a_k \lambda_k}. \tag{20}$$

is true for all $j \in \{0, \ldots, m-1\}$ and we prove that it is also true for $j = m$. First, we have to prove the base case $m = 1$. Since $\boldsymbol{a} \in \mathcal{C}_\nu$, then for $j = 0$ ($n = N$)

$$\begin{aligned} \frac{\phi_N}{P_N^\delta} \frac{a_N}{p_N - a_N} &< \frac{1 - \sum_{k=1}^{N-1} \phi_k a_k \lambda_k}{\sum_{k=1}^{N-1} P_k^\delta a_k \lambda_k + P_N^\delta p_N \lambda_N} \\ &= \frac{1 - \sum_{k=1}^{N} \phi_k a_k \lambda_k + \phi_N a_N \lambda_N}{\sum_{k=1}^{N} P_k^\delta a_k \lambda_k + P_N^\delta (p_N - a_N) \lambda_N}. \end{aligned}$$

Then, using simple manipulations, we can show that the above inequality is equivalent to

$$\frac{\phi_N}{P_N^\delta} \frac{a_N}{p_N - a_N} < \frac{1 - \sum_{k=1}^{N} \phi_k a_k \lambda_k}{\sum_{k=1}^{N} P_k^\delta a_k \lambda_k}.$$

Thus, the base case $m = 1$ is true. Now, for $j = m$ and $\boldsymbol{a} \in \mathcal{C}_\nu$, we know that

$$\begin{aligned} &\frac{\phi_{N-m}}{P_{N-m}^\delta} \frac{a_{N-m}}{p_{N-m} - a_{N-m}} \\ &\quad < \frac{1 - \sum_{k=1}^{N-m-1} \phi_k a_k \lambda_k}{\sum_{k=1}^{N-m-1} P_k^\delta a_k \lambda_k + \sum_{k=N-m}^{N} P_k^\delta p_k \lambda_k} \\ &\quad = \frac{1 - \sum_{k=1}^{N} \phi_k a_k \lambda_k + \sum_{k=N-m}^{N} \phi_k a_k \lambda_k}{\sum_{k=1}^{N} P_k^\delta a_k \lambda_k + \sum_{k=N-m}^{N} P_k^\delta (p_k - a_k) \lambda_k}. \end{aligned}$$

Through simple manipulations we can show that the above inequality is equivalent to

$$\begin{aligned} &\frac{\phi_{N-m}}{P_{N-m}^\delta} \frac{a_{N-m}}{p_{N-m} - a_{N-m}} \\ &\quad < \frac{1 - \sum_{k=1}^{N} \phi_k a_k \lambda_k + \sum_{k=N-m+1}^{N} \phi_k a_k \lambda_k}{\sum_{k=1}^{N} P_k^\delta a_k \lambda_k + \sum_{k=N-m+1}^{N} P_k^\delta (p_k - a_k) \lambda_k}. \end{aligned}$$

Now, we only need to verify that

$$\begin{aligned} &\frac{1 - \sum_{k=1}^{N} \phi_k a_k \lambda_k + \sum_{k=N-m+1}^{N} \phi_k a_k \lambda_k}{\sum_{k=1}^{N} P_k^\delta a_k \lambda_k + \sum_{k=N-m+1}^{N} P_k^\delta (p_k - a_k) \lambda_k} \\ &\quad < \frac{1 - \sum_{k=1}^{N} \phi_k a_k \lambda_k}{\sum_{k=1}^{N} P_k^\delta a_k \lambda_k}. \end{aligned}$$

Again, simple manipulations lead to the equivalent inequality

$$\frac{\sum_{k=N-m+1}^{N} \phi_k a_k \lambda_k}{\sum_{k=N-m+1}^{N} P_k^\delta (p_k - a_k) \lambda_k} < \frac{1 - \sum_{k=1}^{N} \phi_k a_k \lambda_k}{\sum_{k=1}^{N} P_k^\delta a_k \lambda_k},$$

which is true from the base case. This can be seen by multiplying (20) by $P^{\delta}_{N-j}(p_{N-j} - a_{N-j})$ at both sides of the inequality and summing over $j \in \{0, \ldots, m-1\}$. Thus, $\mathcal{C}_\nu \subset \mathcal{R}$ for the mapping $\nu : n \longmapsto n$. The extension for another instances of $\nu \in \mathcal{V}$ is analogous. This concludes the proof that $\bigcup_{\nu\in\mathcal{V}} \mathcal{C}_\nu \subset \mathcal{R}$.

However, we still need to prove the converse, that is $\mathcal{R} \subset \bigcup_{\nu\in\mathcal{V}} \mathcal{C}_\nu$. Note that the set of arrival rates that makes the system stable in Theorem 1 requires that at least one a_n ($n \in \mathcal{N}$) satisfies

$$\frac{\phi_n}{P^{\delta}_n}\frac{a_n}{p_n - a_n} < \frac{1}{\sum_{k=1}^{N} P^{\delta}_k p_k \lambda_k}. \tag{21}$$

Let us show that $\mathcal{R}$ requires the same restriction by contradiction. Suppose that there exist $\boldsymbol{a} \in \mathcal{R}$ such that

$$\frac{\phi_n}{P^{\delta}_n}\frac{a_n}{p_n - a_n} \geq \frac{1}{\sum_{k=1}^{N} P^{\delta}_k p_k \lambda_k} \quad \forall n \in \mathcal{N}. \tag{22}$$

Multiplying (22) by $P^{\delta}_n(p_n - a_n)\lambda_n > 0$ at both sides and summing over $n \in \mathcal{N}$ we have

$$\sum_{n=1}^{N} \phi_n a_n \lambda_n \geq \frac{\sum_{n=1}^{N} P^{\delta}_n (p_n - a_n)\lambda_n}{\sum_{k=1}^{N} P^{\delta}_k p_k \lambda_k},$$

which is equivalent to

$$\left(\sum_{n=1}^{N} \phi_n a_n \lambda_n\right)\left(\sum_{k=1}^{N} P^{\delta}_k p_k \lambda_k\right) \geq \sum_{n=1}^{N} P^{\delta}_n (p_n - a_n)\lambda_n. \tag{23}$$

Since $\boldsymbol{a} \in \mathcal{R}$, then

$$\frac{\phi_n}{P^{\delta}_n}\frac{a_n}{p_n - a_n} < \frac{1 - \sum_k \phi_k a_k \lambda_k}{\sum_k P^{\delta}_k a_k \lambda_k} \quad \forall n \in \mathcal{N}. \tag{24}$$

Again, multiplying (24) by $P^{\delta}_n(p_n - a_n)\lambda_n > 0$ at both sides, summing over $n \in \mathcal{N}$ and performing some manipulations we have

$$\left(\sum_{n=1}^{N} \phi_n a_n \lambda_n\right)\left(\sum_{k=1}^{N} P^{\delta}_k a_k \lambda_k\right) < \left(1 - \sum_{k=1}^{N} \phi_k a_k \lambda_k\right)\left(\sum_{n=1}^{N} P^{\delta}_n (p_n - a_n)\lambda_n\right). \tag{25}$$

Then, through some manipulations on (23) and (25), we have

$$0 \leq \sum_{n=1}^{N} P^{\delta}_n a_n \lambda_n - \left(1 - \sum_{k=1}^{N} \phi_k a_k \lambda_k\right)\left(\sum_{n=1}^{N} P^{\delta}_n p_n \lambda_n\right) < 0,$$

which clearly is a contradiction, since $\mathcal{R}$ is a non-empty set. Thus, there exists at least one a_n, $n \in \mathcal{N}$ that satisfies (21).

For simplicity of exposition, let us suppose that the arrival rate a_n that satisfies this restriction is from the first user class ($n = 1$). The next step is to show that as in the set $\bigcup_{\nu\in\mathcal{V}} \mathcal{C}_\nu$, the set $\mathcal{R}$ also requires that we have at least one a_n, aside from a_1, that satisfies

$$\frac{\phi_n}{P^{\delta}_n}\frac{a_n}{p_n - a_n} < \frac{1 - \phi_1 \lambda_1 a_1}{P^{\delta}_1 a_1 \lambda_1 + \sum_{k=2}^{N} P^{\delta}_k p_k \lambda_k}.$$

We can also prove this by contradiction and then, for simplicity of exposition, suppose that a_2 is the one that satisfies this restriction. We repeat this procedure until we reach all user classes. Let us show the j-th step for completeness, $j \in \mathcal{N}$. Suppose that for all $n \in \{j, j+1, \ldots, N\}$,

$$\frac{\phi_n}{P^{\delta}_n}\frac{a_n}{p_n - a_n} \geq \frac{1 - \sum_{k=1}^{j-1} \phi_k a_k \lambda_k}{\sum_{k=1}^{j-1} P^{\delta}_k a_k \lambda_k + \sum_{k=j}^{N} P^{\delta}_k p_k \lambda_k}. \tag{26}$$

Multiplying (26) by $P^{\delta}_n(p_n - a_n)\lambda_n > 0$ at both sides, summing over $n \in \{j, j+1, \ldots, N\}$ and manipulating we have

$$\left(\sum_{n=j}^{N} \phi_n a_n \lambda_n\right)\left(\sum_{k=1}^{N} P^{\delta}_k a_k \lambda_k + \sum_{k=j}^{N} P^{\delta}_k (p_k - a_k)\lambda_k\right) \geq \left(1 - \sum_{k=1}^{j-1} \phi_k a_k \lambda_k\right)\left(\sum_{n=j}^{N} P^{\delta}_n (p_n - a_n)\lambda_n\right). \tag{27}$$

Once again, multiplying (24) by $P^{\delta}_n(p_n - a_n)\lambda_n > 0$ at both sides, summing over $n \in \{j, j+1, \ldots, N\}$ and manipulating we have

$$\left(\sum_{n=j}^{N} \phi_n a_n \lambda_n\right)\left(\sum_{k=1}^{N} P^{\delta}_k a_k \lambda_k\right) < \left(1 - \sum_{k=1}^{N} \phi_k a_k \lambda_k\right)\left(\sum_{n=j}^{N} P^{\delta}_n (p_n - a_n)\lambda_n\right). \tag{28}$$

Then, through some manipulations on (27) and (28), we have

$$0 \leq \left(1 - \sum_{k=1}^{N} \phi_k a_k \lambda_k\right)\left(\sum_{n=j}^{N} P^{\delta}_n (p_n - a_n)\lambda_n\right) - \left(\sum_{n=j}^{N} \phi_n a_n \lambda_n\right)\left(\sum_{k=1}^{N} P^{\delta}_k a_k \lambda_k\right) < 0.$$

As expected, we have a contradiction. Then, we must have at least one a_n, $n \in \{j, j+1, \ldots, N\}$ that satisfies

$$\frac{\phi_n}{P^{\delta}_n}\frac{a_n}{p_n - a_n} < \frac{1 - \sum_{k=1}^{j-1} \phi_k a_k \lambda_k}{\sum_{k=1}^{j-1} P^{\delta}_k a_k \lambda_k + \sum_{k=j}^{N} P^{\delta}_k p_k \lambda_k}. \tag{29}$$

We assume that this is satisfied by the j-th class and in this case, $\mathcal{R} \subset \mathcal{C}_\nu$ for $\nu : n \longmapsto n$. Without the assumption of the ordering in which (29) is satisfied, we conclude that (29) must hold for at least one permutation of $\mathcal{N}$. This region is exactly $\bigcup_{\nu\in\mathcal{V}} \mathcal{C}_\nu$. Therefore, $\mathcal{R} \subset \bigcup_{\nu\in\mathcal{V}} \mathcal{C}_\nu$. Finally, $\mathcal{R} = \bigcup_{\nu\in\mathcal{V}} \mathcal{C}_\nu$. □

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