DOI: 10.1364/OE.25.023517

Research Article | Vol. 25, No. 19 | 18 Sep 2017 | OPTICS EXPRESS 23517
Optics EXPRESS

Full three-dimensional isotropic carpet cloak designed by quasi-conformal transformation optics

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Abstract: A fully three-dimensional carpet cloak presenting invisibility in all viewing angles is theoretically demonstrated. The design is developed using transformation optics and three-dimensional quasi-conformal mapping. Parametrization strategy and numerical optimization of the coordinate transformation deploying a quasi-Newton method is applied. A discussion about the minimum achievable anisotropy in the 3D transformation optics is presented. The method allows to reduce the anisotropy in the cloak and an isotropic medium could be considered. Numerical simulations confirm the strategy employed enabling the design of an isotropic reflectionless broadband carpet cloak independently of the incident light direction and polarization.

OCIS codes: (230.3205) Invisibility cloaks; (130.3120) Integrated optics devices; (000.3860) Mathematical methods in physics.

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#304672 https://doi.org/10.1364/OE.25.023517
Journal © 2017 Received 11 Aug 2017; revised 6 Sep 2017; accepted 6 Sep 2017; published 15 Sep 2017

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1. Introduction

Transformation Optics (TO) is a powerful technique used to control the propagation of the electromagnetic waves and to design the most varied types of complex optical devices. According to the theory of TO, Maxwell's equations are invariant under coordinate transformations. Thus, this technique provides accurate permeability and permittivity tensors used to design the desired devices with specific functionality [1].

In the last years, the TO application for invisibility devices has attracted much attention, due to the ability to hide and make objects invisible to an outside viewer. One of these devices is the carpet cloak or ground plane cloak that generates great scientific interest due to its simplicity of development and also to mitigate the difficulties and restrictions imposed by the free-space cloak [2]. The carpet cloak hides objects that are bellow a reflective deformed ground and it does not require singular values for the medium constitutive parameters. Its mathematical principle is based on a coordinate transformation, whose transformed reflective surface over the plane creates a space where the object can be hidden. In contrast, in the free-space cloak, a point is mapped to create a hole where the object will be hidden, however, this operation represents a source of singularities [3]. The scattering of the reflected waves caused through the deformation is restored by the cloak action, leading to an effect as if no deformation is present on the ground simulating a perfect flat mirror.

Although the invisibility cloak is highlighted as the most recognized application in the field of TO, several interesting different applications have made use of this approach, such as perfect lenses [4], optical waveguides [5], compressors [6], among others. Unfortunately, in order to obtain the desired wave propagation control, TO could result in unusual physical medium requirements not commonly found in nature, such as negative refractive index or inhomogeneous anisotropic permeability and permittivity [1].

Artificial materials, such as electromagnetic metamaterials [7], have been proposed as a solution to achieve these properties required by TO. Nevertheless, these materials have the disadvantage to be limited by a narrow bandwidth, since absorption and constitutive properties are frequency dependent. Additionally, they also have restrictions regarding the resolution and the number of layers due to the manufacturing processes.

The quasi-conformal mapping application is an efficient alternative to overcome the mentioned TO difficulties, resulting in a medium with broadband operation and reduced anisotropy that can be approximated as an isotropic material [8]. This type of mapping transforms the original medium into a different one and it can be obtained by different techniques [2, 8–11]. However, most of them are applied only in two-dimensional (2D) coordinate transformations [2, 5, 9]. A three-dimensional (3D) device designed with TO can be obtained by extrusion or revolution process of a 2D refractive index mapping [12–14]. In this case, the propagation control is only achieved for waves traveling in the symmetry planes obtained by these processes. Consequently, the previous 3D carpet cloaks designed with 2D quasi-conformal coordinate transformation [2,11] do not work for arbitrary wave propagation directions and only provides invisibility effects for waves traveling along those planes of symmetry. In other words, the invisibility effect is limited to a small subset of viewing angles.

Alternatively, the methods proposed in [8] and [10] indicate that a 3D quasi-conformal mapping in TO is also possible. The devices designed with such technique do not present the disadvantages previously mentioned when 2D quasi-conformal mapping is used in TO.

In the same way, different proposals have been used to design 3D carpet cloak [13–15]. The design in [13] stands out as the first practical 3D carpet cloak implementation in microwave regime. However, it was implemented with metamaterials, and the 3D refractive index map was obtained by rotating a 2D mapping. Similar implementation was carried out in [14], whose 3D device was obtained from the 2D plane extrusion. These approaches have limitations in the invisibility effect for different incident waves directions, as previously discussed. Recently, [15] described the construction of a 3D carpet cloak using multi-layer dielectrics with performance for all the viewing angles, but operating only in the microwaves regime.

In this work, the technique proposed in [8] is applied to design a full 3D isotropic carpet cloak with quasi-conformal mapping, in optical frequency. The coordinate transformation is performed using parametrization technique and the coefficients are optimized with a quasi-Newton method. The invisibility effect is achieved for any wave propagation directions, independent of the incident wave angle and the light polarization. This functionality is demonstrated through simulations using the finite element method.

2. Development

In TO, the coordinate transformation determines the desired device functionality. The boundary conditions are responsible for maintaining this functionality and the continuity at the interfaces, which is a necessary and sufficient condition to avoid undesired reflections [16]. Therefore, the method proposed in [8] is appropriate for a carpet cloak design, applying the coordinate transformation in a domain with $|x| \leq \frac{w_1}{2}$, $|y| \leq \frac{w_2}{2}$, and $0 \leq z \leq h$, corresponding to the non-transformed cloak region. The variables w_1, w_2, and h are the cloak width, depth, and height, respectively. The coordinate transformation functions used in the design are:

$$x' = x + b(x,y,z)\sum_{i=0}^{p}\sum_{j=0}^{q}\sum_{k=0}^{r} A_{ijk}x^i y^j z^k \qquad y' = y + b(x,y,z)\sum_{i=0}^{p}\sum_{j=0}^{q}\sum_{k=0}^{r} B_{ijk}x^i y^j z^k$$

$$z' = z + c\left(1 - \frac{z}{h}\right)\cos\left(\frac{\pi x}{w_1}\right)\cos\left(\frac{\pi y}{w_2}\right) + b(x,y,z)\sum_{i=0}^{p}\sum_{j=0}^{q}\sum_{k=0}^{r} C_{ijk}x^i y^j z^k \tag{1}$$

in which the terms before $b(x, y, z)$ represent the initial transformation that determines the device functionality in the design, c is the carpet deformation height, the indexes p, q and r are the power series orders, and the power series coefficients A_{ijk}, B_{ijk} and C_{ijk} are optimized in order to minimize the anisotropy using the quasi-Newton method.

The initial transformations in (1) are affected by the parametrization and optimization process, except in the points belonging to the boundary conditions where the boundary function $b(x, y, z)$ is chosen to vanish, i.e., in the point $(0, 0, 0)$, and planes $x = \pm\frac{w_1}{2}$, $y = \pm\frac{w_2}{2}$, $z = h$. The choice for $b(x, y, z)$ is:

$$b(x,y,z) = (x^2 + y^2 + z)(z - h)cos\left(\frac{\pi x}{w_1}\right)cos\left(\frac{\pi y}{w_2}\right) \tag{2}$$

The vanishing property of $b(x, y, z)$ is responsible for preserving the invisibility cloak functionality described by the map $(0, 0, 0) \to (0, 0, c)$, that represents the effect of raising the carpet, as well as for ensuring the continuity of the coordinate transformation at the interfaces, resulting in a reflectionless medium at those boundaries.

After anisotropy minimization, the material can be considered approximately isotropic and the magnetic response of materials can be omitted without changing the dynamics of the light rays [10]. Therefore, the graded refractive index profile for the transformed region is calculated as:

$$n(x,y,z) = \frac{n_b}{\sqrt{\left(\frac{\partial x'}{\partial x}\right)^2 + \left(\frac{\partial y'}{\partial x}\right)^2 + \left(\frac{\partial z'}{\partial x}\right)^2}} \tag{3}$$

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where n_b is the refractive index of the background medium. Consequently, any object situated below the deformed reflective surface with apex at $(0, 0, c)$ will become invisible to an external observer, if a carpet cloak with index given by (3) is used above this surface.

3. Results and discussion

The carpet transformation parameters used in the design are $c = 0.2\,\mu\text{m}$, $h = 1.5\,\mu\text{m}$, and $w_1 = w_2 = 4.0\,\mu\text{m}$. The background medium has a refractive index $n_b = 1.5$, and the lower boundary of device is a Perfect Electric Conductor (PEC). Several optimization runs with different values for p, q and r were analyzed, and a reduction of anisotropy was observed with the increasing of the polynomial series orders. The quasi-conformal mapping of the 3D carpet cloak with optimization parameters $p = q = r = 3$ is shown in Fig. 1(a). The grid lines of the transformed coordinates intersect at approximately 90°, indicating an angle-preserving transformation and the anisotropy minimization effect.

Full-wave numerical simulations using the Finite Element Method (FEM) were performed in the COMSOL Multiphysics to verify the device behavior at 750 nm wavelength and evaluate its performance in hiding 3D objects. Since the TO technique is frequency independent, the carpet cloak performance is not affected by the light wavelength [2]. In contrast, an anisotropic carpet cloak would require metamaterial to be implemented, which present both constitutive properties and absorption properties highly frequency dependent [12].

The refractive index distribution resulting from optimization with variation between 1.45 and 2.24 is presented in Fig. 1(b) in perspective view, and $x = 0$ and $y = 0$ planes. This index range could guarantee the device fabrication in a silicon-on-insulator (SOI) platform using the standard complementary metal-oxide-semiconductor (CMOS) process [17]. Additionally, due to limitations in resolution for longer wavelengths, a possible alternative of manufacturing could be with a mixture of simple isotropic materials available in nature, using 3D printing techniques [18]. By increasing the values of p, q, and r, the number of degrees of freedom (DoF) and the contrast of refractive indexes are increased along with the reduction in anisotropy, in agreement with [2, 5, 6, 8]. For example, using the parameters $p = q = r = 6$, the refractive index ranges from 1.42 to 2.46 and the anisotropy value only reduces about 30% when compared to $p = q = r = 3$, while the number of DoF increases from 192 to 1029.

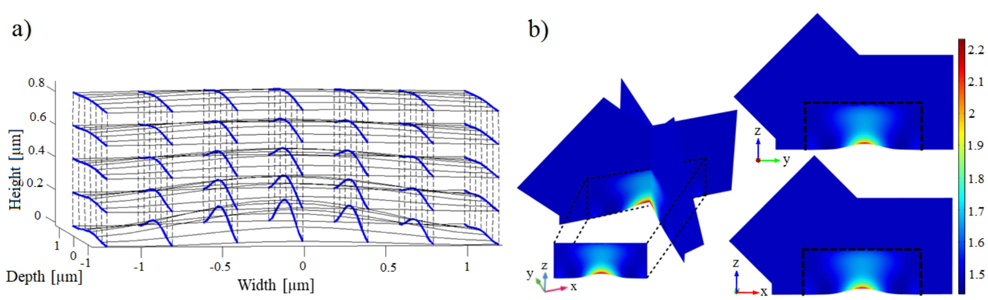

Fig. 1. Simulation results for 3D carpet cloak with $p = q = r = 3$: (a) optimized coordinate transformation, and (b) refractive index profile in perspective view, and $x = 0$ and $y = 0$ planes.

The anisotropy reduction enables the approximation for an isotropic carpet cloak, and the 3D numerical simulations confirm the device invisibility functionality. Two Gaussian beams with different incidence angles, one in the (y, z) plane and other in a orthogonal (x, z) plane, were used as sources to excite the device under test. The waves travel towards the origin at 45° of incidence, and reflect at the lower boundary of the carpet. The perpendicular polarization of incident waves was used in the simulations, i.e., the incident electric field is in the plane of incidence. The same functionality for parallel wave polarization is also confirmed by numerical simulations, since in quasi-conformal mapping both polarizations can be applied [2].

The first 4 columns of Fig. 2 represent the normalized electric field propagation in perspective, frontal views of each vertical planes of incidence, and a horizontal cut plane at $z = 2.31\,\mu$m. The last column of Fig. 2 presents the electromagnetic power flow in the same horizontal plane. As a reference, Fig. 2(a) shows the reflection from a perfect flat mirror, i.e., a flat PEC surface. The presence of the PEC deformation defined by the boundaries of (1) distort the reflected wave in Fig. 2(b), which means an external observer easily sees the deformations on the reflected wavefront. Invisibility is provided by the introduction of the optimized isotropic material that constitutes the cloak in Fig. 2(c), represented by the dashed lines indicating the cloak limits. The reflected field in the cloak limits is similar to the case where there is no deformation in the PEC, confirmed by the almost equal mode profiles of the normalized electric field of Fig. 2(a) and Fig. 2(c), highlighted by the cuts of the electric field in the (y, z) planes. The preservation of electric field propagation for both Gaussian excitations, in each plane, indicates the possibility of achieving invisibility in different directions independently of the incident wave angle. The electromagnetic power flow, in Fig. 2(a), is uniformly distributed along the device outputs, while in Fig. 2(b) the bump caused by the PEC deformation results in destructive interference. Once the cloak is introduced in the medium, Fig. 2(c), the power flow of reflected wave is restored and uniformly distributed, as in the perfectly flat mirror.

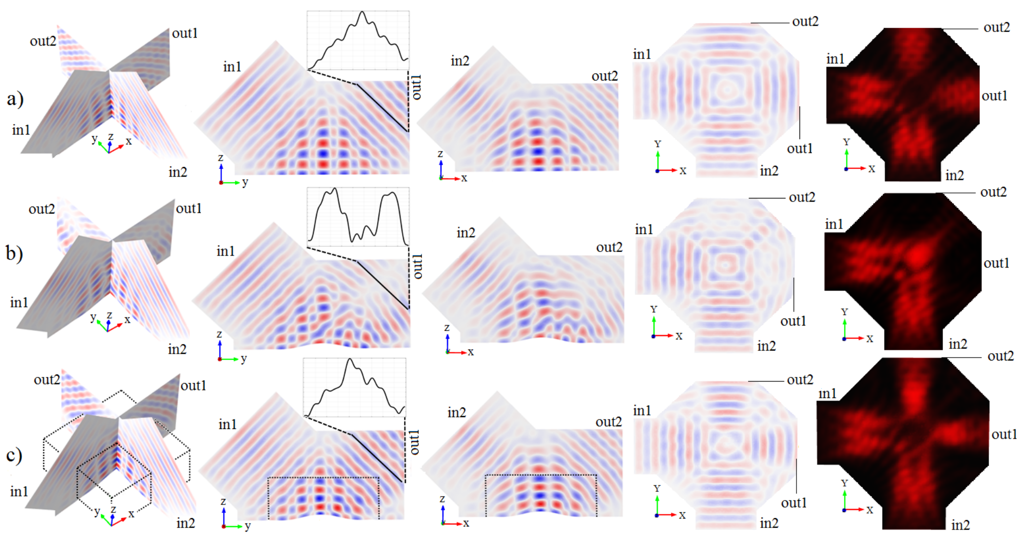

Fig. 2. Instantaneous normalized electric field plot in perspective view, $x = 0$, $y = 0$ and $z = 2.31\,\mu$m planes, and electromagnetic power flow in the $z = 2.31\,\mu$m plane, respectively, for (a) a perfectly flat mirror, (b) deformed mirror without cloak, and (c) deformed mirror with cloak whose limits are indicated by the dashed lines.

The effect of increasing the height of carpet deformation was also investigated. By changing the respective parameter to $c = 0.4\,\mu$m, a greater difficulty in reducing anisotropy and a higher refractive index contrast was verified when compared with $c = 0.2\,\mu$m. With the increase of c and maintaining the optimization parameters used in the previous case, $p = q = r = 3$, a four times greater anisotropy was obtained and the refractive index variation increases to the range between 1.45 to 4.53. For the case $p = q = r = 6$, the maximum anisotropy value also reduces approximately 30% when compared to $p = q = r = 3$, and the refractive index distribution remains between 1.31 and 11.3. However, the increase in refractive index contrast implies a great difficulty in finding materials with the required properties for fabrication and possibly

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more complex and precise manufacturing processes. Fig. 3 presents the normalized electric field propagation for $c = 0.4\,\mu\text{m}$ without and with cloak. By covering the deformation with cloak, as shown in Fig. 3(b), the scattered field was recovered and the distortion of the reflected wave was minimized when compared to the uncloaked deformed ground in Fig. 3(a). It was also observed that the mode profile in Fig. 3(b) approximates to the one obtained in a perfectly flat mirror (Fig. 2(a)). Nevertheless, a lower performance was observed when comparing to the device with $c = 0.2\,\mu\text{m}$, and it can be explained due to the lower degree of anisotropy reduction. Regarding the electromagnetic power flow for $c = 0.4\,\mu\text{m}$, in the right corner of Fig. 3(a), a higher power gap was perceived due to the greater deformation and the presence of higher anisotropy. This effect is minimized with the cloak insertion in Fig. 3(b). However, the wavefront reconstruction is less effective when compared to Fig. 2(c), i.e., for $c = 0.2\,\mu\text{m}$.

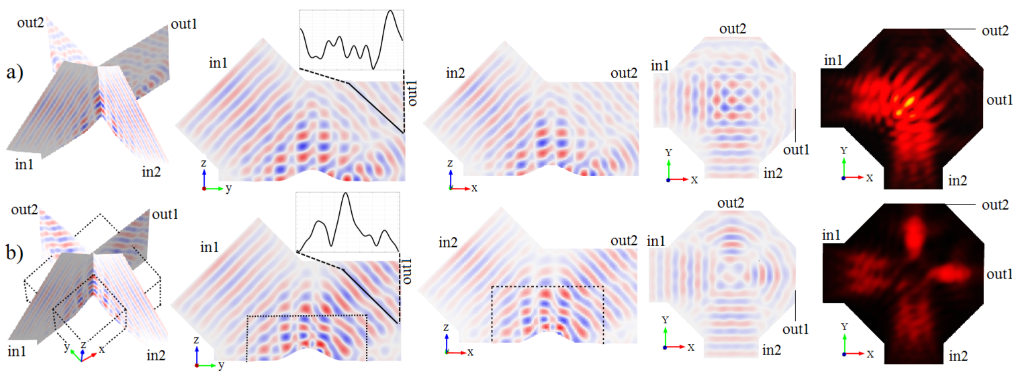

Fig. 3. Instantaneous normalized electric field plot for $c = 0.4\,\mu\text{m}$ in perspective view, $x = 0$, $y = 0$ and $z = 2.31\,\mu\text{m}$ planes, and electromagnetic power flow in $z = 2.31\,\mu\text{m}$ plane, respectively, for (a) deformed mirror without cloak, and (b) deformed mirror with cloak whose limits are defined by the dashed lines.

It is important to note that, in general, the 3D transformation anisotropy cannot be arbitrarily reduced even with an increase in the degrees of freedom in the optimization algorithm. If we consider any 2D surface inside the transformed domain bounded by the reflectionless edges of the 3D domain in a closed path, i.e., where $x' = x$, $y' = y$, and $z' = z$, only the identity transform would have zero anisotropy over that surface. Thus, to reduce the anisotropy in the 3D domain, we would have to approach the identity transform for every possible choice of 2D surface, which is impossible if we consider that the functionality of the device is intrinsically linked to the deformation imposed by the original transformation. In other words, there is a finite distance between the Riemman Map with zero anisotropy and the 2D surfaces inside the domain responsible for the device properties. This distance, which varies with the transformation, dictates the minimal reachable anisotropy.

4. Conclusion

This paper confirms the effectiveness of the parametrization and optimization technique to design a full 3D broadband carpet cloak with TO. An isotropic medium with negligible anisotropy and reflection at the boundaries was obtained, and the invisibility effect was confirmed by the full-wave simulations. Finally, the results present the successful invisibility for all viewing angles of the incident light, and the device is insensitive to the light wavelength and wave polarization.

Funding

CAPES; CNPq; FAPEMIG.